Published in final edited form as:

Int J Imaging Syst Technol. 2012 June ; 22(2): 114–120. doi:10.1002/ima.22011.

Filtered Backprojection Algorithm Can Outperform Iterative Maximum Likelihood Expectation-Maximization Algorithm

Gengsheng L. Zeng

Utah Center for Advanced Imaging Research (UCAIR), Department of Radiology, University of Utah, Salt Lake City, UT 84108

Abstract

The iterative maximum-likelihood expectation-maximization (ML-EM) algorithm is an excellent algorithm for image reconstruction and usually provides better images than the filtered backprojection (FBP) algorithm. However, a windowed FBP algorithm can outperform the ML-EM in certain occasions, when the least-squared difference from the true image, that is, the least-squared error (LSE), is used as the comparison criterion. Computer simulations were carried out for the two algorithms. For a given data set the best reconstruction (compared to the true image) from each algorithm was first obtained, and the two reconstructions are compared. The stopping iteration number of the ML-EM algorithm and the parameters of the windowed FBP algorithm were determined, so that they produced an image that was closest to the true image. However, to use the LSE criterion to compare algorithms, one must know the true image. How to select the optimal parameters when the true image is unknown is a practical open problem. For noisy Poisson projections, computer simulation results indicate that the ML-EM images are better than the regular FBP images, and the windowed FBP algorithm images are better than the ML-EM images. For the noiseless projections, the FBP algorithms outperform the ML-EM algorithm. The computer simulations reveal that the windowed FBP algorithm can provide a reconstruction that is closer to the true image than the ML-EM algorithm.

Keywords

image reconstruction; ML-EM algorithm; filtered backprojection algorithm; Poisson noise

I. INTRODUCTION

The iterative maximum-likelihood expectation-maximization (MLEM) algorithm (or its ordered-subset version) is the most widely used iterative image reconstruction algorithm in nuclear medicine (Shepp and Vardi, 1982). Many studies have been carried out to compare the ML-EM algorithm with the analytical filtered backprojection (FBP) algorithm (Wilson and Tsui, 1993; Prvulovich et al., 1997; Wang et al., 1998; Gutman et al., 2003). Except for some parametric imaging studies, the ML-EM algorithm output is usually considered to be better than the FBP algorithm's output (Carson et al., 1994; Oda et al., 2001). However, image evaluation and comparison is always task based, and there is not a universal criterion.

Using one criterion, algorithm A can be better than algorithm B. While using another criterion, algorithm B can be better and algorithm A. If the task is lesion detection, an algorithm that can provide a smooth background and a high-contrast lesion wins (Tourassi et al., 1991). Some tasks rely on the image variance and bias tradeoff (Xu et al., 1993), or resolution and signal-to-noise trade-off.

The FBP algorithm's performance depends on the selection of the window function that is used to regulate the ramp filter. A new window function is suggested in this article, and the motivation for the design of the window function is presented in the Appendix.

If we adopt the criterion that the best image is the one closest to the true image in the sense of the pixel-by-pixel least-squared-error (LSE), then the proposed windowed FBP algorithm can outperform the ML-EM algorithm. The windowed FBP algorithm may not work well in a lesion detection task; but it may find application in quantitative image reconstruction, which is important in, for example, kinetic parameter estimation in dynamic studies.

As the true image must be known to use this criterion, only computer simulations are performed in this article. It is impractical to use real data for the comparison studies, because the true image is unknown.

II. METHODS

The Shepp–Logan head phantom is used in the computer simulations (Shepp and Logan, 1974). The projection data are calculated analytically as line-integrals of the phantom (without pixelization of the phantom). Poisson noise is added to the projections, and 100 noise realizations are used for each simulation study. The one-dimensional (1D) detector array contains 128 bins. No scattering, attenuation, system blurring, and motion are considered in data generation. The detector stops at 120 views uniformly distributed over 180°. The image is reconstructed into a two-dimensional (2D) 128 × 128 array, using the ML-EM algorithm and the new windowed FBP algorithm, respectively.

No modification is made to the conventional ML-EM algorithm that models the Poisson noise. The number of iterations is chosen, when the reconstructed image is closest to the true image, that is, when the pixel-by-pixel squared error

$$\text{Squared error} = \sum_{k \in \text{All pixels}} [\text{Reconstruction}(k) - \text{True}(k)]^2 \quad (1)$$

reaches a minimum.

