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Golden-Ratio Rotated Stack-of-Stars Acquisition for Improved Volumetric MRI

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Abstract

Purpose—To develop and evaluate an improved stack-of-stars radial sampling strategy for reducing streaking artifacts.

Methods—The conventional stack-of-stars sampling strategy collects the same radial angle for every partition (slice) encoding. In an under-sampled acquisition, such an aligned acquisition generates coherent aliasing patterns and introduces strong streaking artifacts. We show that by rotating the radial spokes in a golden-angle manner along the partition encoding direction, the aliasing pattern is modified, resulting in improved image quality for gridding and more advanced reconstruction methods. Computer simulations were performed and phantom as well as *in-vivo* images for three different applications were acquired.

Results—Simulation, phantom, and *in-vivo* experiments confirmed that the proposed method was able to generate images with less streaking artifact and sharper structures based on under-sampled acquisitions in comparison with the conventional aligned approach at the same acceleration factors. By combining parallel imaging and compressed sensing in the reconstruction, streaking artifacts were mostly removed with improved delineation of fine structures using the proposed strategy.

Conclusion—We present a simple method to reduce streaking artifacts and improve image quality in 3D stack-of-stars acquisitions by re-arranging the radial spoke angles in the 3D partition direction, which can be used for rapid volumetric imaging.

Keywords

Stack-of-stars; golden-ratio; 3D gridding; parallel imaging; compressed sensing

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Introduction

Radial sampling is a widely used k-space sampling trajectory for fast MRI. It offers several advantages compared with Cartesian k-space sampling, including improved robustness in the presence of motion (1) due to the continuous update of the k-space center, and its suitability for high under-sampling factors due to its relatively benign under-sampling artifacts (2). The disadvantages of radial sampling include decreased SNR efficiency (3), increased sensitivity to system imperfection and trajectory errors (4–6), and more complex image reconstruction (7). Radial sampling has been used in many MRI applications, such as MR angiography (2,8), cardiac imaging (9), phase-contrast imaging (10) and abdominal imaging (11). To acquire the 3D k-space data using radial sampling, two acquisition strategies are commonly used: 3D stack-of-stars (SOS) (2) and 3D radial (Koosh-Ball) (8,9).

In the conventional 3D SOS acquisition strategies (2,11–13), the azimuthal angle of the radial spokes does not vary in the partition (i.e. 3D Cartesian phase encoding) direction such that if a radial spoke with angle θ is sampled in one partition, it is also sampled in all the other partitions. Recently, Chen L et al. (14) and Wech T et al. (15) proposed to rotate the radial spokes along the partition (slice) direction, but both did not provide sufficient implementation details on how the radial spokes were rotated. Other approaches such as SOS-CAIPIRINHA (16) and rotated stack-of-spirals (17) vary the angle of the radial spokes or spiral interleaves in a linear fashion in the partition direction; however, we show in this report that such a strategy is sub-optimal. We propose a rotated SOS (RSOS) sampling method in which the radial spokes are rotated in a golden-ratio (18) fashion in the partition encoding direction, i.e. RSOS-GR. We show that our RSOS-GR strategy creates varying aliasing patterns along the partition direction, and such a varying aliasing pattern improves the condition of the inverse problem in a parallel imaging reconstruction and improves the incoherence of the sampling function in a compressed sensing reconstruction. We compare the conventional aligned SOS (ASOS), RSOS with a linear rotation in the partition direction (RSOS-Linear), and the proposed RSOS-GR using computer simulation, phantom and *in-vivo* studies.

Method

For the sake of simplicity and clarity, we only consider two common cases of in-plane view order in radial sampling: linear and golden angle (18), as our work mainly pertains to the angle variations in the partition direction. It is straightforward to generalize the proposed RSOS approach to more sophisticated in-plane radial sampling strategies such as tiny golden angle radial sampling (19) and segmented golden angle radial sampling (20).

