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Multi-Frequency Interpolation in Spiral Magnetic Resonance Fingerprinting for Correction of Off-Resonance Blurring

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Abstract

Magnetic resonance fingerprinting (MRF) pulse sequences often employ spiral trajectories for data readout. Spiral k-space acquisitions are vulnerable to blurring in the spatial domain in the presence of static field off-resonance. This work describes a blurring correction algorithm for use in spiral MRF and demonstrates its effectiveness in phantom and *in vivo* experiments. Results show that image quality of T1 and T2 parametric maps is improved by application of this correction. This MRF correction has negligible effect on the concordance correlation coefficient and improves coefficient of variation in regions of off-resonance relative to uncorrected measurements.

Keywords

magnetic resonance fingerprinting; off-resonance; spiral trajectory; relaxometry

1. Introduction

Magnetic resonance fingerprinting (MRF) using undersampled spiral k-space readouts has been shown to be an efficient method to estimate T1 and T2 using balanced steady-state free precession (bSSFP)[1] and unbalanced steady-state free precession (SSFP)[2]. While the bSSFP sequence allows concomitant fitting of static magnetic field (B0) off-resonance frequencies, it is subject to banding artifacts caused by off-resonance[3]. The SSFP sequence circumvents banding, but has lower signal than bSSFP and does not fit for B0. The application of the unbalanced SSFP MRF sequence is expanding to anatomical regions outside the brain that may have more prevalent B0 heterogeneity[4,5].

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It is well known that as spiral imaging acquisition time increases, B0 inhomogeneity increases image blurring in proportion to the magnitude of the off-resonant frequency. This blurring is present in both bSSFP and SSFP implementations of MRF. Previous MRF work has shown accuracy despite B0 non-uniformity in phantom and *in vivo*[2]. However, neither the effect of spatial blurring due to B0 variations on parameter maps has been explored in detail, nor have explicit corrections been applied to compensate for the effect. Parametric maps generated by spiral-based MRF pulse sequences without blurring correction may have inaccurate values due to the off-resonance induced point spread function. This may have clinical relevance for regions close to air-tissue interfaces, tissue-tissue boundaries, or other regions of significant magnetic susceptibility gradients.

Many works have addressed correction of spatial blurring from off-resonance in spiral trajectories[6–9], and the work of Man et al. provides such a solution using multi-frequency interpolation (MFI)[10]. The MFI technique relies on a series of basis frequencies to express phase deviations in k-space caused by off-resonance. For each frequency in the basis, each point in k-space is phase corrected according to the sample time after radiofrequency excitation. This basis set of phase-corrected k-spaces is transformed to the spatial domain, and a linear combination of the basis images are formed for each voxel according to the off-resonant frequency of that voxel obtained from a B0 map. This linear combination of basis images closely approximates the image of that voxel without blurring. Recent work has begun to assess whether established blurring correction schemes can be used in MRF[11,12].

This work shows that unbalanced SSFP MRF phantom and *in vivo* data acquired using a spiral k-space trajectory may contain T1 and T2 parameter map distortions in the presence of spatial modulations in B0. Results demonstrate that MFI corrections applied to the MRF images diminish those artifacts in the relaxation parameter maps and improve uncertainty in region of interest measurements.

2. Theory and General Approach

As briefly stated in the Introduction, the goal of MFI is to approximate the deblurred (conjugate phase) image as a linear combination of basis images that are formed from the measured image. Similar to the original MFI work[10], we write the deblurred solution of the image $I_{cp} \in \mathbb{C}$ at position $\mathbf{r}$ with an associated off-resonance frequency f as

$$I_{cp}(\mathbf{r};f) \approx I_{MFI}(\mathbf{r};f) = \sum_{m=1}^M c_m(f) I_{f_m}(\mathbf{r}) \quad (1)$$

where $c_m(f) \in \mathbb{C}$ is the weighting coefficient for $I_{f_m} \in \mathbb{C}$, which is the basis image formed from the f_m frequency. The basis image I_{f_m} is given as

$$I_{f_m}(\mathbf{r}) = \mathcal{F}^{-1} \left\{ S(\mathbf{k}) e^{i2\pi(\mathbf{r} \cdot \mathbf{k} + f_m t(\mathbf{k}))} \right\} \quad (2)$$

Where $S(\mathbf{k})$ is the gridded and sample-density compensated k-space signal at discretized spatial frequency position $\mathbf{k}$, and $t(\mathbf{k})$ is the time following radiofrequency excitation for the given point in k-space. The accuracy of Eq. (1) is dependent on the spacing in the frequency basis and the number of frequencies used in the basis.

