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Physiological Motion Modeling for Organ-Mounted Robots

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Abstract

Background—Organ-mounted robots passively compensate heartbeat and respiratory motion. In model-guided procedures, this motion can be a significant source of information that can be used to aid in localization or to add dynamic information to static preoperative maps.

Methods—Models for estimating periodic motion are proposed for both position and orientation. These models are then tested on animal data and optimal orders are identified. Finally, methods for online identification are demonstrated.

Results—Models using exponential coordinates and Euler-angle parameterizations are as accurate as models using quaternion representations, yet require a quarter fewer parameters. Models which incorporate more than 4 cardiac or 3 respiration harmonics are no more accurate. Finally, online methods estimate model parameters as accurately as offline methods within 3 respiration cycles.

Conclusions—These methods provide a complete framework for accurately modelling the periodic deformation of points anywhere on the surface of the heart in a closed chest.

Introduction

Organ-mounted robots represent an alternative approach for minimally invasive beating-heart surgery which avoids the requirement of high dexterity and actuation effort that characterizes other surgical robotic systems in favor of miniature robots that are not mechanically grounded to the world outside the patient and that adhere directly to the anatomical site using vacuum pressure (1–3). By adhering directly to the operating site, these robots provide a platform approaching zero relative motion with no actuation effort and provide a stable platform from which interventions may be launched. Due to their small size, they may also be introduced to the heart through a single subxiphoid port, obviating differential ventilation and lung deflation (4).

Numerous interventions are possible using these devices, shown in Fig. 1, including cell transplantation (5), gene therapy for angiogenesis (6), intrapericardial drug delivery (7),

epicardial electrode placement for resynchronization (8), epicardial atrial ablation (9,10), and ventricle-to-coronary artery bypass (VCAB) (11). Interventions already demonstrated preclinically with these robots include myocardial injection (1,12–14), epicardial pacing (15), and epicardial ablation (16).

The capabilities of these robots, however, do come at a cost. The subxiphoid approach used to access the heart with these robots restricts line of sight to the operating arena, requiring-model guided techniques to navigate the robot. In this paradigm, we know that we are moving in unison with the heart, but our exact location on the heart must be determined. While a naive approach to registration may filter out the periodic motion of the heart, this motion is in fact a significant source of information and must be considered in order to achieve optimal performance.

This work is the first stage in addressing the registration and localization of organ-mounted robots on the beating heart. Although the approach we have taken to beating-heart surgery is vastly different, the question we must first address is the same that has been addressed by some traditional approaches: how points move on the surface of the beating heart. The nature of our robots, however, requires addressing this question in ways which have yet to be explored.

This paper presents methods for modelling the periodic motion of robots crawling on the beating heart in $SE(3)$ (the space of motions in six degrees of freedom). These models, which capture the periodic translational and rotational deformation of points on the surface of the heart, enable predictions of the future motion of the observed points that can be used to localize the robot in preoperative models of the static heart. The work takes as its starting point an amplitude-modulated (AM) Fourier series model, as in Bachta et al. (17), and through exploration of multiple representations of rotational motion, with their corresponding distance metrics, accomplishes modeling of the rotational periodic motion in the same framework as translational motion, with optimization using least squares. The work then identifies the best model for representation of rotation in this application, determines the optimal model order to limit overfitting and model size, and demonstrates online estimation of the model parameters. This work is the first to use data from a closed-chest procedure, making the work of greater relevance for clinical applications. The closed-chest procedure was enabled by the use of a subxiphoid approach, which in turn was enabled by the highly flexible miniature organ-mounted robots used, and by the use of 6-degree-of-freedom (6-DOF) microBird magnetic tracking sensors (Ascension Technologies, Burlington, VT). This work is also the first to address the orientation of the robot. The inclusion of rotational motion is potentially a significant source of information, providing information on how the shape of the heart is changing locally. Also, as these robots are mobile, data from the entire heart must be considered, as opposed to just a few points as in previous literature.

Related work

The vast majority of work on the modeling of the motion of the freely beating heart is in the area of active motion cancellation (AMC). AMC aims to provide a virtual stable operating platform to the surgeon by automatically moving robotically controlled tools in unison with

the surface of the beating heart. Due to the large displacements and high velocities of the heart (18), feedforward controls are often implemented which rely on predictions of the future position of the surgical site. A wide variety of methods and measurement techniques has been used to provide these predictions, from which relevant work is discussed below. Further information can be found in reviews of these methods (17,19).

Several methods for prediction of motion of the heart do not attempt to fit models to the motion. Copies of previous measurements have been used for prediction (20–22). Autoregressive models, in which future states are linear combinations of previous measurements, similarly eschew directly modeling the motion (23–27).

