Comparison of a fiber-gel finite element model of vocal fold vibration to a transversely isotropic stiffness model

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(Received 15 July 2016; revised 4 August 2017; accepted 9 August 2017; published online 12 September 2017)

A fiber-gel vocal fold model is compared to a transversely isotropic stiffness model in terms of normal mode vibration. The fiber-gel finite element model (FG-FEM) consists of a series of gel slices, each with a two-dimensional finite element mesh, in a plane transverse to the tissue fibers. The gel slices are coupled with fibers under tension in the anterior-posterior dimension. No vibrational displacement in the fiber-length direction is allowed, resulting in a plane strain state. This is consistent with the assumption of transverse displacement of a simple string, offering a wide range of natural frequencies (well into the kHz region) with variable tension. For low frequencies, the results compare favorably with the natural frequencies of a transversely isotropic elastic stiffness model (TISM) in which the shear modulus in the longitudinal plane is used to approximate the effect of fiber tension. For high frequencies, however, the natural frequencies do not approach the string mode frequencies unless plane strain is imposed on the TISM model. The simplifying assumption of plane strain, as well as the use of analytical closed-form shape functions, allow for substantial savings in computational time, which is important in clinical and exploratory applications of the FG-FEM model. © 2017 Acoustical Society of America. [<http://dx.doi.org/10.1121/1.5001055>]

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Pages: 1376–1383

I. INTRODUCTION

Recent studies on the biomechanical properties of vocal fold tissues have focused on tissue anisotropy (Miri *et al.*, 2012; Kelleher *et al.*, 2013; Yin and Zhang, 2013; Zhang, 2014; Rohlfes *et al.*, 2013). Anisotropy requires more than a single Young's modulus and a single Poisson's ratio to characterize an elastic material. The number of independent elastic constants that characterize an elastic material in general anisotropy is 18 (Lekhnitskii, 1981). When there are some degrees of symmetry or isotropy this number decreases. For example, for a single plane of symmetry it drops to 13, and with three planes of symmetry (orthotropy) it drops to 9. For a plane of isotropy (transverse isotropy) it drops to 5.

The critical source of anisotropy in vocal fold tissues is fiber tension in the vocal ligament and the thyroarytenoid muscle, which was recognized and quantified about 40 years ago. Following the first two computational models of vocal fold tissue vibration that did not include fiber tension (Flanagan and Landgraf, 1968; Ishizaka and Flanagan, 1972), anisotropy was introduced shortly thereafter by way of fiber tension in muscle and ligament (Titze, 1973, 1974; Titze and Strong, 1975; Titze, 1976; Titze and Talkin, 1979). Fiber tension was further embedded into a finite-element implementation by Alipour *et al.* (2000), while more recently anisotropic models with three-dimensional

continuum approaches have been introduced (Rosa *et al.*, 2003; Tao and Jiang, 2006; Luo *et al.*, 2008; Cook *et al.*, 2009). Yin and Zhang (2013) have modeled both fiber and stiffness anisotropy. They introduced the concept of hyperelasticity into the field of voice science, writing a constitutive equation that embedded nonlinear stress-strain relations of vocal fold tissues into the elastic constants. These elastic constants were related to intrinsic muscle activations of the larynx. Yin and Zhang suggested that vocal fold stiffness is at least as important as vocal fold tension in determining vocal fold eigenfrequencies, particularly for speech-like fundamental frequencies (up to 281 Hz in their study). This conclusion is in agreement with an earlier analytic approach to eigenfrequency analysis (Berry and Titze, 1996). Our approach here is to go far beyond speech fundamental frequencies, as in singing and calling by a variety of birds and mammals. Beyond 500 Hz, the dominant eigenfrequencies will then become string-like (Titze, 2006, Chap. 4).

Some investigators, specifically Alipour *et al.* (2000), Zheng *et al.* (2011), Xue *et al.* (2012), have assumed plane strain in vocal fold vibration, based on studies with x-ray pellettography that showed vibrational displacement primarily in the coronal plane (Saito *et al.*, 1981; Fukuda *et al.*, 1983; Saito *et al.*, 1983) and computation that showed vibrational displacements along the fiber direction an order of magnitude smaller than those in the transverse plane (Rosa *et al.*, 2003). The plane strain assumption is continued into the current modeling.