Coefficient calculation of $\{c_m(f)\}$ for Eq. (1) can be performed on an ideal (noiseless) synthetic signal and solved with a linear system. The column vector $\mathbf{j}(f) \in \mathbb{C}^N$ is the complex exponential with row elements at discretized time t_n given as

$$j_n = \frac{1}{\sqrt{N}} e^{i2\pi t_n f} \quad (3)$$

and is related to the MFI coefficients $\mathbf{c}(f)$ as a column vector and system matrix $\mathbf{A} \in \mathbb{C}^{N \times M}$ via the following linear relation

$$\mathbf{j}(f) = \mathbf{A} \mathbf{c}(f) \quad (4)$$

The n^{th} row and m^{th} column entry of $\mathbf{A}$ is given as

$$A_{nm} = \frac{1}{\sqrt{N}} e^{i2\pi t_n f_m} \quad (5)$$

The coefficients may be solved as

$$\mathbf{c}(f) = \mathbf{A}^\dagger \mathbf{j}(f) \quad (6a)$$

or

$$\mathbf{c}(f) = \mathbf{A}^H \mathbf{j}(f) \quad (6b)$$

Where $\mathbf{A}^\dagger$ and $\mathbf{A}^H$ are the Moore-Penrose pseudoinverse and the complex conjugate of $\mathbf{A}$, respectively. The latter case may be used when the columns of $\mathbf{A}$ are orthonormal to each other. The off-resonance frequency range and its map can be determined from an independently derived B0 map. The coefficients for a finely spaced array of off-resonance frequencies may be calculated and stored in a lookup table, and then applied with the discretized form of Eq. (2) to Eq. (1) to solve for the unblurred image. The timing information used in Eq. (2) can be solved analytically or mapped and is further discussed in previous work[6,10].

In the case of MFI applied to MRF, the images may be highly undersampled, creating spatial aliasing of the image. Despite this aliasing, spiral MRF has shown accuracy in parameter estimation, even up to undersampling factors of 48[2]. Treating the aliased component as a

noise term, we show in the Appendix that if the basis frequencies $\{f_m\}$ are chosen so that the columns of $\mathbf{A}$ are orthonormal, the aliasing error will be minimized relative to other system matrix choices that may not be as well conditioned (i.e. matrices with columns that are not orthogonal). The upper limit to the noise for MRF MFI is not known, but the Appendix shows it consists of linear combinations of aliasing noise that have been blurred/deblurred from the original image and is approximately bounded by the largest aliasing term. Like the standard MRF sequence, the MRF MFI aliasing at a given position are assumed to be incoherent with the sought-after signal.

3. Materials and Methods

3.1 Phantoms and Human Subjects

Three phantoms were used in this study. The first was composed of ten 50 mL conical centrifuge tubes filled with deionized water and variably doped with manganese chloride or gadopentate diglumine to provide a range of T2 and T1 contrast (10-tube phantom). The second phantom was a commercially available quantitative MRI system phantom (High Precision Devices, Inc., Colorado) with temperature calibrated T1 and T2 contrast spheres. As relaxation can be temperature dependent, the MRI system (Msys) phantom relaxation values were temperature corrected before comparing to measurements[13]. A Periodic Image Quality Test (PIQT) phantom (Philips Healthcare, The Netherlands) was used to determine geometric blurring improvement over the field-of-view.

One healthy subject underwent scanning after informed consent and with approval of the local institutional review board. The subject was scanned in the transverse plane superior to the orbits and nasal sinuses. Regions outside of the brain were masked because correction near the skull were difficult to identify as having sufficient signal to produce reliable parameter estimation and were also found to produce less robust B0 corrections.

3.2 Data Acquisition and Experimental Design

Data were acquired on a Philips Achieva 3 Tesla scanner (Philips Healthcare, The Netherlands) with an eight-channel receive head coil for phantom and cranial *in vivo* acquisitions. MRF scans used an RF excitation of a Gaussian windowed sinc function with a time-bandwidth product of 10 and a minimum TR of 17 ms (27.5 ms *in vivo*) (with TR extensions detailed below) and TE of 3.25 (3.5 *in vivo*) ms. A numerically designed variable density spiral[14–16] with zero net gradient over a single TR and minimum/maximum undersampling factor of 24/48 (12 *in vivo*) was used for k-space encoding with a data acquisition time of ~3 to 7 ms per TR (15 ms *in vivo*). The spiral was designed to not exceed an instantaneous gradient frequency greater than 1 kHz. The transition radius from least undersampled to most undersampled k-space sampling was 0.7 of the maximum k-space radius. The calculated spiral was rotated by an angle of 111.254° between successive TRs. For all phantom and *in vivo* scans, a field of view of 240×240 mm² was used with an in-plane resolution and slice thickness of 1×1 mm² and 5 mm, respectively. A single MRF planar acquisition consisted of 1,000 TRs. B0 maps were generated using a Cartesian sampled gradient-echo scan with two echo times spaced 1 ms apart and then reconstructed from the raw data and masked using `multithresh`, a multi-threshold implementation of