Approaches which attempt to fit predictive models to the periodic motion of the heart include the use of splines (28), but the vast majority involve frequency-based models. Frequency-based models used have included single Fourier series (29,30), dual Fourier series (31–35), discrete Fourier transforms (36), amplitude-modulated Fourier series (17), and dual Fourier series with a quadratic coupling term (37). Transformations were used to model the point motion of the heart in simulation in (38). This work assumed that measurements of diaphragm position and cardiac phase were available and used a convolution of triangular basis functions to model the motion due to both heart contraction and respiration. This is the only work which attempted to model the orientation of the heart surface, but the technique was not demonstrated on real data, nor were measurement modalities presented which enable orientation measurements.

Measurement modalities used in the previous works include sonomicrometry (22–27,37,39), stereo vision (20,32–35), monocular vision (17,28,36), fiber optic laser (31), and ultrasound (29,30). Each of these measurement modalities, excluding ultrasound, requires direct access to the heart through a sternotomy or thoracotomy, either to place sonomicrometry crystals or for direct line of sight for visual methods. While endoscopes may be used for imaging in minimally invasive procedures, the cramped space around the heart limits the ability of the camera to provide satisfactory views without requiring large access ports.

Materials and Methods

Animal Data

A. Collection—Data used in this work were collected during experiments in the porcine model *in vivo* under an IACUC board-approved protocol. Access to the intrapericardial space was achieved through a subxiphoid skin incision and an incision in the pericardium near the apex of the heart. Lamprey, Fig. 1(a), was introduced through these incisions and manually guided over the surface of the heart using preoperatively generated models for visual feedback. At approximately 100 points distributed over the surface of the heart, the robot was attached using vacuum pressure, and data were logged for 30 seconds at each point. A diagram of the experimental setup is shown in Fig. 2.

Measurements, sampled at 100 Hz, include the 6-DOF pose of the robot and a reference probe placed on the chest wall, flow rates of forced inspiration and expiration, and the ECG signal. Sensors used to provide these measurements were: (1) microBird magnetic tracking

sensor (Ascension Technologies, Burlington, VT), (2) two non-contact MEMS airflow sensors (Omron D6F-50A-000) placed in line with the inspiration and expiration lines of the ventilator, and (3) 0–1 V analog output from a LIFEPAK 12 defibrillator/monitor (Medtronic, Minneapolis, MN) using a Physio-Control LIFEPAK 12/15 Analog ECG Output Cable (Part #: 3010484-00).

B. Data Processing—These raw measurement signals were then processed to yield the signals used in the remainder of this paper. The pose of the robot in the measurement frame, T_H^W , and the pose of the reference probe in the measurement frame, T_P^W , yield the pose of the robot in the reference frame, $T_H^P = [T_P^W]^{-1} T_H^W$. Time traces of these poses are shown in Fig. 3(a)–(b), where unit quaternions are used for orientation. While it is assumed that the patient and measurement frame are fixed, working with the relative poses ensures that motion of the patient or measurement frame does not corrupt the data. Cardiac phase, ϕ_C , was extracted from the raw ECG signal by first using a Pan-Tompkins QRS detection algorithm (40) to detect the QRS complexes and then linearly interpolating between them. Cardiac phase is then defined as

$$\phi_C = 2\pi \frac{t(k) - t_{QRS}}{t_{QRS+1} - t_{QRS}} \quad (1)$$

where $t(k)$ is the current time, t_{QRS} is the time of the last QRS detection, and t_{QRS+1} is the time of the next QRS detection. Respiration phase, ϕ_R , is similarly extracted from the flow rate measurements by first detecting the start of inspiration using a simple threshold on the flow sensor output and linearly interpolating between successive events. Respiration phase is then defined as

$$\phi_R = 2\pi \frac{t(k) - t_{INSP}}{t_{INSP+1} - t_{INSP}} \quad (2)$$

where $t(k)$ is the current time, t_{INSP} is the time of the last detected start of inspiration, and t_{INSP+1} is the time of the next inspiration detection. These phase values, ϕ_C and ϕ_R , are periodic signals that characterize the proportion of the physiological cycle completed and are used as the basis for the periodic models of motion presented. Mean heart and respiration rates for this experiment were 1.34 Hz (80 bpm) and 0.2 Hz (12 bpm) respectively.

C. Analysis—As previously noted by many researchers, the motion of the heart is the combination of a low-frequency component due to respiration, and a high-frequency component due to heartbeat. These components can be seen clearly in the frequency response plots of both displacement and rotation in Fig. 3(c), where peaks due to respiratory motion are denoted with blue dotted lines from the primary frequency through the fifth harmonic, and peaks due to the contraction of the heart are denoted with red dotted lines. A few prior works have also noted that significant peaks occur at integer combinations of the cardiac and respiration frequencies ($j\omega_C + j\omega_R$) (17). These peaks, denoted by green dotted

lines in Fig. 3(c), capture the effect of lung volume on the contraction of the heart, namely that when the lungs are full the motion of the heart is constricted.

