Sparse dictionary for synthetic transmit aperture medical ultrasound imaging

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It is possible to recover a signal below the Nyquist sampling limit using a compressive sensing technique in ultrasound imaging. However, the reconstruction enabled by common sparse transform approaches does not achieve satisfactory results. Considering the ultrasound echo signal's features of attenuation, repetition, and superposition, a sparse dictionary with the emission pulse signal is proposed. Sparse coefficients in the proposed dictionary have high sparsity. Images reconstructed with this dictionary were compared with those obtained with the three other common transforms, namely, discrete Fourier transform, discrete cosine transform, and discrete wavelet transform. The performance of the proposed dictionary was analyzed via a simulation and experimental data. The mean absolute error (MAE) was used to quantify the quality of the reconstructions. Experimental results indicate that the MAE associated with the proposed dictionary was always the smallest, the reconstruction time required was the shortest, and the lateral resolution and contrast of the reconstructed images were also the closest to the original images. The proposed sparse dictionary performed better than the other three sparse transforms. With the same sampling rate, the proposed dictionary achieved excellent reconstruction quality. © 2017 Acoustical Society of America.

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I. INTRODUCTION

In recent years, compressive sensing (CS) has been proposed as a method to resolve high-speed sampling and large-capacity data storage problems. In this algorithm, the signal can be reconstructed from partial samplings by reconstruction algorithms if the original signal is sparse within a certain sparse domain.^{1–3} Since it was proposed in 2006, CS has been studied extensively and applied in various fields, such as information theory, image processing, microwave imaging, pattern recognition, wireless communications, etc.^{4–7}

The purpose of this study was to investigate the application of CS in the digital ultrasound imaging field, particularly in synthetic aperture imaging, which was proposed by Passman and Ermert in 1996 to improve resolution and imaging quality.⁸ The essential idea of the synthetic aperture is that a single element transmits a pulse signal gradually and then all elements receive the echo signals from the detecting area. Therefore, the process will lead to a high sample rate and inevitably result in a large memory requirement and complexity of the hardware system. However, the application of CS in the field of ultrasound imaging can reduce the amount of acquired data needed to construct an image, and the problem of hardware complexity will be correspondingly resolved.

In the CS algorithms, the sparse coefficients are first constructed in a certain sparse domain and then the original

signal can be reconstructed when needed. Consequently, the sparsity of those coefficients, or the sparse transform, determines the reconstruction quality. With the same reconstruction condition, the sparser the coefficients the better the reconstruction quality. Therefore, the design of an efficient sparse dictionary is crucial in CS techniques. Currently, sparse transforms mainly includes the discrete Fourier transform (DFT), discrete cosine transform (DCT), and discrete wavelet transform (DWT). Researchers have introduced these sparse transforms to medical ultrasound imaging.^{9–11} Quinsac *et al.* used the DCT and wavelet transform to achieve CS.¹² Liebgot *et al.* combined the DFT and DWT to sparsely represent the original echo signal, and the application of CS in ultrasound was theoretically realized.¹³ Although this technique can reconstruct the ultrasonic echo signal, the process of image reconstruction is too complicated to be realized because this approach can form only a compact support domain and cannot form a sparse basis. These transforms do not take advantage of the ultrasound echo signal's features, such as attenuation, repetition, and superposition. Therefore, the efficacy of sparse representation with these approaches is low and the reconstruction quality cannot achieve satisfactory results. This paper proposes a sparse dictionary according to the ultrasound echo signal's features. The number of received echo pulses determines the sparsity of the sparse coefficients, and the echo signal can be maximally sparsely represented in the dictionary.

The paper is organized as follows. Section II briefly reviews the method of CS and describes the proposed dictionary. Section III presents the simulation results, including

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scattering point targets and cyst phantoms. The experimental results are given in Sec. IV, and the conclusion is provided in Sec. V.