The windowed FBP algorithm is almost the same as the standard FBP algorithm except that the ramp filter is regulated by a newly proposed window function. Using window functions in an FBP algorithm is not new. One popular window function is the rectangular function suggested by Ramachandran and Lakshminarayanan (1971). Another popular window is smoother than the rectangular function and was proposed by Shepp and Logan (1974). Many other lowpass filters, such as the Hamming window (Hamming, 1977), the Hann window

(which is the raised cosine function) (Chesler and Riederer, 1975), the Butterworth window (Butterworth, 1930), and so on, are also widely utilized. Our proposed window function is designed and inspired by the iterative Landweber algorithm (Strand, 1974). A symbolic derivation of this window function is given in the Appendix and in Zeng (2012a). The proposed window function is expressed in the Fourier domain as:

$$\text{Window}(\omega) = 1 - \left(1 - \alpha \frac{(0.5 + 0.5 \cos(\omega))^\gamma}{|\omega|^k}\right)^k, \quad (2)$$

where ω is the frequency variable, and γ and k are the parameters selected by the user. In Eq. (2), $\alpha > 0$ is a small constant that corresponds to the step size in the iterative Landweber algorithm, k corresponds to the number of iterations in the Landweber algorithm, and γ is number of times to use the raised cosine lowpass filter $0.5 + 0.5 \cos(\omega)$. In computer simulations, the discrete-signal frequency ω is in the range of $(-\pi, \pi]$. If the data in the discrete Fourier domain have M samples, the smallest (i.e., closest to the direct current (DC)) nonzero frequency is at $2\pi/M$. Thus, α can be chosen in the interval $0 < \alpha < 2\pi/M$ to

ensure the requirement of $\left|1 - \alpha \frac{(0.5 + 0.5 \cos(\omega))^\gamma}{|\omega|^k}\right| < 1$ when $\omega \neq 0$. This requirement guarantees that the window function [Eq. (2)] is always non-negative. In our computer simulations, $\alpha = \pi/M = \pi/128$ is adopted. When $\omega = 0$ (DC), the window function is not effective and not used, because at this DC frequency the ramp-filter $|\omega| = 0$.

In this article, the 1D filtering step in the windowed FBP algorithm is implemented in the Fourier domain. The ML-EM algorithm has a non-negativity constraint, whereas the FBP algorithm does not have this constraint. In order to have a fair comparison, after the image is reconstructed with the windowed FBP algorithm, all negative valued pixels are set to zero. In fact, there exist better ways to process the negative reconstruction values as suggested by O'Sullivan et al. (1993). The resulting image is scaled by a constant, so that the total sum of the forward projections of the image is the same as the total sum of the projection photon counts.

For a given set of projections, the parameters k and γ are chosen such that the reconstructed image and the true image have the minimum squared error as defined in Eq. (1).

III. RESULTS

In Table I, the optimal ML-EM images and the optimal windowed FBP images are compared.

By optimal we mean that the images reach the least squared difference from the true image or LSE. The LSE listed in Table I is the average over 100 noise realizations. In Table I, if the ratio $\text{LSE}(\text{FBP})/\text{LSE}(\text{ML})$ is less than 1, then the FBP image has a smaller LSE than the ML-EM image.

Table I shows consistently that FBP images are superior to MLEM images for different noise levels. However, we do not have a mathematical proof that for any projection data set, a set

of parameters k and γ may always be determined, such that their associated FBP algorithm will outperform the ML-EM algorithm. One cannot draw definite conclusions just from examples. In other words, we still cannot say that the windowed FBP algorithm is always better than the ML-EM algorithm.

Figure 1 shows some examples of the frequency-domain window functions that have been used in the computer simulations reported in Table I. The trend of the window function appears to be to suppress the high-frequency components more and more as the projection data noise gets larger (i.e., the counts become lower). It seems that every noise level corresponds to an optimal shape of the window function, but we have not yet established a mathematical relation between them.

Finally, Table II presents the same comparison results as in Table I, except that the windowed FBP algorithm is replaced by the regular FBP algorithm, which in fact, uses the rectangular window function (Ramachandran and Lakshminarayanan, 1971). When using very-noisy Poisson projections, the ML-EM images outperform the regular FBP images. Tables I and II imply that optimization of the window function parameters does make a difference.

