Two-step inversion with a logarithmic transformation for microwave breast imaging

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Purpose: The authors have developed a new two-step microwave tomographic image reconstruction process specifically designed to incorporate logarithmic transformed microwave imaging algorithms as a means of significantly improving spatial resolution and target property recovery. Log transform eliminates the need for *a priori* information, but spatial filtering often integrated as part of the regularization required to stabilize image recovery, generally smooths image features and reduces object definition. The new implementation begins with this smoothed image as the first step, but then utilizes it as the starting estimate for a second step which continues the iterative process with a standard weighted Euclidean distance regularization. The penalty term of the latter restricts the new image to a multi-dimensional location close to the original but allows the algorithm to optimize the image without excessive smoothing.

Methods: The overall approach is based on a Gauss-Newton iterative scheme which incorporates a log transformation as a way of making the reconstruction more linear. It has been shown to be robust and not require *a priori* information as a condition for convergence, but does produce somewhat smoothed images as a result of associated regularization. The new two-step process utilizes the previous technique to generate a smoothed initial estimate and then uses the same reconstruction process with a weighted Euclidean distance penalty term. A simple and repeatable method has been implemented to determine the weighting factor without significant computational burden. The reconstructions are assessed according to conventional parameter estimation metrics.

Results: We apply the approach to phantom experiments using large, high contrast canonical shapes followed by a set of images recovered from an actual patient exam. The image improvements are substantial in regards to improved property recovery and feature delineation without inducing unwanted artifacts. Analysis of the residual vector after the reconstruction process further emphasizes that the minimization criterion is efficient with minimal biases.

Conclusions: The outcome is a novel synergism of an established stable reconstruction algorithm with a conventional regularization technique. It maintains the ability to recover high quality microwave tomographic images without the bias of *a priori* information while substantially improving image quality. The results are confirmed on both phantom experiments and patient exams. © 2017 American Association of Physicists in Medicine [https://doi.org/10.1002/mp.12384]

Key words: breast, clinical, log transformation, microwave imaging, two-step, tomography

1. INTRODUCTION

Microwave tomographic imaging has been studied for several decades for biomedical applications and is now beginning to appear in some clinical studies.^{1–3} Interest has stemmed from multiple reports demonstrating that actionable diagnostic information can be derived from non-invasive dielectric property recovery. Studies have ranged from dielectric contrast between breast tumor and benign tissue,^{4–6} bone property changes as a function of mineral density,^{7,8} and differences in properties between blood and normal brain tissue for stroke detection.⁹ In addition, tissue property changes as a function of temperature have informed the development of techniques for monitoring temperature during thermal therapy.^{10,11}

One of the more vexing challenges in medical microwave imaging is convergence of iterative image reconstruction to unwanted local minima.^{12–18} Common strategies for averting local minima have been to incorporate *a priori* information or invoke frequency hopping.^{19–21} *A priori* information has been considered in various forms.^{12,14,17,22–24} For example, Fhager and Persson¹⁴ investigated abnormality detection within a normal, homogeneous organ, and provided accurate property values of a large object (presumably an organ) as the initial estimate to reconstruct internal aberrations. Catapano et al.¹⁷ used a linear sampling method to recover target morphology, and then imposed a uniform property distribution across the target as the starting point for their reconstruction process. Dorn et al.²⁵ applied a level set algorithm to recover

the shape of a pollutant plume within a homogeneous ground sample. They used a simple inversion to generate a low frequency (20 MHz), low resolution property distribution which was subsequently used as the starting estimate for their level set process. In practice, priors require estimates of structure or dielectric properties or both within the target, and are not always available. Frequency hopping starts by recovering an image at a low frequency, where the computed and measured data are sufficiently close,^{19,20} then utilizes that information as the starting estimate for the next higher frequency to maintain closeness between the measured and computed data.