A series of applications has derived from the Alipour *et al.* (2000) finite element model, which we now call the *fiber-gel finite element model* (FG-FEM). Alipour and Scherer (2015) included the ventricular folds in self-sustained oscillation, Titze and Riede (2010) simulated

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the vocalization of Rocky Mountain elk, [Palaparthi et al. \(2014\)](#) used the model to optimize functional vocal fold morphology, and [Mau et al. \(2015\)](#) studied the effect of resection depth of early glottis cancer on vocal outcome computationally.

The primary purpose of this paper is to compare normal mode frequency calculations of the FG-FEM, which uses only two elastic constants for a given tissue layer, to both an ideal string model and a transversely isotropic stiffness model (TISM) that does not embed fibers explicitly. The ideal string model is conceptually and quantitatively important for comparison because it explains the wide frequency ranges observed across species and the relation of f_o to body size and vocal fold length ([Titze et al., 2016](#)). It has been shown that biological tissue containing a dense fiber structure, like a ligament, can be classified as hyperelastic ([Weiss et al., 1996](#)). Vocal fold tissue, which contains collagen fibers in a gelatinous ground substance, can thus be reasonably approximated by such a hyperelastic model. The hyperelasticity in our FG-FEM model is represented with vibrating strings embedded in a ground substance (the gel). Two analytical solutions, one with simple string theory and one with three-dimensional elasticity theory, are provided as comparisons to the computational FG-FEM and TISM solutions. The analytic solutions also serve as asymptotic approximations. We consider none of the solutions to be a standard because all of them contain simplifying assumptions that are only partially justified at this point. The comparison is for vibrational displacement only, not for postural movement of the vocal folds. To keep the focus on tissue anisotropy, the boundary conditions are kept simple (rectangular geometry) and the tissue morphology is a single homogeneous layer. A secondary purpose of this paper is to explore the reduction in computation time achieved with the simplifying assumptions of the FG-FEM.

II. ASSUMPTIONS MADE IN THE FIBER-GEL FINITE ELEMENT MODEL

Given the frequent use of the FG-FEM (primarily because of its speed of computation), an update on the assumptions and parameter validation is given here. Greater detail has been provided elsewhere ([Alipour et al., 2000](#); [Titze, 2006](#), Chap. 4). The assumptions are as follows.

- (1) Vibrational tissue displacements can be separated from postural displacements; the two solutions are obtained independently.
- (2) All vocal fold tissue is incompressible at sonic vibrational frequencies.
- (3) Along the length of the vocal folds, M coronal-plane slices of thickness Δy are used for spatial discretization. In these slices, the displacement vector for vibration is two-dimensional such that only medial-lateral (x) and inferior-superior (z) deformations are computed.
- (4) Planar-isotropy is assumed for each of an arbitrary number of tissue layers within each slice. The shear modulus in this planar-isotropic tissue layer is μ , the planar Poisson's ratio is near 1.0, and the Young's modulus E is approximately 4μ . Thus, only one elastic constant per

tissue layer is needed for stiffness of the gel in the transverse plane, either μ or E . A two-dimensional finite element mesh with triangular elements is constructed in each slice. The same number of elements is chosen for each slice, but the element size would change if non-parallel boundaries were chosen.

- (5) In the anterior-posterior (y) direction, fiber tension connects the three corresponding nodes of each triangular element of adjacent slices. The fiber tension T is derived from axial stress-strain curves measured on mucosa, ligament, and muscle tissue. The curves are highly nonlinear ([Min et al., 1995](#)). The axial stress is recalculated dynamically with postural movement of the anterior-posterior boundaries. If σ_y is the axial stress, then $T = A\sigma_y$, where A is the element cross-sectional area.
- (6) [Weiss et al. \(1996\)](#) have shown that for a hyperelastic fibrous material, the strain energy density of the fibers can be added to the elastic strain energy density of the ground substance. The strain energy density for an elastic material is

