A capacitive ultrasonic transducer based on parametric resonance

Sushruta Surappa, Sarp Satir, and F. Levent Degertekin

G. W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, USA

(Received 4 May 2017; accepted 12 July 2017; published online 24 July 2017)

A capacitive ultrasonic transducer based on a parametric resonator structure is described and experimentally demonstrated. The transducer structure, which we call capacitive parametric ultrasonic transducer (CPUT), uses a parallel plate capacitor with a movable membrane as part of a degenerate parametric series RLC resonator circuit with a resonance frequency of f_0 . When the capacitor plate is driven with an incident harmonic ultrasonic wave at the pump frequency of $2f_0$ with sufficient amplitude, the RLC circuit becomes unstable and ultrasonic energy can be efficiently converted to an electrical signal at f_0 frequency in the RLC circuit. An important characteristic of the CPUT is that unlike other electrostatic transducers, it does not require DC bias or permanent charging to be used as a receiver. We describe the operation of the CPUT using an analytical model and numerical simulations, which shows drive amplitude dependent operation regimes including parametric resonance when a certain threshold is exceeded. We verify these predictions by experiments with a micromachined membrane based capacitor structure in immersion where ultrasonic waves incident at 4.28 MHz parametrically drive a signal with significant amplitude in the 2.14 MHz RLC circuit. With its unique features, the CPUT can be particularly advantageous for applications such as wireless power transfer for biomedical implants and acoustic sensing. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4995564>]

Parametric-effect devices are characterized by the presence of a nonlinear or time varying reactance and have found numerous applications in electrical,^{1–4} mechanical,^{5–8} and optical systems.^{9–11} Electrical parametric amplifiers and frequency up/down converters, which used varactor diodes as the time-varying reactance, came into prominence in the late 1900s as they had the inherent advantage of low noise as compared to semiconductor-based amplifiers.^{3,4,12} More recently, Raskin *et al.* used surface micromachined, membrane-based capacitors as the time varying capacitance to develop a non-degenerate MEMS electrical parametric amplifier.¹³ Parametric amplification in the mechanical domain was first demonstrated by Rugar and Grütter who electrostatically varied the stiffness of a micro-cantilever, thereby achieving improved force sensitivity, and demonstrated thermal noise squeezing in the cantilever.⁵ Zhang and Turner¹⁴ demonstrated high resolution mass sensing using MEMS based parametric resonators by making use of fringing electrostatic fields. As such, mechanical parametric oscillators have been used for a variety of applications such as sensing, as resonators, filters, and many others as described by Moran *et al.*¹⁵

The most common method of obtaining parametric amplification or resonance in electrical and mechanical devices is by modulating the reactance of the system by electrical means. In varactor-based parametric amplifiers, the capacitance is changed by applying a voltage across the depletion layer of the varactor diode.⁴ Similarly, electrostatic fields are used to change the stiffness of structural members to obtain mechanical parametric amplification in MEMS devices.^{6,14} However, mechanical energy can also be directly transferred to an electrical circuit by mechanical modulation of the reactance. Most recently, an inductor based parametric resonator is demonstrated for low frequency energy harvesting utilizing

a mechanically modulated inductor in an RLC circuit.¹⁶ Here, we explore a method of mechanically modulating the capacitance of a variable capacitor with the use of ultrasound. By this technique, we remotely drive an electrical circuit into parametric resonance and convert the ultrasound energy to electrical energy, thereby utilizing the parametric resonator as an electromechanical transducer.

Capacitive ultrasonic transducers exhibit large bandwidth and high sensitivity and can be micromachined on silicon in CMOS compatible processes,¹⁷ but unlike piezoelectric transducers, a DC bias or a permanent charge on the plates of the capacitor is required to operate the transducer efficiently as a receiver.^{18,19} Parametric resonance based capacitive transducers build on the advantages of conventional capacitive transduction methods as they can be fabricated by standard micromachining techniques, and additionally, they do not need a DC bias or permanent charge to operate. As shown later, when driven to an unstable regime, these devices can get started with negligible external electrical stimuli such as RF interference signals or potentially with the thermal noise in the electrical system providing the initial conditions required for parametric resonance.