IV. DISCUSSION AND CONCLUSIONS

In this article, we suggest a new window function to be used with the traditional FBP algorithm. The purpose of the window function is to emulate a linear iterative algorithm, characterized by parameters k and γ . By carefully choosing these two parameters, the windowed FBP can outperform the ML-EM algorithm, using the criterion of LSE. We understand that using LSE as the figure-of-merit (FOM) is problematic in practice and is not very meaningful for many tasks (Barrett and Myers, 2004). This article merely presents a mathematical case: if LSE is chosen as the FOM, the ML-EM algorithm may not always give the optimal solution even with Poisson data. If a different criterion is used, the windowed FBP algorithm may not perform as well as the ML-EM algorithm. A potential application of the windowed FBP algorithm is in quantitative imaging, for example, in dynamic studies of compartment modeling.

Why is a new window function needed? Why not use one of the many existing window functions (e.g., the Hann window or Butterworth window)? Unlike so many existing window functions that are specified by the cut-off frequencies, the new window functions are closely related to the iterative Landweber algorithm's iteration number, which, in turn, is related to many other iterative algorithms including the ML-EM algorithm, if an appropriate noise weighting model is modeled. For each noise level in the phantom study, the best iteration number of the ML-EM algorithm should be chosen, and the best parameters k and γ of the new FBP window function should be determined. One Hann window or one Butterworth window (which may be approximated by one set of k and γ) cannot be the optimal function for all situations.

In the computer simulations, we observe that the FBP results are much better than the ML-EM results, when there is no noise or the projection data's count rate is extremely high. This

is because the ML-EM algorithm uses nonoverlapped, piece-wise constant, square pixels to represent the image, and the curved edges in the object cannot be accurately represented. The image misrepresentation errors are amplified, as the iteration number increases.

The optimal parameters in both algorithms depend on not only the noise level but also the object. A practical open problem associated with the windowed FBP algorithm is the difficulty in determining the optimal values of k and γ for a given noisy projection data set, because in an actual patient or phantom scan the true image is never available.

The iterative ML-EM algorithm is an excellent and widely used reconstruction method for image reconstruction, especially in nuclear medicine imaging. Its projection and backprojection matrices can readily model the imaging geometry, system blurring, photon attenuation and scattering, and other effects. However, when the counts are low, it may provide biased reconstruction (Carson et al., 1994). Also, different frequency components have different convergence rates. Thus, when the iterative algorithm is stopped early, the reconstructed image may have a nonuniform point spread function (Zeng, 2010).

On the other hand, the FBP algorithm is a linear method and does not introduce image bias. As pointed out by Hanson (1980), the FBP algorithm is an optimal estimator for the task of determination of an object's amplitude (bias). Its point spread function is shift-invariant throughout the entire image.

No scattering, attenuation, blurring, and motion are simulated in the computer simulations. Poisson noise is the only factor that degrades the image quality. In the real world, when scattering, attenuation and blurring effects must be compensated, the projector in the ML-EM algorithm is able to model these effects, and it is challenging for the FBP algorithm to model these effects. In this case, the ML-EM algorithm will outperform the FBP.

Iterative Bayesian algorithm uses some prior information to specify a desired and useful solution, by adding an extra Bayesian term in the objective function. Our most recent Medical Physics paper (Zeng, 2012b) developed a Bayesian-FBP algorithm, by adding a Bayesian term $\beta h(\omega)$ to the ramp filter in a special way, that is, the frequency-domain ramp filter $|\omega|$ becomes $1/[1/|\omega| + \beta h(\omega)]$, where β is small weighting factor for the Bayesian contribution. This Bayesian-FBP algorithm can give the same result as an iterative Bayesian algorithm. Although no noise-weighting is used in the new FBP algorithm of this article, noise weighting can be incorporated in the window function, and the new windowed FBP algorithm can be significantly improved (Zeng, 2012b).

The current version of the windowed FBP algorithm cannot handle data attenuation, scattering, system blurring, and irregular imaging/sampling geometries. An FBP algorithm that can compensate for nonuniform attenuation in single photon emission computed tomography is available (Novikov, 2002). It is likely that this new FBP algorithm with attenuation correction capability can be modified with some window function, and its noise performance gets improved. These open problems will be left for further investigation.

Acknowledgments

The author thanks Dr. Roy Rowley of the University of Utah for English editing.

Grant sponsors: This work was supported in part by the Margolis Foundation and NIH grant NIH R01 HL108350.