$$W_e = [e]^T [C] [e], \quad (1)$$

where $[e]^T$ is the transpose of the strain vector $[e]$ and $[C]$ is the elasticity matrix. The matrix operation produces a scalar quantity with an algebraic sum of terms. As shown in [Titze \[2006, p. 191, Eq. \(4.180\)\]](#), one strain energy density term for a transversally isotropic material is

$$W = \frac{1}{2} \mu'_o \left[(\partial \xi / \partial y)^2 + (\partial \zeta / \partial y)^2 \right], \quad (2)$$

where μ'_o is the shear modulus in a plane perpendicular to the transverse plane, ξ is the x displacement in the transverse plane, and ζ is the z displacement in the transverse plane. The strain components are $e_{xy} = \partial \xi / \partial y$ and $e_{zy} = \partial \zeta / \partial y$.

The differential strain energy density for a string (fiber) vibrating with constant tension in two directions in the transverse plane is

$$W_f = \frac{1}{2} (T/dxdz) \left[(\partial \xi / \partial y)^2 + (\partial \zeta / \partial y)^2 \right], \quad (3)$$

where T is the fiber tension and $dxdz$ is the fiber cross-section in the transverse plane ([Morse and Feshbach, 1953](#), p. 127, extended here to two directions of motion and expressed as energy per unit volume). Given that the terms in brackets in Eqs. (2) and (3) are identical and the strain energy density terms are additive according to [Weiss et al. \(1996\)](#), a combined effective shear modulus becomes

$$\mu' = \mu'_o + T/dxdz = \mu'_o + \sigma_y. \quad (4)$$

The stress σ_y is not a dynamic stress, but the magnitude of a static postural stress in the longitudinal fiber direction. This postural stress effectively becomes a hyperelastic property of the inhomogeneous material for the purpose of tissue vibration. It varies slowly and is highly nonlinear with changing boundaries in postural movements, but is

constant for a given posture of the vocal folds. The value of μ'_0 is generally set to the transverse gel shear modulus μ such that isotropy exists when $\sigma_y = 0$. However, additional stiffness anisotropy can be introduced if $\mu'_0 \neq \mu$. In the finite-element programming of our fiber-gel model, the two components are additive as Eq. (4) indicates.

- (7) A viscosity η defines the viscous energy dissipation for deformation in the transverse plane.

III. METHODS

For the current study of anisotropy, the geometry of the vocal folds and its boundary conditions were simplified. A rectangular vocal fold shape was assumed, with length = 1.5 cm (anterior-posterior), thickness = 0.5 cm (inferior-superior), and depth = 0.7 cm (medial-lateral). Anterior, posterior, and lateral boundaries were fixed, whereas inferior, superior, and medial boundaries were free.

A. Analytical solutions

The analytic solution for the normal mode frequencies of the TISM with plane strain is obtained from two bi-harmonic equations (Titze, 1976),

$$(\mu \nabla^2 + \mu' \partial^2 / \partial y^2 - \rho \partial^2 / \partial t^2) \nabla^2 \xi = 0, \quad (5)$$

$$(\mu \nabla^2 + \mu' \partial^2 / \partial y^2 - \rho \partial^2 / \partial t^2) \nabla^2 \zeta = 0, \quad (6)$$

where ξ and ζ are the transverse (planar) displacements in the x and z directions, respectively, and ∇^2 is the Laplace operator in the transverse plane. The normal modes for Eq. (5), which we call the x-modes, were found by applying all the boundary conditions (zero displacement at three fixed boundaries and zero stress at all the free boundaries), following the Titze (1976) analysis.

