Sound speed, attenuation, and reflection in gassy sediments

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A predictive model for acoustic dispersion and attenuation in gassy sediments is proposed. The model combines the linear solution for gas-bubble pulsations in a viscoelastic medium with corrected Biot equations involving gas-bubble pulsations. Numerical results for sound speed and attenuation are compared with predictions from Anderson and Hampton's model to demonstrate the advantages of the proposed model. The most important advantage of the current model is that it combines the dispersion regimes associated with gas-bubble pulsations and relative motion between the pore water and solid framework. The reflection coefficient at the water/gassy-sediment interface is derived based on the current model, and numerical results show that gas-bubble resonance can lead to the highest reflection. This model can also be used with a full acoustic inversion to estimate gas-bubble size distributions. © 2017 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>). [<http://dx.doi.org/10.1121/1.4996440>]

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I. INTRODUCTION

The Biot model^{1,2} is commonly used to predict the dispersion and attenuation of sound in saturated marine sediments and has been consistently validated through experimental work.^{3–5} However, a completely different acoustic response in gassy marine sediments than that in sediments, which are fully saturated with water, is observed because of the presence of gas bubbles.

Three types^{6,7} of gas bubbles in gassy sediments are generally considered as illustrated in Fig. 1 (i.e., interstitial, reservoir, and sediment-displacing bubbles).

The interstitial bubbles for type I are smaller than the interstitial spaces of the sediment framework. The gas is entirely contained in the interstitial bubbles. Gas bubbles may be free floating within the pore fluid in this instance. Zheng and Huang⁸ developed a model to predict the acoustic response in gassy sediments of this type based on the Biot model and gas-bubble pulsations in pore water.

The reservoir bubbles for type II displace only the pore water, whereas the sediment grains remain in place. Brandt⁹ presented a theoretical approach to predict the compressional wave velocity of this type of gassy sediment by considering a random stacking of spherical grains of four different sizes. Lee¹⁰ provided alternative formulations for the fluid bulk modulus and density by modifying Biot Gassmann¹¹ theory to predict the acoustic responses of types I and II.

The structure of solid grains for type III is modified because of the presence of sediment-displacing bubbles. An alternative description is that some isolated gas bubbles are surrounded by saturated marine sediments. Anderson and Hampton^{12,13} proposed the now prevalent predictive model (abbreviated A&H model) in gassy marine sediments to

predict sound speed dispersion and attenuation based on the theory of Spitzer.¹⁴ The main limit of this model is that it leads to an ambiguous inverse problem because of the plus/minus sign that enters the expression for the complex sound speed.

In a related work, Bedford and Stern¹⁵ derived a model for marine sediments that contain few gas bubbles by using a variational method and analyzed the variation in sound speed dispersion and attenuation at near frequencies close to the bubble resonance. Leighton¹⁶ presented a theory that considers liquid compressibility and thermal damping based on the non-stationary nonlinear dynamics of gas bubbles in marine sediments. Mantouka *et al.*¹⁷ developed a model to predict acoustic response in gassy sediments, which combines the bubble pulsations in a viscoelastic medium¹⁸ with the method to obtain complex speed in bubbly liquids developed by Commander and Prosperetti.¹⁹ This model predicts higher attenuation at resonance than the results predicted by the A&H model, which is caused by the additional damping terms unlike in the A&H model.

Additionally, many scholars have paid much attention on the prediction of backscatter^{20–23} from gassy sediments with bistatic models developed for sandy and mud sediments, several of which include resonance effects. Boyle and Chotiros^{20,21} modeled the propagation of two compressional waves in sediment that contains gas bubbles. However, they attributed viscoelastic damping to only the pore water viscosity, which may result in the attenuation caused by gas bubbles being underestimated.¹⁷

Two main techniques can be applied to measure the bubble size distribution and void fraction, namely, density-based techniques and acoustic-based techniques. The first class of techniques²⁴ (x-ray computed tomography scan) provides a reliable manner to measure gas bubbles in gassy sediments. The second class of techniques^{25–36} has garnered many achievements in the measurement of sound speed and

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FIG. 1. (Color online) Generic bubble classification for bubbles in sediments.

attenuation coefficient in gassy marine sediments. Lyons *et al.*²⁵ employed the scattering cross-section to predict gas bubble contents in gassy sediments. Wilkens and Richardson²⁶ performed acoustic experiments with different measurement techniques to obtain the frequency dependence of the gassy sediments of Eckernförde Bay (Baltic Sea) in a frequency range of 5–400 kHz. Best *et al.*²⁷ performed an *in situ* acoustic experiment to observe the frequency shift of gas-bubble resonance at an intertidal site. Tóth *et al.*²⁸ utilized the seismoacoustic data in ten frequency bands that cover a frequency range of 0.2–43 kHz to investigate the shallow free gas in the Bornholm Basin (Baltic Sea).

The present study proposes a model to predict the speed and attenuation of sound for two types of longitudinal wave and one shear wave in type III gassy sediments. It can also be used in an inversion model that requires simultaneous data on sound speed and attenuation. The remainder of this paper is organized as follows. Section II reviews the linear solution for gas-bubble pulsations in a viscoelastic medium derived by Yang and Church.¹⁸ Combining the linear results provided in Sec. II with the modified Biot equations derived by Zheng and Huang,⁸ a model for acoustic dispersion and attenuation in a gassy unsaturated porous medium is proposed in Sec. III. Section IV analyzes the speed and attenuation of the longitudinal and shear waves in gassy sediments. The reflection coefficient of the water/gassy-sediment is derived based on the proposed model, and associated numerical results are presented in Sec. V. Finally, Sec. VI discusses the results and presents the conclusions of this study.