References

- Barrett, HH., Myers, K. Foundations of image science. Wiley; Hoboken, NJ: 2004.
- Butterworth S. On the theory of filter amplifiers. *Exp Wireless Eng.* 1930; 7:536–541.
- Carson RE, Yuchen Y, Chodkowski B, Yap TK, Daube-Witherspoon ME. Precision and accuracy of regional radioactivity quantitation using the maximum likelihood EM reconstruction algorithm. *IEEE Trans Med Imaging.* 1994; 13:526–537. [PubMed: 18218528]
- Chesler DA, Riederer SJ. Ripple suppression during reconstruction in transverse tomography. *Phys Med Biol.* 1975; 20:632–636. [PubMed: 1187793]
- Gutman F, Gardin I, Delahaye N, Rakotonirina H, Hitzel A, Manrique A, Guludex D, Véra P. Optimisation of the OS-EM algorithm and comparison with FBP for image reconstruction on a dual-head camera: A phantom and a clinical ^{18}F -FDG study. *Eur J Nucl Med Mol Imaging.* 2003; 30:1510–1519. [PubMed: 14579091]
- Hamming, RW. Digital filters. Englewood Cliffs; Prentice Hall: 1977.
- Hanson KM. On the optimality of the filtered backprojection algorithm. *J Comput Assist Tomogr.* 1980; 4:361–363. [PubMed: 7372867]
- Novikov RG. An inversion formula for the attenuated X-ray transformation. *Ark Math.* 2002; 40:145–167.
- Oda K, Toyama H, Uemura K, Ikoma Y, Kimura Y, Senda M. Comparison of parametric FBP and OS-EM reconstruction algorithm images for PET dynamic study. *Ann Nucl Med.* 2001; 15:417–423. [PubMed: 11758946]
- Prvulovich EM, Lonn AHR, Bomanji JB, Jarritt PH, Ell PJ. Effect of attenuation correction on myocardial thallium-201 distribution in patients with a low likelihood of coronary artery disease. *Eur J Nucl Med Mol Imaging.* 1997; 24:266–275.
- Ramachandran GN, Lakshminarayanan AV. Three-dimensional reconstruction from radiographs and electron micrographs: Application of convolutions instead of Fourier transforms. *Proc Natl Acad Sci USA.* 1971; 68:2236–2240. [PubMed: 5289381]
- Schafer RW, Mersereau RM, Richards MA. Constrained iterative restoration algorithms. *Proc IEEE.* 1981; 69:432–450.
- Shepp LA, Logan BF. The Fourier reconstruction of a head section. *IEEE Trans Nucl Sci.* 1974; NS-21:21–43.
- Shepp LA, Vardi Y. Maximum-likelihood reconstruction for emission tomography. *IEEE Trans Med Imaging.* 1982; MI-1:113–122.
- Strand ON. Theory and methods related to the singular-function expansion and Landweber's iteration for integral equations of the first kind. *SIAM J Numer Anal.* 1974; 11:798–825.
- O'Sullivan F, Pawitan Y, Haynor D. Reducing negativity artifacts in emission tomography: Post-processing filtered backprojection solutions. *IEEE Trans Med Imaging.* 1993; 12:653–663. [PubMed: 18218459]
- Tourassi GD, Floyd CE Jr, Munley MT, Bowsher JE, Coleman RE. Improved lesion detection in SPECT using MLEM reconstruction. *IEEE Trans Nucl Sci.* 1991; 38:780–783.
- Wang CX, Snyder WE, Bilbro G, Santago P. Performance evaluation of filtered backprojection reconstruction and iterative reconstruction methods for PET images. *Comput Biol Med.* 1998; 28:13–25. [PubMed: 9644571]
- Wilson DW, Tsui BMW. Noise properties of filtered-backprojection and ML-EM reconstructed emission tomographic images. *IEEE Trans Nucl Sci.* 1993; 40:1198–1203.
- Xu XL, Liow JS, Strother SC. Iterative algebraic reconstruction algorithms for emission computed tomography: A unified framework and its application to positron emission tomography. *Med Phys.* 1993; 20:1675–1684. [PubMed: 8309440]

- Zeng, GL. Medical image reconstruction—A conceptual tutorial. Beijing: Springer; 2010.
- Zeng GZ. A filtered backprojection algorithm with characteristics of the iterative Landweber algorithm. *Med Phys.* 2012a; 39:603–607. [PubMed: 22320769]
- Zeng GZ. A filtered backprojection MAP algorithm with non-uniform sampling and noise modeling. *Med Phys.* 2012b; 39:2170–2178. [PubMed: 22482638]

APPENDIX

The proposed window function is motivated and inspired by the iterative Landweber algorithm, which solves a system of linear equations $\mathbf{A}\mathbf{X} = \mathbf{P}$ by an iterative procedure:

