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MOTION OF A RIGID CYLINDER BETWEEN PARALLEL PLATES IN STOKES FLOW:

PART 2: POISEUILLE AND COUETTE FLOW

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Abstract

A numerical solution is presented for the motion of a neutrally buoyant circular cylinder in Poiseuille and Couette flows between two plane parallel boundaries. The force and torque on a stationary particle are calculated for a wide range of particle sizes and positions across the channel. The resistance matrix calculated in Ref. [1] (henceforth referred to as Part 1) is utilized to find the translational and angular velocity for a drag- and torque-free particle. The results are compared with analytical perturbation solutions for a small cylindrical particle situated on the channel centerline, and for the motion of a spherical particle in a circular tube or between plane parallel boundaries. It is found the behavior of flow around a cylindrical particle in a channel is qualitatively similar to the behavior of flow around a spherical particle in a tube, while the flow around a spherical particle in a channel frequently exhibits different trends from the above two cases.

1. INTRODUCTION

This paper presents a numerical solution for creeping flow around a circular cylinder in Poiseuille and Couette flows; the cylinder is either held fixed in the stream, or is drag- and torque-free and is carried by the flow. The numerical method used in the present investigation is described elsewhere [2, 3]. The solution utilizes the resistance matrix described in Part 1.

The earlier analytic solutions for a quiescent cylindrical particle situated midway between channel walls in a Poiseuille flow were obtained by Harrison [4], Faxen [5] and Takaisi [6]. These authors used the method of reflections, which limited the results to small ratios of particle radius to channel width. The results of Harrison [4] and Takaisi [6] generally agree to the order computed; the results of Faxen [5] contain higher order terms and, therefore, are more accurate.

The mathematical formulation of the problem is given in Section 2 of Part 1; additional details are given in Section 2 below. Section 3 of the present paper presents the solution for a cylindrical particle in 2-D Poiseuille flow and Section 4 the solution for Couette flow. The solutions are obtained for a wide range of particle radii and positions across the channel.

In Section 5 we discuss the characteristics of the 2-D case and two cases of 3-D motion reported in the literature (a sphere in a circular tube and a sphere between parallel plates).

2. MATHEMATICAL FORMULATION

In this section we briefly outline the formulation given in detail in Part 1 and specify additional features of the problem pertaining to the 2-D Poiseuille and Couette flows. Consider the creeping flow around a circular cylindrical particle of radius R in a channel formed by two plane parallel boundaries a distance H apart. All variables are rendered dimensionless using H as the characteristic length scale and U as the characteristic velocity; the velocity will be specified for each particular case considered below.

The geometry of the problem is shown in Fig. 1. In the case of Poiseuille flow a parabolic velocity profile $u = 6y(1 - y)$ is specified at the inlet of the channel. The characteristic velocity is defined as the mean velocity in the channel. The no-slip boundary conditions are imposed on the lower and upper boundaries, $u = 0$ at $y = 0$ and $y = 1$.

In the case of Couette flow the velocities of the boundary plates are specified to be equal in magnitude but opposite in sign; the magnitude of the plate velocity is chosen as the characteristic velocity U . Thus, $u = 1$ at $y = 1$ and $u = -1$ at $y = 0$, and at the inlet of the channel the velocity profile is $u = 2(y - 0.5)$ so that the volumetric flow rate is zero.

As was outlined in Part 1, the Stokes equations are solved numerically for a given flow configuration using an iterative algorithm for solution of the finite-difference equations in numerically generated boundary-conforming coordinates. When the flow field is obtained, the net force and torque on the particle are calculated by integrating over the surface of the particle. These calculations result in a linear relationship between the force-torque vector (F , T) and velocity vector (U_c , Ω):

$$\begin{pmatrix} F_1 \\ T \end{pmatrix} = \begin{pmatrix} A_{11} & A_{13} \\ A_{31} & A_{33} \end{pmatrix} \begin{pmatrix} U_c \\ \Omega \end{pmatrix} + \begin{pmatrix} F_{10} \\ T_0 \end{pmatrix}, \quad (2.1)$$

Due to the symmetry of the problem and the assumption of zero Reynolds number, the force and velocity in the direction perpendicular to the walls are zero and do not appear in the relationship (2.1) (Ref. [7]). The coefficients A_{11} , A_{33} , and $A_{13} = A_{31}$ as functions of particle position, y and particle radius, R , were reported in Part 1. In the present paper the functions $F_{10}(y, R)$ and $T_0(y, R)$ and velocity distributions $U_c(y, R)$ and $\Omega(y, R)$ for drag- and torque-free particles will be calculated for both Poiseuille and Couette flows.