In this letter, we present an ultrasound driven MEMS based degenerate capacitive parametric transducer for applications in immersion and in air, which we call CPUT (capacitive parametric ultrasonic transducer). We start by describing the principle of operation of the CPUT with a simple mathematical model. We then simulate this concept for a parallel plate capacitor immersed in water in the time domain and explore some of its operational characteristics. Finally, we perform proof of principle experiments using a micromachined membrane based capacitor as the time varying reactance and demonstrate that ultrasound power from the piezoelectric transducer is harnessed

across the receiver load resistance as electrical power without having to bias the capacitor. The use of ultrasound to drive a parametric transducer opens an exciting domain of possibilities such as wireless ultrasound power transfer and acoustic sensing.

To understand the operation of the degenerate parametric oscillator, consider a linear-time varying capacitor in series with an inductor and a resistor to form an RLC circuit having a resonance frequency of f_o (Fig. 1). The capacitance of the capacitor is varied at a pump frequency $f_p = 2f_o$ by the incoming ultrasound.

Applying the Kirchhoff Voltage Law and Kirchhoff Current Law to the above circuit, we obtain two equations

$$-i = \frac{d}{dt}Q = \frac{d}{dt}CV, \quad (1)$$

$$V = R_L i + L \frac{d}{dt}i, \quad (2)$$

where V is the voltage across the capacitor, i is the current in the circuit, L is the inductance, and R_L is the load resistance. If we eliminate current from the first equation, we obtain

$$\left[\frac{d^2}{dt^2} + \frac{R_L}{L} \frac{d}{dt} + \frac{1}{LC} \right] V = 0. \quad (3)$$

The solution of Eq. (3) is similar to the response of a damped harmonic oscillator. Now, if we modulate the capacitance at the pump frequency $\omega_p = 2\pi f_p$

$$C = C_o \left[1 - \frac{\Delta C}{C_o} \sin(\omega_p t) \right], \quad (4)$$

where $\omega_o = \frac{1}{\sqrt{LC_o}}$, and assume $\Delta C \ll C_o$, Eq. (3) can be written as

$$\left[\frac{d^2}{dt^2} + \frac{R_L}{L} \frac{d}{dt} + \omega_o^2 \left[1 + \frac{\Delta C}{C_o} \sin(\omega_p t) \right] \right] V \cong 0. \quad (5)$$

Equation (5) is a damped Mathieu equation which has been previously studied in great detail.^{20,21} Assuming that there is a non-zero initial oscillation in the circuit around the frequency ω_o , the pump signal generates a drive signal around the resonance frequency when $\omega_p \cong 2\omega_o$. From the solution of Eq. (5), it is observed that when $\frac{\Delta C}{C_o} > \frac{2R_L}{\omega_o L} = \frac{2}{Q_{el}}$, where Q_{el} is the quality factor of the electrical circuit, the system becomes unstable and one obtains a growing voltage across

FIG. 1. Equivalent circuit of a degenerate parametric oscillator, which forms the CPUT.

the capacitor. Here, $\frac{\Delta C}{C_o}$ can be understood as the normalized electrical energy pumped into the system from the external source and $\frac{2}{Q_{el}}$ is the normalized energy lost in the oscillator in every cycle. It is important to note that in a real system, the changing capacitance is a function of both time and gap between the two plates of the capacitor, for which one obtains a nonlinear Mathieu equation having a stable solution. However, for small values of $\Delta C/C_o$, the level of forcing required to drive the system into parametric resonance holds for both the linear and nonlinear cases.

