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Determining geometric error model parameters of a terrestrial laser scanner through Two-face, Length-consistency, and Network methods

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Abstract

Terrestrial laser scanners (TLS) are increasingly used in large-scale manufacturing and assembly where required measurement uncertainties are on the order of few tenths of a millimeter or smaller. In order to meet these stringent requirements, systematic errors within a TLS are compensated in-situ through self-calibration. In the Network method of self-calibration, numerous targets distributed in the work-volume are measured from multiple locations with the TLS to determine parameters of the TLS error model. In this paper, we propose two new self-calibration methods, the Two-face method and the Length-consistency method. The Length-consistency method is proposed as a more efficient way of realizing the Network method where the length between any pair of targets from multiple TLS positions are compared to determine TLS model parameters. The Two-face method is a two-step process. In the first step, many model parameters are determined directly from the difference between front-face and back-face measurements of targets distributed in the work volume. In the second step, all remaining model parameters are determined through the Length-consistency method. We compare the Two-face method, the Length-consistency method, and the Network method in terms of the uncertainties in the model parameters, and demonstrate the validity of our techniques using a calibrated scale bar and front-face back-face target measurements. The clear advantage of these self-calibration methods is that a reference instrument or calibrated artifacts are not required, thus significantly lowering the cost involved in the calibration process.

Keywords

Terrestrial laser scanners; Self-calibration; Geometric error model; Network method; Two-face method; Length-consistency method; Uncertainty

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1 Introduction

Terrestrial laser scanners (TLS) are commonly used in historical preservation and archiving, digitization and reverse engineering, surveying and geographic modeling, etc. More recently, there has been increasing interest in applying TLS to engineering and other manufacturing industries, particularly in dimensional metrology and assembly of large structures such as aircraft components. While TLS offers large amounts of data in a short period of time, one of its disadvantage is the problem of relatively larger measurement errors when compared to other 3D measurement instruments. Since improving the manufacturing and assembly accuracy of TLS parts may be very difficult and expensive, researchers and engineers in industry and academia have explored other ways to improve the accuracy of TLS; one such method is self-calibration. In this paper, we discuss three self-calibration methods, the Network method, the Two-face method, and the Length-consistency method. The latter two methods are new methods proposed in this paper. The advantage of self-calibration methods is there is no requirement for a reference instrument or calibrated artifacts, thus significantly lowering the cost involved in the calibration process.

General overview and information about laser scanners, in particular, about TLS, can be found in [1,2]. In the Network method, numerous targets distributed in the work-volume are measured from multiple locations with the TLS to determine geometric error model parameters. These model parameters are then applied to subsequent measurements to improve accuracy. The Network method of self-calibration has been explored by several researchers. Gielsdorf [3] presented a network approach based on measuring 15 planar targets from four different TLS locations. Lichti [4] described the results of long term TLS parameter monitoring obtained using planar (circular) targets that can be reduced to a single point. Reshetyuk [5] investigated the correlations between parameters obtained from the Network method. Gonzales-Auguilera *et al.* [6] described the determination of model parameters for different TLS. Abbas *et al.* [7] described self-calibration of a TLS using 130 black and white targets using the Network method. Abbas *et al.* [8] attempted to lower the network station (number of TLS locations) and target configuration requirements in the Network method of TLS calibration. In that study, they determined the minimum number of scan stations, the minimum number of real-world planes, and the minimum number of point targets per plane that are required. Lichti and Licht [9] and Holst *et al.* [10] presented a review of TLS models and self-calibration methods. Chow [11] compared point-based and plane-based network self-calibration. Hughes *et al.* [12] described the Network method for the calibration of TLS using sphere targets based on a model developed by Muralikrishnan *et al.* [13].

While estimating the model parameters is important, it is also important to quantify the uncertainty in the parameters. In general, in literature surveyed above, the uncertainties in the parameters are estimated from the residuals in the bundle-adjustment process of the Network method. While that captures some of the error sources from within the TLS, some systematic errors are not accounted for in that process. Also, the Network method generally is performed using measurements from one face of the TLS. Because several model parameters are sensitive to front-face and back-face measurements, measuring a single target using both faces of the TLS will provide better estimates of those parameters, i.e., with

lower uncertainties. It is assumed that the random component of the error is much smaller than the systematic error in determining the coordinates of a target. The objective of this work is therefore to determine the geometric error model parameters so as to reduce or eliminate the systematic errors in the measurements. The uncertainty in the estimated model parameters is determined by propagating the random component of the measurement error in each measured target coordinate.

In this context, the aim of this paper is threefold:

1. Describe an improved geometric error model of a TLS, in which couplings between some of the parameters described by Muralikrishnan *et al.* [13] are eliminated.
2. Propose two self-calibration methods to determine the model parameters, namely, the Two-face method and the Length-consistency method, and compare the uncertainty in the model parameters with those obtained through the Network method.
3. Demonstrate the validity of our techniques through two-face and calibrated scale-bar measurements.

The rest of this paper is organized as follows. The geometric error model of TLS is presented in Section 2 along with our proposed improvement to eliminate coupling in the parameters. In Section 3, the Network method is briefly described followed by a detailed discussion of the Two-face method and the Length-consistency method. Our simulation based procedure to determine parameter uncertainties is also discussed in this section. In Section 4, the TLS measurement experiment including the targets are described and the self-calibration results of these methods are presented and compared. Conclusions are presented in Section 5.

2 Geometric error model of a TLS

2.1 Introduction

A schematic of an ideal TLS considered in this study is shown in Fig. 1. The TLS has a laser source, and a spinning prism mirror mounted on a platform that rotates about the vertical Z axis where XYZ is the fixed coordinate frame attached to the base of the TLS. The spinning mirror rotates about the T axis. The intersection of the two axes is the origin O . The laser transmitter position is denoted by O . The laser beam from the laser transmitter along the T axis reaches the point O of the spinning mirror surface. Then, based on the rotational position of the spinning mirror, the laser beam is directed to the target represented by P . Consequently, the path of the laser beam is OO and OP . The axis ON is, by definition, always perpendicular to both the T axis and the Z axis. During a measurement, the spinning platform rotates slowly about the vertical Z axis as the spinning prism mirror rotates rapidly about the T axis. The coordinates of the target P are represented in the spherical coordinate system (R, H, V) , where R is the radial distance between the target P and the point O , H is the horizontal angle between the N axis and the X axis, and V is the vertical angle (the zenith angle) between the laser beam OP and the Z axis.