3. CYLINDRICAL PARTICLE IN POISEUILLE FLOW

First, we determine the drag on a particle held fixed in a Poiseuille flow, $u = 6y(1 - y)$. Harrison [4], Faxen [5] and Takaisi [6] obtained approximate solutions to the problem for a particle situated on the centerline of the channel calculations in Fig. 2. All solutions are in agreement for particle radius $R \leq 0.15$. Above this value the solutions of Harrison and Takaisi are inaccurate as is manifested by their qualitatively incorrect behavior. Our solution

agrees with Faxen's up to $R \cong 0.25$. Faxen's equation for the force has a singularity at $R = 0.313$ where the force goes to infinity. Clearly, the singularity should be located at $R = 0.5$, i.e. when the particle blocks the channel. Thus, the numerical solution agrees with the approximate perturbation solutions within their range of validity.

Figure 3 shows the horizontal force on a stationary particle across the channel, $F_{10}(y, R)$, for particles of different sizes. The particle experiences maximum force when it is located in the middle of the channel, and the force decreases monotonically as the particle is displaced towards the wall. When the particle touches the wall, i.e. $y = 1 - R$, the force has a finite value that is higher for larger particles.

In addition to the drag, a particle eccentrically situated in the channel experiences a torque. Figure 4 depicts the torque on a quiescent particle as a function of particle position. The torque vanishes midway between the channel walls, as can be expected from the symmetry of the problem, and increases monotonically as the particle is displaced towards the wall. When the particle touches the wall, the torque has a finite magnitude.

These results complete the calculation of the coefficients in eqn (2.1) for plane Poiseuille flow. The information obtained can now be utilized in determining the translational velocity $U_c(y, R)$ and angular velocity $\Omega(y, R)$, for a neutrally-bouyant force- and torque-free particle in Poiseuille flow. The velocities are found from eqn (2.1) by setting $F_1 = T = 0$ and solving the system of two linear equations with respect to U_c and Ω .

We can compare the obtained centerline velocity, $U_{co} = U_c(0.5, R)$, with the values resulting from Faxen's perturbation solution. Although Faxen did not report the free particle velocity in his papers, it can be easily obtained by superposition of his solutions for the force on a quiescent particle in Poiseuille flow and therefore on a particle translating with constant velocity parallel to the channel walls in a quiescent fluid [5]. Our velocity for $R = 0.15$ is somewhat lower than the value predicted by Faxen; the relative error is 1.9%. For $R = 0.2$ the predictions practically coincide; for $R > 0.2$ Faxen's values decrease sharply and his solution becomes inapplicable. It is useful to derive an empirical expression for the particle translational velocity, $U_{co}(R)$. The expression:

$$U_{co}(R) = \frac{3}{2} \frac{1}{1+R^\gamma}, \quad (3.1)$$

is chosen by analogy with the form proposed for the case of a spherical particle in a circular tube [8]. A nonlinear fit of the data (Faxen's results for $0 < R \leq 0.2$ and the present results for $0.2 < R \leq 0.45$) yields $\gamma = 1.91$.

Figure 5(a) shows the profile of translational velocity across the channel for a free particle. Curve 1 represents the parabolic velocity profile, $u = 6y(1 - y)$. When the particle approaches the wall, $y \rightarrow 1 - R$, its translational velocity vanishes.

