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Accounting for the spatial observation window in the two-dimensional Fourier transform analysis of shear wave attenuation

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Abstract

Recent measurements of shear wave propagation in viscoelastic materials have been analyzed by constructing the two-dimensional Fourier transform (2DFT) of the shear wave signal and measuring the phase velocity $c(\omega)$ and attenuation $\alpha(\omega)$ from the peak location and full width at half maximum (FWHM) of the 2DFT signal at discrete frequencies. However, when the shear wave is observed over a finite spatial range, the 2DFT signal is a convolution of the true signal and the observation window, and measurements using the FWHM can give biased results. In this study, we describe a method to account for the size of the spatial observation window using a model of the 2DFT signal and a nonlinear, least squares fitting procedure to determine $c(\omega)$ and $\alpha(\omega)$. Results from the analysis of finite element simulation data are in agreement with $c(\omega)$ and $\alpha(\omega)$ calculated from the material parameters used in the simulation. Results obtained in a viscoelastic phantom demonstrate that the measured attenuation is independent of the observation window and are in agreement with measurements of $c(\omega)$ and $\alpha(\omega)$ obtained using the previously described progressive phase and exponential decay analysis.

Keywords

shear wave; phase velocity; attenuation; viscoelasticity*

Introduction

Shear wave elasticity imaging (Sarvazyan, et al., 1998) is a quantitative, non-invasive technique that can be used to measure the shear modulus of materials from the observation of shear wave propagation. In viscoelastic materials, these measurements are complicated because the shear modulus is a complex, frequency dependent quantity $\mu(\omega)$. Fourier transform techniques are required to decompose the shear wave signal into discrete frequency components, and two independent measurements are required at each frequency to determine the real and imaginary parts of $\mu(\omega)$. For example, as given by Rouze, et al.

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(2015), the phase velocity $c(\omega)$ and attenuation $\alpha(\omega)$ can be used to determine the complex wavenumber

$$\kappa(\omega) = k(\omega) - i\alpha(\omega) = \frac{\omega}{c(\omega)} - i\alpha(\omega) \quad (1)$$

and the shear modulus

$$\mu(\omega) = \frac{\rho\omega^2}{\kappa^2(\omega)} = \frac{\rho\omega^2}{(k(\omega) - i\alpha(\omega))^2}. \quad (2)$$

Measurements of $c(\omega)$ and/or $\alpha(\omega)$ in viscoelastic materials have been reported by many authors including Deffieux, et al.(2009), Nenadic, et al.(2017), Nightingale, et al.(2015) and Budelli, et al.(2017). In addition, direct measurements of $\mu(\omega)$ have been reported by several authors including Henni, et al.(2010) and Manduca, et al.(2001). Techniques to measure $c(\omega)$ and $\alpha(\omega)$ can be derived from analytic models of shear wave propagation such as those given by Graff(1991), Nenadic, et al.(2017), and Rouze, et al.(2016). These models describe shear wave propagation by solving the Navier equation of motion in an incompressible material at positions outside a localized, cylindrically symmetric source with infinite axial extent. The model from Rouze, et al.(2016) describes shear wave propagation in a viscoelastic material and gives the spatial-temporal particle velocity signal $v(x, t)$ expressed in the temporal frequency domain as

$$\tilde{v}(x, \omega) = i\omega \frac{\tilde{W}_{ex}(\omega)}{\mu(\omega)} S_0(\omega) K_0(i\kappa x) \quad (3)$$

where the tilde represents the temporal Fourier transform, $W_{ex}(t)$ gives the time dependence of the excitation, $S_0(\omega)$ is a function of the shape and size of the localized excitation, K_0 is a modified Bessel function, and $\kappa(\omega)$ and $\mu(\omega)$ are given by (1) and (2), respectively. The description from Graff(1991) is equivalent to (3) for the case of an elastic material, and the description from Nenadic, et al.(8) is equivalent to (3) for a plane-wave, source-free model.