2.2 Front-face and back-face measurements

The spinning mirror in the TLS system spins continuously in the same direction. As the zenith angle ranges from 0° to 180° , the beam emerges from the front face of the instrument; measurements of targets that are in front of this face are considered to be made in front-face mode. As the spinning mirror spins past 180° and towards 360° , the beam emerges from the back face of the instrument; measurements of targets that are in front of this face are considered to be made in back-face mode.

A two-face measurement involves measuring a single target first in front-face mode and subsequently in back-face mode. This is accomplished by first positioning the TLS system in such a manner that the target can be seen from the front-face of the instrument. After the front-face measurement, the scanner head is rotated by 180° about the vertical axis so that the same target is now in front of the back-face. The spinning mirror now has to spin past 180° to sight the target, hence this measurement of the target would constitute as a back-face measurement.

2.3 Geometric/optical misalignments and error model

Due to the existence of fabrication and assembly errors, geometric errors in TLSs cannot be avoided completely. Therefore, improving the measurement accuracy by factory calibration or self-calibration is necessary and is undoubtedly beneficial for both TLS manufacturing companies and TLS users. Recently, researchers at the National Institute of Standards and Technology (NIST) have developed a geometric error model for TLS[13].

A detailed discussion of the error model is beyond the scope of this paper. We describe very briefly the terms in our geometric error model followed by the model itself. The model addresses parameters that primarily affect angular errors. The ideal laser beam emerges from O and is assumed to be along the axis OO . However, in reality, the beam may be offset along ON or along OZ , these offsets are captured by two parameters x_{1z} and x_{1n} , respectively. The laser beam may also be tilted, i.e., not parallel to OO . This tilt is captured by two parameters x_{5z} and x_{5n} . If the horizontal T axis and the vertical Z axis do not intersect, that offset is captured by the parameter x_2 . If the two axes do intersect but are not orthogonal, that non-orthogonality (tilt) is captured by the parameter x_7 . If the origin O (intersection of the ideal horizontal T axis the vertical Z axis) does not lie on the face of the spinning mirror, that mirror offset is captured by the parameter x_3 . If the face of the spinning mirror is tilted so that the beam OP does not lie in the ZON plane, that mirror tilt is captured by the parameter x_6 . If the zero of the vertical angle encoder is shifted away from the zenith position (directly above the scanner), that vertical index offset is captured by the parameter x_4 . First order encoder errors (i.e., eccentricity with respect to the corresponding axes) are captured by the four parameters x_{8x} , x_{8y} , x_{9n} , and x_{9z} . Second order scale errors in the encoder (i.e., periodic errors in the scale) are captured by the four parameters x_{11a} , x_{11b} , x_{12a} , and x_{12b} . Finally, constant errors in the range are captured by the parameter x_{10} .

The parameters of that geometric error model are listed in Table 1 and the model itself is listed in the following equations.

$$R=R_m+\Delta R_m \quad (1)$$

$$H=H_m+\Delta H_m \quad (2)$$

$$V=V_m+\Delta V_m \quad (3)$$

$$\Delta R_m=k(x_2 \sin V_m)+x_{10} \quad (4)$$

$$\begin{aligned} \Delta H_m=k \left[\frac{x_{1z}}{R_m \tan V_m} + \frac{x_3}{R_m \sin V_m} + \frac{x_{5z}}{\tan V_m} + \frac{2x_6}{\sin V_m} - \frac{x_7}{\tan V_m} - x_{8x} \sin H_m + x_{8y} \cos H_m \right] \\ + \left[\frac{x_{1n}}{R_m} + x_{5n} + x_{11a} \cos 2H_m + x_{11b} \sin 2H_m \right] \end{aligned} \quad (5)$$

$$\begin{aligned} \Delta V_m=k \left[\frac{x_{1n} \cos V_m}{R_m} + \frac{x_2 \cos V_m}{R_m} + x_4 + x_{5n} \cos V_m + x_{9n} \cos V_m \right] \\ + \left[-\frac{x_{1z} \sin V_m}{R_m} - x_{5z} \sin V_m - x_{9z} \sin V_m + x_{12a} \cos 2V_m + x_{12b} \sin 2V_m \right], \end{aligned} \quad (6)$$

where R , H , and V are the true (corrected) range, the true (corrected) horizontal angle, and the true (corrected) vertical angle respectively; R_m , H_m , and V_m are the measured range, the measured horizontal angle, and the measured vertical angle by the TLS, respectively; and R_m , H_m , and V_m are the corrections to the measured range, the measured horizontal angle, and the measured vertical angle, respectively. The coefficient k is +1 for front-face and -1 for back-face measurements. The eighteen model parameters x_{1n} to x_{12b} are listed in Table 1. In this model, the horizontal angle H varies from 0° to 360° while the vertical angle V only varies from 0° to 180° .

2.4 Improved model

In the model described by Muralikrishnan *et al.* [13] and summarized in Eqs. (1) to (6), there is a problem of parameter coupling whereby some parameters cannot be effectively separated from the others. From a physical standpoint, it is important to be able to distinguish the parameters effectively so that we may then understand the precise mechanical cause of the errors. But from the perspective of improving the accuracy of the

TLS, we show in this paper that we can effectively combine certain parameters without degrading the effectiveness of the model.