It would be useful for future applications to derive an empirical relationship for the normalized translational velocity distribution. We introduce a new transverse coordinate:

$$z = \frac{y - 0.5}{0.5 - R}, \quad (3.2)$$

such that when y varies between R and $1 - R$, z varies between -1 and 1 . An empirical relationship is sought in the form:

$$U_c/U_{co} = 1 - \sum_k a_k(R) z^{2k} \quad (k=1, 2, 3 \dots) \quad (3.3)$$

with no slip condition imposed on the boundaries $z = \pm 1$. The asymptotic behavior of the coefficients a_k can be predicted for $R \rightarrow 0$: $a_1 \rightarrow 1$, $a_k \rightarrow 0$ ($k \geq 2$); and for $R = 0.5$: $a_k \rightarrow 0$ ($k \geq 1$). Table 1 lists the values a_k for particles of different radii determined by the method of least squares. Only the calculated values corresponding to the interval $0 \leq y \leq 0.95 - R$ were used, i.e. the gap between the particle and the wall was not smaller than 5% of the channel width. For smaller gaps, the calculated velocity profile becomes very steep and an increasingly large number of terms in eqn (3.3) would be required to describe the profiles accurately. For the interval of y considered, the relative error in approximating the calculated profiles by relationship eqn (3.3) with three terms does not exceed 5.5%, with the error increasing towards the wall.

Figure 5(b) shows the angular velocity of a free particle in Poiseuille flow as a function of particle position. Curve designated $R \rightarrow 0$ represents the angular velocity of an infinitesimally small particle in an unbounded shear flow. The angular velocity is zero at the channel centerline, as expected from symmetry. It reaches a maximum at some lateral position and steeply decreases to zero as the particle approaches the wall.

When a free particle is present in the channel, the pressure difference required to maintain a certain flow rate is larger than in the case of fluid flow without particle. The additional pressure drop, Δp , is shown in Fig. 6 as a function of particle position in the channel. For a given particle, the pressure drop increases if the particle is closer to the wall; when the particle touches the wall, Δp is maximal. The additional pressure drop in a flow with a symmetrically located particle is a monotonic function of particle radius and increases from zero for $R = 0$ to infinity as radius approaches 0.5.

4. CYLINDRICAL PARTICLE IN COUETTE FLOW

In this section solutions are presented for the motion of a neutrally bouyant cylindrical particle in Couette flow. The flow is generated by the motion of plane channel walls in opposite directions with equal velocity (top wall moves to the right) U ; U is chosen as the characteristic velocity with respect to which dimensionless quantities are defined. The total volumetric flow rate through any cross section of the channel is kept at zero.

First, we will calculate the force, F_{10} and torque, T_0 , on a quiescent particle for different positions of the particle in the channel and for particles of different sizes. Then we will use

relationship eqn (2.1), in which the coefficients of the resistance matrix have been reported in Part 1, to calculate the translational and angular velocities, $U_c(y, R)$ and $\Omega(y, R)$ for a free particle in Couette flow.

Figure 7 gives the force on a quiescent particle in Couette flow as a function of the position of the particle. As could be expected from the symmetry of the problem the force is zero at the centerline and monotonically increases to infinity as the particle approaches the wall. The calculations predict an inflection point on each curve. In order to gain more insight in the behavior of the force acting on a body placed in Couette flow, an analytical expression for the drag on a long quiescent slab of thickness $2d$ is obtained. The force per unit length of the slab is given by:

$$F_1 = -(1 - 2y) \left[\frac{1}{(y - d)(1 - y - d)} + \frac{3(1 + 2d)}{(1 - 2d)(1 - 3y + 3y^2 + d^2 - d)} \right] \quad (4.1)$$

The behavior of the force is qualitatively very similar to the force on a cylindrical particle. In particular, an inflection point is present on each curve.

