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New approach to establish an object reference frame for dental arch in computer-aided surgical simulation

J. Li^{1,*}, P. Yuan^{1,*}, C.-M. Chang^{1,2,*}, D. C.-Y. Ho^{1,3,*}, Y.-F. Lo^{1,2}, S. Shen^{1,4,5}, D. Kim¹, J. F. Teichgraber⁶, D. M. Alfi^{7,8}, J. Gateno^{7,8}, and J. J. Xia^{1,4,7,8}

¹Surgical Planning Laboratory, Department of Oral and Maxillofacial Surgery, Houston Methodist Research Institute, Houston, TX, USA

²Department of Oral and Maxillofacial Surgery, Taichung Veterans General Hospital, Taichung, Taiwan, R.O.C

³Department of Oral and Maxillofacial Surgery, Taipei Municipal Wan Fang Hospital, Taipei, Taiwan, R.O.C

⁴Department of Oral and Craniomaxillofacial Surgery, Shanghai Ninth People's Hospital Shanghai Jiaotong University School of Medicine, Shanghai, China

⁵Shanghai Key Laboratory Stomatology, Shanghai, China

⁶Department of Pediatric Surgery, The University of Texas Houston Health Science Center, Houston, TX, USA

⁷Department of Clinical Surgery (Oral and Maxillofacial Surgery), Weill Medical College, Cornell University, New York, NY, USA

⁸Institute for Academic Medicine, Houston Methodist Hospital, Houston, TX, USA

Abstract

The purpose of this study was to develop a principal component analysis-based adaptive minimum Euclidean distances (PAMED) approach to establish an optimal object reference frame for symmetrical alignment of the dental arch during computer-aided surgical simulation (CASS). It was compared with our triangular methods and the standard principal component analysis (PCA)

Correspondence: James J. Xia, Department of Oral and Maxillofacial Surgery, Houston Methodist Research Institute, 6560 Fannin Street, Suite 1280, Houston, TX 77030, JXia@HoustonMethodist.org.

*These authors contributed equally

Competing interests

Dr. James Xia, Dr. Jaime Gateno, and Dr. John Teichgraber receive a patent royalty from Medical Modeling Inc. through the University of Texas Health Science Center at Houston. Dr. Xia and Dr. Gateno receive a second patent royalty from Medical Modeling Inc. through the Houston Methodist Hospital.

Ethical approval

Houston Methodist Hospital IRB Approval: #IRB(2)1011-0187x.

Patient consent

Not required.

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method. Thirty sets of maxillary digital models were used. Midsagittal and occlusal planes were ranked by three experienced evaluators based on their clinical judgment. The results showed that for the midsagittal plane, all three evaluators ranked “ideal” for all 30 models with the PAMED method, 28 with the triangular method, and at least 11 with the PCA method. For the occlusal plane, one evaluator ranked all 30 models “ideal” with both the PAMED and the PCA methods while the other two evaluators ranked all 30 models “ideal” with the triangular method. However, the differences among the three methods were minimal. In conclusion, our PAMED method is the most reliable and consistent approach for establishing the object reference frame for the dental arch in orthognathic surgical planning. The triangular method should be used with caution because it can be affected by dental arch asymmetry. The standard PCA method is not recommended.

Keywords

object reference frame; dental arch; symmetrical alignment; midsagittal plane; occlusal plane; orthognathic surgery; computer-aided surgical simulation; surgical planning

Introduction

An important step in orthognathic surgical planning is to restore the symmetrical alignment of a dental arch with reference to the whole face¹⁻⁴. Analyzing dental arch symmetrical alignment requires an object reference frame, previously called a local coordinate system¹ or a local reference frame⁴. Like the global reference frame for the whole face, the object reference frame for a dental arch is composed of three orthogonal planes. The axial plane divides the dental arch into upper and lower halves; the coronal plane divides the arch into front and back halves; and the midsagittal plane evenly divides the arch into right and left halves evenly. By comparing the object reference frame for the dental arch to the global reference frame for the whole face, the symmetrical alignment of the dental arch can be calculated as a transverse difference in the central incisal midpoint (dental midline), and orientational differences in yaw and roll (cant)¹⁻⁴.