Regularization techniques are important to success as well, and include projection schemes²⁶ and wavelet methods,^{27,28} among others. Most approaches provide initial property distribution estimates or explore penalty terms to improve image quality and robustness. In an alternate but related strategy to *a priori* information and frequency hopping techniques, log transformed minimization^{29,30} maintains measured and computed signal phases on the same Riemann sheet, and by doing so, does not require *a priori* information. Indeed, recent analysis suggests that the form of non-uniqueness often found in microwave tomography can be overcome by ensuring that the measured and computed field values are present on the same Riemann sheet.³¹ The idea is similar to radar interferometry where phase data from synthetic aperture radar (SAR) is unwrapped to derive accurate altitude information.³²

In this paper, log transformed image reconstruction is reconsidered but extended to a two-step method. Here, the first step proceeds from a homogeneous initial property estimate and converges to a smoothed image of the target that acts as prior information for the second step, but with the advantage that it does not impose structural or property value assumptions at the outset, and normalcy of the resulting residual errors demonstrates the property distribution estimate is unbiased — a claim not available to most other techniques. The final images obtained after both the first and second steps are completed are also unbiased which is an important image attribute for a standalone microwave imaging system. Investigations focus on 2D reconstructions in part because computational costs are substantial when forming 3D images,^{33,34} and often involve parallel computers or multi-GPU systems,³⁴ whereas 2D images can be recovered in 5–10 s on a laptop computer,³⁵ and allow important parameters to be studied rapidly which provides insights into their selection in the computationally more challenging 3D problem.^{29,36} In addition, many more 2D images have been generated from patient exams relative to 3D reconstructions which can be revisited,^{3,37,38} for example, for breast cancer diagnosis³⁷ and monitoring tumor progression during neoadjuvant chemotherapy.³ A technique for determining the second-step regularization parameter and application of the algorithm in two phantoms comprised of large objects with moderate to high contrast are presented. These results are followed by a clinical example demonstrating the image quality improvement associated with the new two-step approach. Well-established parameter estimation metrics are applied to assess the quality of the new reconstruction process and the appropriateness of its minimization criterion as an unbiased estimator.

2. METHODS

2.A. Theory

2.A.1. First step

The first step of the algorithm utilizes Gauss-Newton optimization and implements a log transform to partially linearize the process.^{29,30} Equation (1) shows the minimization statement for the log transform Gauss-Newton iterative imaging algorithm

$$\min \|\Gamma^m - \Gamma^c(k^2)\|^2 + \|\Phi^m - \Phi^c(k^2)\|^2 \quad (1)$$

where Γ^m , Γ^c , Φ^m , and Φ^c are the log magnitudes and phases of the measured and computed signals, respectively. k^2 is effectively the reconstructed image which incorporates both the relative permittivity and electrical conductivity according to:

$$k^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_r + j\omega \mu_0 \sigma \quad (2)$$

where ω is the frequency in radians, μ_0 is the free space magnetic permeability, ϵ_0 is the free space permittivity, ϵ_r is the relative permittivity, σ is the electrical conductivity and j is the square root of -1 . Levenberg-Marquardt regularization^{39,40} is implemented as a damping function

$$[J^T J + \lambda I] \{\Delta k^2\} = [J^T] \left\{ \begin{array}{l} \Gamma^m - \Gamma^c(k^2) \\ \Phi^m - \Phi^c(k^2) \end{array} \right\} \quad (3)$$

where J and J^T are the Jacobian matrix and its transpose, $J^T J$ is the Hessian matrix, λ is the damping coefficient which is typically reduced at each iteration, I is the identity matrix, Δk^2 is the image update at each iteration, and matrix elements are scaled so that diagonal terms are unity.³⁹ In this case, the image is updated at each iteration according to:

$$\{k_i^2\} = \{k_{i-1}^2\} + S \{\Delta k^2\} \quad (4)$$

where S is the iteration step size ranging from 0 to 1 and was determined empirically.

Here, methods described in Fang et al.⁴¹ and Meaney et al.²⁹ are implemented which illustrate how to construct the log transform Jacobian through efficient adjoint techniques. One challenging aspect of the approach is the need to unwrap both measured and computed phases, and the techniques we use are described below (in Section 2.A.2 and Section 2.A.3). In all cases presented in this paper, initial property estimates are always the property distribution of the homogeneous background, and no other *a priori* information related to the distribution of target properties is included.