Figure 8 presents the torque on a quiescent particle in Couette flow as a function of particle position. Due to the symmetry of the problem, the torque is symmetric with respect to the centerline of the channel. The torque is negative (clockwise direction) when the center of the particle is situated on the channel centerline, $y = 0.5$. As the particle is displaced toward the wall, the torque should tend to negative infinity due to the increasing velocity gradients in the gap between the particle and the wall. The behavior of the torque between these two extremes is not monotonic however: first, the magnitude of the torque decreases as the particle is shifted from the channel centerline, and then goes to negative infinity as the particle approaches the wall. For larger particles ($R \geq 0.25$) the torque even becomes positive, i.e. it tends to rotate the particle in the counterclockwise direction even though the motion of the boundaries appear to generate a flow that would rotate the particle in the clockwise direction. To unravel this paradox, we examine the flow pattern around a quiescent eccentrically situated particle of $R = 0.2$ (Fig. 9). Clearly, most of the flow is blocked by the particle causing the streamlines to turn back. The shear forces on the particle, generated by this backflow, tend to rotate the particle in the counterclockwise direction. Therefore, the net torque on the particle, T_0 , is a result of two competing actions: the flow along the walls tends to rotate the particle in the clockwise direction but the flow turning back (blocked flow) tends to rotate the particle in the counterclockwise direction. The particle radius $R \cong 0.2$, corresponding to a particle blocking about 40% of the channel width, is sufficient for these actions to cancel one another at a certain position of the particle, $y \cong 0.77$, so that the net torque vanishes. For larger particles the turning flow prevails for a range of particle positions, and the net torque is positive.

Once the force and torque on a quiescent particle in Couette flow are determined, the resistance matrix eqn (2.1) can be resolved with respect to U_c and Ω for a neutrally-bouyant force- and torque-free particle by setting the net force, F_1 and torque, T to zero. The resulting profiles of translational velocity are shown in Fig. 10(a). The straight line

represents the velocity profile in undisturbed Couette flow, or the velocity of an infinitesimally small particle, $R \rightarrow 0$. The velocity is zero at the centerline of the channel and gradually increases as the particle is displaced towards the wall. As the particle touches the wall it reaches the maximum velocity of $U_c = 1$. As could be expected, a larger particle is affected by the wall to a larger degree than a smaller particle at the same position and thus it moves faster than a smaller particle.

Figure 10(b) presents the angular velocity of a particle in Couette flow. The angular velocity is maximal at the centerline of the channel and monotonically approaches zero as the particle approaches the wall. As the radius of the particle decreases, the angular velocity of the particle located at the centerline of the channel approaches the angular velocity of a neutrally buoyant particle in an unbounded shear flow.

The angular velocity of a free symmetrically situated particle is plotted vs particle radius in Fig. 11. The angular velocity is negative and its magnitude decreases as the particle radius increases from $R = 0.15$ – 0.45 . When the radius decreases, the angular velocity should approach unity. The dotted line in Fig. 11 is a smooth extension of the calculated curve.

Figure 12 shows the distribution of pressure across the channel due to a free particle in Couette flow. The pressure drop is defined as the pressure at the entrance section of the channel minus pressure at the exit. The pressure difference is zero for a particle located at the centerline, as would be expected from symmetry. It increases as the particle is displaced towards the wall, reaches a maximum, then monotonically decreases. The pressure difference is negative when the particle touches the wall. To interpret this behavior of the pressure difference, we notice that when the particle center is close to the channel centerline its translational velocity is almost the same as that of the unperturbed fluid (Fig. 10a), but the angular velocity is considerably smaller (Fig. 10b) than the angular velocity of fluid particles; thus the fluid in the gap between the particle and the nearer wall will be hindered by the particle pressure. As a result, the pressure upstream (to the left of the particle when the particle is located in the upper half of the channel) will be higher than the pressure downstream, hence a positive pressure difference. When the particle is close to the wall its translational velocity approaches unity, whereas the angular velocity approaches zero; thus the particle acts as a “paddle” driving the fluid in the same direction as the motion of the nearer wall. This flow generates a higher pressure downstream and hence a negative pressure difference.

5. DISCUSSION

We have presented a complete investigation of the motion of a cylindrical particle in 2-D Poiseuille and Couette flows between plane parallel boundaries. The only particular solution in the literature that we can compare our results with is the solution of Faxen [5] for a quiescent cylindrical particle midway between two parallel plates.

