Frequency domain synthetic aperture focusing technique for variable-diameter cylindrical components

Haoran Jin,¹ Eryong Wu,^{1,a)} Ye Han,¹ Keji Yang,¹ and Jian Chen²

¹State Key Laboratory of Fluid Power and Mechatronic Systems, Zhejiang University, 38 Zheda Road, Westlake District, Hangzhou 310027, China

²School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, 639798, Singapore, Singapore

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Ultrasonic non-destructive testing (UNDT) plays an important role in ensuring the quality of cylindrical components of equipment such as pipes and axles. As the acoustic beam width widens along propagation depths, the diffraction of acoustic wave becomes serious and the images of defects will be interfered with. To precisely evaluate the dimensions of defects and flaws concealed in components, the synthetic aperture focusing technique (SAFT) is introduced to enhance the image resolutions. Conventional SAFTs have been successfully implemented for the ultrasonic imaging of normal cylinders, while solutions for complex ones, such as variable-diameter cylinders, are still lacking. To overcome this problem, a frequency-domain SAFT for variable-diameter cylindrical components is proposed. This algorithm is mainly based on acoustic field extrapolation, which is modified from cylindrical phase shift migration with the aid of split-step Fourier. After a series of extrapolations, a high-resolution ultrasound image can be reconstructed using a particular imaging condition. According to the experimental results, the proposed method yields low side lobes and high resolutions for flat transducers. Its attainable angular resolution relies on the transducer diameter D and scanning radius R and approximates $D/(2R)$. © 2017 Acoustical Society of America. [<http://dx.doi.org/10.1121/1.5003650>]

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I. INTRODUCTION

Cylindrical components of equipment, such as petroleum pipelines, rail axles, and turbine shafts, are widely used in industrial fields. To guarantee the functionality of these products, the use of ultrasonic non-destructive testing (UNDT) makes significant sense for quality assurance.^{1–3} In typical cylindrical ultrasonic testing, a transducer moves over the surfaces of cylinders along circular trajectories, recording the reflection echo signals and imaging the inner defects. The ultrasonic beam is assumed to be very narrow and to only illuminate regions of the object located directly in front of the transducer. Off-axis scatterers can then be ignored. This is the exact same problem that sidescan sonar makes, and sidescan systems have inherently lower maximum resolutions versus synthetic aperture based systems.^{4,5} Therefore, the synthetic aperture focusing technique (SAFT) method is also introduced to enhance the spatial resolution of images in UNDT.

Early versions of the SAFT were time-domain implementations based on the principle of delay and sum (DAS),⁶ followed by frequency-domain ones.^{7–10} Unfortunately, given the drawback of low-efficiency computations of refraction points, time-domain SAFT is not suitable for the inspection of multilayer materials. Such cases always appear in immersion and contact inspections, as the combination of coupling materials (transducer wear or water) and specimens will form multilayer structures. On the other hand, the frequency-domain SAFT method starts from homogenous materials using a frequency

wavenumber method called Stolt migration and omega- k migration.^{11,12} In recent years, further research has expanded frequency-domain SAFT to layered structures¹³ and successfully applied it to immersion ultrasonic testing.¹⁴ As this research was based on the migration of a phase factor, it was called phase shift migration (PSM). After that, many studies about PSM improvements were proposed including efficiency boosting,¹⁵ application broadening,^{16–18} model improvement,^{19,20} and so on. Taking advantage of the computationally efficient fast Fourier transform, PSM-type methods are well able to address three-dimensional (3D) ultrasound signals.

The implementation of PSM in cylindrical UNDT was first proposed by Skjelvareid *et al.*,²¹ where it is called cylindrical phase shift migration (CPSM) and applied to interior pipe inspection.²² CPSM was then developed for exterior cylindrical scanning²³ and helical scanning,²⁴ its implementation on multilayer structures is also provided in this paper.²³ In CPSM, cylindrical components are decomposed into many cylindrical layers, and then their acoustic field is recursively extrapolated through phase shift migration. The variations of acoustic velocities between different planes are acceptable, and therefore, CPSM is suitable for regularly layered structures.