II. BUBBLE DYNAMICS IN A VISCOELASTIC MEDIUM

Yang and Church¹⁸ proposed an equation to model the radial motion of a spherical gas bubble of radius $R(t)$ with the equilibrium radius R_0 in viscoelastic tissues,

$$\begin{aligned}
 & \rho_0 \left(1 - \frac{\dot{R}}{c_0} \right) R \ddot{R} + \frac{3}{2} \rho_0 \left(1 - \frac{\dot{R}}{3c_0} \right) \dot{R}^2 \\
 &= \left(1 + \frac{\dot{R}}{c_0} + \frac{R}{c_0} \frac{d}{dt} \right) \left(p_g - \frac{2\sigma}{R} - p_0 + p_A g(t) \right. \\
 & \quad \left. - \frac{4\eta_0 \dot{R}}{R} - \frac{4G_0}{3R^3} (R^3 - R_0^3) \right), \quad (1)
 \end{aligned}$$

where the dots in Eq. (1) denote the total time derivatives, ρ_0 is the density of saturated porous medium, G_0 is the real part of the shear modulus of saturated porous medium, η_0 denotes the shear viscosity, c_0 is the phase velocity of saturated porous medium, p_0 is the static pressure outside the bubble, p_g is the pressure within the bubble, and σ denotes the surface tension at the bubble wall. The equilibrium gas pressure p_{g0} can be expressed in terms of p_0 , Laplace pressure when the bubble is at equilibrium radius ($p_{\sigma,0} = 2\sigma/R_0$) and vapour pressure p_v as

$$p_{g0} = p_0 + \frac{2\sigma}{R_0} - p_v. \quad (2)$$

Equation (1) is also applicable to porous media provided that the appropriate viscoelastic constitutive relation is employed. The Voigt model is appropriate for most porous media (e.g., marine sediments) because the acoustic excitations induce small strains.^{37,38} The common approach in terms of modeling the viscoelastic behavior of marine sediments is to treat the medium as a lossy elastic solid by utilizing a complex shear modulus.³⁹ The shear viscosity of saturated porous medium is deduced from the imaginary part of the shear modulus G^* as follows:

$$G^* = G_0 + iG'_0, \quad (3a)$$

$$\eta_0 = \frac{G'_0}{\omega}, \quad (3b)$$

where ω is the angular frequency. We apply a self-consistent method introduced by Berryman^{40,41} to calculate the effective modulus of a composite medium G_0 as follows:

$$\frac{1}{K_0 + 4G_0/3} = \frac{1 - \beta}{K_s + 4G_0/3} + \frac{\beta}{K_w + 4G_0/3}, \quad (4a)$$

$$\frac{1}{G_0 + Q} = \frac{1 - \beta}{G_b + Q} + \frac{\beta}{G_w + Q}, \quad (4b)$$

$$Q = \left(\frac{G_0}{6} \right) \frac{9K_s + 8G_0}{K_s + 2G_0}. \quad (4c)$$

In Eq. (4), β is the porosity of the porous medium, K_s is the grain bulk modulus, G_b is the shear modulus of the solid

framework, and G_w is the shear modulus of pore water, which is related to the pore water viscosity with $G_w = i\omega\eta$.

The linearized solution is derived by Yang and Church,

$$R = R_0 \left(1 - \frac{p_A}{\rho_0 R_0^2 (\omega_0^2 - \omega^2 + 2ib_{\text{tot}}\omega)} \right), \quad (5)$$

where ω_0 is the resonance frequency of gas bubble pulsations in a viscoelastic medium,

$$\omega_0^2 = \left(3p_{g0} \text{Re}(\phi) - \frac{2\sigma}{R_0} + 4G_0 \right) / \left(\rho_0 R_0^2 + \frac{4\eta_0 R_0}{c_0} \right). \quad (6)$$

The damping term b_{tot} , which is sum of the viscous damping, thermal damping, radiation damping, interfacial damping, and elastic damping. These damping terms are denoted as follows:

$$b_{\text{tot}} = b_{\text{vis}} + b_{\text{p-th}} + b_{\text{rad}} + b_{\text{int}} + b_{\text{el}}, \quad (7a)$$

$$b_{\text{vis}} = 2\eta_0 / \left(\rho_0 R_0^2 + \frac{4\eta_0 R_0}{c_0} \right), \quad (7b)$$

$$b_{\text{p-th}} = \left(\frac{3p_{g0} \text{Im}(\phi)}{2\omega} \right) / \left(\rho_0 R_0^2 + \frac{4\eta_0 R_0}{c_0} \right), \quad (7c)$$

$$b_{\text{rad}} = \left(\frac{\rho_0 \omega^2 R_0^3}{2c_0} \right) / \left(\rho_0 R_0^2 + \frac{4\eta_0 R_0}{c_0} \right), \quad (7d)$$

$$b_{\text{int}} = -\sigma / (\rho_0 c_0 R_0^2 + 4\eta_0 R_0), \quad (7e)$$

$$b_{\text{el}} = 2G_s / (\rho_0 c_0 R_0 + 4\eta_0 R_0), \quad (7f)$$

where the complex polytropic function ϕ is denoted as follows:

$$\phi = \frac{3\gamma_g}{1 - 3(\gamma_g - 1)i\chi \left[(i/\chi)^{1/2} \coth(i/\chi)^{1/2} - 1 \right]}, \quad (8)$$

where $\chi = D/\omega R_0^2$, D is the thermal diffusivity of gas, and γ_g denotes the ratio of specific heats.