Currently, there are no reports on how to establish an object reference frame for a dental arch for digitally planning orthognathic surgery. A common practice is to place a transparent ruled grid over the dental cast and align it to the midpalatal raphe, so that a clinician can visually spot asymmetry of the arch form^{5,6}. The median palatal suture is used as the center of the maxillary dental arch and the symmetry axis. However, the dental midline rarely coincides with the median palatal suture even after orthodontic treatment. Ferrario et al.⁷ used the line connecting the centers of gravity of the anterior and poster teeth to determine the axis of symmetry. However, this method can only be used in patients with perfectly symmetrical dental arches⁸. Nonetheless, all of these methods only define the median line for the arch and none can be used to construct an object reference frame for the dental arch. Without using an object reference frame, the midline deviation may only be corrected by using the median line of the dental arch; the correction of the yaw and roll misalignments of the dental arch remains problematic.

In the past, our Surgical Planning Laboratory developed a triangular method in computer-aided surgical simulation (CASS) to analyze the symmetrical alignment for the dental arches

(Fig. 1). This method utilizes three points: the midpoint between the two central incisal embrasures (U0), and the right and left mesiobuccal cusps of the first molars (U6)¹⁻³. In this method, the origin of the object reference frame is U0. The axial plane is the occlusal plane constructed using these three landmarks. The midsagittal plane is the plane that passes through U0 and evenly divides the arch into right and left halves. The coronal plane is perpendicular to both the axial and the midsagittal plane. This triangular method is simple and easy to implement. However, it was developed based on the authors' experience during the utilization of CASS planning and has not been formally evaluated. In addition, we found that the triangular method was not reliable when there was dental arch asymmetry of any etiology, for example, unilateral edentulism, or individual tooth misalignment. Any of the above conditions can skew the triangular method and cause errors in defining the object reference frame.

It is the authors' belief during CASS planning that the symmetrical alignment of the dental arch can only be correctly analyzed when the object reference frame, especially the midsagittal plane, is correctly established. Our hypothesis is that the use of more dental landmarks will improve the accuracy of establishing an object reference frame for the dental arch. To this end, we developed a principal component analysis-based adaptive minimum Euclidean distances (PAMED) method to estimate an optimal object reference frame for a dental arch. This PAMED method was then compared with our triangular method¹⁻⁴ and the standard principal component analysis (PCA) method^{9,10}.

Materials and methods

Thirty sets of patients' maxillary digital and stone dental models were used in this study. Using a random number table, the models were randomly selected from our dental model archive for patients with dentofacial deformity. The inclusion criteria were (1) patients who had received preoperative orthodontic treatment and had undergone either one-piece or multipiece Le Fort I osteotomy; (2) the maxillary models had been used to establish a stable final occlusion during the surgical planning. In the case of the multipiece Le Fort I osteotomy, the models had been segmentalized and hand articulated by surgeons at the time of CASS planning. The research protocol was approved by our institutional review board (IRB(2)1011-0187x).

Three methods were used to establish the object reference frame for each dental arch, including PAMED (Figs 2 and 3), and the triangular (Fig 1) and the standard PCA (Fig 4) methods. The computation was programmed using MATLAB 2014a (The MathWorks, Inc, Natick, MA), and the calculation was completed in real time. The landmarks used in these three methods are listed in Table 1.

Evaluation

The object reference frames established using the three methods were evaluated by three evaluators, including two oral and maxillofacial surgeons who were experienced in orthognathic surgery (C.-M.C. and D.C.-Y.H.) and an orthodontist (Y.-F.L.). A random number was assigned to each method within each model so that the evaluators were blinded

to the methods used for establishing the object reference frames. They were also blinded to each other's evaluation results.