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \alpha \mathbf{A}^T \mathbf{S}(\mathbf{P} - \mathbf{A}\mathbf{X}^{(k)}), \quad (\text{A1})$$

where $\mathbf{A}$ is the projection matrix, $\mathbf{A}^T$ is the backprojection matrix, $\mathbf{P}$ is the projection data vector, $\mathbf{X}$ is the image vector, $\mathbf{X}^{(k)}$ is the estimated image at the k th iteration, and $\alpha > 0$ is the step size. In Eq. (A1), $\mathbf{S}$ is a lowpass filter, and this lowpass filter is new, unlike in the original Landweber algorithm. This recursive relation can be rewritten as a nonrecursive expression as

$$\begin{aligned} \mathbf{X}^{(k+1)} &= \mathbf{X}^{(k)} + \alpha \mathbf{A}^T \mathbf{S}(\mathbf{P} - \mathbf{A}\mathbf{X}^{(k)}) \\ &= \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) \mathbf{X}^{(k)} \\ &= \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) [\alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) \mathbf{X}^{(k-1)}] \\ &= \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^2 \mathbf{X}^{(k-1)} \\ &= \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^2 [\alpha \mathbf{A}^T \mathbf{S} \mathbf{P} \\ &\quad + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) \mathbf{X}^{(k-2)}] = \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} \\ &\quad + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^2 \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^3 \mathbf{X}^{(k-2)} \\ &= \dots = [\mathbf{I} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A}) + \dots + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^k] \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} \\ &\quad + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^{k+1} \mathbf{X}^{(0)} = \left[\sum_{n=0}^k (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^n \right] \alpha \mathbf{A}^T \mathbf{S} \mathbf{P} + (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^{k+1} \mathbf{X}^{(0)}. \end{aligned} \quad (\text{A2})$$

If the initial image $\mathbf{X}^{(0)}$ is set to zero, the result from k iterations of the Landweber algorithm is

$$\mathbf{X}^{(k)} = \alpha \left[\sum_{n=0}^{k-1} (\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^n \right] \mathbf{A}^T \mathbf{S} \mathbf{P}. \quad (\text{A3})$$

It can be verified that $(\mathbf{I} - \alpha \mathbf{A}^T \mathbf{S} \mathbf{A})^n \mathbf{A}^T \mathbf{S} = \mathbf{A}^T (\mathbf{I} - \alpha \mathbf{S} \mathbf{A}^T \mathbf{A})^n \mathbf{S}$. Thus, Eq. (A3) becomes

$$\mathbf{X}^{(k)} = \mathbf{A}^T \alpha \left[\sum_{n=0}^{k-1} (\mathbf{I} - \alpha \mathbf{S} \mathbf{A}^T \mathbf{A})^n \right] \mathbf{S} \mathbf{P}. \quad (\text{A4})$$

Notice that when $\alpha > 0$ is small enough, as $k \rightarrow \infty$ we have

$$\alpha \left[\sum_{n=0}^{k-1} (\mathbf{I} - \alpha \mathbf{S} \mathbf{A}^T \mathbf{A})^n \right] \rightarrow (\mathbf{S} \mathbf{A}^T \mathbf{A})^{-1} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{S}^{-1}, \quad (\text{A5})$$

and

$$\mathbf{X}^{(k)} \rightarrow \mathbf{A}^T (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{S}^{-1} \mathbf{S} \mathbf{P} = \mathbf{A}^T (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{P}. \quad (\text{A6})$$

From Eq. (A6) we observe that as the iteration number tends to infinity, the Landweber algorithm resembles an FBP algorithm, with $(\mathbf{A}^T \mathbf{A})^{-1}$ being the ramp-filter. This observation motivated us to design a window function for the FBP algorithm, making the windowed FBP act like an iterative algorithm, so that the performance of the FBP algorithm may be similar to an iterative algorithm.

For a finite k , we have

$$\alpha \left[\sum_{n=0}^{k-1} (\mathbf{I} - \alpha \mathbf{S} \mathbf{A}^T \mathbf{A})^n \right] \mathbf{S} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{S}^{-1} [\mathbf{I} - (\mathbf{I} - \alpha \mathbf{S} \mathbf{A}^T \mathbf{A})^k] \mathbf{S}. \quad (\text{A7})$$

The proofs of the above equations are available in a review paper by Schafer et al. (1981).