To gain more insights into the complex nonlinear operation of the CPUT, a 1-dimensional (1D) lumped parameter system is simulated in the time domain using Simulink (The MathWorks, Inc., Natick, MA). The target ultrasound frequency, $2f_o$, is 2 MHz, and the area is 1 mm^2 for a practical implementation. The time varying capacitor is modelled as two parallel plates where one plate is fixed and the other plate is movable with a mass of $6.33 \times 10^{-7} \text{ kg}$ and a spring constant of $1 \times 10^8 \text{ N/m}$. These system parameters can be achieved with different equivalent spring mass structures. In this 1D setting, the effect of the fluid medium is included using the acoustic impedance for a plane wave in water²² ($Z_f = \rho c A$, ρ : density, c : speed of sound, and A : area of the moving plate), which is a good approximation for the radiation impedance of a circular baffled piston with a similar area at 2 MHz. With this acoustic load, the mechanical quality factor is roughly around 5. Assuming 120 nm gap between the plates to form the capacitor, C_o , the values of the inductance ($L = 340 \mu\text{H}$) and load resistance ($R_L = 50 \Omega$) are chosen such that the undisturbed resonant frequency (f_o) of the electric circuit is 1 MHz with a Q_{el} value of 43. With this idealized model, the parametric resonance threshold behavior is investigated with the results summarized in Fig. 2. First, a short unipolar pulse is applied across the

FIG. 2. (a) Applied ultrasound forcing on the CPUT plate at $2f_o$. (b) A unipolar pulse is applied to excite oscillations in the RLC circuit. (c) When the forcing level is below the threshold required for parametric resonance, there is no buildup of voltage across the load resistance. (d) When the forcing is above the required threshold, the voltage across the load resistance increases until the forcing is stopped.

electrical circuit to induce some initial oscillation while applying a sinusoidal forcing at $2f_o$ directly on the face of the plate, simulating a normally incident ultrasound pump field. Figures 2(c) and 2(d) show the effect of the forcing level on the response of the system, indicated by the voltage on the load resistor. When the forcing does not produce the change in capacitance required for parametric resonance, the oscillations decay [Fig. 2(c)]. As the forcing is increased, the oscillations are sustained for a longer time. Once the criterion for parametric resonance is satisfied, the current in the circuit phase locks with the forcing and a growing voltage across the load resistance is observed [Fig. 2(d)]. Energy is transferred from the ultrasonic pump signal input to the load resistance. The nonlinearities present in the system eventually cause the voltage to attain a steady-state value. When the forcing is stopped, the voltage decays.

Due to its nonlinear nature, the CPUT performance for converting acoustic energy to electrical energy varies with the level of forcing, as well as receiver dynamics, forcing frequency, incident ultrasonic field distribution, and CPUT construction. Within the limitations of this letter, the variation of efficiency with frequency and forcing level for a fixed load resistance is explored using the 1D model. Here, the acousto-electrical conversion efficiency is defined as the ratio of the time averaged power dissipated across the resistor to the available acoustic power

$$\text{Acousto - electrical conversion efficiency} = \frac{\frac{1}{T} \int i^2 R_L dt}{I_{ac} \times A} \times 100 (\%). \quad (6)$$

Here, i is the current in the circuit, R_L is the load resistance, I_{ac} is the acoustic intensity incident at the surface of the capacitor plate, and A is the area of the moving plate as defined above. The 2D color map in Fig. 3 shows the variation of conversion efficiency with incident acoustic intensity and ultrasound frequency for a fixed load resistance of 50Ω . The acoustic intensity threshold for parametric resonance depends on the frequency, and it is minimized at the mechanical resonance frequency. The maximum efficiency is over 99% (corresponding to an insertion loss of ~ 0 dB) at 1.99 MHz for 4 mW/mm^2 , indicating that with proper selection of parameters, CPUT

FIG. 3. 2D color map plot of acousto-electrical conversion efficiency of the CPUT operating around 2 MHz vs input intensity and forcing frequency based on a 1-dimensional model. The efficiency is defined in Eq. (6).