Currently, industrial products are becoming increasingly complex, leading to an ever-increasing demand for variable-diameter cylinders, such as fan main shafts, railcar axles, variable-diameter pipes, and so on. However, CPSM assumes that the acoustic velocity is constant within each cylindrical layer, which is not true in the case of variable-diameter cylinders and seriously restricts CPSM's applications to complex cylinders. As shown in Fig. 1, there is a

^{a)}Electronic mail: wueryong@zju.edu.cn

z -dependence of material at a given radius in the cylindrical coordinate system. Therefore, a frequency-domain SAFT method for variable-diameter cylinders is proposed in this paper. This method also divides the specimen into extremely thin cylindrical layers and then extrapolates the acoustic field in each plane, as CPSM does. During the extrapolation in the planes that have z -dependent acoustic velocity, the Step-Split Fourier (SSF)²⁵ algorithm will be introduced to compensate for the phase vibration caused by irregular diameters. Finally, a high-resolution image can be reconstructed from these acoustic fields by a particular imaging condition. Since the proposed method combines SSF with CPSM, it is named cylindrical step-split migration (CSSM).

This paper is organized as follows. Section II will introduce the basic concepts of acoustic field extrapolation in cylindrical coordinates, the SSF algorithm, and the exploding reflector model (ERM). Section III will explain the detailed implementation of CSSM including surface detection and the flow chart of CSSM. Experiments with multi-diameter and variable-diameter specimens are used to verify CSSM's performance in Sec. IV. Finally, we state some discussions and conclusions in Sec. V.

II. THEORY

A. Acoustic field extrapolation

The wave equation in cylindrical coordinates is defined as²⁶

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) p(t, \phi, z, r) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} p(t, \phi, z, r). \quad (1)$$

FIG. 1. (Color online) An irregular stratification surface with variable-diameter shafts.

Based on the cylindrical wave equation's solution, the sound pressure can be decomposed into

$$p(t, \phi, z, r) = \sum_{n=-\infty}^{\infty} e^{in\phi} \iint_{z, \omega} (A_n(k_z, \omega) H_n^{(1)}(k_r r) + B_n(k_z, \omega) H_n^{(2)}(k_r r)) e^{ik_z z} e^{-i\omega t} d\omega dk_z, \quad (2)$$

where $H_n^{(1),(2)}(k_r r)$ denotes a Hankel function of the first or second kind with order n ; k_z , k_r , and n denote the wave numbers along the z , r , and ϕ axes, respectively; ω denotes angular frequency; and $A_n(k_z, \omega)$ and $B_n(k_z, \omega)$ denote complex amplitudes for the individual wave components. The relationship between k_z , k_r , and ω is defined as

$$k_r^2 + k_z^2 = \left(\frac{\omega}{c} \right)^2. \quad (3)$$

The Fourier-domain representation of sound pressure is defined as

$$p(t, \phi, z, r) = \sum_{n=-\infty}^{\infty} \exp(in\phi) \iint_{z, \omega} P_n(\omega, k_z, r) \times \exp(ik_z z) \exp(-i\omega t) d\omega dk_z. \quad (4)$$

As Eq. (4) is similar to Eq. (2), the Fourier-domain sound pressure could be represented as

$$P_n(\omega, k_z, r) = A_n(k_z, \omega) H_n^{(1)}(k_r r) + B_n(k_z, \omega) H_n^{(2)}(k_r r). \quad (5)$$