Applying the boundary conditions results in a transcendental equation

$$\frac{1}{2}(qr + q^{-1}r^{-1}) \sin \gamma T \sin \beta T + \cos \gamma T \cos \beta T = 1, \quad (7)$$

where γ and β are wave propagation constants in the x and z directions, respectively, and

$$r = 1 + \frac{1}{2}(\beta^2 - \gamma^2)/\gamma^2, \quad (8)$$

$$q = \frac{1}{2}(\beta/\gamma + \gamma/\beta). \quad (9)$$

With the simultaneous solution of γ and β from the above equations, the normal mode frequencies are found from the equation

$$f = \{[1/(2\pi)](\mu/\rho)[\beta^2 - \gamma^2 + (\mu'/\mu)(m^2\pi^2/L^2)]\}^{1/2}, \quad (10)$$

where ρ is the tissue density (1.04 g/cm³) and m is the mode integer in the anterior-posterior (y) direction, which is set to 1 for the important (1,0) and (1,1) modes. For a fully compressible tissue, Titze and Strong (1975) showed that $\gamma = (2n - 1)\pi/(2D)$, which for this case ($n = 1$ and $D = 0.7$ cm) results in a value $\gamma = 2.24$ cm⁻¹. It was also shown that for compressible tissue $\beta = \pi/T$ for a half-wavelength mode in the z direction, which for $T = 0.5$ cm results in a value $\beta = 6.28$ cm⁻¹. Using these values as seed values for Eq. (7) for the incompressible material resulted in $\gamma = 2.1395$ and $\beta = 4.4217$. Note that incompressibility reduces both propagation constants in the transverse plane, which means that the mode patterns are not described by exact half or quarter sinusoids (wavelengths) in this transverse plane. This makes the mode patterns not as obvious visually as for a compressible tissue. (Many of the mode patterns are shown in Titze, 2006, pp. 192–197.) When $\mu' \gg \mu$, and γ and β are small in comparison to the third term in brackets in Eq. (10), the TISM approaches the simple string model, for which the lowest mode frequency ($m = 1$) reduces to

$$f = \frac{1}{2L} \sqrt{\mu'/\rho}. \quad (11)$$

Figure 1 shows a comparison between the string-mode frequency and the plane-strain TISM mode frequency ($m = 1$) derived analytically as a function of μ' , which ranges from 0.5 to 500 kPa on the horizontal axis. Alipour-Haghighi et al. (1989) showed that thyroarytenoid muscle fibers in canines could produce up to 140 kPa stress (active plus passive) under maximum *in vitro* contraction at 30% strain. Min et al. (1995) measured axial ligament stress in humans as high as 550 kPa at 60% strain. Hence, the range of μ' in Fig. 1 has been confirmed empirically. In Fig. 1(a), a transverse shear modulus $\mu = 0.5$ kPa is chosen, which is typical of lax mucosal tissue (Chan and Titze, 1999, 2000). It is seen that a longitudinal μ' on the order of 1.0 kPa can produce frequencies below 100 Hz. In this region, the tissue is nearly isotropic. The string model diverges from the TISM model because it lacks the gel

FIG. 1. (Color online) Mode frequencies for the analytic TISM with plane strain (curved line) and the simple string model (straight line) for a large range of μ' : (a) for $\mu = 0.5$ kPa and (b) for $\mu = 5.0$ kPa.

FIG. 2. (Color online) Lateral (x) deflection of the medial surface showing the two dominant mode shapes. Left is the (1,0) mode and right the (1,1) mode.

properties. For high frequencies, however, with large values of μ' the two models merge asymptotically.

In Fig. 1(b), the transverse shear modulus μ is chosen to be 5.0 kPa. This represents a stiff gel, typically used for augmentation of vocal fold tissue (Chan and Titze, 1998; Klemuk and Titze, 2004). While μ' values on the order of 500 kPa still produce phonation around 500 Hz, no frequencies below 120 Hz are possible with the stiff gel in the TISM. The string model diverges even more at lower frequencies. These analytic solutions will now be used to interpret computational finite-element solutions.

B. Computational finite element solutions

To make both the computational TISM model and the FG-FEM model nearly transversely isotropic, ν' was set to 0.01, ν was set to 0.99, and E' was set to 25 E . The exact value of E' is not important because it was assumed that there is no vibrational displacement along the fibers, but E' needs to be high enough such that the Lempriere condition for transversely isotropic materials [Lempriere, 1968, Eq. (11)] is satisfied

$$\nu < 1 - 2\nu'^2 E/E'. \quad (12)$$

Graphical representation of the equation suggests that for large values of ν such as 0.95, ν' should be smaller than 0.15. With the values given above, the inequality was easily satisfied. In addition, for the TISM model in ANSYS, all element nodes were constrained in the y direction to impose plane strain condition, consistent with the FG-FEM model assumptions.