In Eq. (5), x_{5z} and x_7 are indistinguishable (both have the same coefficient $1/\tan V_m$), while in Eq. (6), x_{5n} and x_{9n} are indistinguishable as are x_{5z} and x_{9z} . Because x_7 does not affect the vertical angle measurements, x_{5z} and x_7 can be effectively combined in Eq. (5), resulting in a new parameter x_{5z7} , given by $x_{5z} - x_7$. Because x_{9z} does not influence horizontal angle measurements, x_{5z} and x_{9z} can be combined in Eq. (6), resulting in a new parameter x_{5z9z} , given by $x_{5z} + x_{9z}$. Similarly, because x_{9n} does not influence horizontal angle measurements, x_{5n} and x_{9n} can be combined in Eq. (6), resulting in a new parameter x_{5n9n} , given by $x_{5n} + x_{9n}$.

In addition to this modification, we also note that the parameter x_{5n} produces a rotation about the vertical axis. Estimating this parameter is only possible if there are specific constraints introduced during the measurement or optimization. Because it is not otherwise feasible to estimate this parameter, we do not consider this parameter in our improved model. The new parameter set is described in Table 2.

Based on the above discussion, Eqs. (5) and (6) are reduced to the model described by Eqs. (7) and (8) below:

$$\Delta H_m = k \left[\frac{x_{1z}}{R_m \tan V_m} + \frac{x_3}{R_m \sin V_m} + \frac{x_{5z7}}{\tan V_m} + \frac{2x_6}{\sin V_m} - x_{8x} \sin H_m + x_{8y} \cos H_m \right] + \left[\frac{x_{1n}}{R_m} + x_{11a} \cos 2H_m + x_{11b} \sin 2H_m \right] \quad (7)$$

$$\Delta V_m = k \left[\frac{x_{1n} \cos V_m}{R_m} + \frac{x_2 \cos V_m}{R_m} + x_4 + x_{5n9n} \cos V_m \right] + \left[-\frac{x_{1z} \sin V_m}{R_m} - x_{5z9z} \sin V_m + x_{12a} \cos 2V_m + x_{12b} \sin 2V_m \right] \quad (8)$$

where $x_{5z7} = x_{5z} - x_7$, $x_{5n9n} = x_{5n} + x_{9n}$, and $x_{5z9z} = x_{5z} + x_{9z}$. Therefore, determining the 16 model parameters in Eqs. (1) to (4), (7), (8) is the key aspect in the calibration method for TLSs.

3 Self-calibration methods

As described in an earlier section, TLS calibration can be performed using a variety of methods. One approach involves the use of calibrated artifacts and/or a reference instrument such as a laser tracker that has higher accuracy than the TLS. These are often not available to most users of TLSs.

Self-calibration methods, on the other hand, do not require the use of reference instruments or other standard reference artifacts. One of the most popular existing self-calibration methods is the Network method, which is described in Section 3.1. Two new self-calibration

methods, the Two-face method and the Length-consistency method, are described in Sections 3.2 and 3.3, respectively. In all cases, we use the method of least-squares to determine model parameters from the measured data.

3.1 Network method

The well-known Network method [12] for TLS calibration is realized as follows:

1. A geometric model of the systematic errors for TLSs such as the model given in Section 2, is first developed.
2. The TLS under calibration is then used to measure the positions of several (tens or hundreds) fixed targets with different R , H and V coordinates. The targets are measured again with the TLS placed in several different physical locations.
3. The model parameters are estimated from the condition that the coordinates of all the targets measured from a given location should be overlap with the corresponding targets measured from the first location after transformation. This is framed as an optimization problem where the unknown parameters are not only the 16 error model parameters but also the translation/rotation parameters for each of the TLS locations (except the first location which is considered as the reference frame).

In general, the Network method is realized as a bundle adjustment process with appropriate weights assigned to the residuals during the optimization. In this paper, we have adopted a rigid body transform method without considering weights. The detailed description of the Network method can be found in Refs. [3, 8, 12]. In existing studies of the Network self-calibration method for TLSs, typically only front-face measured data of the targets are used for deriving the model parameters. However, considering the geometric error model of TLSs in Eqs. (1) to (8), using both front-face and back-face measured data for the self-calibration calculation may be advantageous as we show in this paper. We therefore explore both approaches, i.e., using only front-face data and again recalculating the parameters using both front-face and back-face data. We refer to the former approach as 'Network method (FF only)' where FF stands for front-face, and the latter as 'Network method (Both faces)'.

3.2 Two-face method

The Two-face method is a two-step process. In the first step, many model parameters are determined directly from the difference between front-face and back-face measurements of targets distributed in the work volume. In the second step, all remaining model parameters are determined through the Length-consistency method. The experimental set-up is the same as for the case of the Network method, however, all the targets are scanned in both front-face and back-face by the TLS from the different locations. The procedure to estimate the model parameters is described below.

Let the spherical coordinate data of the i -th target measured in the front-face by the TLS from the j -th location be denoted as $(R_{mf,ij}, H_{mf,ij}, V_{mf,ij})$ and the corresponding back-face coordinates be denoted as $(R_{mb,ij}, H_{mb,ij}, V_{mb,ij})$, where $i=1, 2, \dots, M$, and $j=1, 2, \dots, N$ for M targets and N TLS locations.

In the first step of the Two-face method, we estimate parameters by considering the fact that the corrected coordinates in the front-face must be equal to the corrected coordinates in the back-face. For example, in Eq. 4, the corrected range to a target measured in the front-face must equal the corrected range to the same target measured in the back-face. This provides the condition equation necessary to evaluate the parameter x_2 . From Eq. 7, the corrected horizontal angle to a target measured in the front-face must equal the corrected horizontal angle to the same target measured in the back-face. This provides the condition equation necessary to evaluate the six parameters x_{1z} , x_3 , x_{5z7} , x_6 , x_{8x} , and x_{8y} . Finally, from Eq. 8, the corrected vertical angle to a target measured in the front-face must equal the corrected vertical angle to the same target measured in the back-face. This provides the condition equation necessary to evaluate the three parameters x_{1n} , x_4 , and x_{5n9n} .