Otsu's method[17] in MATLAB (v 8.5.0 Mathworks, Inc., Nattick, MA). In phantom cases where the off-resonance exceeded the available B0 map bandwidth, an additive correction was applied above or below threshold to avoid frequency wraparound and confirmed by visual inspection. For the *in vivo* study, an inversion prepared T1-weighted 3D spoiled gradient echo sequence with TE/TR of 4.6/8.3 ms, nutation angle of 9°, 1×1×1 mm³ voxels, with 884 Hz per voxel was acquired. For anatomical comparison with the thicker MRF slice, the T1-weighted slice nearest the center of the MRF and nearest 4 slices were summed before display.

Phantom experiments consisted of evaluating geometric improvement in a standard image quality phantom and in relaxation contrast phantoms without and with B0 heterogeneity, as well as evaluating the effect of MFI on MRF T1 and T2 maps. The PIQT phantom was imaged in the coronal plane over a rectangular grid of water-filled pins. Linear shim settings were tuned to provide a wide range of off-resonance frequencies over the field of view. The total acquisition time of the spiral was 3.76 ms (R = 48, uniform spiral). In order to evaluate spatial dependence and off-resonance magnitude on MFI correction, the 10-tube phantom was acquired with manually set shim values in longitudinal, transverse, and in-plane oblique directions to provide heterogeneous B0 over a wide range of frequencies. The Msys phantom was imaged with a coronal orientation to cover the T1 and T2 contrast planes under both well shimmed and poorly shimmed conditions. Fourteen regions of interest (ROIs) were drawn on the magnitude image of each contrast slice of the well shimmed case and applied to all B0/correction variations of the Msys experiment. These were numbered from 1 to 10 beginning at the 12 o'clock position moving clockwise, and from 11 to 14 beginning from the top left of the inner four contrast insets moving counter clockwise. Measurements were compared using boxplots to determine the effect of MFI on relaxometry values without and with B0 heterogeneity. Concordance correlation coefficients (CCC) with confidence intervals[18] were calculated for all B0/correction variations using all ROIs and the Msys specifications as a reference.

3.3 MRF Sequence, Dictionary Creation, and Matching

The original MRF SSFP sequence with an inversion time of 7 ms using the same variable repetition time extensions and relative flip angle evolution as described by Jiang et al. was used in this study, with a maximum nutation angle of 70°[2]. Two dictionaries were generated using an extended phase graph algorithm with the following T1 and T2 range and spacing (minimum:step:maximum), depending on the experiment. The first dictionary with T1 values from 20:10:3000, 3200:200:5000 ms; and T2 10:5:300, 350:50:2000 ms was used for all experiments except the Msys phantom. The Msys dictionary spanned T1 from 10:10:100, 120:20:1000, 1040:40:2000, 2100:100:3000 ms; and T2 from 2:2:10, 15:5:100, 110:10:300, 350:50:800[19]. Prior to matching, the dictionary and acquired MRF signal were normalized and compressed using a singular value decomposition method[20], preserving 99.99% of the dictionary energy. The inner product was formed between the compressed signal with all dictionary entries, and the highest magnitude of the inner product determined the dictionary entry that defined T1 and T2 for each image voxel. The preceding and following reconstruction and analysis was performed in MATLAB unless otherwise noted.

3.4 MRF and MFI Image Reconstruction

Raw spiral k-space was first reconstructed without B0 correction using gridding with iterative sample density compensation[15,21–23] and independently measured k-space coordinates[24,25]. Individual channel images for each TR were combined using adaptive reconstruction described by Walsh et al.[26]. Adaptive coil combination coefficients were derived from the complex sum of the 1,000 MRF image TRs for each channel. For each MRF TR the derived coefficients were used to adaptively combine the different coil images. Each voxel in the coil-combined MRF image was then fit for T1 and T2 as discussed above.

The MFI correction coefficients were generated as previously described in [10] and in Section 2. To generate the lookup table of coefficients, time t_n was discretized from 0 to $\alpha(TE + T_{acq})$ in N steps, with N set to the number of read points in the spiral, T_{acq} is the spiral acquisition time, and α a scale factor set to 1.2 in this work. The scale factor increases the time range in the columns of $\mathbf{A}$. It was used because it was observed that regions near the endpoints in time of the modeled phase were least accurate and is similar to the time expansion used in the original work. As discussed by Man et al., this system can be approximated by extracting $M - 1$ columns closest to $f_0 = 0$ from the DFT matrix and normalizing according to Eq. (5). So, $\mathbf{A}$ is composed of orthonormal columns of complex exponentials equal to the total number of basis frequencies chosen to represent all possible discretized off-resonant frequencies in the image. M was chosen to be 21 (31 *in vivo*) for reconstructions in this study, as it was found to keep the average root-mean-square error below approximately 1.5% for frequency deviations within ± 700 Hz for the echo and acquisition times used here.