Since there was no "gold standard" for establishing the object reference frame for a dental arch, the evaluators were instructed to evaluate the results using their best clinical judgment as if they were performing dental arch analysis in their routine clinical practice. The midsagittal (YOZ) plane and the occlusal plane (either XOY or $X'O'Y'$ plane) were evaluated separately. The coronal plane was not evaluated since it was naturally derived from the other two. It is important to note that the occlusal plane is either the XOY plane of the object reference frame in the triangular method, or the $X'O'Y'$ plane which is parallel to the XOY plane of the object reference frame in the PAMED and the standard PCA methods. In the last two methods, $X'O'Y'$ is located superiorly to XOY planes with a mean of 0.2 mm and a standard deviation (SD) of 0.4 mm (the value was obtained based on our computational result of the 30 models used in this study). The reason for evaluating the $X'O'Y'$ occlusal plane instead of the XOY axial plane of the object reference frame is that the occlusal plane is clinically more meaningful.

Three ranks were given for each method: rank 1 represented an "ideal" result; rank 2 represented an "acceptable but less than ideal" result; and rank 3 represented an "unacceptable" result. Ideal indicated that the method achieved the best possible result and could be used for surgical planning. Acceptable indicated that the method achieved the best possible result if the ideal result did not exist, and it could also be used for surgical planning. An unacceptable ranking indicated that the method should not be used for surgical planning. If the results achieved with the different methods were the same, the same rank could be given. During the evaluation, the evaluators looked at the dental arch and its object reference frame established by each method on a 27" computer screen. At the same time, they could manually manipulate the corresponding stone dental model in order to help them better evaluate the computer-generated object reference frame. Prior to the formal evaluation, a training session was held for each evaluator on the use of the ranking system. Once the evaluation was completed, the methods for establishing the object reference frame were unblinded. The ranking scores determined by the three evaluators were paired and summarized descriptively.

Finally, the differences among the three methods were computed quantitatively. The orientational differences (deltas) of the object reference frames were directly measured using Tait-Bryan angles (pitch, roll, and yaw) in a pairwise fashion: PAMED-triangular methods, PAMED-PCA methods, and triangular-PCA methods. This was because only the difference between two reference frames could be measured mathematically using Tait-Bryan angles. As such, only descriptive statistics, including root mean square deviations (RMSDs)¹¹, means, and SDs, were used to present the data. The computations of Tait-Bryan angles were also completed using a custom MATLAB program.

Results

Three object reference frames were established using three methods for each maxillary arch, resulting in a total of 90 sets of data for each evaluator. In the evaluation of the midsagittal

plane using the PAMED method, the three evaluators ranked all 30 models “ideal”. Using the triangular method, all three evaluators consistently ranked 28 models “ideal”, and the remaining two “acceptable but less than ideal”. There were no “unacceptable” ranked using the triangular method. Using the standard PCA method, at least 11 models were ranked “ideal” by all three evaluators. In addition, one evaluator ranked 15 models “acceptable but less than ideal” and two models “unacceptable”; a second evaluator ranked 12 models “acceptable but less than ideal” and three models “unacceptable”; a third evaluator ranked three models “acceptable but less than ideal” and 16 models “unacceptable”.

The results were less consistent among the three evaluators in evaluating of the occlusal plane. One evaluator felt that an occlusal plane should evenly pass through all the edges and cusps, and thus ranked all 30 occlusal planes established using PAMED and standard PCA methods “ideal”. He also ranked 25 occlusal planes established using the triangular method “acceptable but less than ideal”. The other two evaluators believed that an occlusal plane should only pass through U0 and the right and left U6. Therefore, they both ranked all 30 occlusal planes established using the triangular method “ideal”. In addition, one evaluator ranked 24 “ideal” for PAMED and 22 “ideal” for the standard PCA methods, while the other ranked 24 “ideal” for both PAMED and the standard PCA methods. The rest were ranked as “acceptable but less than ideal”. Nonetheless, none of the three evaluators gave any “unacceptable” ranks.

The quantitative differences in Tait–Bryan angles for the three methods are summarized in Table 2 in a pairwise fashion. The results showed that there was only a minimal difference in the midsagittal planes (yaw) between PAMED and the triangular methods (RMSD 0.9°). The differences in all but two patients were within 1°. The remaining two patients showed a large difference in yaw (−2.2° and −3.7°). However, the differences in the midsagittal planes were large between the standard PCA methods and the other two methods (RMSD 3.4° to PAMED method, 4.2° to the triangular method). Finally, there was a minimal difference in the occlusal planes among all three methods (RMSD <1° in both pitch and roll).