The first order Hankel function represents an outward travelling wave, while the second order represents an inward travelling wave. In cases of exterior cylindrical scanning, only the outward wave is needed, so the second order Hankel function will be neglected. Equation (5) is simplified to

$$P_n(\omega, k_z, r) = A_n(k_z, \omega) H_n^{(1)}(k_r r). \quad (6)$$

Then, the formula of acoustic field extrapolation is obtained as

$$P_n(\omega, k_z, r) = P_n(\omega, k_z, R) \cdot \frac{H_n^{(1)}(k_r r)}{H_n^{(1)}(k_r R)}, \quad (7)$$

where the ratio of Hankel functions represents the transfer function of cylindrical layers from radius R to radius r . Correspondingly, the sound pressure of a cylindrical layer of arbitrary radius r is calculated by

$$p(t, \phi, z, r) = \mathfrak{F}_{t, \phi, z}^{-1} \left\{ \mathfrak{F}_{t, \phi, z} (p(t, \phi, z, R)) \cdot \frac{H_n^{(1)}(k_r r)}{H_n^{(1)}(k_r R)} \right\}, \quad (8)$$

where $\mathfrak{F}_{t, \phi, z}$ and $\mathfrak{F}_{t, \phi, z}^{-1}$ denote the Fourier and inverse Fourier transform about z , ϕ , and t , respectively. Note that $P_n(\omega, k_z, R)$ is the Fourier form of $p(t, \phi, z, R)$, which is the sampled signal. In the case of a cylindrical layer with constant acoustic velocity, the relationship of wavenumbers

satisfies Eq. (3) and hence, the sound pressure can be correctly obtained with Eq. (8).

B. Compensation of z-dependent acoustic velocity

Sometimes, the material is z-dependent within a radius as shown in Fig. 2, and the acoustic velocity will change in an extrapolation plane. Therefore, the performance of acoustic field extrapolation does not hold. To expand the implementation of extrapolation, a phase shift compensation technique named SSF is introduced to compensate for the diameter variance.

The bulk longitudinal velocity varies with the r-axis and z-axis, and its reciprocal, called slowness, is defined as

$$s(z, r) = 1/c(z, r). \quad (9)$$

SSF splits the space-variant slowness into two terms²⁵

$$s(z, r) = s_0(r) + s_v(z, r), \quad (10)$$

where $s_0(r)$ and $s_v(z, r)$ represent the constant terms and perturbation terms, respectively.

Substituting Eq. (10) into the cylindrical wave equation gives

$$\begin{aligned} & \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{\partial r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) p_s(t, \phi, z, r) \\ &= (s_0(r) + s_v(z, r))^2 \frac{\partial^2}{\partial t^2} p_s(t, \phi, z, r). \end{aligned} \quad (11)$$

Here, $p_s(t, \phi, z, r)$ is used to express the sound pressure extrapolated considering the acoustic variation along the z-axis and to distinguish it from $p(t, \phi, z, r)$. The wave equation Eq. (11) is Fourier-transformed along the time axis to become

$$\begin{aligned} & \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{\partial r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) P_s(\omega, \phi, z, r) \\ &= (s_0(r) + s_v(z, r))^2 \omega^2 P_s(\omega, \phi, z, r). \end{aligned} \quad (12)$$

Then, Eq. (12) is rewritten as

$$\begin{aligned} & \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{\partial r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} - s_0(r)^2 \omega^2 \right) P_s(\omega, \phi, z, r) \\ &= \left[2s_0(r)s_v(z, r) + s_v(z, r)^2 \right] \omega^2 P_s(\omega, \phi, z, r), \end{aligned} \quad (13)$$

whose homogenous equation is a standard cylindrical wave equation with acoustic velocity $1/s_0(r)$. According to the theory of SSF,¹⁹ Eq. (13) can be integrated over a thin depth layer Δr by ignoring the contribution of $s_v(z, r)^2$. Its solution form is

$$\begin{aligned} P_s(\omega, \phi, z, r) &= \frac{\exp(j\omega s_v(z, r)\Delta r)}{2\pi} \int p(t, \phi, r, z) \\ &\times \exp(i\omega t) dt, \end{aligned} \quad (14)$$

where $\exp(j\omega s_v(z, r)\Delta r)$ represents the compensation factor of phase shift and can be rewritten as

$$\exp(j\omega s_v(z, r)\Delta r) = \exp\left(j\omega \left(\frac{\Delta r}{c(z, r)} - \frac{\Delta r}{c_0(r)} \right)\right), \quad (15)$$

where $c_0(r) = 1/s_0(r)$. The deviation between $\Delta r/c(z, r)$ and $\Delta r/c_0(r)$ can be considered as a time delay caused by velocity variation.