II. METHODS

A. Synthetic aperture ultrasound imaging based on CS

CS can reduce the amount of acquired echo data needed to reconstruct the final image. Suppose $x(t)$ is the echo signal received by one element and $\mathbf{x}$ is the corresponding length- N vector after discrete sampling. If Ψ is chosen as the sparse basis and α represents the sparse coefficients, $\mathbf{x}$ can then be expressed as follows:

$$\mathbf{x} = \Psi\alpha. \quad (1)$$

Compressive sampling of echo signal $\mathbf{x}$ is conducted to acquire a measurement vector $\mathbf{y}$ with length- M ($M < N$) according to the measurement matrix $\Phi \in \mathbb{C}^{M \times N}$. The corresponding mathematical expression is

$$\mathbf{y} = \Phi\mathbf{x}. \quad (2)$$

M/N is the sampling compressive ratio. A delta matrix is often chosen as the measurement matrix of the ultrasound. The delta matrix is a row full rank matrix in which all elements are composed of "0" and "1," with each row having only one 1 element, while the others are 0. The delta matrix has randomness in the column where the element 1 is located in each row; that is, signals are randomly selected from the sampling signal of the ultrasonic element to form a measuring vector for realizing the random sampling of the ultrasonic echo signals. The delta matrix can be generally expressed as follows:

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}_{M \times N}. \quad (3)$$

Substituting the solution of Eq. (1) into Eq. (2) gives

$$\mathbf{y} = \Phi\mathbf{x} = \Phi\Psi\alpha = \Theta\alpha. \quad (4)$$

In Eq. (4), $\Theta = \Phi\Psi$ is the sensing matrix. This matrix transforms the problem of solving the original signal $\mathbf{x}$ by measuring vector $\mathbf{y}$ into the problem of solving the sparse coefficient α . When the number of nonzero elements s in the sparse coefficient α is very small, that is, α is sparse enough, the original signal $\mathbf{x}$ can be reconstructed from the measure vector $\mathbf{y}$ by CS. The number of nonzero elements s is also called the sparsity of the vector.

The solution of Eq. (4) is an underdetermined problem; therefore, the solution of α is not unique. The sufficient condition for the accurate reconstruction of $\mathbf{x}$ from $\mathbf{y}$ is that the sensing matrix satisfies the Restricted Isometry Property (RIP).^{14,15}

$$(1 - \delta_{2k})\|\mathbf{x}\|_2^2 \leq \|\Theta\mathbf{x}\|_2^2 \leq (1 + \delta_{2k})\|\mathbf{x}\|_2^2. \quad (5)$$

The parameter δ_{2k} is the minimum value that satisfies the upper expression. When $\delta_{2k} \in (0, 1)$, the sensing matrix satisfies the $2K$ order RIP. The solution of Eq. (4) can be transformed into an optimization problem with minimum l_0 norm,

$$\tilde{\mathbf{v}} = \min_{\mathbf{v}} \|\mathbf{v}\|_{l_0} \text{ subject to } \mathbf{y} = \Theta\mathbf{v}. \quad (6)$$

The l_0 norm is $\|\mathbf{v}\|_{l_0} = |\{i, v_i \neq 0\}|$, which is equivalent to solving the number of nonzero elements in the original signal. However, this problem is an NP-hard problem, and the solution process requires considerable computation time. When the sensing matrix satisfies the $2K$ order RIP and $\delta_{2k} < \sqrt{2} - 1 \approx 0.4142$, the optimization problems based on the l_0 norm and the l_1 norm are equivalent. The reconstruction of the original signal $\mathbf{x}$ can be translated to an optimization problem as follows:

$$\arg \min \|\alpha\|_{l_1} \text{ s.t. } \Phi\Psi\alpha = \mathbf{y}. \quad (7)$$

The l_1 norm is $\|\mathbf{v}\|_{l_1} = \sum_{i=1}^n |v_i|$, which is the sum of the absolute values of all elements in the original signal.

Substituting the solution of Eq. (7) into Eq. (1), we obtain the original signal $\mathbf{x}$. It should be noted that all the reconstruction algorithms used in this paper are the YALL1_group, which is fast and efficient.¹⁶

B. The sparse dictionary and its application

The measurement matrix, sparse dictionary, and reconstruction algorithm are the three most important factors in CS. In the case in which the measurement matrix and reconstruction algorithm are known, the sparse dictionary mainly determines the reconstruction effect. Under the same reconstruction condition, the sparser the sparse coefficients, that is, the fewer the nonzero elements, the higher the reconstruction accuracy and the better the efficiency of the sparse dictionary. The commonly used sparse transform matrices are DFT, DCT, and DWT. Considering the ultrasound echo signal's characteristics of attenuation and superposition, this paper proposes a high-efficiency sparse dictionary. The four sparse transforms used in this paper are analyzed as follows.