Natural frequencies and mode shapes for the TISM model were obtained from modal analysis with ANSYS. For the FG-FEM, natural frequencies were taken from the results of a transient analysis, for which a pressure impulse was applied to the medial surface to excite the normal modes. A transient analysis was also performed for the TISM model with ANSYS, but only to compare computation time with the FG-FEM model.

Two mode shapes are dominant in phonation, according to Berry *et al.* (1994) and Berry and Titze (1996). The plots of those two mode shapes are shown in Fig. 2, based on the modal analysis of the TISM with ANSYS. The first mode has been labeled the (1,0) mode. This mode has a half-wavelength variation along the y direction (left to right in the figure) and little variation along the z direction (up to down in the figure). The second mode is the (1,1) mode, which has nearly a half-wavelength variation in both y and z directions, with a nodal line mid-z.

An example of the FG-FEM impulse response for mode-frequency analysis is shown in Fig. 3. Note that the dominant low-frequency mode is a gel mode, but higher in the spectrum are the (1,0) and the (1,1) string mode frequencies in close proximity, often not more than a few Hz apart. Two additional peaks between the dominant gel mode and the string modes are modes that we could not consistently label or track because they were not uniquely excited by the pressure impulse. The reason for the close proximity of the string modes is that the fibers control the frequency more than the gel for the example chosen here ($\mu = 0.5$ kPa and

FIG. 3. (Color online) Impulse response spectrum from the FG-FEM model. The elastic constants were $\mu = 0.5$ kPa and $\mu' = 50$ kPa.

FIG. 4. (Color online) Mode frequencies for FG-FEM (dots), plane-strain TISM computed with ANSYS (dashed curves), and the plane-strain analytic TISM (solid black curve) for the (1,1) mode. The horizontal axis represents a large range of μ' . (a) $\mu = 0.5$ kPa and (b) $\mu = 5.0$ kPa.

$\mu' = 50$ kPa). The extra shear restoring force from the gel in the z-direction for the (1,1) mode makes only a small contribution to the overall restoring force. It should also be mentioned that for an incompressible tissue, there is strong interaction between the modes because every horizontal displacement results in a vertical displacement and the mode patterns are not based on simple half or quarter wavelengths (recall the above analytical analysis with a bi-harmonic fourth-order differential equation). In our experiments, it was never possible to excite a single mode with a surface impulse.

IV. RESULTS

A. FG-FEM performance across a range of μ' values

Each computational solution was obtained for μ' values ranging from 0.5 to 500 kPa while μ values were held constant at 0.5 kPa in one data set (representing a soft gel) and at 5.0 kPa in another set (representing a stiff gel). The range of μ' spanned the range of measured values as discussed above for the analytic solutions. The natural frequencies for the (1,0) mode and the (1,1) mode were extracted as described, but for clarity only the (1,1) frequencies are plotted (Fig. 4). (A complete spectrum of modes has been published in [Titze, 2006](#), Chap. 4.) Here, a three-way comparison is shown between mode frequencies for the FG-FEM (data points), the analytical plane-strain TISM (solid lines), and the plane-strain TISM computed with ANSYS (dashed lines).

FIG. 5. (Color online) ANSYS (1,1) mode frequencies with and without assuming plane strain. (a) $\mu = 0.5$ kPa and (b) $\mu = 5.0$ kPa. Solid lines are for the no plane strain assumption and dotted lines for plane strain assumption.

There is a slight discrepancy between the analytical solution and the computational finite element solutions at low frequencies (lower values of μ'). Identifying the modes consistently was not trivial because of the strong interaction between vertical and horizontal mode displacements; hence, for one case of μ' (50 kPa) we show two data points that seemed equally valid. Even for the ANSYS modal analysis, many modes looked similar and were closely spaced in frequency when the gel properties dominated over the fiber properties (low values of μ'). Recall also that the Poisson ratio ν was set exactly to 1.0 in the analytical solution and to 0.99 in the computational solutions. Figure 4 reflects these differences.