III. MODELING GASSY UNSATURATED POROUS MEDIA

A. Corrected Biot wave equation

The corrective poro-elastic equations derived by Zheng and Huang⁸ are shown in Eqs. (9) and (10), and the detailed derivation is given in Appendix A,

$$G_b \nabla^2 \mathbf{u} + [H - G_b] \nabla(\nabla \cdot \mathbf{u}) - C \nabla(\nabla \cdot \mathbf{v}) - \beta C \nabla \beta_g = \frac{\partial^2}{\partial t^2} (\rho \mathbf{u} - \rho_w \mathbf{v}), \quad (9)$$

$$C \nabla(\nabla \cdot \mathbf{u}) - M \nabla(\nabla \cdot \mathbf{v}) - \beta M \nabla \beta_g = \frac{\partial^2}{\partial t^2} \left(\rho_w \mathbf{u} - \frac{\alpha \rho_w}{\beta} \mathbf{v} \right) - \frac{\eta F}{\kappa} \frac{\partial \mathbf{v}}{\partial t}, \quad (10)$$

where H , C , and M denote the Biot elastic moduli, $\mathbf{u}$ denotes the absolute displacement vector of the solid framework, $\mathbf{v}$ denotes the relative displacement vector of the pore fluid

relative to the solid framework, ρ is the density of the porous medium mixture, ρ_w is the density of pore fluid, η denotes the viscosity coefficient of the pore water, β is the porosity, κ denotes the permeability, α is the tortuosity, F denotes the high frequency complex correction factor, and β_g is the dynamic gas bubble void fraction.

Comparing these variables with the original Biot wave equation, two terms that involve $\nabla \beta_g$ have been introduced, which are again related to gas-bubble pulsations. In order to eliminate this gradient of the gas-bubble void fraction (i.e., $\nabla \beta_g$), the equation of motion of the effective fluid, which involves the effective sound pressure p_{eff} and the effective displacement $\mathbf{u}_{\text{eff}} = \mathbf{u} - \mathbf{v}$, is written as

$$-\nabla p_{\text{eff}} = \rho_{\text{eff}} \ddot{\mathbf{u}}_{\text{eff}} = \rho_{\text{eff}} (\ddot{\mathbf{u}} - \ddot{\mathbf{v}}), \quad (11)$$

where the effective density is

$$\rho_{\text{eff}} = \frac{\rho \tilde{\rho}_0 - \rho_w^2}{\rho + \tilde{\rho}_0 - 2\rho_w}, \quad (12a)$$

$$\tilde{\rho}_0 = \frac{\alpha \rho_w}{\beta} - \frac{iF\eta}{\kappa\omega}. \quad (12b)$$

The gas-bubble void fraction can be denoted as $\beta_g = N \frac{4}{3} \pi R^3$ by using N to denote the bubble number per unit volume. The following equation is then obtained by combining this equation with Eq. (5) and ignoring higher-order perturbations:

$$\nabla \beta_g = - \frac{4\pi N R_0}{\rho(\omega_0^2 - \omega^2 + 2ib_{\text{tot}}\omega)} \nabla p_{\text{eff}}. \quad (13)$$

Substituting Eqs. (11) and (13) into Eqs. (9) and (10), the following equations are obtained:

$$G_b \nabla^2 \mathbf{u} + [H - G_b] \nabla(\nabla \cdot \mathbf{u}) - C \nabla(\nabla \cdot \mathbf{v}) = \frac{\partial^2}{\partial t^2} ((\rho + \gamma \tilde{\rho}) \mathbf{u} - (\rho_w + \gamma \tilde{\rho}) \mathbf{v}), \quad (14)$$

$$C \nabla(\nabla \cdot \mathbf{u}) - M \nabla(\nabla \cdot \mathbf{v}) = \frac{\partial^2}{\partial t^2} \left((\tilde{\rho} + \rho_w) \mathbf{u} - \frac{\alpha(\rho_w + \tilde{\rho})}{\beta} \mathbf{v} \right) - \frac{\eta F}{\kappa} \frac{\partial \mathbf{v}}{\partial t}, \quad (15)$$

where

$$\tilde{\rho} = \frac{4\pi N R_0 M \beta}{(\omega_0^2 - \omega^2 + 2ib_{\text{tot}}\omega)} \frac{\rho_{\text{eff}}}{\rho}, \quad \gamma = 1 - \frac{K_w}{K_s}.$$

Equations (14) and (15) can be written in another form,

$$\frac{G_b}{\text{ratio}_2} \nabla^2 \mathbf{u} + \frac{[H - G_b]}{\text{ratio}_2} \nabla(\nabla \cdot \mathbf{u}) - \frac{C}{\text{ratio}_2} \nabla(\nabla \cdot \mathbf{v}) = \frac{\partial^2}{\partial t^2} \left(\frac{(\rho + \gamma \tilde{\rho})}{(\rho_w + \gamma \tilde{\rho})} \rho_w \mathbf{u} - \rho_w \mathbf{v} \right), \quad (16)$$

$$\frac{C}{\text{ratio}_1} \nabla(\nabla \cdot \mathbf{u}) - \frac{M}{\text{ratio}_1} \nabla(\nabla \cdot \mathbf{v}) = \frac{\partial^2}{\partial t^2} \left(\rho_w \mathbf{u} - \frac{\alpha \rho_w}{\beta} \mathbf{v} \right) - \frac{\eta F}{\text{ratio}_1 \kappa} \frac{\partial \mathbf{v}}{\partial t}, \quad (17)$$

where $\text{ratio}_1 = (\rho_w + \tilde{\rho})/\rho_w$ and $\text{ratio}_2 = (\rho_w + \gamma \tilde{\rho})/\rho_w$.