A linearly interpolated B0 map was generated using the masked B0 map using Delaunay triangulation[27] via MATLAB's `scatteredInterpolant` method. This helped to overcome the missing off-resonance values in low signal regions of the image. For instance, signal from a voxel near a phantom or tissue surface may blur into adjacent voxel of air. If the air voxel's off-resonance is considered zero, this blurred information will not be returned to the correct voxel. The linear system was then solved for all frequencies in the range of the median adjusted interpolated B0 map discretized to 0.1 Hz over the expanded discretized time. Times before TE and after TE + T_{acq} were apodized as described in the original work to improve accuracy over the true acquisition window. The resulting interpolated coefficients were stored in a lookup table, and a coefficient map was created according to the linearly interpolated B0 map.

MFI correction was applied before coil combination. In order to avoid gridding M basis k-space representations for 1,000 MRF TRs for each coil, the previously saved coil images for all TRs were transformed back to gridded k-space. An acquisition time map[6] for gridded k-space was constructed, and each gridded k-space point was modulated in phase according to this time map for a given basis frequency (Section 2. Eq. (2)). This was repeated for all basis frequencies and transformed to the image domain to form M basis images. The coefficient map was then applied to the basis images to produce a single corrected frame (Section 2. Eq. (1)). The MFI-corrected undersampled images were then adaptively coil-combined as described previously.

4. Results

4.1 Reconstruction of PIQT phantom

The PIQT results show significant blurring correction using MFI over the field of view as seen in Figure 1. The B0 heterogeneity over the image spanned approximately 700 Hz asymmetrically about 0 Hz. This produced substantial blurring in regions of off-resonance that exceed approximately 200 Hz. The degree of blurring increases in proportion with off-resonance. The MFI corrected image shows very little if any residual blurring for any of the signal point sources across the field of view. Reduced signal of one edge of the phantom is observable in both the uncorrected and corrected images due to low coil sensitivity at the edge of the phantom.

4.2 MRF with B0 inhomogeneity

Figure 2 demonstrates MFI correction of T1 and T2 parameter maps under a variety of off-resonance patterns across the coronal section. Four examples of different off-resonance maps are given: less than ± 100 Hz variations in B0, trans-axially spanning ~ 700 Hz, longitudinally spanning ~ 600 Hz, and ~ 700 Hz across an oblique axis. The uncorrected T1 and T2 maps show distortion of the phantom tube boundaries in all cases. The geometric integrity of these boundaries appears restored in three out of four cases. The fourth set of corrected off-resonance parameter maps (Fig. 2d) demonstrates partial restoration: some corrected cylinders still appear slightly distorted, though less than in the uncorrected maps. The T1 values in the uncorrected maps relative to the corrected maps appear largely consistent. However, the T2 maps show nonuniformity across phantom tubes in both uncorrected and corrected cases. The pattern of T2 heterogeneity within a given phantom tube changes between different B0 heterogeneity patterns and between the uncorrected and MFI corrected reconstructions.

4.3 MRI system phantom results

The Msys phantom imaged with poorly shimmed B0 exhibits decreased blurring in magnitude images after MFI correction (Figure 3). The B0 maps from both the T1 and T2 contrast slices show that the contrast spheres span a range of ~ 300 Hz off-resonance asymmetrically about zero. Ring artifacts around the contrast insets are not apparent in the corrected image, and three fiducials outside the circumference of the spheres are more clearly resolved after MFI reconstruction. Additionally, susceptibility artifacts around the central T1 spheres, which contain the highest concentrations of contrast agent, are improved in the MFI corrected image. The effects of the blurring and correction are also visible in the parameter maps (Figure 4). In the uncorrected contrast maps, signal from the phantom background spreads into contrast spheres that are in regions of high off-resonance at the bottom and top portion of the phantom image. After correction, this background signal contamination is reduced.

The boxplots from ROI measurements of the T1 and T2 contrast spheres are shown in Figure 5. Here, the well shimmed (ws) and poorly shimmed (ps) results are shown without (ws/ps) and with (ws+/ps+) MFI correction for each of the 14 contrast spheres of each relaxation type. The temperature-corrected specification value for each ROI is also shown for

reference. The ws and ws+ show general agreement across all ROIs for T1 and T2. The ps and ps+ measurements have larger T2 medians than ws and ws+ cases for the smaller T2 ROIs (8 through 14). The T1 and T2 concordance correlation coefficients (CCCs) with 95% confidence intervals are shown in Table 1.