On the basis of the above discussion, we first estimate the transit offset x_2 from the differences between front-face and back-face range as follows. The equations for the true (corrected) range in the front-face $R_{f,ij}$ and in the back-face $R_{b,ij}$ are given by:

$$R_{f,ij} = R_{mf,ij} + x_2 \sin V_{mf,ij} + x_{10} \quad (9)$$

$$R_{b,ij} = R_{mb,ij} - x_2 \sin V_{mb,ij} + x_{10} \quad (10)$$

Because the true (corrected) range value to any target in the front-face should equal the corresponding range value in the back-face, i.e., $R_{f,ij} = R_{b,ij}$, subtracting Eq. (10) from Eq. (9), we get:

$$0 = R_{mf,ij} - R_{mb,ij} + x_2 \sin V_{mf,ij} + x_2 \sin V_{mb,ij} \quad (11)$$

That is:

$$R_{mb,ij} - R_{mf,ij} = (\sin V_{mf,ij} + \sin V_{mb,ij}) x_2 \quad (12)$$

Because there are M targets measured from N locations of the TLS, we have $M \times N = Q$ equations from which we estimate the parameter x_2 in a least squares sense, given as follows:

$$x_2 = ([\sin V_{mf,ij} + \sin V_{mb,ij}]_{Q \times 1})^T ([\sin V_{mf,ij} + \sin V_{mb,ij}]_{Q \times 1})^{-1} [\sin V_{mf,ij} + \sin V_{mb,ij}]_{Q \times 1}^T [R_{mb,ij} - R_{mf,ij}]_{Q \times 1} \quad (13)$$

We next estimate six parameters (x_{1z} , x_3 , x_{5z7} , x_6 , x_{8x} , and x_{8y}) from the difference between front-face and back-face horizontal angle measurements of the M targets from $NTLS$ locations. In a manner similar to that described in Eqs. (9) and (10), we equate the true (corrected) front-face and back-face horizontal angle to obtain the following:

$$H_{mf,ij} + \left[\frac{x_{1z}}{R_{mf,ij} \tan V_{mf,ij}} + \frac{x_3}{R_{mf,ij} \sin V_{mf,ij}} + \frac{x_{5z7}}{\tan V_{mf,ij}} + \frac{2x_6}{\sin V_{mf,ij}} - x_{8x} \sin H_{mf,ij} + x_{8y} \cos H_{mf,ij} \right] \\ \approx H_{mb,ij} - \left[\frac{x_{1z}}{R_{mb,ij} \tan V_{mb,ij}} + \frac{x_3}{R_{mb,ij} \sin V_{mb,ij}} + \frac{x_{5z7}}{\tan V_{mb,ij}} + \frac{2x_6}{\sin V_{mb,ij}} - x_{8x} \sin H_{mb,ij} + x_{8y} \cos H_{mb,ij} \right] \quad (14)$$

We estimate the six parameters from the Q equations represented in Eq. (14) above. Solving in a least-squares sense, we have:

$$[x_{1z} x_3 x_{5z7} x_6 x_{8x} x_{8y}]^T = (A^T A)^{-1} A^T Y \quad (15)$$

where:

$$A = \begin{bmatrix} \frac{1}{R_{mf,ij} \tan V_{mf,ij}} + \frac{1}{R_{mb,ij} \tan V_{mb,ij}} & \frac{1}{R_{mf,ij} \sin V_{mf,ij}} + \frac{1}{R_{mb,ij} \sin V_{mb,ij}} & \frac{1}{\tan V_{mf,ij}} + \frac{1}{\tan V_{mb,ij}} \\ \frac{2}{\sin V_{mf,ij}} + \frac{2}{\sin V_{mb,ij}} & -\sin H_{mf,ij} - \sin H_{mb,ij} & \cos H_{mf,ij} + \cos H_{mb,ij} \end{bmatrix}_{Q \times 6} \quad (16)$$

$$Y = [H_{mb,ij} - H_{mf,ij}]_{Q \times 1} \quad (17)$$

We then estimate three parameters (x_{1n} , x_4 , and x_{5n9n}) from the differences between front-face and back-face vertical angle measurements of the M targets from $NTLS$ locations. In a manner similar to that described in Eqs. (9) and (10), we equate the true (corrected) front-face and back-face vertical angle to obtain the following:

$$\begin{aligned}
 V_{mf,ij} + \left[\frac{x_{1n} \cos V_{mf,ij}}{R_{mf,ij}} + \frac{x_2 \cos V_{mf,ij}}{R_{mf,ij}} + x_4 + x_{5n9n} \cos V_{mf,ij} \right] \\
 \approx V_{mb,ij} - \left[\frac{x_{1n} \cos V_{mb,ij}}{R_{mb,ij}} + \frac{x_2 \cos V_{mb,ij}}{R_{mb,ij}} + x_4 + x_{5n9n} \cos V_{mb,ij} \right] \quad (18)
 \end{aligned}$$

Because the parameter x_2 is already known based on Eq. (13), we estimate the three parameters from the Q equations represented in Eq. (18) above. Solving in a least-squares sense, we have:

$$[x_{1n} x_4 x_{5n9n}]^T = (B^T B)^{-1} B^T Z \quad (19)$$

where:

$$B = \left[\begin{array}{cc} \frac{\cos V_{mf,ij}}{R_{mf,ij}} + \frac{\cos V_{mb,ij}}{R_{mb,ij}} & 2 \cos V_{mf,ij} + \cos V_{mb,ij} \end{array} \right]_{Q \times 1} \quad (20)$$

$$Z = \left[V_{mb,ij} - V_{mf,ij} - \frac{x_2 \cos V_{mf,ij}}{R_{mf,ij}} - \frac{x_2 \cos V_{mb,ij}}{R_{mb,ij}} \right]_{Q \times 1} \quad (21)$$

After these calculations, the remaining six parameters x_{5z9z} , x_{10} , x_{11a} , x_{11b} , x_{12a} , and x_{12b} still remain unknown in the geometric error model presented by Eqs. (1) to (4), (7) and (8). The estimates of these unknown parameters may be performed by the Network method described earlier or the Length-consistency method as shown below. In this study, we estimate the remaining parameters through the Length-consistency method. As mentioned in Section 3.1, it is possible to estimate these remaining parameters using only front-face data or using both front-face and back-face data. We refer to the approach involving only front-face data as ‘Two-face and LC FF only’ and the approach involving both front-face and back-face data as ‘Two-face and LC both faces’, where LC stands for Length-consistency.