B. Effect of not imposing plane strain on the TISM model

The effect of not imposing plane strain in the gel slices was assessed using the TISM model in ANSYS. Recall that plane strain was imposed by setting the anterior-posterior (y) displacements of all nodes to zero. Without this constraint, which we claim to be part of the hyperelasticity model, internal dynamic elongations and contractions will occur in the longitudinal direction and an “effective fiber tension” cannot be considered a constant in vibration, nor can it easily be related to measured stress-strain curves. In Fig. 5, the plane-strain solution is compared to a no-plane strain solution. Without the plane-strain constraint, the model does not approach the asymptotic string model. We are not ready to discard the traditional concept that vocal folds with a ligament composed of dense collagen fibers tend to vibrate more

like vocal “cords” than vocal “folds.” If speech with relatively low f_o is the primary (or perhaps the only) objective in voice simulation, the hyperelastic model without plane strain (Yin and Zhang, 2013) produces an adequate frequency range with the range of measured elastic parameters. For high f_o in singing and vocalization across different species (Titze and Riede, 2016), the hyperelastic properties of the fibers with plane strain offer the appropriate extension of the stiffness model.

For example, for $\mu' = 500$ kPa, the natural frequency was increased from less than 200 Hz without plane strain to 700 Hz with plane strain for a soft gel [Fig. 5(a)]. For a stiff gel [Fig. 5(b)], the corresponding increase in natural frequency was from 500 to 750 Hz. Fundamental frequencies in singing go well beyond 1000 Hz, suggesting that fiber tension dominates over gel stiffness at high f_o .

C. Mesh density analysis

A convergence study was conducted to determine required mesh densities for further analyses. Five different densities were evaluated. For the (1,1) mode, results showed convergence to within 1% for ANSYS with a $24 \times 50 \times 17$ element mesh (x, y, z dimensions, respectively). Convergence with the FG-FEM was more gradual. A $36 \times 50 \times 26$ mesh showed convergence to within 5%. The more gradual convergence resulted from the fact that finite element interpolation functions are not used in the y direction. Rather, second-order differential equations are solved to obtain the string restoring forces between slices, which reduces computational accuracy but slows the convergence. For higher modes of vibration, more elements are needed for convergence.

D. Use of non-converged solutions

The use of a fully converged mesh is always warranted for testing the validity of a solution. For multiple exploratory runs, however, some accuracy is often sacrificed for speed of computation. In voice research, thousands of runs with hundreds of cycles of vibration are sometimes attempted in optimization protocols (Palaparthi et al., 2014). A high mesh density requires a high sampling rate for stability in the solutions of partial differential equations. The criterion of Courant et al. (1967) is that a spatial increment Δy must be on the order of $f\lambda\Delta t$, where f is the mode frequency, λ is the wavelength of the mode, and Δt is the sampling interval. If mode frequencies up to 1000 Hz are of interest in vocalization and Δt is chosen to be 1/44 100 s, the criterion suggests that Δy should not be much smaller than about $\lambda/40$. For half-wavelength mode patterns illustrated in Fig. 1, at least 20 elements along the length appear to be reasonable for computational stability at 44 100 Hz sampling. The amount of error introduced by making the mesh coarser is shown in Table I.

For exploratory runs, the relative change in frequency of oscillation is often more important than the absolute frequency. In these cases, a coarse mesh density is justified, as long as the final simulation is not contaminated by a non-convergent mesh.

TABLE I. Percentage error in frequencies of normal mode (1, 1) for various reduced mesh sizes compared to the frequency obtained for 36, 50, 26 mesh size.