Discussion

It is critical in orthognathic surgical planning to establish a correct object reference frame of the dental arch during symmetrical alignment. Owing to the nature of the dental arch, the occlusal plane is often used as the axial plane. Once the midsagittal plane is correctly defined, it is not difficult to define the coronal plane. It is always mutually perpendicular to both the axial and midsagittal planes and passes through U0. However, as described earlier, there is no gold standard for correctly defining the midsagittal plane.

Defining the midsagittal plane is the key to establishing the object reference frame for the dental arch. The result of this study showed that the PAMED method is the most consistent method of creating the midsagittal plane for the dental arch, even in the presence of a unilateral missing tooth or individual tooth misalignment. The triangular method performs reasonably well in generating the midsagittal plane because the two posterior landmarks are digitized “dynamically”. Instead of statically using the two mesiobuccal cusps of the first molars, the evaluators may have to change landmarks in order to form a hypothetical

isosceles triangle representing an arch, for example using either the mesiobuccal cusps of the second molars or the second premolars². As expected, when using the triangular method, the midsagittal plane is affected by the presence of a unilateral missing tooth (1/30) and individual tooth misalignment (1/30). In the quantitative analysis, there were two patients with major differences in yaw (-2.2° and -3.7° , respectively) using the triangular and PAMED methods. Nonetheless, the midsagittal planes for both patients, calculated using the triangular method, are consistently ranked as “acceptable but less than ideal” by all three evaluators. Finally, the standard PCA method is the least reliable method.

This is because PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables, the principal components (vectors). Thus, the origin of the three orthogonal principal components is located in the middle of the dental arch. Although two principal components (Y - and Z -axes) are assigned to be the midsagittal plane, it may not necessarily pass through $U0$. When used in CASS surgical planning, the origin must be translated to $U0$, causing the midsagittal plane to be shifted towards one side. In addition, the standard PCA method is sensitive to the landmarks used for the computation because it only uses up to 13 dental landmarks. Any outlier may significantly skew the result. Although our PAMED is also based on the PCA method to determine the occlusal plane, the Y -axis for the midsagittal plane is iteratively recomputed by minimizing the Euclidian distances between the right and left dental curves. The PAMED method also has solved the outlier problem by resampling the 13 dental landmarks to 1,399 points.

There are two definitions to define an occlusal plane. Traditionally, an occlusal plane passes through the central incisal edges and the mesiobuccal cusps of the first molars. This fits the definition of our triangular method. However, it is sensitive to the landmarks used to construct the triangle. The object reference frame can be affected by outliers in the triangular method if an overerupted or impacted tooth is used. The authors believe that an occlusal plane is better defined when it evenly passes through all edges and cusps. This fits the definition of PAMED and the standard PCA methods: the $X'O'Y'$ plane is constructed by the first and second principal components. Nonetheless, this evaluation was usually completed while a dental arch was enlarged to fill the entire 27" computer screen. Once the image of the dental arch was reduced to the life size, none of the evaluators could visually distinguish the differences of occlusal planes among the three methods. The quantitative differences in the occlusal planes between PAMED and the triangular methods were minimal (RMSD less than 0.8° in pitch and 0.4° in roll) and not clinically significant in CASS planning. However, the differences of the midsagittal planes remained significant.

It is important to give a computer program a consistent definition on how an object reference frame should be established. The clinical application used by the evaluators serves to support this mathematical premise. The results of this study demonstrate that the PAMED method is the most reliable approach to establish the object reference frame for the dental arch. The triangular method also performs reasonably well most of the time. However, it can be affected by the presence of a unilateral missing tooth or individual tooth misalignment. Although there is no “unacceptable” rank for the triangular method, it still should be used with caution. The standard PCA method is not recommended because it is unreliable in

establishing the object reference frame for the dental arch. Finally, it is important to note that the PAMED method is designed for symmetrical alignment of the dental arch that is ready for orthognathic surgery. In this case, U0 usually corresponds to the facial midsagittal plane and is always considered to be the origin of the object reference frame. This method should not be used to analyze object intrinsic symmetry or to plan the treatment of forming a new symmetrical dental arch by orthodontics or multi-piece Le Fort osteotomy. Our laboratory is currently working on completely different approaches to tackle these problems.