3.3 Length-consistency method

As discussed in Section 3.1, the basic idea of the Network method is that the measured coordinates of the same target by the TLS from the different locations should be as close as possible after the coordinates are transformed into the reference coordinate frame. However, the related coordinate transformation parameters are also unknown and are determined by the optimization step of the Network method described in Section 3.1. This increases the complexity of the Network method.

The proposed Length-consistency method described here also involves measurement of a common set of M targets from N TLS locations. The estimates of all parameters in the

geometric error model are determined through an optimization procedure. However, unlike the Network method, the optimizing criterion of the proposed Length-consistency method is that the distances (i.e., the lengths) between any pair of two targets determined from the different TLS locations should be as close to each other as possible.

This optimization problem is described as follows:

$$\min f(x_{1z}, x_{1n}, x_2, x_3, x_4, x_{5n9n}, x_{5z7}, x_{5z9z}, x_6, x_{8x}, x_{8y}, x_{10}, x_{11a}, x_{11b}, x_{12a}, x_{12b}) = \min \sum_{j=1}^N \sum_{k=1}^{M(M-1)/2} (L_{kj} - \bar{L}_k)^2, \quad (22)$$

where L_{kj} is the distance between two targets from the set of M targets determined from the j -th TLS location, and $\bar{L}_k$ is the mean of the distances of the same two targets scanned from the different locations. Because only the model parameters are determined (and not the coordinate transformation parameters), there is significant reduction in complexity of the problem and therefore improvement in optimizing speed.

When the Length-consistency method is applied to determine the remaining six parameters of the Two-face method, Eq. (22) is modified as

$$\min f(x_{5z9z}, x_{10}, x_{11a}, x_{11b}, x_{12a}, x_{12b}) = \min \sum_{j=1}^N \sum_{k=1}^{M(M-1)/2} (L_{kj} - \bar{L}_k)^2. \quad (23)$$

As mentioned in Sections 3.1 and 3.2, it is possible to estimate the parameters using only front-face data or using both front-face and back-face data. We refer to the approach involving only front-face data as ‘Length consistency (FF only)’ and the approach involving both front-face and back-face data as ‘Length-consistency (Both faces)’.

3.4 Measurement uncertainty

The uncertainties in the model parameters are estimated through a Monte Carlo (MC) simulation as described here. The basic idea is to propagate the uncertainty in the determination of the coordinate of each of the M targets from the N locations to the model parameters for each of the six methods described in the previous sub-section. Because the TLS measures in spherical coordinates, we require an estimate of the uncertainty in the range, horizontal angle, and vertical angle, from each TLS position to each measured target. For this purpose, we perform the following two experiments.

Because the geometric model captures all systematic sources of error in the measured angles, we consider the repeatability as the uncertainty along the angular axes. We therefore measure each of the M targets a total of 10 times, from each of the N TLS locations. From this data, we calculate the one standard deviation repeatability (one sigma) along the horizontal angle and vertical angle axes; the results are presented in Section 4.3.

The uncertainty in the range to each target is somewhat more challenging to determine because there are a number of influence factors to consider such as the inherent ranging errors of the measurement technology, interaction between the laser and the target surface, incidence angle of the laser etc. We estimate the relative range error to be smaller than ± 0.25 mm over a range of 8 m using a procedure similar to that outlined in Ref. [14]. Ignoring the other influence factors here, we propagate a range uncertainty of 0.15 mm (one standard deviation of a ± 0.25 mm rectangular distribution) in our MC simulations.

Our MC simulation proceeds as follows:

- a. We first determine the simulated front-face and back-face coordinates of all the targets from each TLS location. Then, we perturb that data using the model parameters determined by each of the six methods described earlier for the scanner under study.
- b. Next, we add measurement noise generated randomly based on the coordinate uncertainties as described earlier in this sub-section.
- c. We then calculate the 16 model parameters for each of the six self-calibration approaches.
- d. We repeat steps (c) and (d) numerous times (e.g., 1000 times), so that many sets of the 16 parameters may be computed.
- e. We finally calculate the standard deviation of each parameter from the result of the simulation. The standard deviation values are considered as the uncertainties of the 16 parameters in the geometric error model of the TLS.

The results of this simulation are given in Section 4.

4 Experiment and results

4.1 Targets and TLS

The targets used in the calibration process are commercially obtained magnetic checkerboard targets. Twenty-nine targets were used in the calibration process. The targets were distributed uniformly on the four walls, the ceiling, and floor of a 12 m $\times$ 6 m $\times$ 4 m room (Fig. 2). The target center is determined using software provided by the manufacturer of the TLS.

The two-face validation tests are performed using six checkerboard targets mounted in a single vertical column. The scale-bar validation tests are performed using commercially obtained contrast targets mounted on a rotary scale bar, nominally 2.3 m long. The commercially available contrast target is a metallic plate with a partial sphere on the back. On the front-face of the plate is a square that is partitioned into four triangles by its two diagonals. Two opposing triangles are black and the other two are white. The point of intersection of the two diagonals on the front-face of the plate is also the theoretical center of the partial sphere. The offset of the sphere center from the plane of the triangles was measured on a coordinate measuring machine (CMM) to be less than 5 μ m. While we did not measure the in-plane eccentricity between the intersection of the diagonals and the

sphere center, we eliminate this offset by averaging measurements from two orientations of the target that are 180° apart as described in Section 4.2. Fig. 3 shows the views from the front and the back of a contrast target used in the scale-bar validation tests.