1. DFT

DFT transforms a finite-length discrete signal into a frequency domain signal of equal length and expresses it in different frequency amplitudes within the frequency domain. Supposing $\mathbf{x} = (x(0), x(1), \dots, x(N-1))$ is a N -dimension discrete signal, the DFT of $x(n)$ is

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n) \left(\cos\left(-2\pi k \frac{n}{N}\right) + i \sin\left(-2\pi k \frac{n}{N}\right) \right) \\ &= \sum_{n=0}^{N-1} x(n) \exp^{-i(2\pi kn/N)} \quad k = 0, 1, \dots, N-1. \end{aligned} \quad (8)$$

Thus, the DFT of $\mathbf{x}$ expressed by the matrix transform is

$$\begin{pmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{pmatrix} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & \exp^{-i(2\pi/N)} & \dots & \exp^{-i[2\pi(N-1)/N]} \\ \vdots & \vdots & & \vdots \\ 1 & \exp^{-i[2\pi(N-1)/N]} & \dots & \exp^{-i[2\pi(N-1)^2/N]} \end{pmatrix} \times \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{pmatrix}. \quad (9)$$

2. DCT

The DCT is similar to the DFT, but the DCT obtains the real part of the frequency domain signal, the data after the

transformation are relatively concentrated, and the energy is mainly located in the low-frequency part, which ensures that the signal or image can be recovered from the spectrum with fewer data. Therefore, DCT is widely used in compression of signals and images. Supposing $x = (x(0), x(1), \dots, x(N-1))$ is an N -dimension discrete signal, the DCT of $x(n)$ is

$$C(0) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n),$$

$$C(k) = \frac{2}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \cos\left(\frac{(2n+1)k\pi}{2N}\right), k=1, 2, \dots, N-1. \quad (10)$$

Therefore, the DCT of x expressed by the matrix transform is

$$\begin{pmatrix} C(0) \\ C(1) \\ \vdots \\ C(N-1) \end{pmatrix} = \sqrt{\frac{1}{N}} \begin{pmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ \sqrt{2} \cos\left(\frac{(N-1)\pi}{2N}\right) & \dots & \sqrt{2} \cos\left(\frac{(2N-1)(N-1)\pi}{2N}\right) \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{pmatrix}. \quad (11)$$

3. DWT

In contrast to the DFT, with DWT, the signal contains only the frequency domain information of the original signal, the signal after the DWT contains both the frequency domain information and spatial information. This feature also enables the DWT to be widely used in the field of image compression. The scheme of the DWT is shown in Fig. 1.

In Fig. 1, g is a high-frequency filter, h is a low-frequency filter, and $\downarrow 2$ represents a decimation filter. As seen from Fig. 1, the DWT is different from the other transformations, and it is essentially a multi-layer decomposition process. The first decomposition divides the N -length signal into the high-frequency part D_1 and the low-frequency part A_1 , whose length is $N/2$. The second decomposition divides the low-frequency part A_1 into the high-frequency part D_2 and the low-frequency part A_2 and so on.

The mathematical expression of the first-layer decomposition in the DWT is

$$D_1(k) = \sum_{n=0}^{N-1} x(2k-n)g(n),$$

$$A_1(k) = \sum_{n=0}^{N-1} x(2k-n)h(n). \quad (12)$$

The mathematical expression of the i th ($i \geq 2$)-layer decomposition in the DWT is

$$D_i(k) = \sum_{n=0}^{N-1} A_{i-1}(2k-n)g(n),$$

$$A_i(k) = \sum_{n=0}^{N-1} A_{i-1}(2k-n)h(n). \quad (13)$$

4. Proposed sparse transform (sparse dictionary)

Assuming the pulse signal $s(t)$ transmitted by the ultrasound probe is known, the mathematic expression of the echo signal can be expressed as follows:

FIG. 1. The scheme of the DWT.