Faxen’s solution, obtained by the method of reflections, is valid up to ratios of particle diameter to channel width less than 0.25. In this range, our results are in good agreement with Faxen’s as shown in Fig. 2. The relative difference between the two solutions is 1.1% R

$= 0.15, 1.6\%$ for $R = 0.2$, and 5.6% for $R = 0.25$. Two other solutions reported in the literature (Harrison [4] and Takaisi [6]) are less accurate than Faxen's; their behavior is also shown in Fig. 2. Faxen's solution for the quiescent particle in Poiseuille flow can be combined with his solution for a translating particle in otherwise quiescent fluid to yield the translational velocity of a neutrally buoyant free particle in Poiseuille flow. Our solution $U_{co}(R)$ agrees with Faxen's for particle sizes $R \leq 0.25$.

Although no other analytical or numerical solutions for 2-D Poiseuille or Couette flows have been previously reported in the literature, it is instructive to compare the present results with the solutions of related 3-D problems. Our analysis in Part 1 for particle motion in a quiescent fluid shows that in certain cases the behavior in 2-D and 3-D cases was very similar, whereas in other cases there were qualitative differences. Here, we compare our results to those of Bungay and Brenner [9, 10] for a spherical particle in a circular tube in Poiseuille flow, and of Refs [11–13] for a spherical particle between two plane parallel boundaries in Poiseuille and Couette flows. The results in these papers are presented in different forms than the results of the present paper, so some additional calculations were required to rescale the variables.

Bungay and Brenner [10] analyzed the motion of a closely fitting neutrally buoyant sphere suspended in Poiseuille flow in a circular tube using the singular perturbation analysis. Their analysis is applicable to situations in which the sphere occupies almost the entire cross section of the tube, so that the clearance between the particle and the tube wall is everywhere small compared with both the sphere and tube radii. They showed that displacement of the sphere to eccentric positions in the tube decreases its translational velocity only slightly, until the sphere almost touches the wall; in other words, the profile of particle translational velocity is flat for large particles. This result agrees qualitatively with our solution shown in Fig. 5(a); indeed, as the particle radius increases the profile of translational velocity becomes flatter, whereas the velocity gradient sharply increases at positions where the particle almost touches the wall. Bungay and Brenner [10] also predicted that the angular velocity of the sphere attains a maximum value at an intermediate lateral position, and approaches zero as the particle touches the wall. The direction of rotation always corresponds to that for rolling along the nearer side of the tube. Our results for the angular velocity shown in Fig. 5(b) exhibit a similar behavior.

Clearly, when the particle touches the wall both the translational and angular velocity vanish due to the infinite shear stresses that would otherwise be created in the infinitely small gaps between the particle and the wall. For the particle positions very close to the wall, the ratio $\Omega R/U_c$ characterizes a degree of slip, since $\Omega R/U_c = 1$ for pure rolling motion along the wall. Bungay and Brenner [9] predicted a high degree of slip, i.e. small values of the ratio in the case of large particles. To see if the slip occurs in the 2-D case, we calculated the ratio of $\Omega R/U_c$ for $R = 0.2$ when the gap between the particle and the wall is small. The ratio varies between 0.2 and 0.25 when y is between 0.780 and 0.795; hence our calculations demonstrate the slip at the boundary in the 2-D case.

The additional pressure drop due to a sphere suspended in Poiseuille flow was calculated in Ref. [10] for a closely-fitting sphere and in Ref. [9] for a small sphere. Both studies yield a

maximum pressure drop when the particle touches the wall, and the minimum when its center is located at the tube centerline, with a monotonic increase between these positions. A similar pattern is exhibited in the 2-D case, Fig. 6.

Ganatos *et al.* [11–13] studied the motion of a sphere between two plane parallel boundaries using the collocation technique. It was found that the force on a rigidly held sphere in Poiseuille flow decreases monotonically to a finite value as the particle is displaced from the midplane towards the wall; a similar behavior is observed in the 2-D case, Fig. 3. In addition, it was shown that the torque on a rigidly held sphere monotonically increases towards the wall. However, in the 3-D case there is no evidence of a plateau region as in the 2-D case, Fig. 4. In both cases the torque goes to a finite value as the particle approaches the wall.