4.2 Experimental procedure

The experimental procedure is as follows:

1. The TLS' factory determined model parameters are turned off so that the data output by the TLS are not corrected for any geometric misalignments and eccentricities.
2. *Self-calibration experiments:* Calibration experiments to determine TLS model parameters are performed by scanning 29 targets from four TLS locations. At each location, the TLS is oriented in a different height with respect to the targets. The TLS is placed at a height of about 1 m from the ground for two of the locations while it is placed at a height of about 2.5 m for the other two locations. Also, the TLS is rotated by 90° about the vertical axis from one location to the next. At each location, one scan is first performed in front-face of the TLS and another scan is subsequently performed in the back-face. The model parameters are estimated from these measurements
3. *Repeatability tests:* In order to calculate the one standard deviation repeatability along the horizontal and vertical angle axes, each of the 29 targets is measured 10 times from each of the four TLS locations. The height and the rotation of the TLS at each location are as described in Step 2 above. This experiment provides information that is used in the MC simulation to estimate the uncertainty in the model parameters determined in the previous step.
4. *Two-face validation tests:* Two-face validation tests are performed using six contrast targets in one vertical column 4 m high. The TLS is initially placed about 2 m away from the targets. After the targets are scanned in front face and again in back face, the TLS is rotated by 90° and the targets are scanned again. This process is repeated for four different azimuth angles 0°, 90°, 180°, and 270°. This entire process is again repeated for a far position, i.e., with the TLS placed about 6 m away from the targets. The purpose of these tests is to determine if the model parameters have been estimated accurately. Only one scan is performed in each face and from each location/orientation of the TLS.
5. *Scale-bar validation tests:* Scale-bar validation tests are performed using a 2.3 m bar mounted on a rotary stage fixed to a wall. The scale bar is first measured using the interferometer of a laser tracker placed in-line with the bar. Five measurements are averaged to determine the reference length. The TLS is centrally placed about 3 m away from the scale bar. One contrast target is placed at each end of the bar (where the laser tracker target was previously located). Measurements are performed in the horizontal, vertical, left-diagonal, and right-diagonal orientations of the bar. For each orientation of the bar, measurements are performed with four azimuth positions of the TLS – 0°, 90°, 180°, and 270°. Thus, 16 scale bar measurements are performed in all. For completeness, we note

that at each position of the scale bar, two length measurements are actually performed. The first length measurement involves measuring the scale bar with the current orientation of the targets on the bar. The second measurement is performed after each of the targets has been rotated by 180° in their respective nests. These two length measurements are averaged to remove any target eccentricities in the results. The purpose of the scale-bar tests is also to determine if the model parameters have been estimated accurately.

4.3 Results of model parameters and their uncertainties

Before we discuss the model parameters and their uncertainties, we first present results from the repeatability experiment. Fig. 4 shows the one standard deviation repeatability along the horizontal angle and the vertical angle axes. In general, the repeatability depends on the distance and orientation of the target with respect to the TLS. It can be seen from Fig. 4 that the one standard deviation repeatability of most targets is on the order of $10 \mu\text{rad}$. The repeatability of some targets is slightly worse due to relatively large incidence angles of the laser beam (for example, the 12th target from the first location shows poor horizontal angle repeatability).

The model parameters estimated from the experimental data are presented in Table 3. The one-standard deviation uncertainty of the model parameters determined from the MC simulation is presented in Table 4, and a comparison of the parameters along with their uncertainties is shown in Fig. 5.

As shown in Tables 3 and 4, and Fig. 5, the confidence intervals of the obtained model parameters from the six different self-calibration approaches overlap for the most part. This shows the consistency of the six self-calibration approaches. Also, the model parameters obtained from both measured front-face and back-face data have smaller uncertainties, when compared with those obtained from only measured front-face data. This indicates the necessity to use both front-face and back-face measurement data in order to obtain more accurate results of the model parameters. For some parameters such as x_{1n} and x_{5n9n} , the confidence interval does not overlap well indicating that the uncertainties we have estimated may not be adequate.

4.4 Results of two-face validation test

After calculating the model parameters shown in Table 3, the two-face validation test is performed as described in Section 4.2. In this test, the measured data corresponding to the front-face and back-face of the six contrast targets are obtained, scanned by the TLS from the two locations, each with four azimuth positions. As mentioned earlier, the six targets are arranged in a vertical column 4 m high, and are approximately equally spaced apart from each other. The TLS is at a height of about 2 m from the ground. As in the case of the calibration experiment, the TLS' factory determined model parameters are turned off so that the data acquired are not corrected for any geometric misalignments.

After acquiring the raw uncorrected data, we then correct the data using the model parameters obtained from each of the six self-calibration approaches described in Section 3. For each of those corrected sets of data, we calculate the difference between the front-face

and back-face horizontal angles, and the difference between the front-face and back-face vertical angles. These differences should be close to zero assuming random errors are substantially smaller than the systematic errors.

In addition, we also correct the data using the OEM software based on the factory determined model parameters. We then calculate the difference between the front-face and back-face horizontal angles, and the difference between the front-face and back-face vertical angles.

The mean values and standard deviations of the two-face angle errors for the seven methods (OEM model and the six approaches described in Section 3) are listed in Table 5. Figs. 6 and 7 show the actual differences in the angles for the seven methods. The differences in the angles are larger for measurement # 1 through 24 compared to that of # 25 through 48 because the first set of 24 measurements are performed at the near position and therefore the targets closer to the floor and those closer to the ceiling are at steeper vertical angles. For some model parameters, steeper vertical angles result in larger differences in horizontal and vertical angles in a two-face test.

Based on the data shown in Figs. 6, 7 and Table 5, it can be seen that the two-face self-calibration method is able to reduce angle differences between front-face and back-face measurements. The differences of the vertical angles in front-face and back-face are even smaller than that of the OEM model. Based on Fig. 7, the Length-consistency method (both faces) and the Network method (both faces) also produce small angle differences.

However, it can be seen from Fig. 6 that the Length-consistency method (FF only) and the Network method (FF only) failed to correct the raw data properly in the two-face validation test. This is not surprising because no back-face measured data are used for deriving the model parameters in these two self-calibration methods. In fact, using only front-face data has introduced a significant two-face vertical angle error that is larger than the error in the raw data. This indicates the necessity to include back-face measurements in the self-calibration procedure.

4.5 Results of scale-bar validation test

Based on the five repeated measurements of the laser tracker, the mean length of the scale bar was determined to be 2328.990 mm with an expanded uncertainty of 0.01 mm ($k = 2$). The repeatability of the five measurements was 0.002 mm (one standard deviation).