It should be noted that ratio_1 and ratio_2 are both complex, with the moduli larger than 1, as long as the bubble volume fraction is not zero. Equations (16) and (17), show that the bubble volume vibration can be seen to alter the fluid compressibility, which leads to a decrease in the effective elastic modulus of the porous medium. Although the effective density of the porous medium is effectively unchanged by the presence of extremely small bubbles, there is a significant decrease in the speed of sound in the medium. Moreover, additional dissipation mechanisms are involved in the imaginary part of $\tilde{\rho}$ in Eqs. (14) and (15), which are due to the radiation, interfacial, viscous, thermal, and elastic damping of the pulsating air bubbles.

Note that if $\beta_g = 0$, there are no bubbles in the porous medium; the bubble number per unit volume is $N = 0$. In that case, Eqs. (14) and (15) become identical to the Biot equations. Thus, the present theory is tantamount to Biot theory when the gas-bubble pulsation is unaccounted for.

B. Plane waves

The following equations are derived by utilizing the potentials ϕ_s , ϕ_f , ψ_s , and ψ_f to denote the displacement vectors $\mathbf{u}$ and $\mathbf{v}$:

$$\mathbf{u} = \nabla \phi_s + \nabla \times \psi_s, \quad (18)$$

$$\mathbf{v} = \nabla \phi_f + \nabla \times \psi_f. \quad (19)$$

Substituting Eqs. (18) and (19) into Eqs. (14) and (15), the following equations can be obtained:

$$H\nabla^2 \phi_s - C\nabla^2 \phi_f = -\omega^2(\rho + \gamma\tilde{\rho})\phi_s + \omega^2(\rho_w + \gamma\tilde{\rho})\phi_f, \quad (20)$$

$$\begin{aligned} C\nabla^2 \phi_s - M\nabla^2 \phi_f \\ = -\omega^2(\rho_w + \tilde{\rho})\phi_s + \omega^2\left(\frac{\alpha}{\beta}\rho_w + \tilde{\rho}\right)\phi_f + \frac{i\omega F\eta}{\kappa}\phi_f, \end{aligned} \quad (21)$$

$$G_b\nabla^2 \psi_s = -\omega^2(\rho + \gamma\tilde{\rho})\psi_s + \omega^2(\rho_w + \gamma\tilde{\rho})\psi_f, \quad (22)$$

$$-\frac{i\omega F\eta}{\kappa}\phi_f = -\omega^2(\rho_w + \tilde{\rho})\psi_s + \omega^2\left(\frac{\alpha}{\beta}\rho_w + \tilde{\rho}\right)\psi_f. \quad (23)$$

Consider that the longitudinal waves are denoted by the following:

$$\phi_s = e^{i\mathbf{k}_q \cdot \mathbf{r}}, \quad q = 1, 2, \quad (24)$$

$$\phi_f = \gamma_q e^{i\mathbf{k}_q \cdot \mathbf{r}}, \quad q = 1, 2, \quad (25)$$

where $q = 1$ denotes the fast wave, whereas $q = 2$ denotes the slow wave.

When Eqs. (24) and (25) are inserted in Eqs. (20) and (21), as well as considering that the corresponding determinant must be zero, the following dispersion equation of longitudinal waves is obtained:

$$\begin{aligned} \left(Hk_q^2 - \rho\omega^2 - \gamma\tilde{\rho}\omega^2\right) \left(\left(\frac{\alpha\rho_w}{\beta} + \tilde{\rho}\right)\omega^2 - Mk_q^2 + i\frac{F\eta\omega}{\kappa}\right) \\ + \left(Ck_q^2 - \rho_w\omega^2 - \tilde{\rho}\omega^2\right) \left(Ck_q^2 - \rho_w\omega^2 - \gamma\tilde{\rho}\omega^2\right) = 0. \end{aligned} \quad (26)$$

The ratio of the two longitudinal displacement potentials ϕ_f/ϕ_s is as follows:

$$\gamma_q = \frac{(Hk_q^2 - \rho\omega^2 - \gamma\tilde{\rho}\omega^2)}{(Ck_q^2 - \rho_w\omega^2 - \gamma\tilde{\rho}\omega^2)}, \quad q = 1, 2. \quad (27)$$

The following equations can be similarly obtained for the shear wave:

$$\psi_s = \mathbf{e}_h(\mathbf{K})e^{i\mathbf{k}_q \cdot \mathbf{r}}, \quad (28)$$

$$\psi_f = \gamma_t \mathbf{e}_h(\mathbf{K})e^{i\mathbf{k}_q \cdot \mathbf{r}}, \quad (29)$$

$$\begin{aligned} (G_b k_t^2 - (\rho + \gamma\tilde{\rho})\omega^2) \left(\omega^2\left(\frac{\alpha\rho_w}{\beta} + \tilde{\rho}\right) + i\frac{F\eta\omega}{\kappa}\right) \\ + (\rho_w + \gamma\tilde{\rho})(\tilde{\rho} + \rho_w)\omega^4 = 0, \end{aligned} \quad (30)$$

$$\gamma_t = \frac{G_b k_t^2 - (\rho + \gamma\tilde{\rho})\omega^2}{(\rho_w + \gamma\tilde{\rho})\omega^2}. \quad (31)$$

Solving Eq. (26), the wave numbers of longitudinal waves can be obtained,

$$k_q = \sqrt{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}, \quad q = 1, 2. \quad (32)$$