In this study, five different sampling strategies were studied: 1) the conventional ASOS where all the partitions have the same radial spoke angles; 2) $\text{RSOS}_{(\text{Lin})}$ -Linear where the angle arrangement is linear both in-plane and across the partitions; 3) $\text{RSOS}_{(\text{Lin})}$ -GR, which is the proposed GR rotation in the partition direction applied to linear in-plane radial angles; 4) $\text{RSOS}_{(\text{GR})}$ -Linear, which includes GR in-plane radial spokes and linear rotation in the partition direction; 5) $\text{RSOS}_{(\text{GR})}$ -GR, which includes GR in-plane radial spokes and the proposed GR rotation in the partition direction. An example of the radial spoke angle

arrangement for all five strategies is shown in Figure 1. For in-plane linear view order, the angle θ_i of the i^{th} spoke out of N_r total spokes is calculated as: $\theta_i^L = \frac{\pi}{N_r} * (i - 1)$, $i = 1, 2, \dots, N_r$. For in-plane GR view order, the angle θ_i of the i^{th} spoke out of N_r spokes is calculated as: $\theta_i^G = \text{mod} \left((i - 1) * \pi * \frac{\sqrt{5} - 1}{2}, \pi \right)$, $i = 1, 2, \dots, N_r$. In the proposed GR rotation in the partition direction, non-zero azimuthal angle offsets that change across the partitions are introduced as follows:

$$\varphi_G(j) = \text{mod} \left((j - 1) * \frac{\pi}{N_r} * \frac{\sqrt{5} - 1}{2}, \frac{\pi}{N_r} \right), j = 1, 2, \dots, N_{PE}, \quad [1]$$

where N_{PE} is the total number of partitions and j is the partition index. For the linear rotation in the partition direction, which is used in the $\text{RSOS}_{(\text{Lin})}$ -Linear and $\text{RSOS}_{(\text{GR})}$ -Linear

strategies, the azimuthal angle offsets are: $\varphi_L(j) = \frac{j - 1}{N_{PE}} * \frac{\pi}{N_r}$, $j = 1, 2, \dots, N_{PE}$. Based on the above definitions, the angle for the i^{th} radial spoke in the j^{th} partition will be θ_i^L for ASOS, $\theta_i^L + \varphi_L(j)$ for $\text{RSOS}_{(\text{Lin})}$ -Linear, $\theta_i^L + \varphi_{GR}(j)$ for $\text{RSOS}_{(\text{Lin})}$ -GR, $\theta_i^G + \varphi_L(j)$ for $\text{RSOS}_{(\text{GR})}$ -Linear, and $\theta_i^G + \varphi_G(j)$ for $\text{RSOS}_{(\text{GR})}$ -GR. To reduce eddy currents and the resultant phase errors, spokes with the same in-plane index i (as defined above) in all partitions were acquired first before acquiring the next subset of spokes with index $i + 1$.

Computer Simulation

The point spread functions (PSFs) of the conventional ASOS, $\text{RSOS}_{(\text{Lin})}$ -Linear, $\text{RSOS}_{(\text{Lin})}$ -GR, $\text{RSOS}_{(\text{GR})}$ -Linear and $\text{RSOS}_{(\text{GR})}$ -GR were first compared using simulations. The PSF for each of the five radial k-space trajectories was calculated following three steps: 1) Each sampled point was set to unit value; 2) The resultant k-space data were subsequently interpolated onto a Cartesian grid using a 3D Kaiser-Bessel kernel (21) in conjunction with appropriate density compensation derived from the Ram-Lak method; 3) The PSF was obtained as the Fourier transform of the gridded Cartesian k-space data. The k-space trajectory consisted of full spokes with base resolution of 256, partition number $N_{PE} = 36$ and was gridded onto a matrix size of $256 \times 256 \times 36$. The spoke angles on each partition were calculated according to the aforementioned five strategies. The PSF calculation was performed for data sets with 20 to 100 radial spokes per partition in increments of 5 spokes to evaluate the effect of the number of spokes on k-space sampling. The following two metrics were used to quantitatively assess the simulated PSFs for conventional gridding reconstructions and compressed sensing algorithms, respectively: 1) The ratio of the main lobe energy to the sum of energy of side-lobes was calculated as a PSF quality metric for conventional gridding reconstructions; 2) The ratio of the main lobe magnitude of the PSF to the standard deviation of the PSF side-lobes magnitude (22) was calculated as a measure of the incoherence of the 3D PSF, which has been shown to affect the image reconstruction quality using compressed sensing algorithms (13,22).

Phantom Experiment

To evaluate the performance of the proposed $\text{RSOS}_{(\text{Lin/GR})}$ -GR sampling strategy and compare it with ASOS and $\text{RSOS}_{(\text{Lin/GR})}$ -Linear, a 3D spoiled gradient recalled echo (GRE) sequence was modified to implement the five acquisition strategies. Phantom imaging and all *in-vivo* studies in this work were performed on a 3.0T MRI scanner (Prisma, Siemens Medical Solutions, Erlangen, Germany). Relevant imaging parameters are listed in Table S1 in the online supporting materials. A fully-sampled reference data set (400 spokes per partition, ASOS) and fifteen additional prospectively under-sampled data sets (20 spokes, 40 spokes, and 80 spokes per partition, using the aforementioned five strategies) were acquired. All acquired data sets were reconstructed with the 3D gridding approach as mentioned in the “Computer Simulation” section.

