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Abstract. The purpose of this study was to extend the concept of weighted CT dose index ($CTDI_w$) to the elliptical phantoms. Based on the published body dimension data, eight body aspect ratios were chosen between 1 (perfectly circular) and 1.72 (extremely elliptical). For each aspect ratio, two elliptical cylinders were created digitally to represent adult and pediatric bodies. Their cross-sectional areas were identical to the standard 32- and 16-cm CTDI phantoms. For each phantom, $CTDI_{100}$ at center and periphery were simulated for tube voltages between 70 and 140 kVp using a validated Monte Carlo program. The simulations also provided the average dose over the cross-sectional area, $CTDI_{xsec}$. Values of $CTDI_{xsec}$ and $CTDI_{100}$ allowed linear systems of equations to be established, from which central and peripheral weighting coefficients were solved. Regardless of phantom shape, only two weighting coefficients were needed: w_1 for the central $CTDI_{100}$ and w_2 for the average of the four peripheral $CTDI_{100}$'s. Over the full range of aspect ratios, w_1 increased linearly from 0.37 to 0.46, whereas w_2 decreased linearly from 0.63 to 0.54, allowing the concept of $CTDI_w$ to be readily extended to the elliptical phantoms. When cross-sectional area (hence volume) was kept constant, all phantoms had the same $CTDI_{xsec}$ regardless of shape. © 2017 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JMI.4.3.031205]

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1 Introduction

Currently, the standard CT dose index (CTDI) phantoms are perfectly circular cylinders. Lately, the elliptical cylinders have been proposed as a potential alternative because they represent the shapes of the human body better, which further permits the evaluation of automatic tube current modulation systems.^{1,2} The elliptical shape, however, calls for a new definition of weighted CTDI ($CTDI_w$). $CTDI_w$ is intended to represent the planar average dose, i.e., the average dose over the cross-sectional area (at the midplane of a large scan volume). In its current definition

$$CTDI_w = \frac{1}{3}CTDI_{100,center} + \frac{2}{3}CTDI_{100,periphery}, \quad (1)$$

the 1/3 and 2/3 weight coefficients approximate to the relative areas represented by the central and peripheral values.³ In other words, it is assumed that a 1/3 of the circular area receives a dose that can be approximated by $CTDI_{100,center}$, whereas 2/3 of the circular areas receives a dose that can be approximated by $CTDI_{100,periphery}$. Because an elliptical phantom has two peripheral values (Fig. 1), it is natural to ask how they should be combined with the central value to calculate $CTDI_w$ so that this concept still approximates to the planar average dose. Furthermore, it is natural to ask whether the weighing coefficients should be different for elliptical phantoms of different aspect ratios (the ratio of major axis to minor axis). The goal of this study was to answer these questions. Toward this goal,

we performed a Monte Carlo study using elliptical phantoms of various aspect ratios created digitally.

2 Methods

2.1 Clinical Relevant Range of Body Aspect Ratios

Body dimensions of adults and children are available in the literature in the form of anteroposterior (AP) and lateral (LAT) thicknesses. Kleinman et al.⁴ measured AP and LAT thicknesses of children from clinical chest, abdomen, and pelvis CT images. From their published data, body aspect ratios were calculated and plotted as a function of pediatric age [Fig. 2(a)]. Similarly, Ogden et al.⁵ measured AP and LAT thicknesses of adult and pediatric patients from clinical chest and abdomen CT scans. Their study provided equations of AP and LAT thicknesses as functions of body weight, which allowed the aspect ratio to be established as a function of weight [Fig. 2(b)]. For the abdomen region only, AAPM Report No. 204 provides equations of effective diameter ($\sqrt{AP \times LAT}$) as a function of AP and LAT thicknesses.⁶ From these equations, the aspect ratio as a function of effective diameter was obtained [Fig. 2(c)]. Body aspect ratios were also calculated for the widely used pediatric and adult mathematical anthropomorphic phantoms developed at Oak Ridge National Laboratory (ORNL).^{7,8} The trunk of each ORNL phantom is an elliptical cylinder with chest, abdomen, and pelvis regions having the same elliptical shape. The body aspect ratio was calculated from the AP and LAT

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Fig. 1 An elliptical phantom has two different peripheral CTDI values.

thicknesses of this elliptical cylinder and plotted as a function of phantom age [Fig. 2(a)], where the adult anthropomorphic phantom was given an age of 20. These data suggested a maximum body aspect ratio of 1.72.

2.2 Elliptical Phantoms of Various Aspect Ratios

With the data in Fig. 2 as guidance, 16 elliptical phantoms were designed with aspect ratios ranging from 1 to 1.72 (Fig. 3). Eight of the phantoms represented adult body. All of them had the same cross-sectional area (and volume) as the standard 32-cm-diameter circular CTDI phantom. Each adult body phantom had a corresponding pediatric body phantom. All the pediatric phantoms had the same cross-sectional area (and volume) as the standard 16-cm-diameter circular CTDI phantom. The dimensions of the 16 phantoms are summarized in Table 1.

To enable the simulation of CTDI using a Monte Carlo program, a voxelized model was created for each phantom using an interactive program (MATLAB®, R2011b; Mathworks, Natick, Massachusetts). The voxelized model consisted of seven objects (Fig. 4): five rods, 100-mm long and 8 mm in diameter, to represent the air cavity of the pencil ion chamber when placed at the central and peripheral hole locations, the middle 100-mm-long region of the phantom (dark region in Fig. 4), and the rest of the phantom. The center of each peripheral hole was 1 cm below the phantom surface to emulate the rod locations in a standard circular CTDI phantom.