The (+) sign corresponds to the fast wave ($q = 1$), whereas the (−) sign corresponds to the slow wave ($q = 2$). The parameters in Eq. (32) are as follows:

$$a = C^2 - HM, \quad (33a)$$

$$b = Ha_2 - Ma_1 - Ca_3 + Ca_4, \quad (33b)$$

$$c = a_1a_2 - a_3a_4, \quad (33c)$$

$$a_1 = -\omega^2(\rho + \gamma\tilde{\rho}), \quad (33d)$$

$$a_2 = \omega^2\left(\frac{\alpha}{\beta}\rho_w + \tilde{\rho}\right) + \frac{i\omega F\eta}{\kappa}, \quad (33e)$$

$$a_3 = \omega^2(\rho_w + \gamma\tilde{\rho}), \quad (33f)$$

$$a_4 = -\omega^2(\rho_w + \tilde{\rho}). \quad (33g)$$

The wave number for the shear wave is obtained by solving Eq. (30),

$$k_t = \sqrt{(a_3a_4/a_2 - a_1)/G_b}. \quad (34)$$

The phase velocity and attenuation in dB/unit length for the shear wave are denoted as follows:

$$c_t = 1/\text{Re}(k_t/\omega), \quad (35)$$

$$\alpha_t^{(m)} = \frac{20}{\ln 10} \text{Im}(k_t). \quad (36)$$

The phase velocity and attenuation in dB/unit length for the longitudinal waves are denoted as

$$c_q = 1/\text{Re}(k_q/\omega), \quad (37)$$

$$\alpha_q^{(m)} = \frac{20}{\ln 10} \text{Im}(k_q). \quad (38)$$

IV. EXAMPLE COMPUTATIONS

This section applies the predictive model of sound speed and attenuation in a gassy unsaturated porous medium to gassy sediments by setting the appropriate input parameters. The physical and geometrical parameters of the gas sediments are presented in Table I. The simulation input parameters are selected in their empirical ranges. The sound speed and attenuation for longitudinal and shear waves are computed by employing these input parameters and Eqs. (35)–(38).

Figure 2 shows a comparison of the sound speed and attenuation for the fast wave between the proposed model and the A&H model. Figure 3 shows the acoustic response to the fast wave according to the proposed model in this paper. The results in Figs. 2 and 3 indicate similar frequency dependencies for sound speed and attenuation.

For the insonifying frequency below the gas-bubble resonance frequency, the phase velocity is lower, and the attenuation coefficient is significantly higher than that in non-

TABLE I. Model parameters.

	Model parameters	Units	Value
Biot model parameters	Grain diameter	d/mm	0.781
	Grain density	$\rho_s/\text{kg m}^{-3}$	2465
	Grain bulk modulus	K_s/Pa	3.6×10^{10}
	Water density	$\rho_w/\text{kg m}^{-3}$	998.2
	Water bulk modulus	K_w/Pa	2.193×10^9
	Water viscosity	$\eta/\text{Pa s}$	1.002×10^{-3}
	porosity	β	0.370
	permeability	κ/m^2	2.54×10^{-10}
	Pore size	a_p/m	1.53×10^{-4}
	Tortuosity	α	1.25
	Frame bulk modulus	K_b/Pa	5.31×10^7
	Frame shear modulus	μ/Pa	5.58×10^6
	Longitudinal logarithmic decrement	δ_l	0.15
	Shear logarithmic decrement	δ_s	0.15
Air bubble parameters	Gas density	$\rho_g/\text{kg m}^{-3}$	1.1691
	Gas sound velocity	$c_g/\text{m s}^{-1}$	340
	Hydrostatic pressure at infinity	P_∞/Pa	1.01×10^5
	Thermal diffusivity of the gas	$D/\text{m}^2 \text{s}^{-1}$	2.4×10^{-5}
	Surface tension	$\sigma/\text{N m}^{-1}$	72.75×10^3
	Ratio of specific heats	γ_g	1.4
	Specific heat of gas at constant pressure	$s_p/\text{J}(\text{kg K})^{-1}$	1001.73
	Thermal conductivity of gas	$C_g/\text{W}(\text{m K})^{-1}$	0.0241

FIG. 2. (Color online) (a) Sound speed and (b) attenuation as frequency functions for a series of mono-dispersed bubble populations, which all have a void fraction of 10^{-5} (i.e., the radius for each bubble labeled in the figure). The solid line denotes the results calculated by the proposed model, whereas the dashed line denotes the results of the A&H model.

gassy sediments. A transition zone near the resonance can be observed, in which the phase velocity rapidly increases. In particular, the phase velocity can highly exceed the non-gassy velocity at the resonance frequency, but this phenomenon was not observed in the experiment. The attenuation is highest at frequencies close to the gas-bubble resonance frequency, which depends mainly on the bubble radius. Above the resonance, the phase velocity approaches a constant value similar to the non-gassy sediment, whereas the attenuation is higher than non-gassy attenuation and increases with the increasing frequency.

Notably, significant differences between the predictions of these two models are shown in Fig. 2(b). The current model predicts higher attenuation than the results predicted by the A&H model. The main reason for this scenario is that the current model involves more dissipation regimes: the relative motion between the pore water and solid frame, elastic damping, and interface damping. The main advantage of the current model is that its expression that predicts the velocity

FIG. 3. (Color online) Variation in the (a) fast wave velocity and (b) attenuation with frequency for a range of gas contents (constant gas-bubble radius of 5 mm).

does not contain sign ambiguity and can be applied to an inversion model, which requires simultaneous information on sound speed and attenuation data.

As previously mentioned in Sec. II, the proposed model can be used to predict the phase velocity and attenuation coefficient of a shear wave (Fig. 4). The shear wave response curves show similar frequency-dependent behavior to that of a fast wave. The phase velocity is lower than that in gas-free sediments because the insonifying frequency is lower than the resonance frequency of the bubbles. The phase velocity at the resonance can significantly exceed the non-gassy velocity, and attenuation is the highest.