provides a near perfect impedance match to the fluid medium. The efficiency reduces gradually with increasing intensity at this frequency but remains over 70% in a wide range of intensity values above 4 mW/mm^2 . It is observed that the average value of the capacitance C_o gradually increases with intensity due to the nonlinear nature of the electrostatic forces, lowering the electrical resonance frequency and leading to this reduction in efficiency. The graph also shows that the bandwidth of the CPUT is narrow, about 70–80 kHz around the maximum frequency of 1.99 MHz at 4 mW/mm^2 , and it broadens as intensity is increased. These results show that when the resonances of the mechanical ($2f_o$) and electrical (f_o) sides are matched and the parametric resonance condition is met with input forcing and quality factor adjustments, the CPUT is potentially a high efficiency energy-conversion device. CPUT operation should not be different in air as long as these conditions are met, albeit at low frequencies where attenuation is reasonable. Further generalizations for CPUT design in different media and for different applications require more in depth analysis in terms of impedance matching and mechanical and acoustic design including incidence angle variations.

We performed proof-of-principle experiments using a micromachined capacitor as the time varying reactance to demonstrate the most critical aspect of the CPUT: ultrasound driven parametric resonance behavior. The micromachined capacitor consists of an array of 80 square membranes, each having an edge length of $46 \mu\text{m}$, fabricated on a silicon substrate (a common structure used for capacitive micromachined ultrasonic transducers or CMUTs). A bottom electrode which is common to all the membranes is made by sputtering chromium, whereas AlSi deposited on the membrane acts as the top electrode. The membranes themselves are made from Si_3N_4 deposited by a plasma-enhanced chemical vapor deposition process and are about $2 \mu\text{m}$ thick. The effective gap between the suspended membrane and the substrate is approximately 120 nm. The membranes are then electrically connected in parallel to increase the total capacitance. The fabrication and characterization of these CMUT arrays are discussed in the literature in great detail.^{23–25} It should be noted that the CMUT was originally fabricated for imaging applications with a center frequency of 7 MHz in water and is not optimized as a CPUT. The CMUT capacitor is wirebonded to a printed circuit board (PCB) and forms the CPUT as part of a series RLC circuit along with a $100 \mu\text{H}$ inductor and a 20Ω resistor. The Q-factor and resonance frequency of the electrical circuit with the CMUT immersed in water were adjusted with an op-amp based negative resistance circuit placed in parallel to the inductor and the resistor, to be 130 and 2.14 MHz (f_o), respectively. A piezoelectric transducer (Unirad 546) is aligned normally to the CMUT element at a distance of $d = 30 \text{ mm}$ from its surface which corresponds to the focal length of the transducer.

The schematic of the experimental setup is shown in Fig. 4. A function generator (Agilent 33250A) is connected to an RF power amplifier (ENI 310L) which is connected to the piezoelectric transducer. The output across the load resistance is recorded by using an oscilloscope (Tektronix TDS5054). A sinusoidal tone burst at 4.28 MHz ($2f_o$) is applied to the piezoelectric transducer. The duration of the tone burst is adjusted such that the end of the burst corresponds to the arrival of the

FIG. 4. Schematic of the experimental setup.

ultrasound waves at the surface of the CMUT. The time taken (t) for the wave to travel the distance d is equal to $d/c \cong 20 \mu\text{s}$, where $c = 1500 \text{ m/s}$ is the speed of sound in water. Hence, the number of cycles in the tone burst is calculated as $t \times 2f_0$, which is 83 cycles. To record the response of the system with and without the ultrasound pump signal, a plastic plate that is transparent to the electro-magnetic (EM) waves but which blocks the ultrasound is introduced between the transmitter and the CPUT.