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Appendix: The algorithms for the three methods

PAMED method

Our PAMED method is completed in the following six major steps (Fig. 2). The key to this approach is to find the optimal minimum for the midsagittal plane, which evenly divides the dental arch into the right and left halves.

1. Digitize landmarks and form right and left dental curves. Thirteen dental landmarks are digitized on a maxillary dental arch, six landmarks on each side with one in the middle (Table 1 and Fig. 3A). The midpoint U0 represents the central dental midpoint. The 13 digitized dental landmarks are then connected to form a right and a left dental curve, seven points (U0, U2–U7) on each side. The first point of both right and left curves is U0. Since U0 is derived from the right and left central incisors (U1), both the right and left U1 are not used in the calculation. In cases of a missing tooth, its landmark is not digitized and the two adjacent landmarks are directly connected (Fig. 3B).
2. Resample digitized landmarks. The Euclidian distances of the right and left dental curves are computed respectively. The distal (molar) end of the longer curve is then trimmed off, making the right and left curves equal-distance (Fig. 3A). The right and left curves are then evenly resampled to 700 points on each side, which yields approximately 0.1 mm of resampling resolution. The first points on each side of the point arrays are joined at U0, resulting in a total of 1,399 resampled points for the entire dental arch.
3. Apply PCA to calculate an initial Cartesian coordinate system. A standard PCA^{9,10} is applied on the 1,399 resampled points, computing the first, second, and third principal components. They are mutually perpendicular to each other. The initial Cartesian coordination system (X' - Y' - Z') is determined as follows. The origin of the three principal components, located in the middle of the dental arch, is the origin O' of the initial Cartesian coordinate system (Fig. 3C). The third principal component, the smallest variance, is defined as the Z' -axis. The first and second principal components are defined as the X' - and Y' -axes. The Y' -axis is the principal component that divides the 1,399 points in to the right and left groups, and the X' -axis is the last principal component. Finally, the $X'O'Y'$ plane represents the occlusal plane, which evenly passes through all the edges and cusps.
4. Define the origin and calculate Z-axis of the object reference frame. The origin O of the object reference frame for the dental arch is defined at U0. Therefore, the initial *Cartesian* coordinate system is translated into the new origin O at U0. Subsequently, the X' -, Y' - and Z' -axes become X'' -, Y'' -, and Z'' -axes, and $X''O''Y''$ plane becomes $X''O''Y''$ plane (Fig. 3C). Finally, the Z'' -axis is assigned as the Z-axis of the object reference frame for the dental arch.
5. Iteratively calculate the Y-axis for the object reference frame.

Step 5.1 (Initialization): The 1399 resampled points are projected onto X "OY" plane along the Z-axis. The last two points at the distal end of the right and left projected point arrays are connected to form Line A. Point P is the intersection point of Line A and Y"-axis (Fig. 3D). The Origin O and Point P are then connected to form Line $\overrightarrow{OP}$. It will be the Y-axis for the object reference frame of the dental arch. During the first iteration, Line $\overrightarrow{OP}$ is the Y"-axis (Fig. 3D).

Step 5.2 (Computing the sum of Euclidean distances): On the X "OY" plane, the right side of the projected point array is the mirror image of the left around Line $\overrightarrow{OP}$. The initial sum of the Euclidean distances between the corresponding points are computed (Fig. 3D).

Step 5.3 (Initialization): Point P is moved 0.1 mm both right and left along line A. The sums of the Euclidean distances for both sides are calculated by repeating step 5.2. They are compared with the initial sum of the Euclidean distances calculated in step 5.2. The direction that results in a smaller sum of Euclidean distances is a "good" direction for step 5.4 (Fig. 3E). If the initial sum of the Euclidean distances calculated in step 5.2 is the smallest among the three, Line $\overrightarrow{OP}$ becomes the Y-axis and the iteration stops.