2.A.2. Unwrapping of measurement phases

For both measured and computed signals, phases are normalized by subtracting values from the measurements when no object is present (homogeneous bath) from values when the target is present:

$$\Phi_{\text{normalized}}(j, k, f) = \Phi_{\text{target}}(j, k, f) - \Phi_{\text{homogeneous}}(j, k, f) \quad (5)$$

where Φ_{target} and $\Phi_{\text{homogeneous}}$ are the phases at frequency f for transmit/receive antenna pairs j,k for cases when a target is present and for the homogeneous bath, respectively. In practice, the most reliable way to acquire the data are to record it in normalized (or calibrated) form which subtracts unwanted variations from unknown cable lengths, receiver differences, etc. In general, the phase difference term in the normal equations used to compute the property updates during the reconstruction process is expressed as:

$$\Phi_{\text{target}}^{\text{measured}}(j,k,f) - \Phi_{\text{target}}^{\text{computed}}(j,k,f) \quad (6)$$

which can similarly be written as:

$$\left[\Phi_{\text{target}}^{\text{measured}}(j,k,f) - \Phi_{\text{homogeneous}}^{\text{measured}}(j,k,f) \right] - \left[\Phi_{\text{target}}^{\text{computed}}(j,k,f) - \Phi_{\text{homogeneous}}^{\text{computed}}(j,k,f) \right] \quad (7)$$

assuming both the measured and computed homogeneous solutions are the same. Note that the left term is identical to that in Eq. (5) and can be used directly in the update equation in the form of Eq. (3).

When the target is the homogeneous bath, normalized phases are zero for each transmit/receive antenna pair because the target and homogeneous bath are electrically the same. As the dielectric properties of the target deviate from those of the bath, phases for the transmit/receive pairs change accordingly — the least for receive antennas closest to the transmitter and the most for receive antennas furthest away (e.g., on the opposite side of the target).

Phase changes generally increase as a function of frequency since the wavelengths become increasingly longer at lower frequencies and phase perturbations are reduced.²⁹ Target phases at the lowest frequencies of operation (our acquisitions range from 500 MHz to about 2 GHz) rarely occur outside the span $-\pi$ to $+\pi$, even when receive antennas are on the opposite side (of the target) from the transmitter. Further, normalized phases rarely change by more than π from one frequency to another relatively close frequency. Thus, if data are acquired at closely spaced frequencies from lowest (typically 500 MHz) to highest, phases for each transmit/receive antenna pair can be unwrapped as a function of frequency. Figure 1 shows normalized phases measured for a single transmitter in a phantom (with properties of X1 in Table I, see Section B) where (a) values have been mapped into a single, $-\pi$ to $+\pi$ Riemann sheet and (b) after being unwrapped using the frequency-based approach. Results show that multi-frequency data are sufficient to unwrap the higher frequency phase acquisitions.

To summarize the procedure, normalized phases for the lowest frequency are adjusted to be within $-\pi$ to $+\pi$ by adding or subtracting multiples of π . For the subsequent (next highest) frequency, phases are compared with those from the previous frequency for each transmit/receive pair

$$\Phi_{\text{normalized}}^{\text{measured}}(j,k,f_i) - \Phi_{\text{normalized}}^{\text{measured}}(j,k,f_{i-1}) \quad (8)$$

If the difference falls outside $-\pi$ to $+\pi$, multiples of 2π are added or subtracted to the phase of $\theta_{\text{normalized}}(j,k,f_i)$ until

the difference is within range. The process is continued until reaching the highest frequency, and after completion, the unwrapped normalized phases are known for all measurements at all frequencies.

2.A.3. Unwrapping of computed phases

Normalized computed phases are unwrapped as a function of reconstruction iteration. By selecting the initial property distribution estimate for the reconstruction process to be the homogeneous bath, normalized computed phases begin at zero for all transmit/receive antenna pairs. At each iteration, forward solutions are computed and phase values are extracted for each transmit/receive antenna pair. In this implementation, phases at iteration n are compared with values from the previous iteration, $n-1$, for each transmit/receive pair:

$$\Phi_{\text{normalized}}^{\text{computed}}(j,k,n) - \Phi_{\text{normalized}}^{\text{computed}}(j,k,n-1) \quad (9)$$

If differences are greater than π , multiples of 2π are added or subtracted to phases from the more recent iteration until differences fall within the $-\pi$ to $+\pi$ range. Iteration step size is adjusted to limit changes in phase between iterations to the $-\pi$ to $+\pi$ range, and typically is only reduced during early iterations when property updates tend to be larger. Reductions in step size do slow down the reconstruction; but the time cost is nominal because they only occur in the first few iterations. Thus, the computed phase unwrapping procedure allows unwrapped normalized phases to be computed exactly with minimal increase in computational overhead.