As mentioned in sub-section 4.2, the scale-bar is measured a total of 16 times using the TLS. As in the case of the calibration experiment and the two-face validation experiment, the TLS' factory determined model parameters are turned off prior to measuring the scale-bar so that the data acquired are not corrected for any geometric misalignments. After acquiring the raw uncorrected data, we then correct the data using the model parameters obtained from each of the six self-calibration approaches described in Section 3 and calculate the length of the bar. We also use the OEM software and the factory determined model to correct the raw data and calculate the length of the bar. The lengths determined by the OEM method and the

six self-calibration methods are shown in Fig. 8. The differences (errors) between the calculated lengths and the reference length are shown in Table 6.

It is clear that there are systematic errors in the raw data based on Fig. 8, demonstrating that the calibration or self-calibration is essential. In particular, there are large errors in the raw data for measurement #s 5 through 12. Measurement #s 5 through 8 are performed with the TLS at azimuth orientations of 90° and measurement #s 9 through 12 are performed with the TLS at azimuth orientation of 180° . The reason for the large errors at these positions are because of a combination of two parameters x_{5n9n} and x_{5z7} .

As shown in Fig. 8 and Table 6, the six self-calibration approaches significantly reduce the measurement errors. The Two-face method yields practically the same errors as the OEM method. This is particularly interesting considering that no laser tracker or calibrated reference artifact is needed in the self-calibration process. Undoubtedly, this would be beneficial for TLS users to improve measurement accuracy while lowering associated cost.

5 Conclusions

Self-calibration methods require a geometric error model whose parameters are determined during the calibration process. In this paper, we present an improved geometric model that eliminates parameter coupling, thus allowing for the determination of all parameters.

We then propose two new methods for self-calibration of a TLS, the Two-face method and the Length-consistency method. We also compare these methods against the more generally adopted Network method. The Network and Length-consistency methods may be applied to data collected only in front-face or to data acquired in both faces. The Two-face method is a two-step process. In the first step, many model parameters are determined directly from the difference between front-face and back-face measurements of targets distributed in the work volume. In the second step, all remaining model parameters are determined through the Length-consistency method. Again, the Length-consistency method may be applied to data collected only in front-face or to data acquired in both faces. We report the model parameters of a TLS calculated using each of the six approaches, along with the uncertainties determined through Monte Carlo simulation. Finally, we present the validation results obtained through two-face and scale-bar tests.

A key contribution of this paper is that we clearly show that using data from both faces help improve the self-calibration process. We also clearly show that the proposed ‘Two-face and LC both faces’ self-calibration method produces results that are comparable or better than the OEM model of the TLS. The fact that the Two-face method does not require a reference instrument such as a laser tracker or calibrated reference artifacts makes it particularly appealing. Instead of having to ship the TLS to the manufacturer for periodic calibration, the geometric model parameters may be determined in-house through the methods described here. This technique may therefore be beneficial for TLS users to improve measurement accuracy with minimal cost.

As future work, we plan on developing methods to determine the parameter x_{5n} (Beam tilt component along the n axis in Fig. 1). This is necessary to further improve the performance

of self-calibration methods. We also plan on exploring the use of weights during the optimization process to improve the solution. Finally, many users may lack a large sized room in which to perform such testing. For that purpose, we are exploring the use of an uncalibrated scale bar measured from multiple positions in the room.

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Fig. 1.
Coordinate system of a terrestrial laser scanner[13]

Fig. 2.
TLS and contrast targets in the self-calibration experiment and the validation test

(a)

(b)

Fig. 3.

(a) Close-up view of the magnetic checkerboard target (b) Contrast targets in the scale-bar validation test

Fig. 4.
Measurement repeatability (one standard deviation) of horizontal angle and vertical angle

Fig. 5.

Model parameters obtained from the different self-calibration methods are shown. The error bars represent the uncertainty ($k=3$) in the parameters

Fig. 6. Front-face and back-face angle differences shown for the raw data and OEM model applied. Also shown are front-face and back-face angle differences obtained from the self-calibration methods when only front-face data are used for the model parameters' calculations for (a) horizontal angle, (b) vertical angle

Fig. 7.

Front-face and back-face angle differences shown for the raw data and OEM model applied. Also shown are front-face and back-face angle differences obtained from the self-calibration methods when both front-face and back-face data are used for the model parameters' calculations for (a) horizontal angle, (b) vertical angle

Fig. 8.

Comparison of scale-bar lengths given by the different approaches, (a) laser tracker reference, raw uncorrected length from the TLS, after applying OEM model, Two-face and LC FF only, Length-consistency (FF only), and Network (FF only) results shown, (b) laser tracker reference, raw uncorrected length from the TLS, after applying OEM model, Two-face and LC both faces, Length-consistency (both faces), and Network (both faces) results shown.

Table 1

Model parameters for the TLS[13]

Parameter	Description	Two-face sensitivity
x_{1n}	Beam offset along n	Vertical angle direction
x_{1z}	Beam offset along z	Horizontal angle direction
x_2	Trasit offset	Vertical angle and ranging direction
x_3	Mirror offset	Horizontal angle direction
x_4	Vertical index offset	Vertical angle direction
x_{5n}	Beam tilt component along n	Vertical angle direction
x_{5z}	Beam tilt component along z	Horizontal angle direction
x_6	Mirror tilt	Horizontal angle direction
x_7	Transit tilt	Horizontal angle direction
x_{8x}	Horizontal angle encoder eccentricity along x	Horizontal angle direction
x_{8y}	Horizontal angle encoder eccentricity along y	Horizontal angle direction
x_{9n}	Vertical angle encoder eccentricity along n	Vertical angle direction
x_{9z}	Vertical angle encoder eccentricity along z	
x_{10}	Constant error in range (bird-bath error)	
x_{11a}	Second order scale error in the horizontal angle encoder	
x_{11b}	Second order scale error in the horizontal angle encoder	
x_{12a}	Second order scale error in the vertical angle encoder	
x_{12b}	Second order scale error in the vertical angle encoder	

Table 2

Revised model parameters for the TLS

Combined parameter	Description	Two-face sensitivity
x_{5n9n}	Combination of beam tilt along n and vertical angle encoder eccentricity along n	Vertical angle direction
x_{5z7}	Combination of beam tilt component along z and transit tilt	Horizontal angle direction
x_{5z9z}	Combination of beam tilt along z and vertical angle encoder eccentricity along z	Vertical angle direction

The remaining parameters x_{1n} , x_{1z} , x_2 , x_3 , x_4 , x_6 , x_{8x} , x_{8y} , x_{10} , x_{11a} , x_{11b} , x_{12a} , and x_{12b} are the same as those in Table 1.