In particular, the larger the gas-bubble void fraction, the higher the resonance frequency. Considering the fast wave acoustical response in Fig. 3, the larger the gas-bubble void fraction, the wider the resonance frequency band. Therefore, the shift in the resonance frequency for a shear wave is consistent with the resonance frequency bandwidth for a fast wave as the gas-bubble void fraction increases.

FIG. 4. (Color online) Variation in the (a) shear wave phase velocity and (b) attenuation coefficient with frequency for a range of gas contents (constant gas-bubble radius of 5 mm).

V. REFLECTION OF ACOUSTIC WAVES AT A WATER/GASSY-SEDIMENT INTERFACE

Consider a homogeneous longitudinal wave in the plane incident at angle θ_i to a gassy sediment half-space (Fig. 5). The reflection coefficient can be solved from the “open pore” boundary condition,⁴² whose detailed derivation can be referenced in Appendix B.

The reflection coefficient V_{ww} at the interface that connects water and gassy sediment can be obtained by solving the simultaneous Eq. (B13) numerically with the parameters in Table I. The reflection coefficient and reflection loss with the incident angle for a range of gas contents below the resonance are shown in Fig. 6. Section IV states that the phase velocity of a gassy sediment is lower than that of a gas-free sediment below resonance. Therefore, the acoustic impedance of a gassy sediment is likewise lower than that of a gas-free sediment. The presence of gas leads to smaller reflection coefficients and more acoustic penetration. The phase

FIG. 5. Diagram of the incident, reflected, and refracted potentials.

FIG. 6. (Color online) Variation in the (a) reflection coefficient and (b) reflection loss with the incident angle for a range of gas contents (constant gas-bubble radius of 5 mm and frequency of 100 Hz).

velocity for a gas-bubble void fraction of 10^{-3} is lower than the velocity of water below the resonance. Thus, a complete wave transmission occurs at the intromission angle [see the dashed-dotted line in Fig. 6(a)].

The variations in the normal reflection coefficient and reflection loss with frequency for a range of gas contents are shown in Fig. 7. A large gas content induces poor acoustic penetration at the gas-bubble resonance. For marine sediments that contain relatively small amount of gas, the variation in the normal reflection coefficient with frequency is similar to the variation in phase velocity.

VI. DISCUSSION AND CONCLUSIONS

This study reviews a nonlinear model for gas-bubble pulsations in a viscoelastic medium. The linear solution was applied into the corrected Biot wave equation to obtain the effective wave equation. This model was used to predict the

FIG. 7. (Color online) Variation in the normal (a) reflection coefficient and (b) reflection loss with frequency for a range of gas contents (constant gas-bubble radius of 5 mm).

sound speed and attenuation of longitudinal and shear waves in the presence of linear bubble pulsations in gassy marine sediments. Numerical results for the phase velocity and attenuation coefficient show similar frequency dependencies for longitudinal and shear waves. By contrast, the results for the attenuation of fast waves show a significant difference due to the additional damping regimes comparing to the A&H model. The present model has two major advantages over the existing A&H model for gassy sediments: (1) this current model can predict sound speeds, attenuation coefficients, and reflection coefficients for fast, slow, and shear waves. (2) It can also be utilized in inversion models that require simultaneous data on sound speed and attenuation.

This study accounts for gas-bubble pulsations in a viscoelastic host medium, whose parameters in Eq. (1) were taken from saturated porous medium. Effective parameters should be employed in Eq. (1) to consider the effect of the other bubbles in the averaging volume of a new bubble. The most important parameter is the effective phase velocity of the host medium, which has been discussed by Kargl⁴³ for linear acoustics in bubbly liquids.

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APPENDIX A: DETAILED DERIVATION OF THE CORRECTIVE PORO-ELASTIC EQUATIONS INVOLVING GAS BUBBLE PULSATIONS

Consider the first Biot equation of motion as that of a porous medium

$$\tau_{ij,j} = \rho \ddot{u}_i - \rho_f \ddot{v}_i, \quad (\text{A1})$$

where $v_i = \beta(u_i - V_i)$ is a component of $\mathbf{v}$, which is the displacement vector of the pore fluid relative to the solid framework in direction i ; β is the porosity; u_i and V_i are the absolute fluid displacements in the skeleton and pores, respectively, in the direction i ; ρ is the density of the porous medium; ρ_f is the density of the pore fluid; and $\tau_{ij,j}$ is the stress on the porous medium.

Another equation of motion describes the relations between the fluid pressure and displacements in the porous medium

$$-\partial_i p = \rho_f \ddot{u}_i - \frac{\alpha \rho_f}{\beta} \ddot{v}_i + \frac{\eta}{\kappa} \dot{v}_i, \quad (\text{A2})$$

where p is the pore fluid sound pressure, η is the viscosity of the liquid, α is the tortuosity, and κ denotes the permeability.

Then we consider the relationship between the stress and strain of the porous medium,

$$\tau_{ij} = \lambda e \delta_{ij} + 2G_b \varepsilon_{ij} - \gamma \delta_{ij} p, \quad (\text{A3})$$

where λ is the Lamé constant, G_b is the shear modulus of the solid frame, and e and ε are the bulk and shear strains, respectively, of the frame. The bulk strain of the pore water is denoted by the following:

$$-d\varepsilon_w = \frac{d\rho_w}{\rho_w} = \frac{dp}{K_w}, \quad (\text{A4})$$

where ρ_w and K_w are the density and bulk modulus, respectively, of the pore water.