C. ERM

In the pulse-echo inspection system for cylinders, ultrasound waves propagate from transducers to defects, then reflect back and are received by corresponding transducers. As shown in Fig. 2, the transducer moves around the tested cylindrical component circularly, then moves one z-axis step for the next scan, and so on. After the whole component has been scanned, the sound pressure $p(t, \phi, z, R)$ is recorded. As the transducer is responsible for both exciting and receiving ultrasound, the propagation routes of transmitting and

FIG. 2. Cylindrical scanning geometry.

FIG. 3. Model transformation from the pulse echo model (a) to the ERM (b).

reflecting are reciprocal. In such cases, the propagation procedure could be simplified with the ERM.^{27,28}

The ERM is a basic assumption of phase shift migration,³⁰ which assumes that all the reflectors in the medium explode simultaneously and become outward emitting acoustic sources (Fig. 3). This assumption converts the pulse-echo process to point-like acoustic emissions that explode at the same time and are received by multiple receiver elements. The resulting ultrasound data acquired by a cylindrical scan can be assembled into a synthetic cylindrical aperture. Note that, to guarantee that the time delay of the outward wave is equal to the back-and-forth wave propagation time, the acoustic velocity should be reduced by half.²⁹

According to the ERM, the sources explode at $t = 0$, which means ultrasound waves have not diverged. Without divergence, the signal recorded has a high spatial resolution and is well for imaging. Therefore, the basic principle of image reconstruction is imaging at $t = 0$. First, the observation plane is assumed to locate on the cylindrical layer of transducer scanning. Then, the observation plane and its acoustic field could be virtually retracted along the radius using extrapolation mentioned above. Finally, a high resolution image is obtained by reconstructing all acoustic fields in different radius at $t = 0$.

III. METHOD

A. Immersion testing and surface echo detection

In practical ultrasonic inspection of variable-diameter cylinders, contact testing with transducer wear is difficult to adapt to the surface variation, and therefore, immersion testing is preferred, as shown in Fig. 1. A cylindrical layer at a particular radius may contain multiple media, and thus multiple acoustic

FIG. 4. Flank model of surface wave detection.

velocities. The assumption is made that the sound speeds of both media, the object and the immersing water, are known. However, the geometry of the object may not be known, and so the distribution of acoustic velocities within each cylindrical layer must be determined. This may be done by determining the shape of the object via the specular echo.

In the practical ultrasonic testing case, the thickness of the water layer is measured through tracing the surface echo, as shown in Fig. 4. By monitoring the time of flight (TOF) of the peak point, the surface topography of the cylindrical component will be determined. Furthermore, the distribution of acoustic velocity $c(z, r)$ can be obtained from this topography.

B. Synthetic aperture focusing

After the sampled data and velocity distribution are prepared, synthetic aperture focusing begins. The flow chart of synthetic aperture focusing is shown in Fig. 5, and it is mainly composed of three sub-steps: CPSM, SSF, and imaging.

FIG. 5. Flow chart of the synthetic aperture focusing technique.

FIG. 6. Experimental setup.