In-vivo Experiment

The five strategies were tested in three different applications: brain imaging, abdominal imaging, and dynamic MR angiography (dMRA) using arterial spin labeling (ASL) (23). The sequence used in the brain imaging and abdominal imaging was identical to the one used in the phantom study, while the sequence used in dMRA-ASL imaging was modified based on a balanced steady-state free precession (bSSFP) sequence. This study was approved by our institutional review board and written informed consent was obtained before each MRI scan.

The brain imaging was performed on one healthy volunteer in axial orientation with a 20-channel head coil. Relevant imaging parameters are listed in Table S1. A fully sampled data set and ten prospectively under-sampled data sets (40 spokes and 80 spokes per partition, using the aforementioned five strategies) were acquired. The five under-sampled data sets with the higher under-sampling factor were acquired with 40 spokes instead of 20, due to the increased complexity of in-vivo structures compared to a static phantom.

The abdominal imaging was performed on the same volunteer with a 12-channel body coil. Relevant imaging parameters are listed in Table S1. Five data sets with 40 spokes per partition using the aforementioned five strategies were acquired with a total scan time of 16s each during breath-holds.

The DMRA-ASL imaging was performed on another healthy volunteer with a 20-channel head coil. Relevant imaging parameters are listed in Table S1. Due to the improved performance of $\text{RSOS}_{(\text{Lin/GR})}$ -GR over $\text{RSOS}_{(\text{Lin/GR})}$ -Linear and comparable performance between $\text{RSOS}_{(\text{Lin})}$ -GR and $\text{RSOS}_{(\text{GR})}$ -GR on previous experiments (shown in Result section), only two data sets were acquired using ASOS and $\text{RSOS}_{(\text{Lin})}$ -GR with 20 spokes per partition per temporal phase, with a total of 10 phases.

For the brain imaging and abdominal imaging experiments, the acquired data sets were reconstructed with 3D gridding. To demonstrate the efficacy of RSOS for advanced image reconstruction methods, especially its advantage in the SNR-limited application, the acquired data sets in the dMRA-ASL imaging experiment were reconstructed with 3D gridding and a parallel imaging-compressed sensing (PI-CS) combined reconstruction method (24) as shown below:

$$\hat{d} = \arg \min \sum_{i=1}^N \|\mathcal{F} S_i d - m_i\|_2^2 + \lambda \|Rd\|_1 \quad [2]$$

where $\mathcal{F}$ is the non-uniform fast Fourier transform operator (NUFFT, 25); S_i are the sensitivity maps estimated from the a fully-sampled pre-scan data using the ESPIRiT method (26); d is the image to be reconstructed; m_i is the acquired under-sampled k-space data from each of the N receiver coil elements; R is spatial wavelets transform; and λ is the corresponding regularization parameter. The number of iterations and the value of λ were empirically set to 40 and 0.05, respectively, which provided the best reconstruction for the ASOS data set based on visual assessment. The same values for these two parameters were used for both the ASOS and the RSOS_(Lin)-GR reconstructions.

All reconstructions (3D gridding and PI-CS) were performed offline using a previously described tool (Berkeley Advanced Reconstruction Toolbox, BART) (27) on a Linux PC (4 Core/4GHz, 32 GB Memory, Nvidia GTX 760). For all the scans (phantom and *in-vivo* experiments), 40 initial calibration spokes along the x and y directions (0°, 180°, 90°, 270°, 10 spokes per angle) were additionally acquired and used in the image reconstruction to correct for system-dependent gradient-delay errors, as described in (6).

Data Analysis

For the phantom and brain imaging experiments, to compare different sampling strategies and different under-sampling situations quantitatively, both normalized root mean square errors (nRMSE) and structural similarity index (SSIM) were calculated between each slice of the reference images and images reconstructed from the under-sampled data sets acquired with the five strategies. The calculated nRMSE and SSIM were averaged across all slices. Whereas reduction in nRMSE indicates greater fidelity to the original image, perfect identity is represented by a SSIM value of 1 and the SSIM value decreases as the images differ.