Parameterizations of 3-D Rotation

A major contribution of the work presented in this paper is the modeling of periodic rotational motion. This section summarizes the parameterizations considered in the remainder of the paper as well as distance metrics in this space. A more comprehensive exploration of these topics may be found in (41). The space of rotations in 3 dimensions is given by:

$$SO(3) = \{\mathbf{R} \in \mathbb{R}^{3 \times 3} : \mathbf{R}\mathbf{R}^T = \mathbf{I}, \det \mathbf{R} = 1\} \quad (3)$$

Rotation matrices, $\mathbf{R}$, use nine scalars to describe three degrees of freedom, requiring the enforcement of six constraints. The standard distance metric in this space, the geodesic distance,

$$d_G(\mathbf{R}_1, \mathbf{R}_2) = \frac{1}{\sqrt{2}} \|\log(\mathbf{R}_1^{-1} \mathbf{R}_2)\|_F \quad (4)$$

(where $\|\cdot\|_F$ is the Frobenius norm), while simply being the magnitude of the rotation angle between the two matrices, does not lend itself to simple optimization techniques. The following are three alternative parameterizations we will consider.

A. Unit Quaternions—Unit quaternions are the extension of the complex numbers to rotations. The quaternion corresponding to a rotation angle, θ , about an axis, $\hat{\mathbf{u}}$, is given by

$$q = \begin{bmatrix} q_w \\ q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} q_w \\ \mathbf{q}_\nu \end{bmatrix} = \begin{bmatrix} \cos \frac{\theta}{2} \\ \hat{\mathbf{u}} \sin \frac{\theta}{2} \end{bmatrix} \quad (5)$$

where $\|\mathbf{q}\| = 1$. The rotation matrix corresponding to the quaternion rotation, $\mathbf{q}$, is given by:

$$R(q) = \mathbf{I} + 2q_w \hat{\mathbf{q}}_\nu + 2\hat{\mathbf{q}}_\nu^2 \quad (6)$$

where the operator $\hat{\cdot}$ maps vector $\mathbf{v} \in \mathbb{R}^3$ to $\mathfrak{so}(3)$, the Lie algebra of $SO(3)$ (42).

$$\hat{\mathbf{v}} = \begin{bmatrix} 0 & -\nu_z & \nu_y \\ \nu_z & 0 & -\nu_x \\ -\nu_y & \nu_x & 0 \end{bmatrix} \quad (7)$$

Once again the unit norm constraint of this representation requires four parameters to describe three degrees of freedom. Quaternions are more space-efficient than rotation matrices and allow directly for manipulation using the quaternion multiplication operator. The geodesic distance between quaternions takes a much simpler form than for rotation matrices.

$$d_G(\mathbf{q}_1, \mathbf{q}_2) = 2 \cos^{-1} |\mathbf{q}_1 \cdot \mathbf{q}_2| \quad (8)$$

An alternative distance metric on unit quaternions which is functionally and boundedly equivalent to the geodesic distance is the Euclidean distance (41). Because quaternions double-cover the space of rotations, $-\mathbf{q}$ and $\mathbf{q}$ represent equivalent rotations, which must be accounted for in the metric.

$$d_q(\mathbf{q}_1, \mathbf{q}_2) = \min(\|\mathbf{q}_1 - \mathbf{q}_2\|, \|\mathbf{q}_1 + \mathbf{q}_2\|) \quad (9)$$

B. Exponential Coordinates—In exponential coordinates, a rotation through an angle, θ , about an axis, $\hat{\mathbf{u}}$, is given by

$$\omega = \theta \hat{\mathbf{u}} \quad (10)$$

The skew-symmetric matrix, $\hat{\omega} \in \mathfrak{so}(3)$, is the Lie algebra of the rotation group, and the rotation matrix corresponding to the rotation ω is found using the exponential map.

$$R(\omega) = e^{\hat{\omega}} = \mathbf{I} + \frac{\hat{\omega}}{\|\omega\|} \sin\|\omega\| + \frac{\hat{\omega}^2}{\|\omega\|^2} (1 - \cos\|\omega\|) \quad (11)$$

Exponential coordinates are a minimal state representation, and as such have discontinuities in the space. For this work we choose to center the space at 0 and place this discontinuity at $\pm\pi$. Also, the Euclidean distance in this space is not functionally equivalent to the geodesic distance.

C. Euler Angles—The final parameterization considered is the Euler angle representation. This family of rotations is parameterized as three successive rotations about a set of three axes. There are 24 different conventions of this form due to the noncommutativity of rotations, an exhaustive list of which can be found in (43).