2.A.4. Second step

The second step can be viewed as a reconstruction with a Tikhonov regularization penalty term where the image from the first step serves as prior information.^{14,16,25,42} Specifically, the minimization criterion is

$$\min \left\| \Gamma^m - \Gamma^c(k^2) \right\|^2 + \left\| \Phi^m - \Phi^c(k^2) \right\|^2 + \lambda \left\| k^2 - k_{\text{int}}^2 \right\|^2 \quad (10)$$

where λ is the regularization parameter, k^2 is the image at a current iteration and k_{int}^2 is the intermediate image from the first step. The Euclidean distance penalty term on the right effectively restricts the image to be within a multi-dimensional distance of the intermediate image with λ determining the radius. The regularization parameter is determined empirically as described in more detail in Section 3.A.

2.B. Phantoms

Two relatively challenging, multi-layer cylindrical phantoms were used to evaluate the imaging algorithm capabilities. In both cases, data were collected with our existing imaging system which consists of a coupling bath and 16 evenly spaced monopole antennas configured on a 15.24 cm diameter circle surrounding the target. For all phantoms, the

FIG. 1. Normalized phase values for the 15 receivers associated with a single transmitting antenna over the range of frequencies from 500 to 1700 MHz: (a) after being mapped into a single Riemann sheet ($-\pi$ to $+\pi$), and (b) after being unwrapped utilizing the multi-frequency data. [Color figure can be viewed at wileyonlinelibrary.com]

TABLE I. Cylinder locations, sizes and dielectric properties for the two experimental phantoms.

Phantom	Object	Center x-coord. (cm)	Center y-coord. (cm)	Diam. (cm)	ϵ_r	σ (S/m)
X1	Large cylinder	0.0	0.0	10.8	12.1	0.75
	Small cylinder	-2.5	0.0	4.2	37.1	1.60
X2	Large cylinder	0.0	0.0	10.8	7.45	0.37
	Smaller cylinder	-2.5	0.0	4.2	37.1	1.60
	Smallest cylinder	1.5	-2.5	1.3	51.1	1.45

wall of the containers were made from polyethylene terephthalate (PET) and had dielectric properties of $\epsilon_r = 3.4$ and $\sigma = 0.2$ S/m at 1300 MHz. The wall thicknesses were 0.75 mm for the larger outer cylinder and 0.15 mm for the smaller internal ones. The liquid medium was an 80% glycerin bath having properties $\epsilon_r = 20.6$ and $\sigma = 1.27$ S/m at 1300 MHz. The first phantom (X1) consisted of a large cylinder (10.8 cm diameter) filled with a 88:12 glycerin:water mixture, and an off-center smaller cylinder (4.2 cm diameter) filled with a 72:28 glycerin:water mixture. The second phantom (X2) consisted of the same large cylinder filled with a 97:3 glycerin:water mixture, an off-center smaller cylinder (4.2 cm diameter) filled with a 72:28 glycerin:water mixture, and a second off-center smaller cylinder (1.3 cm diameter) filled with a 55:45 glycerin:water mixture, respectively. The cylinder wall thicknesses were 0.25 mm for the largest cylinder and 0.50 mm for the smaller ones, respectively. The properties and locations of each object within each phantom are summarized in Table I. Data were acquired across the frequency range 500–1700 MHz to assist in the phase unwrapping process described earlier.

Since the degree of difficulty in reconstructing property distributions increases in proportion to the size and contrast

of inclusions, we examined the amount of phase and amplitude change induced by the presence of each phantom, and refer to these measurements as the phase and amplitude projections since they essentially calibrate out the absolute phase and amplitude values in terms of relative changes. Figures 2(a) and 2(b) show the magnitude and phase projections for a single transmitter for both the X1 and X2 phantoms, respectively.

These results are representative of data from the other antennas at different angles. Projections are greatest for the central receiver antennas and diminish to nearly zero for receiver antennas adjacent to the transmitter. The maximum phase projections are greater for the X2 phantom (as expected) since the permittivity contrast of the large cylinder is greater (relative to the X1 phantom). Thus, the dielectric property images from the X2 phantom are generally more challenging to recover, but with the advent of the 2-step algorithm, images from both phantoms are reconstructed quite well, as illustrated in the next section.

3. RESULTS

3.A. Determination of the 2nd step regularization parameter

Determining the optimal second-step regularization parameter is time consuming because the algorithm needs to be tested for different weights by allowing it to iterate until convergence for each value. However, we found that a single second-step iteration can be used to determine a suitable value for λ . Specifically, Figure 3 shows the L2 norm data error behavior as a function of λ for a single second-step iteration is very similar to when the second-step algorithm is allowed to converge to a final solution, especially in terms of the minimum and inflection point λ -value locations. In these cases, the iteration step size was set to 0.1.