Result

Computer Simulation

Figure 2a shows the simulated PSFs in transversal (x–y plane) and coronal views (x–z plane) of the fully-sampled and the under-sampled ASOS, RSOS_(Lin)-Linear, RSOS_(Lin)-GR, RSOS_(GR)-Linear and RSOS_(GR)-GR acquisitions with 20 and 80 spokes per partition. All PSFs were normalized to the peak of each individual PSF with peak amplitude set to 1. In the transversal view, there was significant PSF sidelobe energy for ASOS, but not for the remaining four strategies when 20 radial spokes are sampled. In the coronal view, there was significant energy in the PSF sidelobe for ASOS and RSOS_(Lin/GR)-Linear with 20 spokes, and these sidelobes were greatly reduced in the RSOS_(Lin/GR)-GR. In both transversal and coronal views, the PSFs corresponding to 80 radial spokes were similar among the five sampling methods and they were all similar to the fully sampled PSF. Figure 2b and 2c shows the quantitative comparison of the five strategies in terms of incoherence

measurements from PSF. The incoherence values (for both gridding and compressed sensing reconstructions) for the $\text{RSOS}_{(\text{Lin/GR})}$ -GR strategies were superior (higher) to the ASOS and the $\text{RSOS}_{(\text{Lin/GR})}$ -Linear strategies regardless of the number of radial spokes per partition. This result confirms the superiority of the proposed GR rotation in the partition direction when compared to the conventional aligned strategy or linear rotation, regardless of the in-plane radial angle arrangement.

Phantom Experiment

Figure 3 shows two representative partitions of the phantom images based on the five sampling strategies, as well as the fully sampled reference. With 20 spokes per partition, severe streaking artifacts (white arrows) and blurring of edge structures (blue arrows) were evident on the images acquired with ASOS and $\text{RSOS}_{(\text{Lin/GR})}$ -Linear; the images acquired with $\text{RSOS}_{(\text{Lin/GR})}$ -GR had much reduced artifacts and improved edge delineation. For 40 spokes per partition, streaking artifacts and blurring (yellow arrows) were reduced but were still significant in the ASOS and $\text{RSOS}_{(\text{Lin/GR})}$ -Linear images and the $\text{RSOS}_{(\text{Lin/GR})}$ -GR images were very similar to the fully sampled reference. With 80 views per partition, all five strategies provided high quality images. Table S2 in the online supporting materials shows the comparative results of the nRMSE and SSIM between the five sampling strategies and the three under-sampling factors. With 20 spokes per partition, the nRMSE and SSIM were 0.788/0.412 for ASOS, 0.702/0.501 for $\text{RSOS}_{(\text{Lin})}$ -Linear, 0.538/0.633 for $\text{RSOS}_{(\text{Lin})}$ -GR, 0.711/0.494 for $\text{RSOS}_{(\text{GR})}$ -Linear and 0.541/0.628 for $\text{RSOS}_{(\text{GR})}$ -GR. Overall, the nRMSE and SSIM for $\text{RSOS}_{(\text{Lin/GR})}$ -GR were both better than $\text{RSOS}_{(\text{Lin/GR})}$ -Linear or ASOS for all the under-sampling factors compared, while $\text{RSOS}_{(\text{Lin})}$ -GR and $\text{RSOS}_{(\text{GR})}$ -GR had similar performance (also true for $\text{RSOS}_{(\text{Lin})}$ -Linear and $\text{RSOS}_{(\text{GR})}$ -Linear).

In-vivo Experiment

Figure 4 shows the representative brain images acquired using the five sampling strategies. Similar to phantom experiment, the level of streaking artifacts (white arrows) and blurring of fine structures (zoom-in boxes) were greater when using ASOS or $\text{RSOS}_{(\text{Lin/GR})}$ -Linear, especially at higher under-sampling factors (Figure 4a); in comparison, images acquired with $\text{RSOS}_{(\text{Lin/GR})}$ -GR had reduced artifacts and improved delineation of small features. Differences between the three strategies diminished as the number of spokes per partition increased from 40 to 80 (Figure 4b). As shown in Table S3 in the online support materials, the nRMSE and SSIM for $\text{RSOS}_{(\text{Lin/GR})}$ -GR of in vivo brain imaging data were better than $\text{RSOS}_{(\text{Lin/GR})}$ -Linear and ASOS, similar to our phantom results. With 40 spokes per partition, the nRMSE and SSIM were 0.521/0.632 for ASOS, 0.494/0.663 for $\text{RSOS}_{(\text{Lin})}$ -Linear, 0.352/0.704 for $\text{RSOS}_{(\text{Lin})}$ -GR, 0.502/0.655 for $\text{RSOS}_{(\text{GR})}$ -Linear and 0.361/0.695 for $\text{RSOS}_{(\text{GR})}$ -GR.