This work uses Z - Y - X , or yaw-pitch-roll, Euler angles. In this parameterization the relative pose of two frames is defined by rotating an angle, α , about the z -axis, then rotating an angle, β , about the rotated y -axis, and finally rotating an angle, γ , about the rotated x -axis. The rotation matrix corresponding to this rotation is constructed as follows.

$$\begin{aligned}
 R(\alpha, \beta, \gamma) &= R(\alpha)R(\beta)R(\gamma) = \begin{bmatrix} C_\alpha & S_\alpha & 0 \\ -S_\alpha & C_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_\beta & 0 & S_\beta \\ 0 & 1 & 0 \\ -S_\beta & 0 & C_\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\gamma & -S_\gamma \\ 0 & S_\gamma & C_\gamma \end{bmatrix} \\
 &= \begin{bmatrix} C_\alpha C_\beta & C_\alpha S_\beta S_\gamma - S_\alpha C_\gamma & C_\alpha S_\beta C_\gamma + S_\alpha S_\gamma \\ S_\alpha C_\beta & S_\alpha S_\beta S_\gamma + C_\alpha C_\gamma & S_\alpha S_\beta C_\gamma - C_\alpha S_\gamma \\ -S_\beta & C_\beta S_\gamma & C_\beta C_\gamma \end{bmatrix} \quad (12)
 \end{aligned}$$

Again, as this is a minimal state representation, it possesses drawbacks which must be managed, including gimbal lock and non-uniqueness of rotations. Also, the Euclidian distance in this space is not functionally equivalent to the geodesic distance.

D. Converting Between Parameterizations—Converting between representations is necessary throughout this work. We will use the following notation to indicate mapping the representation of a rotation from parameterization a to b .

$$\mathcal{M}_a^b(): a \mapsto b \quad (13)$$

The mappings from each of the three representations to rotation matrices are given by Eqs. (6,11,12).

Periodic Motion Modeling in $SE(3)$

A. Amplitude-Modulated Fourier Series—The amplitude-modulated (AM) Fourier series model used in this work, introduced by Bachta et al. (17), is an extension to dual Fourier series models for cardiac Cartesian point motion, which directly accounts for the coupling between cardiac and respiration motion. In this framework, the periodic Cartesian motion of a point, $\mathbf{t}(\phi_C, \phi_R)$, is modeled as the sum of a constant offset term, $\mathbf{t}_0$, a low-frequency respiratory motion component, $\mathbf{t}_R$, a higher-frequency heartbeat motion component, $\mathbf{t}_C$, and a coupling term, $\mathbf{t}_{CR}$. This coupling term directly models the effect lung volume has on cardiac motion.

$$\mathbf{t}(\phi_C, \phi_R) = \mathbf{t}_0 + \mathbf{t}_R(\phi_R) + \mathbf{t}_C(\phi_C) + \mathbf{t}_{CR}(\phi_C, \phi_R) \quad (14)$$

$$\mathbf{t}_R = \sum_{i=1}^{H_R} \mathbf{a}_i \sin(i\phi_R) + \mathbf{b}_i \cos(i\phi_R) \quad (15)$$

$$\mathbf{t}_C = \sum_{i=1}^{H_C} \mathbf{c}_i \sin(i\phi_C) + \mathbf{d}_i \cos(i\phi_C) \quad (16)$$

$$\mathbf{t}_{CR} = \sum_{i=1}^{H_{MC}} \sum_{j=1}^{H_{MR}} \mathbf{e}_{ij} \sin(i\phi_C - j\phi_R) + \mathbf{f}_{ij} \cos(i\phi_C - j\phi_R) + \mathbf{g}_{ij} \sin(i\phi_C + j\phi_R) + \mathbf{h}_{ij} \cos(i\phi_C + j\phi_R) \quad (17)$$

where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$, $\mathbf{f}$, $\mathbf{g}$, and $\mathbf{h}$ are vectors of Fourier series coefficients, and H_R , H_C , H_{MC} , and H_{MR} are the number of respiration, cardiac, and coupled cardiac and respiratory harmonics, respectively. The number of parameters in this model per degree of freedom of the signal is a function of the number of harmonics used.

$$N = 1 + 2H_R + 2H_C + 4H_{MC}H_{MR} \quad (18)$$

This model possesses several attributes that warrant its use, aside from being shown to outperform other approaches (17). First, it directly models the coupling between cardiac and respiration motion, which has significant effect in closed-chest procedures. Also, as we have measurements for both cardiac and respiration phases, the model is linear, can be expressed in matrix form as follows, and can be easily solved using a least-squares approach.

$$\mathbf{t}(\phi_C, \phi_R) = \mathbf{W}^T \phi \quad (19)$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{t}_0^T \\ \begin{bmatrix} \mathbf{a}_i^T \\ \mathbf{b}_i^T \end{bmatrix}_{i=1:H_R} \\ \begin{bmatrix} \mathbf{c}_i^T \\ \mathbf{d}_i^T \end{bmatrix}_{i=1:H_C} \\ \begin{bmatrix} \mathbf{e}_{ij}^T \\ \mathbf{f}_{ij}^T \\ \mathbf{g}_{ij}^T \\ \mathbf{h}_{ij}^T \end{bmatrix}_{\substack{i=1:H_C \\ j=1:H_R}} \end{bmatrix}, \phi = \begin{bmatrix} 1 \\ \begin{bmatrix} \sin(i\phi_R) \\ \cos(i\phi_R) \end{bmatrix}_{i=1:H_R} \\ \begin{bmatrix} \sin(i\phi_C) \\ \cos(i\phi_C) \end{bmatrix}_{i=1:H_C} \\ \begin{bmatrix} \sin(i\phi_C - j\phi_R) \\ \cos(i\phi_C - j\phi_R) \\ \sin(i\phi_C + j\phi_R) \\ \cos(i\phi_C + j\phi_R) \end{bmatrix}_{\substack{i=1:H_C \\ j=1:H_R}} \end{bmatrix} \quad (20)$$