Ganatos *et al.* [13] also calculated the distribution of the slip velocity, $U_c - U_\infty/U_\infty$, where U_∞ is the profile of fluid velocity in undisturbed Poiseuille flow, and of the angular velocity. Their results are qualitatively similar to the profiles shown in Figs 5(a), (b). For this 3-D case, the results were presented by the product of the angular velocity and the particle radius, ΩR ; an interesting feature pointed out by Ganatos *et al.* was that the maximum values of ΩR increase as R increases, reach a maximum at the value of R which is slightly smaller than 0.25 and then decrease. In the 2-D case, if we compare the calculated values of the maxima of ΩR , the largest value among these maxima is $\Omega R \cong 0.22$ for $R = 0.25$ in close agreement with the 3-D case.

Ganatos *et al.* [12] analyzed the motion of a sphere in plane Couette flow generated by translation of the upper boundary of the channel whereas the lower boundary was stationary. In our case, the boundaries translate with equal velocity in the opposite directions so that the resulting undisturbed Couette flow is symmetric with respect to the channel centerline. The case of one boundary motion can be represented as a superposition of the symmetric motion with the plates moving in the opposite directions and of translation of the particle in otherwise quiescent fluid. Since we have obtained both solutions, it is possible to compare the features of our solution with those of Ganatos *et al.* [12]. Calculating the superposition of two solutions, we express the force and torque on a quiescent particle in Couette flow with the upper plate moving with positive unit velocity and the lower plate at rest:

$$F = (F_{10} + A_{11})/2, T = (A_{13} - T_0)/2 \quad (5.1)$$

Here F_{10} and T_0 are the force and torque on a quiescent particle in Couette flow calculated above, and A_{11} and A_{13} are the coefficients of the resistance matrix. Adding the functions shown in Fig. 2, Part 2 and Fig. 3, Part 1, according to eqn (5.1) results in functions not symmetric with respect to the channel centerline, $y = 0.5$.

To facilitate the comparison, we plotted the distribution of $F(y)$ and $T(y)$ across the channel for two particle radii, $R = 0.15$ and 0.3 (Figs 13a,b). As could be expected, the force on the particle, F , becomes infinite as the particle approaches the moving (upper) wall, i.e. when $y \rightarrow 0.5$ for $R = 0.15$ and $y \rightarrow 0.7$ for $R = 0.3$. When the particle approaches the lower

stationary wall, the force approaches a finite value. Interestingly, for $R = 0.15$ the force $F(y)$ increases monotonically with y , with a plateau region at $0.6 < y < 0.75$ where the force is almost constant (at least it was not possible to detect a minimum numerically). For larger particles ($R = 0.3$), the force is distinctly nonmonotonic with y : it has a maximum near the centerline and a minimum closer to the wall, both in the upper half of the channel.

The described behavior of the force in the 2-D case is qualitatively different from that in the 3-D case reported in [12]. In the case of a sphere between parallel plates the force decreases as the sphere is displaced from the lower wall, reaches a minimum and then increases again as the particle approaches the moving wall. In Part 1 we discussed possible sources of qualitative differences in the 2-D and 3-D cases. In particular, we showed that the distributions of A_{11} across the channel in both cases are entirely different. Since $A_{11}(y)$ is an important part of the force, F , in eqn (5.1), it is not surprising that we found significant differences in the force distribution.

The calculated torque, $T(y)$, on a quiescent particle in Couette flow when the upper plate is translating with positive unit velocity and the lower plate is at rest is shown in Fig. 13(b) as a function of y for two particle radii: 0.15 and 0.3. As in the case of the force, the torque becomes infinite when the particle approaches the upper wall due to the infinite stresses in the gap between the particle and the wall; the torque tends to a finite value when the particle approaches the lower wall. The torque decreases monotonically with y , although plateau regions appear on both curves where the torque is almost constant. Again, this behavior is qualitatively different from that reported [12] in the 3-D case, where a minimum and a maximum on each curve can be found, at least for particles of smaller sizes.