Among the assessed B0/correction combinations, there is disagreement in the distribution of the T1 and T2 values for ROIs 5 through 7 (Figure 5), which show more aberrant voxels for the ps case than the ws case. This effect is further quantified by the coefficient of variation (COV) plots (Figure 6), which indicate an increase in COV for ps by a factor of approximately five in these ROIs relative to the COV values for ws. These ROIs (at the bottom of Figures 3 and 4) are in the largest off-resonance region of the phantom. After MFI correction, the COVs of the poorly shimmed ROIs are comparable to the ws cases. Fig. 6 also shows 12 of 14 T2 inserts of the ws+ have lower or equal COV than the ws cases, indicating the MFI correction slightly lowered or did not affect the relative T2 variation in absence of large off-resonance effects.

4.4 *In vivo* MRF MFI

The B0 map, T1-weighted anatomical reference, and T1 and T2 parameter maps for the transverse brain slice are shown in Figure 7. The regions of highest off-resonance within the brain are in the frontal regions. Blurring is evident particularly at the midline, that appears less sharp on the uncorrected T1 and T2 maps relative to the corrected maps and the Cartesian sampled T1-weighted anatomical reference. Figure 8 plots the normalized signal magnitude across all TRs for a voxel in cerebrospinal fluid (CSF) (Fig. 8a) and in white matter (WM) (Fig. 8b). The CSF voxel rests in a region of higher off-resonance compared to that of the WM. The signal plot and parameter estimates from the CSF voxel are substantially different between the corrected and uncorrected cases, while those in the WM are nearly identical.

5. Discussion

The results show that the effects of B0 heterogeneity on unbalanced SSFP MRF parameter map details can be substantial due to spatial blurring, but the central regions of the phantoms are largely unchanged. The MFI correction applied to spiral-based MRF demonstrates good reconstruction in many cases of substantial B0 heterogeneity and seems to fail where there is uncertainty in the B0 map. The corrected PIQT phantom results show little to no blurring of the image. Similarly, the Msys phantom and 10-tube phantom showed dramatic improvement after MFI correction in the image as well as parameter maps. In the 10-tube phantom case of longitudinal or transverse B0 gradient, the MFI correction was able to restore images without noticeable geometric defect. An exception to this is the oblique B0 pattern of heterogeneity in the 10-tube phantom, which showed significant but incomplete restoration of the circular phantom shapes. The more complicated off-resonance patterns seen in Figure 2a do not confound MFI, but the off-resonance magnitudes do not much exceed 100 Hz in this case. MFI correction in the Msys phantom was able to substantially reduce blurring, even in areas of steep susceptibility gradients compounded with underlying low frequency B0 variation (Figure 3a–c). Thus, the regions where MFI fails are not

necessarily at the extremes of off-resonance, but in cases where the linear interpolation of the B0 map may fail and the magnitude of off-resonance is relatively large.

The proposed correction method helps to reduce uncertainty in T1 and T2 measurements and does not by itself alter relaxation values. The 10-tube and Msys phantom results demonstrate that MRF relaxation values in uniform contrast regions are degraded by B0 heterogeneity. MRF quantitation near and inside contrast insert/tube boundaries are improved after MFI correction. This improvement can be seen in lower coefficient of variation and greater accuracy near the contrast edges in the poorly shimmed cases (ps vs ps+). Additionally, MFI correction applied to the Msys phantom in the well shimmed cases (ws vs ws+) did not show changes in the CCCs or worsening of COVs of relaxation values. Thus, MFI apparently does not alter ROI-based estimation of relaxation values by itself. While intra-tube T2 heterogeneity in the 10-tube phantom experiments are evident in both uncorrected and corrected images, these variations without and with MFI suggests this may be flow or diffusion artifacts in the water-filled phantoms. This is also supported by the greater homogeneity of T2 across the Msys T2 contrast inserts seen in Figure 4 relative to the 10-tube phantom. However, it is unclear why there is an observable upward shift in the T2 MRI system phantom results at the shorter end of T2 for both the uncorrected and corrected poorly shimmed measurements. This contributes to the rise in COV of the well shimmed cases relative to the poorly shimmed cases at short T2.

These results are largely consistent with the original MRF unbalanced SSFP report by Jiang et al[2]. They found that phantom measurements were largely unaffected by B0 heterogeneity. This work found that CCC values appear greater than 0.97 in the presence of off-resonance, despite lying slightly outside of the 95% confidence interval for the T2 well shimmed case. However, deviations in B0 do produce blurring, and this blurring can largely be corrected by application of MFI.