B. Application to Rotation—The choice of representation of 3-D rotation for this application requires weighing how 3-D rotations will fit into the existing AM Fourier series framework. Due to the topology of the space, no representations will exactly fit into the existing framework; however, we present models for the three which come the closest: unit quaternions, exponential coordinates, and Euler angles.

The direct application of the AM framework to rotation simply treats quaternions as vectors in $\mathbb{R}^4$.

$$\mathbf{q}_H^P(\phi_C, \phi_R) = \mathbf{W}_{\mathbf{q}}^T \phi \quad (21)$$

As the quaternion is modeled as the sum of vectors, no steps are taken to enforce unit length constraints. To return a unit quaternion the evaluated vector can simply be normalized. As previously discussed in Section IV, while the Euclidean distance is a proper distance metric on unit quaternions, because the quaternions double-cover the space, i.e. $d(\mathbf{q}, -\mathbf{q}) = 0$, special care must be taken when using this model. Enforcing smoothness in the representation can be done simply to ensure that jumps in representation do not occur.

$$\mathbf{q}(k) = \begin{cases} \mathbf{q}(k) & \text{if } \|\mathbf{q}(k-1) - \mathbf{q}(k)\| \leq \sqrt{2} \\ -\mathbf{q}(k) & \text{if } \|\mathbf{q}(k-1) - \mathbf{q}(k)\| > \sqrt{2} \end{cases} \quad (22)$$

Exponential coordinates and Euler-angles parameterizations require modification of the model due to the fact that exponential coordinates have a discontinuity at $\pm\pi$, and Euler representation possess singularities when $\beta = \pm \frac{\pi}{2}$, and we wish to represent arbitrary rotations which may cross these boundaries. The solution to this problem is to place the discontinuities opposite the current operating point. This is done by defining an intermediate rotation, $\mathbf{q}_H^P$, defined as the average rotation.

$$\mathbf{q}_H^P(\phi_C, \phi_R) = \arg \min_{\mathbf{q}} \sum_{i=1}^N d(\mathbf{q}, \mathbf{q}_H^P(i))^2 \quad (23)$$

If we can estimate this value, then the remaining periodic rotation, $\mathbf{q}_H^{\overline{H}}$, is centered about the origin. We can then model this zero-mean periodic rotation using any of the three representations of rotation.

$$\mathbf{q}_H^P(\phi_C, \phi_R) = \mathbf{q}_H^P \otimes \mathbf{q}_H^{\overline{H}}(\phi_C, \phi_R)$$

$$= \mathbf{q}_H^P \otimes \mathbf{W}_{\mathbf{q}}^T \phi \quad (24)$$

$$= \mathbf{q}_H^P \otimes \mathcal{M}_{\omega}^{\mathbf{q}}(\mathbf{W}_{\omega}^T \phi) \quad (25)$$

$$=\mathbf{q}_H^P \otimes \mathcal{M}_e^q(\mathbf{W}_e^T \phi) \quad (26)$$

Using either exponential coordinates or Euler angles in this form is desirable as they both are vectors in $\mathbb{R}^3$ and as such do not require enforcement of unity constraints; however, the Euclidean distance is not a proper distance metric in the space of rotations. This method of using the minimal state representations provided by exponential coordinates or Euler angles for only the deformations, and converting them to quaternions, is significant in that it enables us to decrease the number of parameters required by the model while also avoiding the singularities inherent in the representations. Of the four models for rotation presented in (21), (24), (25), and (26), however, there is no clear optimal choice.

C. Extension to Translation—As three of the presented models for rotation model the rotation in the average rotation frame, we may wish to model the periodic deformation in the same reference frame. The standard model for translational motion is

$$\mathbf{t}_H^P(\phi_C, \phi_R) = \mathbf{W}_t^T \phi \quad (27)$$

Defining the mean position as

$$\mathbf{t}_H^P(\phi_C, \phi_R) = \arg \min_{\mathbf{q}} \sum_{i=1}^N \|\mathbf{t} - \mathbf{t}_H^P(i)\|^2 \quad (28)$$

the periodic model in the transformed frame can be defined as

$$\mathbf{t}_H^P(\phi_C, \phi_R) = \mathcal{M}_q^R(\mathbf{q}_H^P) \mathbf{W}_t^P \phi + \mathbf{t}_H^P \quad (29)$$

Combining (26) and (28), and noting that $\mathbf{t}_H^P$ is equivalent to the $\mathbf{t}_0$ component of $\mathbf{W}_t$, it is clear that the parameters in the rotated frame are equivalent to rotating the base frame parameters by the mean rotation.