Step 5.4 ("Coarse" Iteration): Point P is moved continuously in 1.0-mm steps toward the "good" direction. Step 5.2 is repeated until the sum of the Euclidean distances becomes larger (Fig. 3F).

Step 5.5 ("Fine" Iteration): Point P is moved continuously in 0.1-mm steps opposite to the "good" direction. Step 5.2 is repeated in order to calculate an optimal solution for Line $\overrightarrow{OP}$ until the sum of Euclidean distances becomes larger. Line $\overrightarrow{OP}$ that results in the smallest sum of distances, the optimal solution, is defined as Y-axis of the object reference frame (Fig 3G).

Calculate the X-axis, and XOY, YOZ and XOZ planes of the object reference frame. The X-axis of the reference frame is perpendicular to both Y- and Z-axes (Fig. 3H). The XOY (axial), YOZ (midsagittal), and XOZ (coronal) planes are finally computed based on the X-, Y- and Zaxes.

Triangular method

Our traditional triangular method is completed in the following three steps (Fig. 1).

1. The axial (XOY) plane of the object reference frame is defined by three dental landmarks: U0 and the right and left U6. U0 is used as the origin of the object reference frame. If one U6 was missing unilaterally, the right and left U7, or U5, are used instead.
2. The Z-axis is calculated as the normal vector of the triangle that passes through the origin U0. The Y-axis is defined as the axis that passes through U0 and

bisects the line that connects the right and left U6. The X-axis is mutually perpendicular to the other two axes also passing through U0.

3. The YOZ plane is the midsagittal plane of the object reference frame of dental arch. The XOY plane is the axial and the occlusal plane. The XOZ plane is the coronal plane that is perpendicular to both the axial and midsagittal planes.

Standard PCA method

A standard PCA method is completed in the following three steps (Fig. 4).

1. Digitize Landmarks and form right and left dental curves. Thirteen dental landmarks are digitized on a maxillary dental arch, six on each side (U2–U7) with one in the middle (U0).
2. Apply PCA to calculate an initial Cartesian coordinate system. Standard PCA^{9,10} is applied on the 13 digitized dental landmarks, computing the first, second, and third principal components as an initial Cartesian coordinate system. The three principal components are mutually perpendicular to each other and the origin O' is located in the middle of the dental arch. As the same in the PAMED method, the third principal component is defined as Z' -axis. The first and second principal components are defined as the X' - and Y' -axes. The Y' -axis is the principal component that divides the 13 points to right and left groups, and the X' -axis is the last principal component. The $X'O'Y'$ plane is the occlusal plane, which evenly passes through all the edges and cusps.
3. Define the origin and calculate the object reference frame. The origin O of the object reference frame for the dental arch is defined at U0. Therefore, the initial Cartesian coordinate system is translated into the new origin O at U0. Subsequently, the X' -, Y' - and Z' -axes become the X -, Y - and Z -axes of the dental arch's object reference frame. The three orthogonal planes are calculated accordingly and the YOZ plane serves as the midsagittal plane for the dental arch.

Fig. 1. Triangular method. It utilizes three points: midpoint between the two central incisors (U0), and right and left mesiobuccal cusps of the first molars (U6).

Fig. 2.
Flowchart for PAMED method in six major steps.

Fig. 3a

Fig. 3b

Fig. 3c

Fig. 3d

Fig. 3e

Fig. 3f

Fig. 3g

Fig. 3h

Fig. 3.