The *in vivo* MFI results show expected improvements in the high signal brain areas, but they were masked in the skull region where signal was lower and correction less robust. The steep signal gradient and B0 information of the skull spatially correlates with these effects. We also note that chemical shift of fat is not accounted for by MFI because the correction assumes a single off-resonance frequency at a given position. Even pure fat has a multi-component spectral composition. Accounting for the presence of fat is an important subject of ongoing and future work. Despite areas for potential improvement, the current MRF MFI embodiment successfully demonstrates the positive impact of MFI on parameter map estimation in regions of off-resonance, as well as leaving unaffected white matter that do not need signal modulation (Figs. 7 and 8).

Other potential future improvements in off-resonance correction include accuracy of the interpolated map and possibly eliminating the need for a separately acquired B0 scan. When there are only regions of high signal, a B0 map can readily be calculated over the entire image. However, when there are regions of low signal, this information must be derived from surrounding high signal regions. While linear interpolation has shown much success in this study, under some circumstances this method may not produce the most optimal map for MFI correction. The separate acquisition of the B0 map from the MRF scan reduces its

efficiency and may reduce accuracy in situations where physiological motion may create differences in off-resonance between the MRF scan and B0 map acquisition. Some of these problems may be improved by considering a form of MRF that integrates B0 mapping such as bSSFP[1], or a reconstruction that uses a cost metric to implicitly define a B0 map that minimizes the objective function. The latter may be accomplished using auto-focusing[28].

In support of reproducible research, the source code along with figure reproduction scripts and data are freely available for download at https://github.com/jostenson/MRI_Ostenson_MRF_MFI.

6. Conclusion

MFI correction in conjunction with spiral MRF reduces the effects of off-resonant blurring in images and parameter maps. This method has been demonstrated in phantoms and *in vivo* under a variety of off-resonance magnitudes and patterns. This method can be further improved by incorporating B0 acquisition into the MRF scan and better accounting for off-resonance values in regions of low signal or fat.

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Appendix

In MRF, due to the spatial aliasing, we measure the superposition of multiple positions of the true fully sampled complex image at position $\mathbf{r}$ in the undersampled image. Each MRF repetition time q , at each position, is also weighted by transient MRF contrast $\phi_q(\mathbf{r})$ due to $T_1(\mathbf{r})$ and $T_2(\mathbf{r})$. That is, the measured image at $M_q(\mathbf{r})$ at MRF TR q is given as

$$M_q(\mathbf{r}) = \phi_q(\mathbf{r})I(\mathbf{r}) + \phi_q(\mathbf{r} + \mathbf{a}_{1,q})I(\mathbf{r} + \mathbf{a}_{1,q}) + \phi_q(\mathbf{r} + \mathbf{a}_{2,q})I(\mathbf{r} + \mathbf{a}_{2,q}) + \dots + \phi_q(\mathbf{r} + \mathbf{a}_{L,q})I(\mathbf{r} + \mathbf{a}_{L,q}) + \varepsilon \quad (\text{A. 1})$$

where $\phi_q I(\mathbf{r})$ is the complex image value from the fully sampled image for the given TR. The terms $\{\phi_q(\mathbf{r} + \mathbf{a}_l, q)I(\mathbf{r} + \mathbf{a}_l, q)\}$ represent the complex image components from regions of the fully sampled image that have been aliased to position $\mathbf{r}$ from positions $\{\mathbf{a}_l, q\}$. The aliased terms depend (along with L) on the specific k-space trajectory of the MRF TR, the MRF sequence, and the object's spatial distribution of T_1 and T_2 and their weightings. $M_q(\mathbf{r})$ is also subject to a noise term ε , which we assume to be complex and Gaussian distributed with zero mean.

Eq. (A.1) may be written more compactly as

$$M_q(\mathbf{r}) = I_q(\mathbf{r}) + I_{a,q}(\mathbf{r}) + \varepsilon \quad (\text{A. 2})$$

where $I_q(\mathbf{r}) = \phi_q(\mathbf{r})I(\mathbf{r})$ and $I_{a,q}(\mathbf{r}) = \sum_{l=1}^L I_q(\mathbf{r} + \mathbf{a}_{l,q})$.

A basis image $I_{q,f_m}(\mathbf{r})$ used in MFI at position $\mathbf{r}$, TR q with basis frequency f_m is given by Section 2. Eq. (2). The measured basis image is then given as

$$M_{q,f_m}(\mathbf{r}) = I_{q,f_m}(\mathbf{r}) + I_{a,q,f_m}(\mathbf{r}) + \varepsilon \quad (\text{A. 3})$$

with $I_{a,q,f_m}(\mathbf{r}) = \sum_{l=1}^L I_{q,f_m}(\mathbf{r} + \mathbf{a}_{l,q})$. Deblurring a measured image using MFI (Section 2. Eq. (1)) yields

$$M_{cp,q}(\mathbf{r}; f) \approx M_{MFI,q}(\mathbf{r}; f) = \sum_{m=1}^M c_m(f) M_{q,f_m}(\mathbf{r}) \quad (\text{A. 4})$$