First, the acquired raw data are transformed to the Fourier domain by a Fourier series expansion in ϕ and a Fourier transform in z and t . CPSM extrapolation is employed on the spectra of raw data to compute the acoustic field at the r circular plane by using Eq. (7). The wavenumber k_r calculated by Eq. (3) is based on a mean acoustic velocity $c_0(r)$ that is equivalent to the acoustic velocity of the material in a homogenous layer, but defined by $c_0(r) = 1/s_0(r)$ in a heterogeneous one. When handling a z -dependent velocity, as in a cylindrical layer with heterogeneous materials, SSF is employed to compensate for the velocity deviation. Note that, before using Eq. (15) to compensate, the acoustic field should be transformed into frequency- z spectrum. Finally, the image condition of ERM $t = 0$ is used to reconstruct the image. The acoustic field should be extrapolated and inversely Fourier-transformed layer by layer with steps of Δr to obtain focused images at different radii. Since velocity compensation is only implemented at the interface layer between water and cylinders, it will not increase the computational burden much compared to CPSM.

IV. EXPERIMENT

A. Experimental setup

As shown in Fig. 6, the experimental equipment consists of a mechanical scanning platform and an ultrasonic imaging device. The ultrasonic imaging device is mainly responsible for transmitting high voltage pulses, receiving and magnifying the reflected echoes, digitizing the electrical signals, and subsequent signal processing. The analog-to-digital converter has a 100 MHz sampling frequency and a 16-bit

sampling depth. A 5 MHz flat transducer with a 4 mm diameter (I5P4NF, Guangzhou Doppler Electronic Technologies Co., Ltd., Guangzhou, China) is connected to this equipment. The transducer is a circular piston type with no baffling or shaping. The mechanical scanning platform controls the rotation of the specimen and the motion of the transducer. Scanning starts by rotating the specimen by one full rotation, then moving the immersed transducer one step upward, and iterating until the whole specimen is tested.

B. Measurement of multi-diameter shaft

To verify the performance of CSSM, an aluminum shaft specimen with stages is provided for testing. This specimen has five holes as simulated defects, as shown in Fig. 7. Each FBH is 2 mm in diameter, 30 mm in depth, and regarded as a point-like artificial defect. Here, we marked the numbers of holes from “1” to “5” along with their angular positions. The specimen is immersed in a water tank and placed on the center of a rotary table approximately 20 mm from the transducer. The cylindrical scanning goes from 0 to 125° in ϕ with a 0.5° angular sampling rate and from 20 to 56 mm in z with a 1 mm descent interval. The scanning radius of the transducer is 92 mm. Note that the angular sampling rate should be sufficient for the azimuthal sampling criterion.^{30,31}

The envelopes of the raw 3D data set are presented in the ϕ - t and z - t perspectives in Figs. 8(a) and 8(b), respectively. After being processed by the CPSM and CSSM algorithms, the reconstructed data set is presented in the ϕ - r and z - r perspectives in Figs. 8(c)–8(f). Note that the ϕ - t and ϕ - r images are top-down views, and the z - t and z - r images are side-on looks.

In the raw data in Figs. 8(a) and 8(b), the signals reflected from the defects are almost overshadowed by the background noise and heavy-tailed due to the acoustic diffraction effect, which distorts the shapes of scatters. After processing by CPSM, the defects are still blurred and segmented, as shown in Figs. 8(c) and 8(d). Therefore, CPSM cannot correctly reconstruct the data. In contrast, the defects in Fig. 8(e) have converged to circular dots and are clearly presented as continuous horizontal lines in Fig. 8(f). Through these two reconstructed images, the defects can be intuitively inferred as five FBHs.

A detailed angular resolution analysis based on Figs. 8(a) and 8(e) is illustrated in Fig. 9. It is obvious that the signals of holes “1” and “2” are deteriorated by diffraction, and it is difficult to precisely evaluate their widths, as shown in

FIG. 7. Geometry of the multi-diameter specimen.

FIG. 8. (Color online) (a) and (b) ϕ - t and z - t image of raw data; (c) and (d) ϕ - r and z - r image of CPSM; (e) and (f) ϕ - r and z - r image of CSSM.