Nenadic, et al.(2017) have described a method for measuring $c(\omega)$ and $\alpha(\omega)$ by constructing the two-dimensional Fourier transform (2DFT) of a measured shear wave signal. Using (3) and assuming the shear wave signal is measured in a spatial observation window $W_x(x)$, the 2DFT signal $V(k_x, \omega)$ is given by

$$V(k_x, \omega) = i\omega \frac{\tilde{W}_{ex}(\omega)}{\mu(\omega)} S_0(\omega) \int_{-\infty}^{\infty} W_x(x) K_0(i\kappa x) e^{-ik_x x} dx \quad (4)$$

where k_x is the transform wavenumber. To simplify this expression, Nenadic et al.(2017) used an asymptotic expansion of the Bessel function,

$$K_0(i\kappa x) \sim \sqrt{\frac{\pi}{2i\kappa x}} e^{-i\kappa x} = \sqrt{\frac{\pi}{2i\kappa x}} e^{-i\kappa x} e^{-\alpha x}. \quad (5)$$

In addition, a $\sqrt{x}$ weighting factor was applied to the shear wave signal, and the integration in (4) was evaluated by assuming the shear wave signal was observed over the $+x$ axis so that $W_x(x) = 1$ in the range $0 \leq x \leq \infty$. With these assumptions, the magnitude of the 2DFT signal (4) is given by

$$\begin{aligned} |V(k_x, \omega)| &= |i\omega \frac{\tilde{W}(\omega)}{\mu(\omega)} S_0(\omega) \int_0^\infty \sqrt{x} \sqrt{\frac{\pi}{2i\kappa x}} e^{-i\kappa x} e^{-\alpha x} e^{-ik_x x} dx| \\ &= A(\omega) \frac{1}{\sqrt{(k(\omega) + k_x)^2 + \alpha^2(\omega)}} \end{aligned} \quad (6)$$

where the factor $A(\omega)$ includes terms that are independent of the transform wavenumber k_x . Then, as described by Nenadic, et al.(2017) and Rouze, et al.(2015), $k(\omega)$ and $\alpha(\omega)$ can be determined from the k_x -dependent shape of the 2DFT signal at discrete frequencies. For example, the peak value of the signal (6) occurs at $k_{x_{max}} = -k(\omega)$, and the phase velocity is given by

$$c(\omega) = \frac{\omega}{k(\omega)} = -\frac{\omega}{k_{x_{max}}}. \quad (7)$$

Similarly, the 2DFT signal (6) is one-half its maximum value at $k_x = -k(\omega) \pm \sqrt{3}\alpha(\omega)$ and the attenuation is proportional to the full width at half maximum (FWHM) of the 2DFT signal,

$$\alpha(\omega) = \frac{1}{2\sqrt{3}} \text{FWHM}(k_x). \quad (8)$$

One difficulty in using (8) to measure the attenuation is that shear wave signals are typically measured over a finite spatial range, and the measured 2DFT signal is a convolution of the true signal and the observation window. For example, a rectangular observation window of length L has a Fourier transform $\text{sinc}(k_x L/2)$ with half-maximum at $k_x L/2 = \pm 1.8955$ rad, and a corresponding FWHM of

$$\text{FWHM}_{\text{window}} = \frac{7.582 \text{ rad}}{L}. \quad (9)$$

This contribution to the FWHM in (8) can be a significant source of bias, for example, in elastic materials with zero attenuation or in viscoelastic materials where material models predict that the attenuation approaches zero in the limit of zero frequency.

In this study, we describe a finite window, two-dimensional Fourier transform (fw2DFT) method to account for the finite size of the spatial window used in the 2DFT analysis of shear wave propagation. Instead of approximating the calculation of the spatial Fourier transform as in (6), our procedure uses (4) with the specific spatial window $W_x(x)$ used in experimental measurements. Values of $k(\omega)$ and $\alpha(\omega)$ are determined at discrete frequencies using a nonlinear least squares fitting procedure to model the k_x -dependent shape of an experimentally measured 2DFT signal. The procedure is applied to data from finite element (FE) simulations and experimental measurements in a viscoelastic phantom. Results demonstrate that the measured attenuation is independent of the spatial window size and agrees with values determined from simulation parameters and with estimates obtained by assessing the exponential decay of the shear wave signal.