FIG. 2. Plots of the 1300 MHz (a) magnitude and (b) phase projections for a single transmitter as a function of receiver number for both the X1 and X2 phantoms. Antennas are numbered in sequential order around the imaging zone with the 1st and 15th antennas closest to the transmitter while the 8th antenna is furthest away. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 3. Normalized (to the value at the start of the first-step) L2 error as a function of regularization parameter when the second step completes 1 and 20 iterations, respectively. The inflection point of each curve is marked by a black circle. [Color figure can be viewed at wileyonlinelibrary.com]

Previous analyses⁴³ show that choosing λ closest to the data error inflection point produced significantly improved images compared to those obtained from the first-step reconstruction (and when λ was selected at the lowest L2 error, which often caused artifacts near the monopole antennas). Figure 4 shows a series of 1300 MHz images for the X2 phantom reconstructed over a range of regularization parameters (with 20 second-step iterations) utilizing an iteration step size of 0.1 for all reconstructions.

For λ greater than $1e-7$, the images do not vary significantly and only one example is shown in this range. However, as λ decreases from $1e-8$ to $1e-9$, edges of the large cylinder appear to be sharper in the permittivity image. Additional improvements occur in the recovery of internal cylinders. In fact, the permittivity estimated in the inner cylinders is closer to the actual values. For the corresponding conductivity images, the large cylinder outline is vague at $\lambda = 1e-8$, but becomes progressively more well defined at $\lambda = 1e-9$. The smoothing between internal cylinders is more apparent in the

conductivity relative to the permittivity images, but these features are still visible. For λ below $1e-10$, an increase in artifacts is evident in both image sets, especially around the periphery where each of the 16 antennas is closest to the domain. By $\lambda = 4.6e-13$, which corresponds to the lowest L2 relative error, the artifact level is high, and by the time λ reaches $1e-13$, the images have diverged. Thus, the λ -value closest to the L2 data error inflection point offers a practical trade-off between image quality improvement and artifact suppression. Similar results (not shown) and conclusions occur with reconstructed data from the X1 phantom.

3.B. Images as a function of iteration and convergence of magnitude and phase projections

Figure 5 shows 1300 MHz images reconstructed for the X1 phantom as a function of iteration number. In this case, the step size was 0.1, and the first-step algorithm was executed for 50 iterations, after which the second step was

FIG. 4. Second step reconstructed permittivity and conductivity images of the X2 phantom at 1300 MHz for increasing λ – (a) $1.0\text{e-}13$, (b) $4.6\text{e-}13$, (c) $1.0\text{e-}10$, (d) $1.0\text{e-}9$, (e) $4.6\text{e-}9$, (f) $1.0\text{e-}8$, and (g) $1.0\text{e-}7$. [Color figure can be viewed at wileyonlinelibrary.com]

performed for an additional 20 iterations. The large and small cylinders are readily visible in both permittivity and conductivity images for all iterations, but they become more refined after 20 first-step iterations with little further change by 50 first-step iterations. The images are smoothed and have fairly broad property gradients across the bath:large cylinder and large cylinder:small cylinder interfaces, respectively. As the second step begins, property gradients between zones become sharper, homogeneous distributions are retained over the large cylinder, and the small cylinder values are recovered more accurately. Minor increases in artifacts in the region outside of the large cylinder (which corresponds to the coupling fluid background) do appear, but do not detract significantly from the interior (target) portions of the images. Figures 6(a) and 6(b) shows the corresponding horizontal transects for the 50th and 70th iteration permittivity and conductivity images, respectively, along with the associated actual values.

The first and second-step images both match the actual values for the permittivity and conductivity reasonably well. For the former case, the second-step images do a better job of

recovering the properties of the breast phantom and surrounding coupling liquid, and even appear to slightly sense the separation on the left between the coupling liquid and internal inclusion despite the small separation. In fact, the recovered permittivity is quite flat across the breast phantom compared with that for the first stage image. Both techniques recover the location of the internal inclusion, whereas the two-step method recovers the actual permittivity values quite well while that for the first-step image considerably under-shoots the value.

The behavior for the conductivity counterpart is consistent with a few minor exceptions. The second-step image still provides a more accurate recovery of the properties within the breast phantom and is more accurate with respect to the inclusion. While both provide reasonable recovery of the coupling bath properties, it does not appear that either senses the short space between the inclusion and the external coupling bath.