D. Parameter Estimation—Parameters for each model can be estimated using ordinary least squares in an offline batch process, or alternatively in a recursive manner for online implementation. The ordinary least squares solution is found using the pseudoinverse:

$$\mathbf{W} = \Phi^\dagger \mathbf{S} \quad (30)$$

$$\Phi = \begin{bmatrix} \phi(1)^T \\ \vdots \\ \phi(n)^T \end{bmatrix}, \mathbf{S} = \begin{bmatrix} \mathbf{s}(1)^T \\ \vdots \\ \mathbf{s}(n)^T \end{bmatrix} \quad (31)$$

where $\mathbf{s}$ is the signal being fitted. For the standard quaternion and translation in the base frame this signal is simply the measurement vector. However, for the models in the mean frame, we must first estimate the mean, then transform the measurement vectors into the desired parameterization, as shown below for the exponential coordinate parameterization.

$$\mathbf{s} = \mathcal{M}_{\mathbf{q}}^{\omega}(\mathbf{q}_H^{p-1} \otimes \mathbf{q}_H^P) \quad (32)$$

Online estimation of the parameters follows a similar process. In the case of the base-frame models all parameters can be estimated in a single stage, whereas the mean-frame models require a multistage approach. This two-stage approach, shown in Fig. 4, first uses a moving-average filter to estimate the mean of the signal, which is then used to transform the input signal. The transformed signal is then mapped to the proper parameterization, and then fitted using recursive least squares.

Experimental Protocol

This section describes the experiments conducted retrospectively on the data described in Section III in order to compare the performance of the presented models.

A. Model Comparison

The purpose of the first experiment is to determine the effects of the different distance metrics and nonlinear mappings between representations on the relative quality of the fit provided by each of the four models presented for periodic rotation. The numbers of harmonics used in each model were $H_C = 5$, $H_R = 4$, $H_{MC} = 2$, and $H_{MR} = 1$. Each model was fitted to a time trace approximately 30 s in length using ordinary least squares. The learned weights were then used to generate predicted measurements for the entire time trace.

For the two models that operate directly on quaternions the lengths of the predicted quaternions before normalization were calculated. RMS error was then calculated over the entire trace for each model relative to the quaternion measurements as well as between prediction models. Geodesic distance given by Eq. (7) was used to compute error.

B. Model Order Selection

A second experiment was conducted to select the model order which best captures the underlying dynamics of the periodic motion realized by the robot over the entire heart. In order to address this question, the 100 time traces collected over the surface of the heart as described in Section III were fitted with each of the proposed models using a varying range of harmonics.

For each test the first 80% of the time traces, approximately 25 s, or 5 respiration cycles, was used to train the models. These models were then tested on the remaining 20% of data, approximately 5 s or 1 respiration cycle. RMS error for both training and testing was recorded, with the distance metric used for rotation being the geodesic distance, and combined across all points to get a single error for each model and set of harmonics. Each model was then classified by the total number of parameters required by the model, and the model with the smallest error for each number of parameters was identified. In order to limit the space of models, as well as to keep the respiration and cardiac harmonics from interfering in frequency, the numbers of harmonics were capped at $H_C = 7$, $H_R = 6$, $H_{MC} = 7$, and $H_{MR} = 3$.

C. Online Estimation and Prediction

Experiments were also conducted in order to demonstrate the ability to estimate these model parameters online. The cascaded filter estimation scheme was used to estimate the model parameters for exponential representation of rotation and translation. The goal of this study was to demonstrate the ability of the presented scheme to provide stable estimates of the model parameters, to characterize the time required to learn the model, and to determine the prediction accuracy. Once again the 100 time traces collected over the surface of the heart were fitted using the described model and online algorithm. Initial estimates for all parameters were set to 0. The moving-average filter used a window length of 500 samples, or approximately 5 s, while the RLS algorithm used a forgetting factor of 0.995.

At each time step, after the online algorithm updated the model parameters, the current model was used to predict the motion for the next 5 s. RMS errors of these predictions were then calculated and combined over the points to obtain RMS prediction error over the entire heart as a function of time.

Results

A. Rotational Model Comparison

Offline estimation performance for a portion of the 30 s of periodic rotation using the exponential coordinates model is shown in Fig. 5. The resulting predictions from each model were visually indistinguishable. For the quaternion-based models, the lengths of the unnormalized quaternions were notably close to unit length, having maximum and minimum lengths of 1.0006 and 0.9996. RMS errors between each of the prediction models, as well as the ground truth data, are shown in Table I. Each of the models had RMS prediction error of 0.016 radians ($\approx 0.9^\circ$). Looking at the RMS errors of the model predictions with respect to each other, of which the largest is 3.1×10^{-4} ($\approx 0.02^\circ$) shows that for the range of rotations the robot experiences on the heart these four models are essentially equivalent, with the exponential and Euler-based models using far fewer parameters to achieve the same predictions.