Distributing the coefficients $\{c_m\}$ over the different terms in the measurement in Eqs. (A.3) and (A.4)

$$\sum_{m=1}^M c_m(f) M_{q,f_m}(\mathbf{r}) = \sum_{m=1}^M c_m(f) I_{q,f_m}(\mathbf{r}) + \sum_{m=1}^M c_m(f) I_{a,q,f_m}(\mathbf{r}) + \sum_{m=1}^M c_m(f) \varepsilon_{f_m} \quad \# \quad (\text{A. 5})$$

so

$$M_{MFI,q}(\mathbf{r}; f) = I_{MFI,q}(\mathbf{r}; f) + \sum_{m=1}^M c_m(f) I_{a,q,f_m}(\mathbf{r}) + \sum_{m=1}^M c_m(f) \varepsilon_{f_m} \quad (\text{A. 6})$$

The deblurred MRF image at $\mathbf{r}$ is equal to the desired solution plus a linear combination of aliased images and linear combination of noise images at $\mathbf{r}$. We assume the aliased and noise images at $\mathbf{r}$ are incoherent with the underlying signal, similar to the assumption made in the “on-resonance” MRF case (Eq. (A.2)). Regrouping under a common noise image

$\varepsilon'_{f_m} = I_{a,q,f_m}(\mathbf{r}) + \varepsilon_{f_m}$ gives

$$M_{MFI,q}(\mathbf{r}; f) = I_{MFI,q}(\mathbf{r}; f) + \sum_{m=1}^M c_m(f) \varepsilon'_{f_m}(\mathbf{r}) \quad (\text{A. 7})$$

For high undersampling factors, the error term is dominated by the aliasing term.

The magnitude of the error between the MRF MFI measurement and the fully sampled MFI image is

$$\left| \Delta M_{MFI,q} \right| = \left| \sum_{m=1}^M c_m(f) \varepsilon'_{f_m}(\mathbf{r}) \right| \quad (\text{A. 8})$$

If the error terms have an upper bound $\varepsilon'_{f_m}(\mathbf{r}) \leq \varepsilon_{max}$, and coefficients and error terms are written as row and column vectors ε_{max}^T and $\mathbf{c}(f)$, respectively,

$$\left| \Delta M_{MFI,q} \right| = \left| \varepsilon_{max}^T \mathbf{c}(f) \right| \quad (\text{A. 9})$$

Then by the Cauchy-Schwarz inequality

$$\left| \Delta M_{MFI,q} \right| \leq \sqrt{M} |\varepsilon_{max}| \|\mathbf{c}(f)\| \quad (\text{A. 10})$$

If the columns of the system matrix $\mathbf{A}$ are an orthonormal basis and $\mathbf{j}$ is normalized (see Section 2. Eq. (3)), then by Parseval's theorem $\|\mathbf{c}(f)\| = 1$. Conversely, if $\mathbf{A}$ is allowed to be ill-conditioned, $\|\mathbf{c}\|$ and its components may be much greater than unity and the noise may be significantly amplified. To make computation practical, higher frequency components (columns) of $\mathbf{A}$ may be removed, or windowed outside the acquisition time range of $\mathbf{j}$, causing unity to be the upper bounds of $\|\mathbf{c}\|$. Both well-conditioned cases of $\|\mathbf{c}\| = 1$ yield

$$\left| \Delta M_{MFI,q} \right| \leq \sqrt{M} |\varepsilon_{max}|. \text{ However, inserting Section 2. Eq. (6b) for } \mathbf{c}(f) \text{ into Eq. (A.10)}$$

$$\left| \Delta M_{MFI,q} \right| = \left| \varepsilon_{max}^T \mathbf{A}^H \mathbf{j}(f) \right| \quad (\text{A. 11})$$

recalls that the coefficients are projections of a mono-frequency complex sinusoids onto the columns of $\mathbf{A}$, which are mono-frequency complex sinusoids. If the columns of $\mathbf{A}$ are again kept orthogonal, only a few entries of $\mathbf{c}(f)$ will take values much larger than zero and the maximum individual coefficient magnitude will be one. The result is the magnitude of $M_{MFI,q}$ approaches $|\varepsilon_{max}|$. If the maximum MFI aliasing noise from a single aliased phase-conjugated image is similar to the standard MRF aliasing noise, i.e. $|\varepsilon_{max}| \sim |I_{a,q}(\mathbf{r})|$, then the noise deviation in the MRF MFI technique is approximately bounded to that of MRF without MFI.

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Figure 1.