In addition, we performed a least square analysis of the recovered properties within the breast phantom but excluding the inclusion. The region was chosen to not only encompass

FIG. 5. Reconstructed permittivity and conductivity images of the X1 phantom at 1300 MHz as a function of iteration number. The first four images correspond to results from iterations during the first-step reconstruction (which reach a total of 50 iterations), while the latter three images represent results that incorporate the second step reconstruction (which reached a total of 20 iterations). [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 6. (a) Relative permittivity and (b) conductivity transects through the images shown in Fig. 5 for the 50th and 70th iteration. [Color figure can be viewed at wileyonlinelibrary.com]

the center portion where the properties would be more likely to be homogeneous, but also the spaces closer to the edge of the zone to capture areas where the properties are changing rapidly. In each case, the error was the least squared error for the entire zone with respect to the actual values and divided by the number of sample points within the region of interest. In this case, there were 1576 image pixels within our region of interest. The errors for the permittivity images were 76.6 and 43.3 for the first and second-step images, respectively, and 6.90 and 5.49 for the corresponding conductivity images. As can be seen, the benefits for using the two-step method are considerable, with the errors decreasing 43.5 and 20.4% for the permittivity and conductivity images, respectively. Figure 7 shows magnitude and phase projections for the same iterations presented in Fig. 5 compared to the measured data for the X1 phantom.

In this case, projections are only included for receivers associated with a single transmitter, but the behavior is representative of the received data associated with all 16 transmitters. The calculated values acquire the general shape of the measured data projections, and gradually approach them during the iterative process. Curves for the intermediate step (50th iteration) are not a perfect fit, but exhibit similar characteristics in terms of positions of nulls and peaks. Computed values for the second-step iterations improve significantly over the intermediate projections, and converge to the measured fields in both magnitude and phase, which confirms the best images correspond to computed fields closely matching the measured values. Figure 8 shows images recovered from the X2 phantom for the same set of iterations.

While little change occurs from the 20th to 50th first-step iterations, a pattern of increasing visibility of the two internal cylinders and the outline of the large outer cylinder emerges in the permittivity images during second-step iterations. Enhancements from the second step are significant as both smaller cylinders become more visible and property estimates move closer to the actual values. The boundary between the

large cylinder and the bath is sharp, and the internal properties are relatively homogeneous. The corresponding conductivity images follow a similar progression, albeit with lower resolution. The first step achieves a reasonable average conductivity distribution for the large cylinder, and the larger of the two small cylinders is visible, but it blurs with an artifact from the image boundary, and the smaller cylinder is hardly visible at all. During the second step, the larger internal cylinder becomes more distinct and the smaller one is more apparent. The boundary of the outer cylinder is considerably better defined and its internal properties are largely homogeneous. In sum, while the first-step images are reasonably accurate renderings of the more complex X2 phantom, the second-step images are improved in terms of overall quality. Figures 9(a) and 9(b) show the corresponding transects through both small cylinder inclusions for the 50th and 70th iteration permittivity and conductivity images, respectively, along with the associated actual values.

The first and second-step images both match the actual values for the permittivity reasonably well, albeit smoothed across steep gradients. For this case, the second-step image does a better job of recovering the properties of the larger inclusion (somewhat overshooting the actual values) while that for the first-step image essentially only senses the location of a higher property object. The first-step image barely distinguishes the smaller object from the breast phantom while the second-step image clearly senses an elevated property zone near the inclusion location, although the recovered properties are considerably lower than the actual values, with a decreased property area between the two inclusions which approaches the correct values for the breast phantom. This mismatch is largely due to the fact that this inclusion is quite small (only 1.3 cm diameter) which is roughly only 1/12th of a wavelength with respect to the properties of the breast phantom.