B. Model Order Selection

Training and test errors for translational and rotational models averaged across all points are shown in Fig. 6. Errors are plotted against the number of parameters per degree of freedom

of the parameterization, meaning the same model order in exponential coordinates requires a quarter fewer parameters than using quaternions.

The results show that although the fit of the training models improves with model order, the predictive ability of the models improves little past the use of 20 parameters. Prediction error is essentially halved for the best model orders, decreasing from 2.0 mm and 0.07 radians for a constant (zeroth-order) model to a minimum of 0.8 mm and 0.035 radians. The results also show that predictive ability does not degrade at higher model orders.

From these results, we can generate an ordered list of models indicating the numbers of harmonics that provide the best performance for a particular model order, as shown in Table II. While the numbers of harmonics for translation and rotation generally agree, there are a few differences. These differences show how the contributions of motion differ between the cardiac and respiratory components of motion.

C. Online Estimation and Prediction

An example of the predictive performance of the translational and exponential models using online estimation is shown in Fig. 7. These plots show the evolution of the estimate of the means up to the current filter iteration, beyond which the current model is projected forward in time. Fourier Series parameter values for one degree of freedom are shown in Fig. 8 over the entire run. These figures demonstrate the feasibility of estimating these parameters in an online manner.

Inspection of both the estimates of the means and the parameter values shows that the estimation of the mean requires approximately 5 s, the length of a respiration cycle, to converge to a steady value, while the Fourier series parameters require approximately another 2 respiration cycles to completely settle. From this point onwards, the parameter values show little variation, meaning the model has converged.

Once again, combining the results over all 100 points over the surface of the heart yields the RMS prediction error for 5-s prediction horizons, as shown in Fig. 9. There are two important things to note about these plots. First is the rise in error which occurs at 5 s. This increase in error corresponds to the length of a full respiration cycle and is due to changes in the parameter estimates which occur due to the convergence of the mean estimates. While not starting the parameter estimation until after the mean has settled will avoid this bump, it was found to not significantly affect performance. Secondly, the prediction errors for both translation and rotation reach approximately the same minimum values as was the offline experiments: 0.8 mm and 0.035 radians RMS. These minimum values are reached at approximately 15 seconds and do not vary significantly for the remainder of the run.

Discussion

This paper deals with the problem of accurately modeling the quasi-periodic motion of the beating heart as it pertains to organ-mounted robots. The models, which accurately capture the periodic motion of the heart, can then be used to aid in localization or to add dynamic information to static preoperative maps. Due to the unique nature of these robots we were

able to address this motion with data collected from a live-animal closed-chest procedure, over all areas of the heart, with orientation as well as position measurements. The framework used is the amplitude-modulated Fourier series model, which was previously shown to outperform other model-based motion-prediction schemes (17). It directly models the coupling between heartbeat and respiratory motion, and can be optimized easily using a least squares approach.

Four models using three different parameterizations of rotation—unit quaternions, exponential coordinates, and Euler angles—were presented as candidates for describing the periodic rotational motion of the heart. Each model possessed some non-ideality which made its suitability questionable. Quaternions have constraints on length. Exponential coordinates and Euler angles have discontinuities and the Euclidean distance in these parameterizations is not a proper distance metric on $SO(3)$. Comparison of the models, however, showed that, for the relatively small rotations seen in the data, the models are essentially equivalent, and the Euclidean distance metrics are suitable for optimization on these parameterizations. The only practical difference in the models is that the exponential and Euclidean parameterizations provide essentially the same results with a quarter fewer model parameters.

The remainder of the work presented in this paper should be viewed from the very specific application of this work relative to other related works. The goal of these models is not to predict where a point will be in a short time as accurately as possible, but rather to describe where the point will be at all times as accurately as possible. This goal, which uses error metrics which cover long durations of time over all areas of the heart lead us to the conclusion that a large number of harmonics does not improve prediction performance in general. Whereas other works have used as many as 13, 6, 7, and 2 cardiac, respiration, and coupled cardiac and respiration harmonics to predict the motion of the heart (17), for this specific application we found no benefit from using any more than 3, 3, 1 and 2 cardiac, respiration, and coupled cardiac and respiration harmonics, respectively. Likely reasons for these differences in findings include measurement frequency and noise characteristics, measurement modalities, and most importantly the fact that our data was from a closed-chest procedure, introducing many more sources for non-idealities.