FG-FEM mesh density (x, y, z)	% error in (1, 1) gel mode frequencies
7, 15, 6	74.4
14, 25, 10	37.2
28, 40, 17	13.9

E. Computation speed

The CPU time required for each solution was recorded for a transient analysis of 100 ms for both the FG-FEM and the TISM in ANSYS on the same computer using the same mesh of $36 \times 50 \times 26$. The runs were performed back-to-back. The FG-FEM model required 2670 CPU seconds while the TISM model in ANSYS required 14 550 CPU seconds, which shows an approximate 5:1 advantage in computational speed for the FG-FEM.

V. DISCUSSION AND CONCLUSIONS

This study was conducted to assess the degree of anisotropy a model can produce to cover a wide phonation frequency range with a range of tissue parameters that has been validated by measurement. Comparisons of the lowest mode frequencies were made between a simple string model (fiber response in isolation), an anisotropic stiffness model without embedded fibers, and a hyperelastic fiber-gel model that includes nonlinearity in the fiber stress-strain curves and a planar motion constraint. The hyperelasticity is not explicit in terms of a single constitutive equation, but an effective shear modulus is defined that incorporates the fiber nonlinearity along with the plane strain condition. Without the planar movement constraint, the TISM does not show the increase in normal mode frequencies that is needed for high- f_o phonation if measured elastic parameters (Min et al., 1995; Chan and Titze, 1999, 2000) are used. A stiffness-only model acts more like a bending beam than an elastic ground substance with embedded fibers. For example, the ANSYS solutions without the plane-strain assumption showed a reaction moment at the longitudinal faces that is not consistent with a tensile restoring force. Anisotropy that includes tension explicitly, in the context of hyperelasticity, appears to be a requirement for vocal fold modeling in order to achieve the observed range of frequencies. The effect of fibers in a stiffness-only model can to a degree be mimicked with modifications to elastic moduli, but longitudinal displacement constraints resulting from the plane strain assumption are advantageous for achieving wide frequency range with realistic tissue parameters. This result underscores previous statements of the importance of differentiating strong anisotropy from weak anisotropy (Thomsen, 1986).

At this stage, the most rigorous way to embed the fibers has not yet been explored. It will likely require an inhomogeneous material construct (similar to steel rods or rebar in concrete) or fiber-embedded elements types. There is precedent for using a rebar formulation in other biomechanical

applications, such as the cartilage in the spine, as shown in Kallemeyn *et al.* (2010). The recent approach taken by Weiss *et al.* (1996) is promising, but the sophistication in inclusion of hyperelasticity has to be weighed against speed of computation in voice simulation, especially in a clinical or exploratory setting where real-time results are important. A pragmatic approach was used here with our FG-FEM approximations.

The FG-FEM code uses analytic calculations of shape function integrals (Alipour *et al.*, 2000), unlike general purpose software that uses numerical integrations. This speeds up computation significantly. As a result of these simplifications, the FG-FEM uses only about 1/5 of the CPU time as a similar solution with ANSYS. This increase in computational speed may not seem important for many engineering applications for which supercomputers are available. However, as simulation is making its way into voice clinics and vocal studios, economy of computation is an issue. Clinicians and voice teachers want to hear sound samples, most of which need to be on the order of one second or more in length. They expect to get an audio sample while they wait, meaning that a second of sound should be able to be produced in a minute or two of computation. A longer delay than that is not practical in a clinical setting. Furthermore, if optimization strategies are pursued to improve parameters of the model, which might require thousands of iterative solutions with different combinations of parameters (Palaparthi *et al.*, 2014), computational efficiency becomes a major issue.

In summary, the FG-FEM is a compromise between (1) an accurate representation of a bounded three-dimensional viscoelastic structure with embedded fibers, (2) the precision of available material properties for biological species, and (3) the speed of computation. A set of reasonable assumptions with regard to tissue incompressibility, transverse isotropy, and planar displacement during vibration has reduced the need for five elastic constants to two, one shear modulus μ' in the plane of the tissue fibers and one shear modulus μ in a plane perpendicular (transverse) to the fibers. Highly nonlinear tension in the ligament and muscle fibers can be part of μ' and can vary with applied muscle forces on the vocal fold boundaries.

ACKNOWLEDGMENT

Support for this research was provided by NIH NIDCD Grant No. R01 DC012045.

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