Shows the MFI correction for the Periodic Image Quality Test (PIQT) phantom. The magnitude image derived from the complex sum of all magnetic resonance fingerprinting TRs (a) exhibits blurring that increases with the magnitude of off-resonance mapped in (c). This geometrical blurring has been corrected by multi-frequency interpolation (b). Low signal regions can be seen at the bottom of the image, in regions of lower coil sensitivity in the uncorrected and corrected images.

Figure 2.

Shows the results before and after multi-frequency interpolation (MFI) correction of magnetic resonance fingerprinting T1 and T2 parameter maps of the 10-tube phantom for four instances of B0 heterogeneities. Each row from left to right shows the B0 map, uncorrected T1, corrected T1, uncorrected T2, and corrected T2 maps. The total off-resonance was minimized using pencil-beam volume shimming (a) and ramped transversely (b), longitudinally (c), and obliquely (d). The scaling for the parameter maps is the same between uncorrected and corrected cases for a given parameter type. MFI corrected reconstructions show geometrical improvement in all cases. The last row (d) exhibits some phantom boundary distortion after MFI correction.

Figure 3.

Presents the magnitude images for the T1 (top row) and T2 (bottom row) contrast slices of the MRI system phantom as well as B0 maps. The uncorrected (a,d) and corrected (b,e) magnetic resonance fingerprinting magnitude images were derived from the complex sum of all MRF TRs. The B0 maps (c,f) demonstrate a slope in off-resonance from the bottom to top of the image. Without multi-frequency interpolation (MFI) correction, the images show blurring in regions of off-resonance. The central spheres in (a) additionally show susceptibility artifacts that manifest as signal voids. The MFI corrected results (b,e) demonstrate a decrease in blurring in all regions.

Figure 4.

Displays the uncorrected and corrected T1 (a,b) and T2 (c,d) parameter maps from their respective slices in the MRI system phantom as shown in Figure 3. Improvements in boundaries of the contrast spheres are visible, as well as reduction of bias from the high T1 and T2 background blurring.

Figure 5.

Shows boxplots for all T1 and T2 contrast spheres in the MRI system (Msys) phantom. The results are divided into the longer half of T1 (a) and T2 (c) on the left and the shorter T1 (b) and T2 (d) on the right. Each numbered group consists of boxplots for regions of interest in the well shimmed (ws) and poorly shimmed (ps; Figure 4) cases, as well as for the multi-frequency interpolation correction for the well shimmed (ws+) and poorly shimmed (ps+) cases. The horizontal dotted lines indicate the value of the Msys specifications.

Figure 6.

Plots the coefficient of variation (COV) for all regions of interest (ROIs) with and without multi-frequency interpolation (MFI). Each ROI COV is plotted for the well shimmed without MFI (ws), well shimmed with MFI (ws+), poorly shimmed (ps), and poorly shimmed with MFI (ps+). ROIs 5 through 7 for the ps cases demonstrate large COVs relative to the ws and ps+ cases. These ROIs correspond to the contrast spheres at the bottom of the images/maps of Figures 3 and 4.

Figure 7.

Displays a transverse slice of the brain superior to the orbits. The B0 map shows increasing off-resonance near the frontal regions, referenced anatomically in the T1-weighted image (d). Without multi-frequency interpolation (MFI), the uncorrected T1 (b) and T2 (c) maps exhibit an increased blurring near the midline (white arrowhead), which appears sharper in the T1 (e) and T2 (f) MFI-corrected maps. The 'x' and '+' in (d) are the voxels shown in Fig. 8(a) and (b), respectively.

Figure 8.

Shows the MRF signal evolution for cerebrospinal fluid (CSF) (a) marked as '×' in Fig. 7(d), and white matter (WM) (b) marked as '+' in Fig. 7(d) without and with multi-frequency interpolation (MFI). The CSF is in a region of greater off-resonance and exhibits larger departures between corrected and uncorrected signals that are reflected in their different T1 and T2 values. The MFI correction in the low off-resonance region of (b) appears to match closely with the uncorrected signal. The T1 and T2 estimates in the WM are nearly identical.

Table 1

Reconstruction	T1 CCC ¹ [CI] ²	T2 CCC ¹ [CI] ²
ws- ³	0.993 [0.982, 0.997]	0.995 [0.986, 0.999]
ws+ ³	0.993 [0.983, 0.997]	0.995 [0.986, 0.999]
ps- ³	0.994 [0.988, 0.997]	0.974 [0.922, 0.992]
ps+ ³	0.995 [0.987, 0.998]	0.992 [0.977, 0.997]

¹ CCC, concordance correlation coefficient

² CI, confidence interval; numbers in brackets are the 95% CI

³ ws, well shimmed; ws+, well shimmed with multi-frequency interpolation (MFI) correction; ps, poorly shimmed; ps+, poorly shimmed with MFI correction