The final contribution of this paper is the demonstration of the online estimation of the presented models. These online methods achieved errors comparable to those found using offline methods. While an argument could be made for using a filtering approach to fitting the model, our argument for the least squares approach is twofold. As the best model found in this work consists of 127 parameters, if we wish the filter to be adaptive, as it is currently, assigning meaningful transition uncertainties is difficult, rendering the uncertainties which the system outputs difficult to make use of. Secondly, the estimation scheme produces satisfactory results and requires only two parameters to be identified, both of which have physical meaning.

As previously mentioned, this work represents the first stage in addressing the registration and localization of organ-mounted robots on the surface of the beating heart. Work is ongoing in utilizing the periodic models presented in this paper in dynamic registration of

preoperative models, as well as localizing in said models. We believe that using a scheme which directly accounts for the quasi-periodic motion of the heart will provide superior positioning accuracy, pushing organ-mounted robot interventions toward clinical viability.

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Figure 1.

A family of organ-mounted robots for minimally invasive cardiac interventions. Each of these devices attach to the beating heart using vacuum pressure to provide a stable platform for interventions. Currently there are three classes of robots which differ in the number of attachment points to the heart and the method of actuation or locomotion. (a) Lamprey consists of a single foot which is manually manipulated by an operator using a flexible catheter-like device. (b) HeartLander is a miniature mobile robot which uses two feet and flexible push-wires to move over the heart surface in an inchworm-like manner. (c) Cerberus is a parallel wire robot which uses three suction bases to form a support structure from which a tool head is controlled using cables.

Figure 2.

Experimental setup for collection of data. The world or measurement frame, **W**, is defined by the electromagnetic tracking unit. In order to account for possible motion of the patient, a reference probe, **P**, was affixed to the chest wall, while the robot, **H**, was inserted into the intrapericardial space via a subxiphoid incision. The raw measurements provided by the electromagnetic tracker are the 6-DOF pose of the robot in the world frame, T_H^W , and the 6-DOF pose of the reference probe in the world frame, T_P^W .

(a)

(b)

Figure 3.

Characterization of the periodic deformation experienced by the Lamprey robot collected using the experimental setup outlined in Figure 2. Measurements of the robot, $\mathbf{H}$, are in the frame of the reference probe, $\mathbf{P}$, affixed to the chest wall and not necessarily aligned with body frames. Time traces of 15 s in length are shown for both (a) Cartesian deformation, $\mathbf{t}_H^P$, and (b) rotational deformation in quaternion form, $\mathbf{q}_H^P$ and (c) the frequency responses for (top) z-axis Cartesian motion and (bottom) y-axis quaternion rotation. The vertical dashed lines correspond to respiration (blue), cardiac (red), and coupled (green) harmonics.

Figure 4.

Online estimation scheme for exponential parameterization of rotation. The average rotation is first estimated using a moving average filter. The measurement is then transformed by the mean rotation, converted from quaternion to exponential coordinates, and fitted using recursive least squares.

Figure 5.

Offline estimation of periodic deformation. Each model for rotation was fit to 30 s of data, shown in black, and the learned parameters were then used to generate predicted measurements, shown in red.

Figure 6.

Training and test errors vs. number of model parameters per degree of freedom for (top) translational models, and (bottom) rotational models. Geodesic distance is used as the distance metric for rotations.

Figure 7.

Online estimation and prediction results for translation and rotation. In each plot the ground truth measurements are shown in black. The current filter time step is denoted by the vertical black line. Blue lines show the evolution of the estimate of the signal means, while the projection of the current model forward in time is shown in red.

Figure 8.

Model parameter values evolution during online estimation. Parameter values change vary significantly until the mean estimates converge at 5 s.

Figure 9. Online estimation and prediction global error for (top) translation and (bottom) rotation. Error is calculated over a 5-s prediction horizon and averaged across all points.

Table I

Relative Prediction Error for Orientation

	$\mathbf{q}$	$\hat{\mathbf{q}}$	$\boldsymbol{\omega}$	e
Data	0.016	0.016	0.016	0.016
$\mathbf{q}$		2.5×10^{-9}	1.2×10^{-5}	3.1×10^{-4}
$\hat{\mathbf{q}}$			1.2×10^{-5}	3.1×10^{-4}
$\boldsymbol{\omega}$				3.1×10^{-4}

Entries are RMS prediction errors (radians) between each model ($\mathbf{q}$, $\hat{\mathbf{q}}$, $\boldsymbol{\omega}$, e) and the data to which it was fitted.

Table II

List of Preferred Models

N^a	H_C	H_R	H_{MC}	H_{MR}
3	0 / 1 ^b	1 / 0	0	0
5	1	1	0	0
7	1 / 2	2 / 1	0	0
9	2	2	0	0
11	3	2	0	0
13	2	2	1	1
15	2 / 3	3 / 2	1	1
17	3	3	1	1
19	4	3	1	1
21	3	3	1	2

^aNumber of parameters per model DOF from Eq. (17).^bMultiple entries denote different preferences for rotation and translation.