Fig. 9(a). The widths of the other three defects measured by their full width at half maximum (FWHM) are 19, 7.5, and 5.5°, respectively. However, these angular widths are seriously magnified compared to the real dimensions of the defects. In contrast, the defects reconstructed by CSSM are well distinguished, and their FWHMs are approximately 1~1.5°. The maximum theoretical angular resolution²¹ is given by

$$\Delta\phi = D/2R(\text{rad}), \quad (16)$$

where D is the transducer diameter and R is the scanning radius. In this case, the FWHMs are approximated to a theoretical resolution of 1.245°. Figure 9(c) is the resolution comparison between the original image and the reconstructed image. Although accurate FWHMs of holes “1” and

“2” in the original image are not obtained, the approximate values should be larger than that of hole “3” according to the uptrend of defects’ widths. Compared with the original image, CSSM markedly enhances the angular resolution of reconstructed images and efficiently eliminates the diffraction. According to Fig. 8(f), the deviations between the measured depth and the real dimensions are less than 1 mm, which is one scan sampling rate in the z direction. In other words, the resolutions of reconstructed images along the z -axis could reach 1 mm.

C. Measurement of variable-diameter shaft

Another experiment was designed for the measurement of a variable-diameter shaft whose structure is shown in Fig. 10. The outline of the shaft varies gradually from 150

FIG. 9. Angular resolution comparison. (a) The normalized amplitude of simulated defects' reflection echo in the original image. (b) The normalized amplitude of simulated defects' reflection echo in the reconstructed image. (c) The angular resolutions measured by the FWHM.

to 160 mm with a curvature of 80 mm. There are six holes aligned in the shaft, and they are sorted into three groups with a between-group angle of 25° and a within-group angle of 5° . These three groups are distributed with a 5 mm depth offset and a 10 mm radial offset between adjacent groups. This measurement scans from 0 to 90° along the ϕ -axis and from 5 to 36 mm along the z -axis, with the same experimental setup and scanning sampling rate. The transducer is 95 mm from the scanning center.

The envelopes of the raw 3D data set are presented in the ϕ - t and z - t perspectives in Figs. 11(a) and 11(b), respectively. The reconstructed images are presented in Figs. 11(c) and Fig. 11(d). As CPSM has been demonstrated to be

invalid for such cylinders, we neglect its reconstruction results here.

As shown in Fig. 11(a), adjacent defects are interfered due to acoustic diffraction and hard to distinguish. In Fig. 11(b), the defects smoothly vary with the reflection echo of the specimen outline. Note that the break of outline in this figure is due to energy loss caused by oblique incidence. Hence, there is a similar break in the reconstructed image. However, the drawback of interference is completely removed after applying the CSSM method, as shown in Fig. 11(c). Meanwhile, the resolutions of the reconstructed image are significantly enhanced. The angular resolutions measured by the FWHM are all approximately 1° , satisfying the

FIG. 10. Geometry of the variable-diameter specimen.

theoretical resolution of 1.206° . In Fig. 11(d), the defects are reconstructed as straight lines that well approximate the shapes of the holes.

V. DISCUSSION AND CONCLUSION

In this paper, cylindrical split-step migration is proposed to broaden the application of frequency domain SAFT to ultrasound images of variable-diameter cylindrical components. CSSM has the resolution-enhancing ability of CPSM without significantly increasing its computational burden. The capability of CSSM is demonstrated by two inspection measurements of shafts with diameter variation.

The resolutions of the reconstructed images reach approximately 1° in the angular direction and 1 mm in depth and are much better than the original ones. According to these results, the performance of CSSM can be summarized in three aspects:

- (1) The diffraction effect of the ultrasound wave is reduced and the imaging resolution is significantly enhanced. The angular resolution is approximately $D/(2R)$, where D denotes the transducer diameter and R denotes the scanning radius.
- (2) The defect echo energy will be focused while noise is suppressed.

FIG. 11. (Color online) (a) and (b) ϕ - t and z - t image of raw data; (c) and (d) ϕ - r and z - r image of CSSM.