In the Fourier domain, $(\mathbf{A}^T \mathbf{A})^{-1}$ can be approximated as the ramp-filter $|\omega|$, the effects of $\mathbf{S}^{-1}$ and $\mathbf{S}$ cancel each other, and $[\mathbf{I} - (\mathbf{I} - \alpha \mathbf{S} \mathbf{A}^T \mathbf{A})^k]$ can be approximated by the window function

$$W_k(\omega) = 1 - \left\{ 1 - \alpha \frac{[0.5 + 0.5 \cos(\omega)]^\gamma}{|\omega|} \right\}^k. \quad (\text{A8})$$

In (A7), a multiple-raised-cosine lowpass filter $[0.5 + 0.5 \cos(\omega)]^\gamma$ is chosen to implement the lowpass filter $\mathbf{S}$, and γ is a non-negative real number. When $\gamma = 1$, $[0.5 + 0.5 \cos(\omega)]$ is the Hann filter (Chesler and Riederer, 1975). In fact, one can use any lowpass filter to implement $\mathbf{S}$.

Figure 1.

Window functions used in the computer simulations. A: $\gamma = 1$, $k = 1 \times 10^8$; B: $\gamma = 1$, $k = 1808$; C: $\gamma = 1$, $k = 195$; D: $\gamma = 3$, $k = 83$; E: $\gamma = 14$, $k = 37$; and F: $\gamma = 61$, $k = 24$.

Table I

ML-EM versus windowed FBP computer simulations

Total Photon Counts in the Projections	ML-EM (Image from One Realization)	Windowed FBP (Image from One Realization)	LSE(FBP)/LSE(ML)
Noiseless (no random noise is added. Projections are calculated analytically)			$0.029/0.169 = 0.17$; FBP is better
	Iterations = 53	$k = 1 \times 10^8, \gamma = 1$	
380,000,000 (a typical value)			$3.815 \times 10^6 / 1.722 \times 10^7 = 0.22$ (over 100 noise realizations); FBP is better
	Iterations = 51	$k = 1 \times 10^8, \gamma = 1$	
38,000,000 (a typical value)			$1.019 \times 10^5 / 2.011 \times 10^5 = 0.51$ (over 100 noise realizations); FBP is better
	Iterations = 45	$k = 1808, \gamma = 1$	
3,800,000 (a typical value)			$3.306 \times 10^3 / 3.799 \times 10^4 = 0.87$ (over 100 noise realizations); FBP is better
	Iterations = 28	$k = 195, \gamma = 1$	
380,000 (a typical value)			$87.141/104.696 = 0.83$ (over 100 noise realizations); FBP is better
	Iterations = 14	$k = 83, \gamma = 3$	

Total Photon Counts in the Projections	ML-EM (Image from One Realization)	Windowed FBP (Image from One Realization)	LSE(FBP)/LSE(ML)
38,000 (a typical value)	Iterations =6	k= 37, γ = 14	$1.755/2.686 = 0.65$ (over 100 noise realizations); FBP is better
3800 (a typical value)	Iterations =3	k= 24, γ = 61	$0.029/0.055 = 0.53$ (over 100 noise realizations); FBP is better

Table II

ML-EM versus regular FBP computer simulations

Total Photon Counts in the Projections	ML-EM (Image from One Realization)	Regular FBP (Image from One Realization)	LSE(FBP)/LSE(ML)
Noiseless (no random noise is added. Projections are calculated analytically)			$0.0293/0.169 = 0.172$; FBP is better
	Iterations = 53		
380,000,000 (a typical value)			$3.828 \times 10^6 / 1.722 \times 10^7 = 0.22$ (over 100 noise realizations); FBP is better
	Iterations = 51		
38,000,000 (a typical value)			$1.234 \times 10^5 / 2.011 \times 10^5 = 0.61$ (over 100 noise realizations); FBP is better
	Iterations = 45		
3,800,000 (a typical value)			$1.010 \times 10^4 / 3.799 \times 10^3 = 2.66$ (over 100 noise realizations); ML-EM is better
	Iterations = 28		
380,000 (a typical value)			$6.430 \times 10^2 / 1.047 \times 10^2 = 6.14$ (over 100 noise realizations); ML-EM is better
	Iterations = 14		

Total Photon Counts in the Projections	ML-EM (Image from One Realization)	Regular FBP (Image from One Realization)	LSE(FBP)/LSE(ML)
38,000 (a typical value)	Iterations = 6		$14.725/2.686 = 5.48$ (over 100 noise realizations); ML-EM is better
3800 (a typical value)	Iterations = 3		$0.193/0.055 = 3.50$ (over 100 noise realizations); ML-EM is better