Finally, we compare the slip velocity, $U_c - 2(y - 0.5)$, for a free particle in Couette flow in the 2-D case which can be inferred from Fig. 10(a) and in the 3-D case as reported by Ganatos *et al.* [13]. In both cases the behavior is qualitatively similar.

The above analysis leads to a conclusion that caution must be exercised in extending the results for 2-D flows with suspended particles to 3-D and vice versa; we have seen that in some cases the behavior of the same variables is principally different, whereas in other cases not only qualitative but also quantitative features of the flows are very close.

The computational technique described in Ref. [3] and utilized in the present work is not limited to the motion of a single cylindrical particle in a plane channel. Other geometries can be easily considered within the same scheme, e.g. particles of arbitrary shape, multiple particles, channels of irregular shape. Some cases of the motion of a cylindrical particle in a bifurcating channel were reported by Dvinsky [2]. The applications include the rheology of suspensions, particularly the mechanics of blood flow through small vessels.

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Fig. 1.
Geometry of channel and particle for 2-D Poiseuille and Couette flows.

Fig. 2. Horizontal force on a stationary cylindrical particle symmetrically situated in a plane channel with Poiseuille flow. (1) Present solution; (2) Faxen [7]; (3) Harrison [11]; (4) Takaisi [13].

Fig. 3.
Distribution across the channel of the drag force on a cylindrical particle held fixed in Poiseuille flow for particles of different radii.

Fig. 4.
Distribution across the channel of the torque on a cylindrical particle held fixed in Poiseuille flow for particles of different radii.

Fig. 5.

Distribution across the channel of the translational velocity of a drag- and torque-free particle in Poiseuille flow; the curve designated $R \rightarrow 0$ represents the parabolic profile of undisturbed flow and can be regarded as particle velocity for an infinitesimal particle. (b). Distribution across the channel of the angular velocity of a free particle in Poiseuille flow; the line designated $R \rightarrow 0$ represents half of the vorticity of undisturbed flow and can be regarded as particle angular velocity for an infinitesimal particle.

Fig. 6.
Additional pressure drop due to a free particle in Poiseuille flow as a function of position of the particle center.

Fig. 7.
Force on a particle held fixed in Couette flow as a function of particle position.

Fig. 8.
Torque on a particle held fixed in Couette flow as a function of particle position.

Fig. 9.
Streamline pattern in Couette flow for a stationary eccentrically situated particle, $R \rightarrow 0.2$.

Fig. 10.

(a) Translational velocity of a free cylindrical particle in Couette flow as a function of particle position. Straight line designated $R \rightarrow 0$ represents velocity distribution of undisturbed flow and can be regarded as translational velocity of an infinitesimal particle.

(b) Angular velocity of a free cylindrical particle in Couette flow as a function of particle position. Straight line designated $R \rightarrow 0$ represents half of the vorticity of undisturbed flow.

Fig. 11. Angular velocity of a free particle symmetrically situated in Couette flow as a function of particle radius; solid line-calculated; dashed line-extrapolated.

Fig. 12.
Pressure drop in Couette flow with a free cylindrical particle as a function of particle position.

(a)

(b)

Fig. 13.

(a) Distribution across the channel of the force on a particle held fixed in Couette flow when the upper wall translates with positive unit velocity and the lower wall is stationary. (b) Distribution across the channel of the torque on a particle held fixed in Couette flow when the upper wall translates with positive unit velocity and the lower wall is stationary.

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Table 1

Coefficients for equation (3.3)

<i>R</i>	0.15	0.20	0.25	0.30	0.35	0.40
<i>a</i> ₁	0.7712	0.6807	0.5839	0.4662	0.2676	0.07345
<i>a</i> ₂	−1.121	−1.299	−1.426	−1.414	−1.040	−0.3569
<i>a</i> ₃	1.350	1.618	1.842	1.947	1.773	1.283