$$x(t) = \sum_{m=1}^n \alpha_m s(t - t_m), \quad (14)$$

where n is the number of reflected pulse signals, t_m is the delay time, and α_m is the attenuation amplitude of the m th reflection echo. In the mathematical model, $x(t)$ can be regarded as a sparse signal and $s(t)$ satisfies the following conditions:

$$s(t - t_m) = 0, \quad \forall t - t_m \notin [0, T). \quad (15)$$

If the sampling frequency of the system is f_s and the sampling period is $T_s = 1/f_s$, the echo signal can be rewritten as follows:

$$x(t) = \sum_{m=1}^n \alpha_m s(t - n_m T_s). \quad (16)$$

Theoretically, the echo signal is the sum of the emission signals with different time delays and attenuation, and the shape of the emission signal does not change after reflection. Therefore, the echo signal is sparse within a certain transform domain constructed by the emission pulse signal. The construction process of the sparse basis function and the sparse dictionary using the emission pulse signal $s(t)$ is as follows:

$$\psi_i(t) = s(t - iT_s), \quad (17)$$

$$\Psi = \{\psi_i(t) | \psi_i(t) = s(t - iT_s)\}, \quad i = 1, 2, \dots, N. \quad (18)$$

Discrete sampling of the sparse basis function by sampling frequency f_s is

$$\begin{aligned} \psi_i &= [0, \dots, 0, s(T_s), s(2T_s), s(3T_s), \dots, s(kT_s), 0, \dots, 0] \\ &= [0, \dots, 0, \psi, 0, \dots, 0] \end{aligned} \quad (19)$$

where

$$\psi = [s(T_s), s(2T_s), s(3T_s), \dots, s(kT_s)]. \quad (20)$$

Substituting Eq. (19) into Eq. (18), the corresponding sparse dictionary matrix $\Psi \in \mathbb{C}^{N \times N}$ can be obtained,

$$\Psi = [\psi_1, \psi_2, \dots, \psi_N] = \begin{bmatrix} \psi & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \psi \end{bmatrix}. \quad (21)$$

According to Eq. (1), using the above sparse dictionary as a sparse matrix, the sparse expression of the sampling echo signal $\mathbf{x}$ is

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} \psi & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \psi \end{bmatrix} \begin{bmatrix} 0 & \cdots & \alpha_1 & \cdots & \alpha_2 & \cdots & \alpha_n & \cdots & 0 \end{bmatrix} \\ &= \Psi \boldsymbol{\alpha}. \end{aligned} \quad (22)$$

As seen from Eq. (22), the echo signal is very sparse on the proposed sparse dictionary. The sparse coefficients $\boldsymbol{\alpha} \in \mathbb{C}^{N \times 1}$ contain only n nonzero elements, and the sparsity

equals the number of the received reflection echo signals. Compared with the common sparse transform methods, the sparsity of the sparse coefficients Ψ is higher than the proposed sparse dictionary. Likewise, the proposed method can achieve better reconstruction effects.

III. SIMULATION EXPERIMENT

The paper conducts simulation experiments by Field II.^{17,18} The basic parameter settings for the simulation experiment are shown in Table I. The dynamic range of all images was 50 dB. For all the sparse transformations, the YALL1_group algorithm for BP was adopted to reconstruct the image. The code was written in MATLAB with no parallelization of fine-tuning yet, and the reconstructed image was obtained on a computer with Intel Xeon E5-2683 v3 CPU running 64-bit Windows 7 and MATLAB R2014a.

A. The simulation experiment of scattering point targets

The simulation experiment sets 16 scattering point targets distributed on average in the range from 30 to 110 mm with an interval of 10 mm. Figure 2 shows the single echo signal received by an element in the synthetic aperture imaging and the corresponding reconstructed signal associated with different sparse transforms calculated by the YALL1_group algorithm after removing 50% of the original samples.

As seen from Fig. 2, the echo signal is the sum of the emission pulse with time delay and attenuation at different point targets. With increasing depth, the attenuation of the echo signal is more serious, but the shape is almost unchanged. The proposed sparse dictionary yields excellent reconstruction quality, and there is no noise in the reconstructed signal. Although the DCT can be precisely reconstructed at the point targets, it produces some noise at other remaining points. In addition, the reconstruction result of the DFT is the worst.