The voltage signals across the load resistance as measured by using the oscilloscope with and without ultrasound incident on the CPUT are plotted in Figs. 5(a) and 5(b),

FIG. 5. (a) Measured voltage across the load resistance in the absence of ultrasound. (b) Voltage across load resistance when ultrasound (above threshold) is present.

respectively. In the region 1 of the graphs, the tone burst at $2f_0$ is applied to the transducer and the signal is picked up at the receiver due to the EM coupling. The ultrasound takes approximately $20 \mu\text{s}$ to travel from the source to the receiver through the fluid. The start of region 2 coincides with the arrival of the ultrasound at the receiver. At this time, only the ultrasonic excitation is present. The ultrasound impinges on the receiver till the end of region 2. In region 3, the receiver is subjected to neither electrical nor ultrasonic excitation.

In the case of Fig. 5(a), the tone burst is applied with the plastic plate placed between the transducer and the receiver. The $2f_0$ component is seen on the output due to the EM coupling, but the ultrasound is blocked by the plate. In region 2, the f_0 component is excited in the RLC circuit due to EM coupling at the end of the tone burst; however, in the absence of ultrasound forcing, the voltage gradually decays without any discontinuity between regions 2 and 3. In Fig. 5(b), we observe the output across the load resistance when the plastic plate has been removed. Once again, the EM coupling excites the circuit. However, instead of decaying, in the presence of ultrasound forcing above the threshold value, the voltage across the load grows until the end of region 2. Once the ultrasonic excitation ends, the voltage begins to decay. It is clear that in the presence of ultrasound, we are able to drive the circuit into parametric resonance at frequency f_0 . Note that no DC bias or charge is used on the CPUT to obtain these signals.

To ensure that the increase in the received signal is due to the ultrasonic pump signal level, the output voltage for different input forcing levels is plotted in Fig. 6. Here, the voltage measured across the load resistance is first filtered using an 8th order low pass Butterworth filter such that only the f_0 component remains. Then, the envelope of the filtered voltage is plotted as a function of time for different forcing levels. Clearly, there exists a necessary level of forcing to drive the circuit in parametric resonance. For forcing levels lower than the threshold value, the output voltage decays over time. As the forcing is increased, the oscillations are sustained longer and the rate of decay reduces. Once the forcing exceeds the threshold value, the output voltage begins to grow until a steady state value is attained or the

FIG. 6. Envelopes of the measured voltage across the load resistance at 2.14 MHz for the increasing level of the incident ultrasound pump signal (input voltage to the transmitter) at 4.28 MHz.

forcing is stopped. Again, the voltage decays when there is no ultrasound on the circuit. The results summarized in Figs. 5 and 6 are in qualitative agreement with the simulation results shown in Fig. 2, showing the predicted CPUT behavior. Quantitative comparisons with experiments will require complex acoustic and nonlinear analysis,²⁶ which is planned as future research.

To conclude, ultrasound driven parametric resonance presents a different approach to electromechanical transduction, where a capacitive transducer is operated in the absence of a DC bias or a permanent charge. Furthermore, simulations indicate that such a device can potentially be used as a high efficiency transducer. Demonstrative experiments were performed in immersion using a micromachined membrane based capacitor, and it was observed that when the ultrasound forcing at $2f_0$ satisfied the threshold for parametric resonance, an increasing voltage at f_0 develops across the load resistance. Since the capacitor used in the experiment was originally designed for imaging applications, only qualitative observations have been possible. Future CPUT designs for power transfer and sensing applications require more extensive modeling and simulations. A potential challenge would be to miniaturize the CPUT, as the current configuration consists of a coil-wound inductor which may be too large for medical implants and other size-sensitive applications. Compact implementations may utilize piezoelectric resonators as inductors.²⁷

This work was partly supported by the NIH Grant No. R21 EB015607.

¹P. K. Tien, *J. Appl. Phys.* **29**, 1347 (1958).

²G. L. Matthaei, *IRE Trans. Microwave Theory Tech.* **9**, 23 (1961).