- (3) The reconstructed images are built in the geometry coordinates, which are closer to real dimensions and friendlier for further quantitative evaluation of defects.

CSSM has the potential to deal not only with variable diameters in the z -direction but also with variation in the ϕ -direction.³² Put another way, CSSM could in theory be widely adapted to arbitrary cylinders for cylindrical scanning. Upon practical inspection, complex geometry may cause intensive energy loss and finally lead to reflection echoes of defects under transducer sensitivity, though this problem has so far been beyond the scope of SAFT research. Our current research mainly supports vertically incident ultrasound waves, and therefore, oblique incidence will be one direction of investigation in the future. Additional research is planned to implement this SAFT in helical imaging. Both of these future research directions are closely associated with practical UNDT.

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- ¹B. Rajani and K. Yehuda, "Non-destructive inspection techniques to determine structural distress indicators in water mains," in *Evaluation Control of Water Loss in Urban Water Networks*, Valencia, Spain (June 21–25, 2004).
- ²K. Makino, J. Yohso, H. Sakamoto, and H. Ishiduka, "Hollow axle ultrasonic crack detection for conventional railway vehicles," *Q. Report RTRI* **46**(2), 78–84 (2005).
- ³G. Y. Zhao and C. Lu, "Time domain/frequency domain SAFT imaging in thin-diameter rod," *Appl. Mech. Mater.* **380**, 3648–3652 (2013).
- ⁴G. C. Eastland, T. M. Marston, and P. L. Marston, "Reversible bistatic and monostatic synthetic aperture sonar filtering," *J. Acoust. Soc. Am.* **129**(4), 2686 (2011).
- ⁵T. M. Marston, J. L. Kennedy, and P. L. Marston, "Coherent and semi-coherent processing of limited-aperture circular synthetic aperture (CSAS) data," in *Oceans IEEE*, Waikoloa, HI (September 19–22, 2011), pp. 1–6.
- ⁶S. R. Doctor, T. E. Hall, and L. D. Reid, "SAFT—The evolution of a signal processing technology for ultrasonic testing," *NDT Int.* **19**(3), 163–167 (1986).
- ⁷K. Nagai, "Fourier domain reconstruction of synthetic focus acoustic imaging system," *Proc. IEEE* **72**(6), 748–749 (1984).
- ⁸K. J. Langenberg, M. Berger, T. Kreutter, K. Mayer, and V. Schmitz, "Synthetic aperture focusing technique signal processing," *NDT Int.* **19**(3), 177–189 (1986).
- ⁹K. Mayer, R. Marklein, K. J. Langenberg, and T. Kreutter, "Three-dimensional imaging system based on Fourier transform synthetic aperture focusing technique," *Ultrasonics* **28**(4), 241–255 (1990).
- ¹⁰L. J. Busse, "Three-dimensional imaging using a frequency-domain synthetic aperture focusing technique," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **39**(2), 174–179 (1992).
- ¹¹R. H. Stolt, "Migration by Fourier transform," *Geophysics* **43**(1), 23–48 (1978).
- ¹²T. Stepinski, "An implementation of synthetic aperture focusing technique in frequency domain," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **54**(7), 1399–1408 (2007).
- ¹³T. Olofsson, "Phase shift migration for imaging layered objects and objects immersed in water," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **57**(11), 2522–2530 (2010).
- ¹⁴T. Olofsson, M. H. Skjeltvareid, and A. Barkefors, "Ultrasonic imaging of immersed objects using migration techniques," in *Proceedings of the 2010 8th European Conference on Synthetic Aperture Radar (EUSAR)*, Aachen, Germany (June 7–10, 2010), pp. 1–4.