Figure 3 presents the sparse representation of the single echo signal under different sparse transforms. The proposed sparse dictionary is obviously better than the three other sparse transforms. The sparsity of the sparse coefficients equals the number of point targets, namely, 16. The results shown in Fig. 2 coincide with the effect of sparse representation in Fig. 3. Under the same conditions, the sparser the sparse coefficients, the better the reconstruction effect.

The reconstruction of a single echo signal cannot represent the influence of different sparse transforms on the whole signals. Figure 4 shows the overall reconstructed images

TABLE I. Simulation experimental parameters.

Parameter		Value
Central frequency	f_s	3.5 MHz
Sampling frequency		30 MHz
Array type	N	Linear
Transducer number		64
Element width		Half of a wavelength
Element height		5 mm

FIG. 2. (Color online) The single echo signal and the reconstructed signal.

FIG. 3. (Color online) The sparse coefficients associated with the four different sparse transforms.

FIG. 4. The reconstructed images associated with the four different sparse transforms.

after removing the same number of samples. From Fig. 4, we can see that the reconstructed images with the proposed dictionary and DCT are almost the same as the original image; there is no distortion in the two images. In contrast, the images reconstructed with DFT and DWT contain some artifacts, which did not exist in the original image and can cause degradation of image quality. Figure 5 shows the lateral variation of the four reconstructed points at a depth of 60 mm. The lateral variation curves of DCT, DFT, and DWT show a visible difference from the original image, whereas the difference is barely discernible for the proposed dictionary, which also yields the same resolution.

In this paper, we take the mean absolute error (MAE) as an indicator to quantify the reconstruction quality between the reconstructed images and the original image. Its mathematical expression is as follows:

$$\text{MAE} = \frac{\left| \sum_{i=1}^N \sum_{j=1}^M (\tilde{x}(i,j) - x(i,j)) \right|}{N \cdot M}. \quad (23)$$

In Eq. (23), $x(i,j)$ represents the sampling data of the original ultrasonic echo signal, $\tilde{x}(i,j)$ is the sampling data of the reconstructed signals obtained by the compressed sensing, M is the length of the detecting depth, and N is the number of array elements. It is important to note that because the amplitude of the ultrasonic echo signal is very small and there is a large amount of data, the ultrasonic echo signals are normalized first, after which compressed sensing is performed. In addition, reconstruction time (RT) is used to measure the computational load of the sparse transformation under the same reconstruction algorithm; RT indicates the iteration time required for the YALL1_group algorithm.

Table II shows the corresponding values of MAE and RT with different sparse transforms. The MAE values of the first three transforms are approximately 1825, 640, and 64 times worse than that of the proposed dictionary. It also can be seen from Table II that the longest time for the reconstruction algorithm occurs using DFT. The RT of the DWT and DCT is less than that of DFT, and the proposed sparse dictionary requires the shortest RT. The proposed sparse basis makes use of the features of the echo signal. Thus, under the same reconstruction conditions, the sparse

FIG. 5. (Color online) The lateral variation of the reconstructed points at a depth of 60 mm.

TABLE II. The values of MAE and RT with different sparse transforms.

The sparse transform	DFT	DWT	DCT	Sparse dictionary
MAE	0.0314	0.0110	0.0011	1.72×10^{-5}
RT (s)	22 294.12	11 560.96	11 411.05	11 402.85

FIG. 6. The reconstructed beam-forming images after removing 50% of the original samples.

coefficients based on the proposed sparse basis are sparser and less RT is required.

B. The simulation experiment of a cyst phantom

The simulation of a cyst phantom consists of ten cysts and five point targets that are distributed in the range between 30 and 90 mm. The phantom contains five hyperechoic areas whose diameters from top to bottom are 6, 5, 4, 3, and 2 mm as well as five anechoic cysts whose diameters from bottom to top are 6, 5, 4, 3, and 2 mm.