Materials and Methods

Finite Element Simulations

Previously-validated FE models were used to simulate the dynamic response of a viscoelastic solid to impulsive, acoustic radiation force excitations as described by Palmeri, et al.(2005) and Palmeri, et al.(2017) The three-dimensional response of the solid was solved through the balance of linear momentum using LS-DYNA3D (Livermore Software Technology Corp., Livermore, CA, USA) with the *MAT_006 material model that describes the shear relaxation behavior as

$$G(t) = G_{\infty} + (G_0 - G_{\infty})e^{-\beta t} \quad (10)$$

where $G_0 = 40$ kPa and $G_{\infty} = 6$ kPa are short-time and long-time (infinite) shear moduli, respectively, and $\beta = 9714.3 \text{ s}^{-1}$ is the decay constant. This material model is equivalent to a standard linear model of viscoelasticity represented by a spring with stiffness $\mu_1 = G_{\infty}$ in parallel with a series combination of spring with stiffness $\mu_2 = G_0 - G_{\infty}$ and dashpot with viscosity $\eta = (G_0 - G_{\infty})/\beta$. The values of $\mu(\omega)$, $\alpha(\omega)$, and $\alpha(\omega)$ were calculated from μ_1 , μ_2 , and η as described by Rouze, et al.(2015) The excitation force used in the FE simulations was modeled as a cylindrically symmetric Gaussian with widths of $\text{FWHM}_x = \text{FWHM}_y = 1.4$ mm and a height extending the length of the mesh. The excitation duration was 180 μs . The simulations were performed using quarter symmetry with lateral $\times$ elevational $\times$ axial mesh dimensions of 40 mm $\times$ 40 mm $\times$ 120 mm and voxel sizes of 0.33 mm $\times$ 0.33 mm $\times$ 0.25 mm. Perfect matching layers were used on the lateral-axial and elevation-axial boundaries to simulate an infinite volume. Shear wave signals were weighted using a $\sqrt{x}$ weighting factor to account for geometric spreading of a cylindrical source as in (5), and 2DFT signals were calculated using a discrete Fourier transformation. Data were analyzed using two spatial observation windows with lateral ranges of 4 $\times$ 10 mm and 4 $\times$ 16 mm. The minimum lateral position of 4 mm used for these ranges was chosen so that the shear wave signal was observed outside of the localized source as required in the model of Rouze, et al.(2016). The 6 mm and 12 mm ranges are typical of the spatial windows used in experimental measurements.

Phantom data

Shear wave data were collected in a viscoelastic phantom (model E2297-A3) from CIRS, Inc. (Norfolk, VA, USA) using a Verasonics™ Vantage Scanner (Verasonics, Inc., Kirkland, WA, USA) and Philips™ C5-2 curvilinear array transducer (Philips Medical Systems, Bothel, WA, USA) as described by Deng, et al.(2017). Briefly, the scanner applied an acoustic radiation force impulse (ARFI) excitation using a 2.36 MHz, 200 μ s pulse with lateral F-number of F/2.0 focused at a depth of 50 mm. Following the excitation, a series of diverging wave transmit-receive tracking signals were applied with a pulse repetition frequency of 5 kHz and a total acquisition time of 40 ms. The particle velocity signal was calculated using the Kasai(1985) algorithm with a $1.5\text{-}\lambda$ kernel and progressive referencing between adjacent tracking signals. These results were averaged over a 5 mm depth-of-field centered at the focal depth and were filtered using a low pass filter with a cutoff of 1000 Hz. The shear wave signals were weighted using a $\sqrt{x}$ weighting factor, and 2DFT signals were calculated using a discrete Fourier transformation. As with the FE simulations, data were analyzed using two spatial observation windows with lateral ranges of 4×10 mm and 4×16 mm.

fw2DFT analysis of measured data

Measured 2DFT signals $V_{\text{meas}}(k_x, \omega)$ were analyzed by modeling the shape of the k_x -dependent 2DFT signal, including effects of the finite observation window, and using a nonlinear least squares fitting procedure to determine the values of $k(\omega)$ and $\alpha(\omega)$ at each frequency considered. We refer to this procedure as the finite window, two-dimensional Fourier transform (fw2DFT) procedure. The functions $f(k, \alpha; k_x)$ used in the fitting procedure were determined by omitting multiplicative factors in the analytic model (4) and including a $\sqrt{x}$ weighting factor to agree with the measured 2DFT signal,