If p' is applied to denote the effective mean pressure on the solid frame, then the bulk strains of the frame and grains can be expressed as follows:

$$-d\varepsilon_v = \nabla \cdot \mathbf{u} = \frac{d\rho_d}{\rho_d} = \frac{dp'}{K_b} + \frac{dp}{K_s}, \quad (\text{A5})$$

$$-d\varepsilon_s = \frac{d\rho_s}{\rho_s} = \frac{dp}{K_s} + \frac{dp'}{(1-\beta)K_s}, \quad (\text{A6})$$

where $\rho_d = (1-\beta)\rho_s$, $d\rho_d = (1-\beta)d\rho_s - \rho_s d\beta$, K_b is the bulk modulus of the frame, K_s is the bulk modulus of the grains, and ρ_s is the grain density.

The Eulerian description is adopted at this point for the porous-medium seepage problems. A control volume Θ in a porous medium of porosity β is saturated with water and gas as well as surrounded by a surface Ω . A surface element $d\Omega$ is selected with a normal vector $\mathbf{n}$ that always points outward. The displacement of the pore fluid through the surface element during a time Δt is $\mathbf{V}$. The mass of the pore fluid through the surface element in time Δt can then be obtained as $\beta \rho_f \mathbf{V} \cdot \mathbf{n} d\Omega$. Hence, the total mass of the pore fluid through the surface Ω is

$$\oint_{\Omega} \beta \rho_f \mathbf{V} \cdot \mathbf{n} d\Omega.$$

Alternatively, a volume element $d\Theta$ is selected. The density of the pore liquid changes with time in a non-steady state. The mass increment of the pore fluid due to a density variation in time Δt is $d(\beta \rho_f) d\Theta$. Therefore, the total mass increment in this volume is

$$\iiint_{\Theta} d(\beta \rho_f) d\Theta.$$

Invoking the conservation of mass, the following equation is obtained:

$$\iiint_{\Theta} d(\beta \rho_f) d\Theta + \oint_{\Omega} \beta \rho_f \mathbf{V} \cdot \mathbf{n} d\Omega = 0, \quad (\text{A7})$$

where $\rho_f = (1-\beta_g)\rho_w + \beta_g \rho_g \approx (1-\beta_g)\rho_w$, and β_g denotes the volume fraction of the gas to pore fluid.

The surface integral in Eq. (14) can be transformed into a volume integral based on divergence theorem, which leads to the following:

$$\iiint_{\Theta} [d(\beta \rho_f) + \nabla \cdot (\beta \rho_f \mathbf{V})] d\Theta = 0. \quad (\text{A8})$$

Given the continuity of the volume selected in this study, the integrand in Eq. (A8) must be equal to 0. Hence, the continuity equation can be cast in differential form as follows:

$$d\beta + \beta \frac{d\rho_f}{\rho_f} - \nabla \cdot \mathbf{v} + \beta \nabla \cdot \mathbf{u} = 0. \quad (\text{A9})$$

Assuming that there is relatively little gas in the pore fluid (i.e., $\beta_g \ll 1$), the effective pore-fluid density can be approximated as $\rho_f = (1 - \beta_g)\rho_w + \beta_g\rho_g \approx (1 - \beta_g)\rho_w$. Substituting this relation in Eq. (A9), we can derive the following:

$$d\beta + \beta \frac{d\rho_f}{\rho_f} - \beta d\beta_g - \nabla \cdot \mathbf{v} + \beta \nabla \cdot \mathbf{u} = 0. \quad (\text{A10})$$

Combining Eqs. (A5), (A6), and (A10), the continuity equation incorporating the bubble volume vibration is

$$-\dot{p} = C \operatorname{div} \dot{\mathbf{u}} - M \operatorname{div} \dot{\mathbf{v}} - M\beta \cdot \dot{\beta}_g, \quad (\text{A11})$$

where C and M are the Biot elastic moduli, and the over-dots denote the total time derivatives. The variable $\dot{\beta}_g$ denotes the time derivative of the bubble volume fraction, which involves the effect of bubble vibration.

Combining Eqs. (A2)–(A4) and (A11), the following corrective poro-elastic equations of motion are obtained:

$$G_b \nabla^2 \mathbf{u} + [H - G_b] \nabla (\nabla \cdot \mathbf{u}) - C \nabla (\nabla \cdot \mathbf{v}) - \beta C \nabla \beta_g = \frac{\partial^2}{\partial t^2} (\rho \mathbf{u} - \rho_w \mathbf{v}), \quad (\text{A12})$$

$$C \nabla (\nabla \cdot \mathbf{u}) - M \nabla (\nabla \cdot \mathbf{v}) - \beta M \nabla \beta_g = \frac{\partial^2}{\partial t^2} \left(\rho_w \mathbf{u} - \frac{\alpha \rho_w}{\beta} \mathbf{v} \right) - \frac{\eta F}{\kappa} \frac{\partial \mathbf{v}}{\partial t}. \quad (\text{A13})$$

APPENDIX B: MODELING REFLECTION OF ACOUSTIC WAVES AT A WATER/GASSY-SEDIMENT INTERFACE

The effective Biot moduli $\tilde{C}$, $\tilde{M}$, and $\tilde{H}$ are as follows:

$$\tilde{C} = \frac{C}{1 + \frac{\tilde{\rho}}{\rho_{\text{eff}}} \frac{\rho_{\text{eff}}(1 - \gamma_v)}{\rho_w - \rho_w \frac{\alpha \gamma_v}{\beta} + \frac{\eta F \gamma_v}{i \kappa \omega}}}, \quad (\text{B1})$$

$$\tilde{M} = \frac{M}{1 + \frac{\tilde{\rho}}{\rho_{\text{eff}}} \frac{\rho_{\text{eff}}(1 - \gamma_v)}{\rho_w - \rho_w \frac{\alpha \gamma_v}{\beta} + \frac{\eta F \gamma_v}{i \kappa \omega}}}, \quad (\text{B2})$$