Figure 5a shows two partitions of the reconstructed abdominal images acquired with the five sampling strategies. On both partitions, streaking artifact in the body organs and outside of the body (yellow arrows) and blurring of edges (orange arrows) can be seen on the image with ASOS and $\text{RSOS}_{(\text{Lin/GR})}$ -Linear acquisitions. With $\text{RSOS}_{(\text{Lin/GR})}$ -GR, the streaking artifacts were greatly reduced and the edges appeared sharper. Figure 5b shows the dMRA-ASL maximum-intensity projection (MIP) images reconstructed with 3D gridding and PI-

CS methods at two phases using ASOS and RSOS_(Lin)-GR. Distinct differences can be appreciated between the two acquisition strategies when using gridding, where strong streaking artifacts appear on the image with ASOS acquisition, but are mostly reduced in the RSOS_(Lin)-GR acquisition. With the PI-CS reconstruction, residual streaking artifacts (blue arrows) can still be clearly seen. On the contrary, RSOS_(Lin)-GR with PI-CS provided much improved image quality. The example in Fig. 5b shows the benefit of RSOS_(Lin)-GR in SNR-limited applications such as ASL.

Discussion

This work presented a golden ratio rotated stack-of-stars sampling strategy for improved 3D (or dynamic 3D) imaging. Our results show that, by rotating the radial spokes along the through-plane direction in a golden-ratio manner, streaking artifacts arising from under-sampling are less structured and much reduced compared with the conventional aligned acquisition strategy or the rotated stack-of-stars with a linear angle rotation in the partition direction. Compared with the previously proposed linear rotation strategy (16,17), rotation in a golden-angle manner along the partition encoding direction provides more uniform local 3D k-space sampling. Since 3D gridding is a local operation that uses a support-limited kernel to interpolate the sampled data points onto a Cartesian grid, more locally uniform 3D k-space sampling leads to better gridding reconstruction results. Our results further show that our GR rotation approach provides improved image reconstruction regardless of the in-plane radial view order of either linear or golden angle. Another choice for radial sampling of the 3D k-space is 3D Koosh-Ball (8,9), which offers variable density sampling in 3D. However, for imaging applications that have non-isotropic FOV, such as abdominal and thoracic imaging, 3D Koosh-Ball is less commonly used since it typically requires similar FOV in all three orthogonal directions.

A drawback of the proposed method is its reconstruction speed and computational cost. Instead of performing highly parallelizable partition-by-partition 2D reconstructions after an initial Fourier transform along the partition encoding direction, a strategy widely used in ASOS acquisitions to reduce computation cost and improve reconstruction speed, RSOS requires 3D volumetric reconstruction to better take advantage of the incoherent 3D k-sampling provided by the spoke rotation in the through-plane direction. Such a 3D whole-volume reconstruction is computationally more expensive since 3D interpolation (gridding) needs to be performed rather than 2D gridding, which reduces reconstruction speed. However, significant gains in reconstruction speed may be obtained by using dedicated reconstruction libraries (27–29) for CPU/GPU. For example, it only takes 15s to perform 3D gridding on a 256*20*64*32 (readout points*spoke number*partition number*coil number) RSOS data set (5s to reconstruct a same size ASOS data set with partition-by-partition 2D gridding) with BART and the Linux PC used in this study.

In most of our imaging experiments, we only performed 3D gridding reconstruction on the highly under-sampled data sets. One will notice that even for RSOS-GR, reconstructed images still suffer from some residual streaking artifacts and noise amplification. With a PI-CS reconstruction, image quality would be greatly improved. For the parallel imaging part, there are several ways to acquire the sensitivity maps without acquiring a fully-sampled data

set if SENSE-type reconstructions are desired. One solution is to use a pre-scan that acquires a few additional calibration lines (13) that can be used to not only estimate the sensitivity maps but also correct the system-dependent gradient-delays errors. Another solution is to use optimization-based strategies (30) to recover the center region of k-space if k-space is highly under-sampled. The recovered center region can then be used to estimate maps. If a GRAPPA-type instead of SENSE-type reconstruction is desired, self-calibration strategies such as GROG (31), SPIRiT (32) can be used to directly fill out the missing k-space points. For the compressed sensing part, different types of regularizations (13,33–34) may be applied to further remove artifacts and recover details. Since radial sampling provides intrinsic variable density sampling and incoherent artifacts when under-sampled, the incorporation of additional regularizations and sparsifying transforms may potentially benefit the image reconstruction.