$$f(k, \alpha, k_x) = \left| \int_a^b \sqrt{x} K_0(i\kappa x) e^{-ik_x x} dx \right| \quad (11)$$

where $W_X(x) = 1$ in the range $a \leq x \leq b$. Each calculation of $f(k, \alpha; k_x)$ was scaled by a factor $\hat{A}$ determined by minimizing the sum of squared differences between the measured signal and the scaled fitting function,

$$\begin{aligned} \hat{A} &= \underset{A}{\operatorname{argmin}} \sum_i (|V_{\text{meas}}(k_{x_i}, \omega)| - A f(k, \alpha; k_{x_i}))^2 \\ &= \frac{\sum_i |V_{\text{meas}}(k_{x_i}, \omega)| f(k, \alpha; k_{x_i})}{\sum_i f^2(k, \alpha; k_{x_i})}. \end{aligned} \quad (12)$$

This scaling accounted for the normalization of the measured 2DFT signal and the multiplicative factors from (4) that are neglected in (11). Thus, it was necessary to adjust only the parameters k and α in the least squares fitting procedure, and characterization of the excitation force is not required. The integrals in (11) were evaluated numerically using Simpson's rule with a step size of 0.25 mm. Fits were performed using 2DFT data in the range $-400 \text{ cyc/m} \leq k_x/2\pi \leq 200 \text{ cyc/m}$. In practice, fitting functions were precomputed

using values of k and α with $0 \leq k/2\pi \leq 1000$ cyc/m and $0 \leq \alpha \leq 1000$ Np/m. The fitting procedure was performed using a coarse-to-fine grid search to locate the values of k and α with the minimum RMS difference between the scaled fitting function and the 2DFT signal. This procedure required a (nonoptimized) computation time of roughly 4 s for fits at 40 discrete frequencies. All calculations were performed using MATLAB (The MathWorks, Natick MA, USA).

Results

Figure 1 shows results obtained using FE simulation of shear wave signals in a viscoelastic material for signals observed using a spatial window with 4 mm $\times$ 10 mm (top row), and in a spatial window with 4 mm $\times$ 16 mm (bottom row). The left column shows the spatial-temporal shear wave particle velocity signal $v(x,t)$. The second column shows the normalized magnitude of the 2DFT signal, $|V(k_x, \omega)|$, plotted as a function of the spatial frequency $k_x/2\pi$ and temporal frequency $f = \omega/2\pi$. The three columns on the right show plots of the 2DFT signal (black lines) from the experimental measurements as a function of spatial frequency at temporal frequencies of 100 Hz, 200 Hz, and 400 Hz, respectively. Also shown on these plots are results (red points) of the fw2DFT procedure. Values of $k(\omega)$, $c(\omega)$, and $\alpha(\omega)$ determined in this analysis are given in Table 1. The signs of the coordinates in these plots are determined by sign convention used in the definition of the Fourier transform in (4). This convention agrees with the Matlab convention and the convention used by Rouze, et al.(2015).

Figure 2 shows results for the phase velocity (left) and attenuation (right) determined in a frequency range 40 Hz $\leq f \leq$ 500 Hz for both the 4–10 mm (top row) and 4–16 mm (bottom row) observation windows considered in Fig. 1. Results obtained using the fw2DFT analysis procedure are plotted as red lines. Results obtained using the original peak location (7) and FWHM (8) analyses are shown as dashed, blue lines. Results obtained with shear wave spectroscopy described below are shown as dashed green lines. The true values of phase velocity and attenuation determined from the material properties used in the simulation are shown as black lines. We observe that all measurements of the phase velocity are in good agreement with the true velocity. For the attenuation measurements, the fw2DFT and shear wave spectroscopy results are in good agreement with the true values. However, we observe that the measurements obtained using the FWHM of the 2DFT signal which assume an infinite spatial window approach constant values of roughly 180 Np/m and 360 Np/m at low frequencies. These values are consistent with values determined using (9) for the 6 mm and 12 mm window sizes without any additional contributions to the width from attenuation.