The corresponding conductivity transects sense the overall trends of the actual values; however, because the inclusions

FIG. 7. Comparison of computed 1300 MHz (a) magnitude and (b) phase projections in the X1 phantom for a single transmitter as a function of receiver number for the same iterations presented in Fig. 5. The actual measured projections are shown as a reference for assessing convergence behavior. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 8. Reconstructed permittivity and conductivity images of the X2 phantom at 1300 MHz as a function of iteration. The first four images correspond to results from iterations during first-step reconstruction (which reach a total of 50 iterations), while the latter three images represent results from the second-step reconstruction (which reach a total of 20 iterations). [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 9. (a) Relative permittivity and (b) conductivity transects through both inclusions for the images shown in Fig. 8 for the 50th and 70th iteration. [Color figure can be viewed at wileyonlinelibrary.com]

are relatively small and the property contrast are quite large, the recovered traces cannot keep pace with the large gyrations of the actual properties. The 70th iteration trace appears to be more accurate in recovering the values for the larger inclusion, but at a diminished level compared with the actual. In addition, the two-step transect appears to exhibit a slight property elevation, where the smaller inclusion is located indicating that it is sensing it while there is no such indication for the one step algorithm. This smoothing across large property gradients is to be expected when utilizing a least squared process in the presence of large property gradients. Figure 10 shows the magnitude and phase projections for the same iterations presented in Fig. 7 compared to the measured data from the X2 phantom.

The behavior of the computed projections relative the measured values is similar to the X1 phantom, except that the overall size of the projections is noticeably greater; yet, the algorithm is still able to converge to the correct values even with the more demanding measurement set.

3.C. Statistical analysis

As previously noted,³⁰ parameter estimation techniques used in microwave inverse scattering problems should meet least squares criteria associated with linear regression models.^{44–46} Specifically, data error terms should be normally distributed and linearly independent. In cases where the final error terms are either heteroscedastic or not normally distributed, the final solution is compromised. In this section, we examine these characteristics for the reconstructed phantom images described in Section 3.B.

Figures 11(a) and 11(b) show histograms of the residual errors for the X1 and X2 phantoms at the end of the second-step reconstruction, respectively. Histograms were constructed from residual error vectors that remain after the last iteration of the reconstruction process, and represent the final differences between the measured and computed phase and

log amplitude values consisting of 480 terms—240 amplitudes and 240 phases (16 antennas $\times$ 15 receivers per each transmitter) expressed as natural log (log amplitude) and radian (phase) values. Histograms were formed by grouping these signed residual error terms into uniform, equally spaced bins from lowest to highest values.

Normal distributions are overlaid and were determined from the actual mean and standard deviations of the residual values. In these cases, the histograms are reasonable fits to the normal distribution with noteworthy variations from the curve. The chi-squared goodness-of-fit values for each graph are 26.7 and 23.5, respectively. The standard deviation for the X1 phantom was considerably less than its counterpart for the X2 case, and is not surprising given the X2 phantom was considerably more challenging to reconstruct because of its higher magnitude and phase projections (see Fig. 10 vs. Fig. 7). Nonetheless, relatively good normal residual error distributions are achieved in both instances, indicating the quality of the minimization process is high in both phantoms.

3.D. Comparison with single-step Tikhonov regularization reconstruction

While improvements were evident with the two-step approach relative to our standard single-step method, the two-step algorithm was also superior to a single-step Tikhonov regularization reconstruction. Figures 12(a) and 12(b) show 1300 MHz permittivity and conductivity images for both of these cases. Here, the phantom imaging problem was identical to the one in Section 3.A, except that the properties of the larger cylinder were $\epsilon_r = 7.45$ and $\sigma = 0.37$ S/m, which constitute even higher contrast with the background medium. As with all previous reconstructions, no *a priori* information was used and the iteration step size was 0.1. For both approaches, the regularization parameter was determined by the point where the least squares error function underwent an inflection when plotted against the

FIG. 10. Comparison of computed 1300 MHz (a) magnitude and (b) phase projections in the X2 phantom for a single transmitter as a function of receiver number for the same iterations presented in Fig. 7. The actual measured projections are shown as a reference for assessing convergence behavior. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 11. Histograms of the final residuals for the 1300 MHz reconstructed images of the (a) X1 and (b) X2 phantoms, respectively. True normal distributions are overlaid for reference. [Color figure can be viewed at wileyonlinelibrary.com]

regularization parameter. While the single-step approach recovers a reasonable rendering of the target, considerable smoothing and blurring of the interior inclusion occurs with the background medium properties in areas where they are in closest proximity. Minor property variations also appear around the perimeter of both images. The two-step images have less boundary smoothing and exhibit more refined recovery of the interior inclusion. Figures 12(c) and 12(d) show computed magnitude and phase projections as a function of receiver number for a single transmitter for both the single-step and two-step methods, respectively. The actual measured projections are also included as a reference. The overall shapes of the single-step curves echo their measured counterparts, but values do not correspond to the measured data nearly as closely as the two-step results. These plots confirm improvements in two-step images shown in Figures 12(a) and 12(b).