- ¹⁵M. H. Skjeltvareid, T. Olofsson, Y. Birkelund, and Y. Larsen, "Synthetic aperture focusing of ultrasonic data from multilayered media using an omega-k algorithm," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **58**(5), 1037–1048 (2011).
- ¹⁶T. Lukomski, T. Stepinski, and J. Kowal, "Synthetic aperture focusing technique with virtual transducer for immersion inspection of solid objects," *Insight-Non-Destruct. Test. Cond. Monitor.* **54**(11), 623–627 (2012).
- ¹⁷H. Wu, J. Chen, K. Yang, and X. Hu, "Ultrasonic array imaging of multi-layer structures using full matrix capture and extended phase shift migration," *Meas. Sci. Technol.* **27**(4), 045401 (2016).
- ¹⁸T. Lukomski, "Full-matrix capture with phased shift migration for flaw detection in layered objects with complex geometry," *Ultrasonics* **70**, 241–247 (2016).
- ¹⁹C. Yang, K. Qin, and Y. Li, "Real-time ultrasonic imaging for multi-layered objects with synthetic aperture focusing technique," in *Proceedings of the 2014 IEEE International Instrumentation and Measurement Technology Conference (I2MTC)*, Montevideo, Uruguay (May 12–15, 2014), pp. 561–566.
- ²⁰K. Qin, C. Yang, and F. Sun, "Generalized frequency-domain synthetic aperture focusing technique for ultrasonic imaging of irregularly layered objects," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **61**(1), 133–146 (2014).
- ²¹M. H. Skjeltvareid, Y. Birkelund, and Y. Larsen, "Synthetic aperture focusing of outwardly directed cylindrical ultrasound scans," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **59**(11), 2460–2469 (2012).
- ²²M. H. Skjeltvareid, Y. Birkelund, and Y. Larsen, "Internal pipeline inspection using virtual source synthetic aperture ultrasound imaging," *NDT & E Int.* **54**, 151–158 (2013).
- ²³S. Wu, M. H. Skjeltvareid, K. Yang, and J. Chen, "Synthetic aperture imaging for multilayer cylindrical object using an exterior rotating transducer," *Rev. Sci. Instr.* **86**(8), 083703 (2015).
- ²⁴H. Jin, J. Chen, E. Wu, and K. Yang, "Frequency-domain synthetic aperture focusing for helical ultrasonic imaging," *J. Appl. Phys.* **121**(13), 134901 (2017).
- ²⁵P. L. Stoffa, J. T. Fokkema, R. M. de Luna Freire, and W. P. Kessinger, "Split-step Fourier migration," *Geophysics* **55**(4), 410–421 (1990).
- ²⁶E. G. Williams, *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography* (Academic Press, New York, 1999), pp. 115–182.
- ²⁷J. F. Claerbout, *Imaging the Earth's Interior* (Blackwell Scientific, New York, 1985), p. 398.
- ²⁸K. Baik, C. Dudley, and P. L. Marston, "Acoustic quasi-holographic images of scattering by vertical cylinders from one-dimensional bistatic scans," *J. Acoust. Soc. Am.* **130**(6), 3838–3851 (2011).
- ²⁹D. Garcia, L. L. Tarnec, S. Muth, E. Montagnon, J. Porée, and G. Cloutier, "Stolt's fk migration for plane wave ultrasound imaging," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **60**(9), 1853–1867 (2013).
- ³⁰F. Simonetti, L. Huang, and N. Duric, "On the spatial sampling of wave fields with circular ring apertures," *J. Appl. Phys.* **101**(8), 083103 (2007).
- ³¹D. S. Plotnick, P. L. Marston, and T. M. Marston, "Fast nearfield to far-field conversion algorithm for circular synthetic aperture sonar," *J. Acoust. Soc. Am.* **134**(5), 4078 (2013).
- ³²H. Jin, S. Wu, and K. Yang, "A frequency synthetic aperture focusing technique for eccentric circular scanning," in *Proceedings of the 2016 IEEE International Ultrasonics Symposium (IUS)*, Tours, France (September 18–21, 2016), pp. 1–4.