The results of the fw2DFT procedure shown in Fig. 2 can also be compared to shear wave spectroscopy measurements of the phase velocity as described by Deffieux, et al.(2009) and attenuation from the decay of the shear wave signal as described by Budelli, et al.(2017). Using the asymptotic form of the Bessel function in (5) and including a $\sqrt{x}$ weighting factor, the temporal Fourier transform shear wave signal has the form

$$\sqrt{x}\tilde{v}(x, \omega) \sim e^{-ik(\omega)x} e^{-\alpha(\omega)x} \quad (13)$$

Then, the phase velocity $c(\omega) = \omega/k(\omega)$ is determined by the linear variation of the signal phase with position and the attenuation $\alpha(\omega)$ is determined by the exponential decay of the signal magnitude with position. Results using this procedure are also plotted in Fig. 2 as dashed, green lines. We observe that these results are in good agreement with the fw2DFT analysis and the true values determined from the material properties used in the FE simulation.

Figures 3 and 4 show sample signals and fw2DFT results from observations of shear wave propagation in a viscoelastic phantom. Figure 3 shows the shear wave and 2DFT signals using 4–10 mm (top row) and 4–16 mm (bottom row). Values of $k(\omega)$, $c(\omega)$, and $\alpha(\omega)$ determined by the fw2DFT fitting procedure are given in Table 1. Fig. 4 shows the frequency dependent phase velocity (left) and attenuation (right) determined using the fw2DFT analysis procedure for frequencies in the range 40 Hz $\leq f \leq$ 500 Hz. We observe that both of these results are independent of the spatial window size, and that the attenuation measurements are not limited by the window size as with the results obtained using the 2DFT width (8). Figure 4 also shows the results from the shear wave spectroscopy analysis procedure using the phase and magnitude in (13) for the same observation windows. We observe relatively good agreement between the two methods with possible differences between the attenuation measurements at frequencies greater than roughly 250 Hz.

Discussion

Measurement of the shear modulus $\mu(\omega)$ in a viscoelastic material is difficult because two measurements are required at each frequency to determine the real and imaginary parts of $\mu(\omega)$ without assuming a specific material model. Typically, the phase velocity $c(\omega) = \omega/k(\omega)$ and attenuation $\alpha(\omega)$ are measured, and $\mu(\omega)$ is given by (2). The fw2DFT method described in this study presents a method to measure $c(\omega)$ and $\alpha(\omega)$ by constructing the 2DFT of a propagating shear wave signal and analyzing the k_x -dependent shape of the 2DFT signal at discrete frequencies. Compared to previously described methods that measure $c(\omega)$ using the 2DFT peak position analysis (7) and $\alpha(\omega)$ using the FWHM analysis (8), the fw2DFT analysis method accounts for the finite size of the spatial observation window, and as shown in Fig. 2, gives measurements of $\alpha(\omega)$ that are independent of the window size and agree with the true values determined from the material properties used in the FE simulation.

Of course, the measurement of attenuation using the FWHM analysis (8) is valid as long as the shear wave observation window is large, and the contribution (9) to the FWHM of the 2DFT signal from the observation window is small. However, it is not always possible to use a large observation window due to limitations of the shear wave imaging system, or when measurements in small or heterogeneous structures are desired.

The results of the fw2DFT analysis for the FE simulation shear wave signals show nearly perfect results of the fitting procedure in Fig. 1 and excellent agreement of $c(\omega)$ and $\alpha(\omega)$ with the true results determined from the FE material properties for both of the window sizes considered. Thus, we conclude that the fw2DFT analysis procedure can account for the finite observation window used in experimental measurements.

Figure 4 shows relatively good, but not perfect, agreement between the results obtained using the two window sizes for experimental measurements in a viscoelastic phantom. One possible source of the differences can be seen in the fitted data in Fig. 3 where it is observed that the experimental 2DFT signals show asymmetries around the peak spatial frequency that are more pronounced at higher spatial frequencies. These asymmetries are not predicted by the shear wave model (4) and thus will not be present in the fitted functions, and are not observed in the FE simulation signals shown in Fig. 1. Such asymmetries are typically observed in experimental phantom measurements and could be introduced by spatial and temporal noise associated with ultrasonic motion tracking (10) and/or the validity of the assumptions in the shear wave model (3). Further investigations are needed to understand the origin and effects of these artifacts.