3.E. Patient exam

Figures 13(a) and 13(b) show reconstructed permittivity and conductivity images at 1300 MHz of the (a) right and (b)

left breasts of a cancer patient obtained from the two-step algorithm utilizing a 0.1 iteration step size. In this case, the woman was 55 yr old, and had radiographically scattered breasts with a centrally located 22 mm diameter tumor. For each image set, the top row contains the en face permittivity images while the bottom row presents the corresponding conductivity images. Image labels indicate the vertical planes of data acquisition spaced 1 cm apart with plane 1 (P1) located closest to the chestwall and plane 4 (P4) closer to the nipple. The size of the breast cross-section is larger for the lower numbered planes representing the shrinking breast diameter in the progression from chestwall to nipple. Consistently with previous clinical findings, permittivity images are good indicators of bulk water, and exhibit elevated properties corresponding to fibroglandular zones.^{37,38} Patterns are bilaterally symmetric, and areas of highest values span a band from the lower lateral to upper medial portions of tissue in both breasts. The corresponding conductivity images differ considerably from left to right. Left breast images are more homogeneous, and have a modestly elevated property concentration to the lower right. Right breast images are distinctly more heterogeneous, and present significantly elevated

FIG. 12. 1300 MHz reconstructed permittivity and conductivity images of the modified X1 phantom using (a) single-step Tikhonov regularization and (b) two-step reconstruction technique. Final (c) magnitude and (d) phase projections from both single-step and two-step methods for all receivers associated with a single transmitter compared to actual measurement data. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 13. Recovered permittivity (top row) and conductivity (bottom row) images at 1300 MHz of the patient's (a) right and (b) left breast for planes P1 through P4 after second step reconstruction. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 14. T1-weighted, contrast-enhanced axial image of the patient showing both breasts and chest in the (a) axial, and (b) anatomically coronal views, respectively, with the tumor clearly visible in the right breast. [Color figure can be viewed at wileyonlinelibrary.com]

property concentrations to the lower left, especially for P1 – P3, which is consistent with previous clinical findings that demonstrated conductivity images were statistically significant in terms of discriminating tumors from normal and benign breast tissue,^{37,47} and compatible with the effects of bound water fraction on tissue conductivity observed in studies by Foster et al.^{37,47–50} Figures 14(a) and 14(b) show corresponding, contrast-enhanced MR images in the axial and anatomically coronal views illustrating the relatively centralized and dispersed fibroglandular tissue along with the cancer which is clearly visible in the right breast. Note that the resolution in the coronal view is not as good as the axial counterpart. For most clinical situations, the MR spatial sampling is more refined in the axial and sagittal views because of radiologist convention. We show the coronal view in this situation because it more closely correlates with the orientation of the microwave images.

Figure 15 shows a histogram of the residuals from the images of the right breast in P3 from Fig. 13. Similar to the phantom cases, these plots illustrate that the residuals exhibit a well-behaved normal distribution. These results are

FIG. 15. Histograms of the final residuals for the reconstructed, second-step patient images from Plane 3 in Fig. 13 with true normal distributions overlaid. [Color figure can be viewed at wileyonlinelibrary.com]

representative of those observed for the other planes which have a generally decreasing standard deviation progressing from P1 to P4.

4. CONCLUSIONS

We have implemented a two-step regularization procedure for improving the quality of recovered microwave tomography images without the aid of *a priori* information. We have devised a robust, repeatable and simple approach to determining the regularization parameter, which does not add appreciably to computation time. The resulting images are improved in terms of overall property determination, feature localization and size definition. The algorithm has been tested on multi-layered phantoms with moderate to high scattering targets to assess its capabilities over a range of conditions. It was also applied to data from an actual breast examination and showed corresponding image quality improvements.

We have examined the final results in terms of well-known and accepted parameter estimation metrics. Specifically, we evaluated the distribution of residuals between the measured and computed field values at the completion of reconstruction for normalcy, which is a key requirement of the Gauss-Markov theorem. In this case, we demonstrate through histograms of the residuals that the final images meet the normal distribution criterion in both the phantom and patient imaging cases, which confirm that the algorithm is converging properly and is not biased.

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DISCLOSURE

Drs. Meaney and Paulsen are co-founders of Microwave Imaging System Technologies, Inc. in Hanover, NH USA. The company maintains several patents related to microwave imaging as does Dartmouth College for which they are also co-inventors.

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