³E. Sard, B. Peyton, and S. Okwit, *IEEE Trans. Microwave Theory Tech.* **14**, 608 (1966).

- ⁴R. D. Weglein and F. Keywell, *IRE Trans. Microwave Theory Tech.* **9**, 39 (1961).
- ⁵D. Rugar and P. Grütter, *Phys. Rev. Lett.* **67**, 699 (1991).
- ⁶K. L. Turner, S. A. Miller, P. G. Hartwell, N. C. MacDonald, S. H. Strogatz, and S. G. Adams, *Nature* **396**, 149 (1998).
- ⁷J. F. Rhoads, N. J. Miller, S. W. Shaw, and B. F. Feeny, *J. Vib. Acoust.* **130**, 061006 (2008).
- ⁸Y. Linzon, B. Ilic, S. Lulinsky, and S. Krylov, *J. Appl. Phys.* **113**, 163508 (2013).
- ⁹J. A. Giordmaine and R. C. Miller, *Phys. Rev. Lett.* **14**, 973 (1965).
- ¹⁰L. E. Myers, R. C. Eckardt, M. M. Fejer, R. L. Byer, W. R. Bosenberg, and J. W. Pierce, *J. Opt. Soc. Am. B* **12**, 2102 (1995).
- ¹¹C. Reimer, M. Kues, L. Caspani, B. Wetzels, P. Roztock, M. Clerici, Y. Jestin, M. Ferrera, M. Peccianti, A. Pasquazi, B. E. Little, S. T. Chu, D. J. Moss, and R. Morandotti, *Nat. Commun.* **6**, 8236 (2015).
- ¹²K. Johnson, *J. Appl. Phys.* **33**, 2826 (1962).
- ¹³J. P. Raskin, A. R. Brown, B. Khuri-Yakub, and G. M. Rebeiz, *J. Microelectromech. Syst.* **9**, 528 (2000).
- ¹⁴W. Zhang and K. L. Turner, *Sens. Actuators, A* **122**, 23 (2005).
- ¹⁵K. Moran, C. Burgner, S. Shaw, and K. Turner, *Nonlinear Theory Appl.* **4**, 198 (2013).
- ¹⁶N. B. Caldwell and M. F. Daqaq, *Appl. Phys. Lett.* **110**, 093903 (2017).
- ¹⁷Ö. Oralkan, J. Johnson, M. Karaman, U. Demirci, K. Kaviani, T. H. Lee, B. T. Khuri-Yakub *et al.*, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **49**, 1596 (2002).
- ¹⁸U. Bartsch, J. Gaspar, and O. Paul, in *IEEE 22nd International Conference on Micro Electro Mechanical System* (2009), pp. 1043–1046.
- ¹⁹K. K. Park, M. Kupnik, H. J. Lee, Ö. Oralkan, and B. T. Khuri-Yakub, in *IEEE Sensor* (2010), pp. 1024–1028.
- ²⁰R. Grimshaw, *Nonlinear Ordinary Differential Equations* (CRC Press, 1991).
- ²¹E. I. Butikov, *Eur. J. Phys.* **25**, 535 (2004).
- ²²L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics*, 4th ed. (Wiley-VCH, 1999).
- ²³X. Jin, Ö. Oralkan, F. L. Degertekin, and B. T. Khuri-Yakub, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **48**, 750 (2001).
- ²⁴B. T. Khuri-Yakub and Ö. Oralkan, *J. Micromech. Microeng.* **21**, 054004 (2011).
- ²⁵T. Xu, C. Tekes, and F. Degertekin, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **61**, 2121 (2014).
- ²⁶S. Satir and F. L. Degertekin, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **62**, 1865 (2015).
- ²⁷B. Ju, W. Shao, L. Zhang, H. Wang, and Z. Feng, *IET Power Electron.* **8**, 2009 (2015).