Finally, it is not necessary to perform an independent measurement of the attenuation if a specific material model (such as the Voigt or Standard Linear material model) is being considered and dispersion of the phase velocity allows each of the model parameters to be determined. However, as shown in the phase velocity measurements in Fig. 4, these measurements are sensitive to noise in the experimentally measured 2DFT signal which will contribute to noise in the model parameters. Adding independent measurements of the attenuation and modeling both the frequency dependent phase velocity and attenuation will improve the estimation of these model parameters.

Conclusions

This paper describes the finite window, two-dimensional Fourier transform (fw2DFT) method to measure the frequency dependent phase velocity $c(\omega)$ and attenuation $\alpha(\omega)$ from the observation of shear wave propagation in a finite spatial window. Compared to previously described 2DFT analysis methods, the fw2DFT method accounts for the finite size of the spatial window used to observe the wave by including the observation window in a model of the 2DFT signal and using a nonlinear least squares fitting procedure to analyze the shape of the 2DFT signal at discrete frequencies. Results from the analysis of finite element simulation data show that $c(\omega)$ and $\alpha(\omega)$ determined using the fw2DFT analysis procedure are in good agreement with true values determined from the material properties used in the simulation. Results measured in a viscoelastic phantom demonstrate that $c(\omega)$ and $\alpha(\omega)$ are independent of the window size and are in agreement with the results from previously described techniques to measure the phase velocity and attenuation.

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Figure 1.

Results from a FE simulation of shear wave propagation in a viscoelastic material for signals observed in a spatial window with 4 mm $\times$ 10 mm (top row), and a spatial window with 4 mm $\times$ 16 mm (bottom row). The first column shows the spatial-temporal shear wave signal and the second column shows the normalized magnitude of the corresponding 2DFT signal. The three columns on the right show the excellent agreement between the 2DFT signal (black line) and least squares fit (red points) at frequencies of 100 Hz, 200 Hz, and 400 Hz, respectively.

Figure 2.

Frequency dependent phase velocity (left) and attenuation (right) from FE simulations of shear wave propagation in a viscoelastic material for spatial observation windows with 4 mm $\times$ 10 mm (top row) and 4 mm $\times$ 16 mm (bottom row). Results obtained using the fw2DFT analysis are plotted as red lines. Results obtained using the non-windowed 2DFT peak location (7) and FWHM (8) analyses are shown as dashed blue lines. Results obtained using the shear wave spectroscopy analysis using the phase and magnitude in (13) are shown

as dashed green lines. True results calculated from the material properties used in the FE simulation are shown as black lines.

Figure 3.

Results from analysis of shear wave propagation in a viscoelastic phantom that was observed in a spatial window with 4 mm $\times$ 10 mm (top row), and a spatial window with 4 mm $\times$ 16 mm (bottom row). The first column shows the spatial-temporal shear wave signal and the second column shows the normalized magnitude of the corresponding 2DFT signal. The three columns on the right show the experimental 2DFT signal (black line) and least squares fit (red points) at frequencies of 100 Hz, 200 Hz, and 400 Hz, respectively.

Figure 4.

Frequency dependent phase velocity (left) and attenuation (right) measurements in a viscoelastic phantom. Results are shown for the fw2DFT analysis method using 4–10 mm (black lines) and 4–16 mm (red lines) spatial observation windows. Also shown are results obtained from shear wave spectroscopy analysis using the phase and magnitude in (13) for the case of 4–10 mm (blue lines) and 4–16 mm (green lines) spatial observation windows.

Table 1

Values of k , c , and α determined using the fw2DFT procedure for results shown in Figs. 1 and 3.

window	f (Hz)	k (rad/m)	c (m/s)	α (Np/m)
Figure 1				
4–10 mm	100	247	2.55	60
4–10 mm	200	439	2.86	125
4–10 mm	400	687	3.66	280
4–16 mm	100	241	2.60	40
4–16 mm	200	427	2.94	120
4–16 mm	400	680	3.70	290
Figure 3				
4–10 mm	100	339	1.85	45
4–10 mm	200	641	1.96	115
4–10 mm	400	1194	2.10	260
4–16 mm	100	332	1.89	60
4–16 mm	200	641	1.96	115
4–16 mm	400	1181	2.13	235