Illustration for PAMED method: the key to this approach is to find the optimal minimum for the midsagittal plane, which evenly divides the dental arch into the right and left halves. (A) Thirteen dental landmarks (red) are digitized on the dental model. They form a right and a left curves (yellow) joined at U0. The Euclidian distances are calculated for each curve. If the right and left Euclidian distances are not equal, the distal (molar) end of the longer curve is then trimmed off, making the right and left curves equal-distance. The entire dental curve is evenly resampled to 1,399 points (black dots on the yellow curves). (B) The two first premolars are missing in a dental arch of an obstructive sleep apnea patient. The landmarks for the missing teeth are not digitized and the 2 adjacent landmarks are directly connected. (C) A standard PCA is applied to an initial Cartesian coordinate system ($X'-Y'-Z'$). The origin O' is located in the middle of the dental arch. The $X'O'Y'$ plane, marked in dark gray, is the occlusal plane. The initial Cartesian coordinate system is then translated to the new origin O at U0. Subsequently, $X'-$, $Y'-$, and $Z'-$ axes become $X''-$, $Y''-$ and $Z''-$ axes, and $X'O''Y''$ plane becomes $X''OY''$ plane (marked in red and green). Finally, the Z'' -axis is assigned as the Z-axis of the object reference frame for the dental arch. (D) The Y-axis of the reference frame for the dental arch is computed iteratively. The resampled points are projected onto $X''OY''$ plane along Z-axis. The right point array is marked in dark gray and the left is marked in green. Line A connects the last two points at the distal end of the right and left projected point arrays. Point P is the intersection point of Line A and Y'' -axis. The

Origin O and Point P are connected to form Line $\overrightarrow{OP}$, which is the Y-axis to be determined. During the first iteration, Line $\overrightarrow{OP}$ is Y''-axis. The right side of the point array is mirror-imaged to the other side around the Line $\overrightarrow{OP}$ on X''OY'' plane (marked in purple). The sum of Euclidean distances between the corresponding points of the left (marked in green) and the mirror-imaged right (marked in purple) point arrays are calculated. (E) To find a "good" direction. Point P is moved 0.1 mm toward the right and left along line A. The sum of the Euclidean distances is calculated as in step 5.2 in the appendix. The direction that can result in a smaller sum of Euclidean distances is a "good" direction for the next step. In this example, the left is the "good" direction. (F) The "Corse" Iteration: Point P is moved continuously toward the "good" direction in 1.0 mm steps. Step 5.2 is repeated until the sum of Euclidean distances becomes larger. (G) The "Fine" Iteration: point P is then moved continuously opposite to the "good" direction in a step of 0.1 mm to find the optimal solution for Line $\overrightarrow{OP}$. Step 5.2 is repeated until the sum of Euclidean distances becomes larger. Line $\overrightarrow{OP}$ that results in the smallest sum of distances is defined as Y-axis of the object reference frame for the dental arch. (H) Finally, the object reference frame of dental arch, marked in red, is established using the PAMED method. A gray axis indicates the original Y''-axis prior to the iterative calculation.

Fig. 4.

The standard PCA method. A standard PCA is applied to 13 dental landmarks to calculate an initial Cartesian coordinate system (X' - Y' - Z' , all are marked in dark gray). The origin O' is located in the middle of the dental arch. $X'O'Y'$ plane is the occlusal plane. The initial Cartesian coordinate system is then translated to the new origin O at $U0$. Subsequently, X' -, Y' - and Z' -axes become X -, Y -, and Z -axes of the object reference frame of the dental arch. They are marked in red.

Table 1

Definition of the landmarks used in the computation

Landmark	Definition
U0	The midpoint between the two central incisal edges
U2	The midpoint on the lateral incisal edge
U3	The tip of the canine
U4	The buccal cusp of the first premolar
U5	The buccal cusp of the second premolar
U6	The mesiobuccal cusp of the first molar
U7	The mesiobuccal cusp of the second molar

Table 2

The results of the orientational differences of the object reference frames using Tait–Bryan angles (pitch, roll and yaw) in a pairwise fashion: PAMED-triangular methods, PAMED-PCA methods, and triangular-PCA methods

	Pitch		Roll		Yaw	
	RMSD	Mean	SD	RMSD	Mean	SD
PAMED vs. triangular methods	0.8	0.2	0.8	0.4	0.0	0.5
PAMED vs. PCA methods	0.4	0.1	0.4	0.3	0.0	0.3
Triangular vs. PCA methods	1.0	0.3	1.0	0.5	0.1	0.5