$$\tilde{H} = \gamma^2 \tilde{M} + K_f + 4\mu/3. \quad (\text{B3})$$

The displacement potentials of the incident and reflected acoustic waves are considered as follows:

$$\phi_i = e^{i\mathbf{k}_i \cdot \mathbf{r}}, \quad (\text{B4})$$

$$\phi_r = V_{w1} e^{i\mathbf{k}_r \cdot \mathbf{r}}, \quad (\text{B5})$$

where $\mathbf{k}_i = \mathbf{i}k_x + \mathbf{j}k_{iz}$, $\mathbf{k}_r = \mathbf{i}k_x - \mathbf{j}k_{iz}$, $k_x = (\omega/c_w) \sin \theta_i$, and $k_{iz} = (\omega/c_w) \cos \theta_i$. The scalar potentials in the gassy sediments are denoted by the following:

$$\phi_s = V_{w1} e^{i\mathbf{k}_1 \cdot \mathbf{r}} + V_{w2} e^{i\mathbf{k}_2 \cdot \mathbf{r}}, \quad (\text{B6})$$

$$\phi_f = \gamma_1 V_{w1} e^{i\mathbf{k}_1 \cdot \mathbf{r}} + \gamma_2 V_{w2} e^{i\mathbf{k}_2 \cdot \mathbf{r}}, \quad (\text{B7})$$

where $\mathbf{k}_q = \mathbf{i}k_x + \mathbf{j}k_{qz} = \mathbf{i}k_q \sin \theta_q + \mathbf{j}k_q \cos \theta_q$. The vector potentials in the gassy sediments are denoted by the following:

$$\psi_s = \mathbf{e}_h^- V_{wt} e^{i\mathbf{k}_t \cdot \mathbf{r}}, \quad (\text{B8})$$

$$\psi_f = \mathbf{e}_h^- \gamma_t V_{wt} e^{i\mathbf{k}_t \cdot \mathbf{r}}, \quad (\text{B9})$$

where $\mathbf{k}_t = \mathbf{i}k_x + \mathbf{j}k_{tz} = \mathbf{i}k_t \sin \theta_t + \mathbf{j}k_t \cos \theta_t$.

The “open-pore” boundary condition⁴² is employed to solve the reflection problem at a water/gassy-sediment interface. The four boundary conditions are the continuity of fluid movement in and out of the solid framework, normal traction, fluid sound pressure, and tangential traction. At the interface $z = 0$, the four boundary conditions are denoted as follows:

$$\frac{\partial \phi_i}{\partial z} + \frac{\partial \phi_r}{\partial z} = \frac{\partial \phi_s}{\partial z} + \frac{\partial \psi_s}{\partial x} - \frac{\partial \phi_f}{\partial z} - \frac{\partial \psi_f}{\partial x}, \quad (\text{B10})$$

$$\tilde{H} \left(\frac{\partial^2 \phi_s}{\partial x^2} + \frac{\partial^2 \phi_s}{\partial z^2} \right) - 2G_b \left(\frac{\partial^2 \phi_s}{\partial x^2} - \frac{\partial^2 \psi_s}{\partial x \partial z} \right) - \tilde{C} \left(\frac{\partial^2 \phi_f}{\partial x^2} + \frac{\partial^2 \phi_f}{\partial z^2} \right) = -\rho_f \omega^2 (\phi_i + \phi_r), \quad (\text{B11})$$

$$\tilde{M} \left(\frac{\partial^2 \phi_f}{\partial x^2} + \frac{\partial^2 \phi_f}{\partial z^2} \right) - \tilde{C} \left(\frac{\partial^2 \phi_s}{\partial x^2} + \frac{\partial^2 \phi_s}{\partial z^2} \right) = \rho_f \omega^2 (\phi_i + \phi_r), \quad (\text{B12})$$

$$2G_b \frac{\partial^2 \psi_s}{\partial x \partial z} - G_b \left(\frac{\partial^2 \psi_s}{\partial z^2} - \frac{\partial^2 \psi_s}{\partial x^2} \right) = 0. \quad (\text{B13})$$

Substituting the displacement-potential Eqs. (B2)–(B9) into these four boundary conditions, the following simultaneous equations are obtained:

$$AX = D, \quad (\text{B14})$$

where

$$\mathbf{X} = \begin{bmatrix} V_{ww} \\ V_{w1} \\ V_{w2} \\ V_{wt} \end{bmatrix}, \quad (\text{B15})$$

$$\mathbf{D} = \begin{bmatrix} k_{iz} \\ \rho_w \omega^2 \\ \rho_w \omega^2 \\ 0 \end{bmatrix}, \quad (\text{B16})$$

$$\mathbf{A} = \begin{bmatrix} k_{iz} & k_{1z}(1 - \gamma_1) & k_{2z}(1 - \gamma_2) & -k_x(1 - \gamma_t) \\ -\rho_w \omega^2 & k_1^2(\tilde{H} - \gamma_1 \tilde{C}) - 2G_b k_x^2 & k_2^2(\tilde{H} - \gamma_2 \tilde{C}) - 2G_b k_x^2 & -2G_b k_{tz} k_x \\ -\rho_w \omega^2 & k_1^2(\tilde{C} - \gamma_1 \tilde{M}) & k_2^2(\tilde{C} - \gamma_2 \tilde{M}) & 0 \\ 0 & 2G_b k_x k_{1z} & 2G_b k_x k_{2z} & G_b(k_{tz}^2 - k_x^2) \end{bmatrix}. \quad (\text{B17})$$

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