Conclusion

We have presented a golden ratio rotated stack-of-stars sampling strategy to efficiently sample the 3D k-space. Image quality is significantly improved in phantom and *in-vivo* experiments comparing with the conventional aligned sampling strategy or stack-of-stars with linear angle rotation. This strategy may be useful for 3D stack-of-stars radial imaging for various clinical applications.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.

An example of the radial spoke angle arrangement using ASOS (a), RSOS_(Lin)-Linear (b), RSOS_(Lin)-GR (c), RSOS_(GR)-Linear (d), and RSOS_(GR)-GR (e). In this example, each partition has four spokes, and a total of four partitions. The calculated spoke angles are shown at the bottom of each sub-figure.

Figure 2.

(a) The PSF of fully-sampled and ASOS, RSOS_(Lin)-Linear, RSOS_(Lin)-GR, RSOS_(GR)-Linear and RSOS_(GR)-GR k-space sampling with 20 spokes and 80 spokes per partition in transversal and coronal views. All PSFs were normalized to the peak of each individual PSF such that all PSF peaks have unit amplitude. The individual normalization factors are noted at each peak in the transversal view. The PSFs are for the central slice of the volume in the respective view orientations. ASOS produces strong streaking artifact when only 20 spokes are sampled per partition and there are essentially no significant difference between the five strategies when 80 spokes are sampled. For the 20 spokes per partition scenario, RSOS_(Lin/GR)-GR has reduce energy in the PSF sidelobes compared to ASOS and RSOS_(Lin/GR)-Linear in the coronal view. (b,c) The calculated incoherence indices for gridding reconstruction (b) and for CS-based reconstructions (c). RSOS_(Lin/GR)-GR has superior PSF incoherence compared with ASOS and RSOS_(Lin/GR)-Linear.

Figure 3.

Phantom images acquired with different sampling strategies and with 20, 40 and 80 spokes per partition. From left to right, each column in (a), (b) and (c) shows two representative axial slices from the 3D images acquired with: fully-sampled, ASOS, RSOS_(Lin)-Linear, RSOS_(Lin)-GR, RSOS_(GR)-Linear, and RSOS_(GR)-GR, respectively. Three under-sampling scenarios are shown: 20 spokes (a), 40 spokes (b) and 80 spokes per partition (c). Streaking artifacts (white arrows) and blurring of edges (blue arrows) are clearly visible on ASOS and

RSOS_(Lin/GR)-Linear acquisitions when 20 spokes/partition was used. By doubling the spoke number (e.g. 40 spokes), residual streaking and blurring (yellow arrows) still exist.

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Figure 4.

Selected brain images acquired with different sampling strategies and number of spokes per partition. Each column in (a) and (b) shows two representative axial slices from the 3D images acquired with: fully-sampled, ASOS, $RSOS_{(Lin)}-Linear$, $RSOS_{(Lin)}-GR$, $RSOS_{(GR)}-Linear$, and $RSOS_{(GR)}-GR$, respectively. Two under-sampling scenarios are shown: 40 spokes per partition (a) and 80 spokes per partition (b). Zoom-in boxes provide detailed comparisons of the five acquisition strategies on fine structures. White arrows highlight the streaking artifact for ASOS and $RSOS_{(Lin/GR)}-Linear$ acquisitions.

Figure 5.

(a) Two representative slices of abdominal images acquired with ASOS, $RSOS_{(Lin)}-Linear$, $RSOS_{(Lin)}-GR$, $RSOS_{(GR)}-Linear$, and $RSOS_{(GR)}-GR$. $RSOS_{(Lin/GR)}-GR$ provides images with less streaking artifacts and sharper structures. All abdominal images were reconstructed using gridding based on a data set with 40 radial spokes per partition. (b) MIP images at two phases of dMRA-ASL images acquired with ASOS and $RSOS_{(Lin)}-GR$. Even with only 20 spokes per partition per phase, $RSOS_{(Lin)}-GR$ is still able to reduce majority of streaking artifacts compared with ASOS using 3D gridding reconstruction. With a PI-CS

reconstruction, overall image quality was improved for both acquisitions. However, blurring of vessels and residual streaking artifacts can still be seen on images with ASOS acquisition while $RSOS_{(Lin)}$ -GR acquisition provides cleaner and sharper images.