Figure 6 compares the reconstructed beamforming images with different sparse transforms after removing 50% of the original samples. The image quality of the original is obviously better than the first two transforms. The original image features are perfectly preserved with the proposed dictionary, and the DCT produces a slightly worse image. Table III shows the MAE and the RT corresponding to Fig. 6. Compared to Table II, it can be observed that whatever the sparse transform uses, the error increases with the increase in complexity of the original image. However, the proposed

dictionary also yields a much smaller error than the other three transforms, and the sparse dictionary also requires the shortest RT.

The algorithm of compressed sensing can reduce the sampling rate of echo data and simplify the complexity of the hardware system. To investigate the influence of different sampling rates on image quality, Fig. 7 shows the reconstructed images after randomly removing 70%, 80%, 90%, and 95% samples and Table IV provides the corresponding MAE and RT. Due to the reduction of sample data, it is obvious that the reconstructed quality degrades regardless of which sparse transform is used, and the RT increases with the decrease of sampling data. The reconstruction errors of the proposed dictionary with 70% and 80% of samples removed approximates to that of the DCT with 50% of samples removed. The reconstruction errors of the proposed dictionary with 90% and 95% of samples removed are similar to those of the DFT and DWT with 50% of samples removed. In the same condition, the proposed dictionary yields better performance. In other words, the proposed dictionary can obtain the same reconstructed image with fewer samples than other three transforms.

TABLE III. The MAEs and RT corresponding to Fig. 6.

The sparse transform	DFT	DWT	DCT	Sparse dictionary
MAE	0.0754	0.0347	0.0039	9.15×10^{-4}
RT (s)	22 204.42	11 676.88	11 659.67	11 398.35

IV. EXPERIMENT

The experimental radio frequency data were acquired with ultrasonic equipment developed by our laboratory. The parameters of the image system were set as follows: a C3.5-

FIG. 7. The reconstructed images of the dictionary with 30%, 20%, 10%, and 5% samplings.

TABLE IV. The corresponding MAE and RT of Fig. 7.

The percentage of samples removed	70%	80%	90%	95%
MAE	0.0027	0.0093	0.0284	0.0422
RT (s)	22 131.10	25 007.72	29 168.03	31 220.94

FIG. 8. The experimental reconstructed images.

TABLE V. The corresponding MAE and RT of Fig. 8.

The sparse transform	DFT	DWT	DCT	The sparse dictionary
MAE	0.0536	0.0148	0.0114	0.0060
RT (s)	704.75	306.40	305.05	290.34

80R60S probe with a central frequency of 3.5 MHz, sampling frequency of 25 MHz, 16 effective channels, 131 linear scanning lines, and a lateral aperture diameter of 13 mm. The experimental reconstructed images are shown in Fig. 8; in these images, 80% of the original samples were removed and the dynamic range of the images is 50 dB.

Figure 8 shows that the reconstructed image of the DFT is the worst. Although the basic information is preserved in the image from the DWT, this transform introduces much noise, which degrades the final reconstruction quality. Compared to the DCT, the edge profile of the dictionary is blurrier, the noise is reduced, and the contrast is also better. Table V presents the corresponding MAE and the RT. The performance of the four transforms becomes worse with an increase in the noise rate, but it is still evident that the proposed dictionary yields the smallest error for the 20% subsampling rate. The error of the dictionary is approximately one-half that of the DCT, one-seventh that of the DWT, and one-sixteenth of the DFT. As the echo data were reduced, the RT required for various sparse transformations was reduced, and the shortest RT was required in the sparse dictionary.

V. CONCLUSION

This paper proposes an efficient sparse dictionary of CS for ultrasound imaging. Compared with the traditional transforms DFT, DWT, and DCT, the echo signal can be more accurately reconstructed with the proposed dictionary. In fact, the sparsity of the echo signal's sparse coefficients associated with the dictionary equals the number of the received echo pulses. The ultrasound echo signal has perfect sparsity in the proposed dictionary, which exhibits the characteristics of attenuation, repetition, and superposition.

The MAE and RT were introduced to quantify the reconstructed quality at different subsampling rates. The experimental results indicate that the proposed dictionary yields the smallest error and requires the shortest RT. Meanwhile, the lateral resolution and contrast of the reconstructed images associated with the proposed dictionary were more similar to the original images than those provided by the traditional transforms. Correspondingly, the same

reconstructed quality can be achieved with a lower subsampling rate than with the traditional transforms.

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