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Table 3

Model parameters estimated by the different self-calibration methods

Parameters	Two-face and LC FF only	Length- consistency (FF only)	Network (FF only)	Two-face and LC both faces	Length- consistency (Both faces)	Network (Both faces)
$x_{1a}(\text{mm})$	-0.36	0.52	-0.08	-0.36	0.29	0.26
$x_{1z}(\text{mm})$	0.06	-0.16	-0.22	0.06	-0.08	-0.02
$x_2(\text{mm})$	0.06	-0.14	-0.09	0.06	0.07	0.05
$x_3(\text{mm})$	0.45	0.41	0.91	0.45	0.43	0.47
$x_4(\text{rad})$	-0.0000130	0.0011048	0.0005785	-0.0000130	-0.0000055	-0.0000113
$x_{5a}(\text{mm})$	0	0	0	0	0	0
$x_{5a9a}(\text{rad})$	-0.0005726	-0.0015538	-0.0009627	-0.0005726	-0.0007380	-0.0007108
$x_{5z7}(\text{rad})$	-0.0006857	-0.0005655	-0.0005420	-0.0006857	-0.0006660	-0.0006725
$x_{5z9z}(\text{rad})$	0.0002704	0.0017323	0.0010868	0.0002463	0.0002825	0.0002670
$x_6(\text{rad})$	-0.0000806	-0.0001212	-0.0001983	-0.0000806	-0.0000603	-0.0000821
$x_{8x}(\text{rad})$	-0.0000272	-0.0000342	-0.0000274	-0.0000272	-0.0000234	-0.0000271
$x_{8y}(\text{rad})$	-0.0000298	-0.0000189	-0.0000215	-0.0000298	-0.0000272	-0.0000322
$x_{10}(\text{mm})$	-0.04	0.07	0.01	-0.05	-0.07	-0.13
$x_{11a}(\text{rad})$	0.0000029	0.0000110	0.0000096	0.0000019	0.0000068	0.0000056
$x_{11b}(\text{rad})$	0.0000092	0.0000100	0.0000072	0.0000087	0.0000090	0.0000071
$x_{12a}(\text{rad})$	-0.0000195	-0.0003227	-0.0002059	-0.0000220	-0.0000401	-0.0000274
$x_{12b}(\text{rad})$	-0.0000335	0.0003986	0.0001541	-0.0000357	-0.0000323	-0.0000205

Table 4

Uncertainties ($k = 3$) of model parameters given by the Monte Carlo simulation of the different self-calibration methods

Parameters	Two-face and LC FF only	Length- consistency (FF only)	Network (FF only)	Two- face(Both faces)	Length- consistency (Both faces)	Network (Both faces)
x_{1a} (mm)	0.09	1.00	0.83	0.08	0.09	0.06
x_{1z} (mm)	0.09	0.31	0.23	0.09	0.15	0.09
x_2 (mm)	0.03	0.91	0.82	0.03	0.04	0.03
x_3 (mm)	0.02	0.99	0.83	0.02	0.08	0.03
x_4 (rad)	0.0000026	0.0016007	0.0009455	0.0000027	0.0000190	0.0000033
x_{5a} (mm)	0	0	0	0	0	0
x_{5a9a} (rad)	0.0000223	0.0009654	0.0006120	0.0000223	0.0000256	0.0000176
x_{5z7} (rad)	0.0000250	0.0001312	0.0001143	0.0000255	0.0000313	0.0000209
x_{5z9z} (rad)	0.0001066	0.0021110	0.0012317	0.0000780	0.0000728	0.0000592
x_6 (rad)	0.0000022	0.0001740	0.0001412	0.0000022	0.0000289	0.0000029
x_{8x} (rad)	0.0000022	0.0000222	0.0000152	0.0000022	0.0000107	0.0000021
x_{8y} (rad)	0.0000021	0.0000203	0.0000129	0.0000022	0.0000107	0.0000022
x_{10} (mm)	0.21	0.93	0.83	0.17	0.16	0.15
x_{11a} (rad)	0.0000062	0.0000058	0.0000044	0.0000045	0.0000043	0.0000029
x_{11b} (rad)	0.0000056	0.0000051	0.0000041	0.0000039	0.0000041	0.0000030
x_{12a} (rad)	0.0000547	0.0005214	0.0002979	0.0000368	0.0000327	0.0000236
x_{12b} (rad)	0.0000787	0.0005142	0.0003192	0.0000598	0.0000592	0.0000460

Table 5

Mean of absolute angle differences for the different methods

	<i>H</i> angle (rad)	<i>V</i> angle (rad)
Raw data	0.0004439	0.0003665
OEM method	0.0000312	0.0000325
Two-face and LC FF only	0.0000821	0.0000211
Length-consistency (FF only)	0.0001751	0.0021274
Network (FF only)	0.0002161	0.0011325
Two-face and LC both faces	0.0000821	0.0000211
Length-consistency (Both faces)	0.0001107	0.0000499
Network (Both faces)	0.0000852	0.0000496

Table 6

Mean error and standard deviation of the 16 measured lengths for the different methods

	Mean Error (mm)	Std. Dev. (mm)
Raw data	0.93	0.81
OEM method	0.09	0.08
Two-face and LC FF only	0.11	0.09
Length-consistency (FF only)	0.22	0.13
Network (FF only)	0.20	0.12
Two-face and LC both faces	0.12	0.09
Length-consistency (Both faces)	0.19	0.11
Network (Both faces)	0.28	0.12