2.3 Monte Carlo Simulations of $CTDI_{100}$ and Its Planar Average

CTDI simulations were performed using a Monte Carlo program previously developed and validated for a 128-slice CT system

Fig. 3 With body aspect ratios of adults and children as guidance, elliptical phantoms were created digitally with aspect ratios ranging from 1 to 1.72.

Table 1 Dimensions of the 16 elliptical phantoms designed for this study.

Aspect ratio	Adult phantom		Pediatric phantom	
	AP (mm)	LAT (mm)	AP (mm)	LAT (mm)
1	320	320	160	160
1.1	308	338	152	168
1.2	294	354	146	176
1.3	282	368	140	182
1.4	272	382	134	190
1.5	260	394	130	198
1.6	254	408	126	202
1.72	246	422	122	210

(SOMATOM Definition Flash, Siemens Healthcare, Forchheim, Germany).⁹⁻¹² The program is based on a benchmarked Monte Carlo subroutine package for photon, electron, and positron transport (PENELOPE, version 2006, Universitat de Barcelona,

Fig. 2 Body dimensions of adults and children are available in the literature in the form of AP and LAT thicknesses. From these data, body aspect ratios were calculated and plotted as functions of age, weight, or effective diameter.

Fig. 4 The computational model of each phantom contained seven objects for the purposes of CTDI simulations.

Spain).^{13,14} It explicitly models the geometry of the CT system and the trajectory of the x-ray source during a CT scan. The x-ray energy spectra and the photon path lengths through the bowtie filter were provided by the manufacturer. Assuming a point source, the program uses an effective beam width to account for the dose delivered by both the umbra and the penumbra regions of the beam. The effect of bowtie filter attenuation is modeled by varying the statistical weight of each simulated photon based on its energy and its path lengths in the bowtie filter materials.

For each adult and pediatric phantom, single axial scans were simulated for five tube voltage settings (70, 80, 100, 120, and 140 kVp) using the widest beam collimation of 38.4 mm and the standard bowtie filter for body imaging. We assumed each phantom to be perfectly aligned at the isocenter of the scanner. The attenuation provided by the patient table was ignored.

During each simulation, all seven objects in a phantom (Fig. 4) were assumed to be made of polymethyl methacrylate (PMMA). For the photon energies encountered in CT, the ranges of secondary electrons in PMMA are much smaller than the dimensions of the seven objects. Therefore, secondary electrons were not transported; their energies were deposited locally immediately after they were produced. Photon cutoff energy was set to 1/1000 of the maximum photon energy (i.e., the kVp energy) per recommendation by the PENELOPE code package.¹⁵ Each simulation used five hundred million (5×10^8) photon histories, yielding relative dose errors of less than 0.5% for all seven objects.

To obtain $CTDI_{100}$ at each hole location, a conversion from PMMA dose to air dose was first performed. Specifically, the dose received by the 100-mm-long PMMA rod was used to calculate the dose received by the air cavity of the pencil ion chamber using the ratio of the mass energy absorption coefficients

$$D_{\text{air}} = D_{\text{PMMA}} \frac{\sum_E \Psi(E) \left(\frac{\mu_{\text{en}}}{\rho} \right)_{\text{air}, E}}{\sum_E \Psi(E) \left(\frac{\mu_{\text{en}}}{\rho} \right)_{\text{PMMA}, E}}. \quad (2)$$

In Eq. (2), $\Psi(E)$ is the spectrum of photon energy fluence in the 100-mm-long PMMA rod tallied during the Monte Carlo simulation, and $\left(\frac{\mu_{\text{en}}}{\rho} \right)$ is the material- and energy-specific mass energy absorption coefficient published on the website of the National Institute of Standards and Technology. $CTDI_{100}$ was subsequently calculated based on its definition as

$$CTDI_{100} = \frac{D_{\text{air}} \times 100 \text{ mm}}{nT}, \quad (3)$$

where nT denotes the nominal collimation of the beam. This approach to simulating and calculating $CTDI_{100}$ was also used in an earlier study,¹² which found good agreement between simulations and measurements.

In addition to $CTDI_{100}$ at individual hole locations, the planar average $CTDI_{100}$ (i.e., an average over the cross-sectional area of the phantom), denoted as $CTDI_{\text{xsec}}$, was also calculated using Eqs. (2) and (3) based on the dose and photon spectrum simulated for the middle 100-mm-long region of the phantom (dark region in Fig. 4). As mentioned in Sec. 1, this planar average dose is the quantity that $CTDI_w$ is intended to represent.

2.4 Definition of $CTDI_w$ for Elliptical Phantoms

In keeping with the traditional definition of $CTDI_w$ for circular phantoms, we considered the following definition for elliptical phantoms:

$$CTDI_w = w_1 \cdot CTDI_c + w_2 \cdot \langle CTDI_p \rangle, \quad (4)$$

where w_1 is the weighting coefficient for the central hole and w_2 is the weighting coefficient for the average of the four peripheral holes. At each kVp and for each aspect ratio, w_1 and w_2 were obtained by solving the following pair of linear equations:

$$\begin{cases} CTDI_{\text{xsec}}^{\text{adult}} = w_1 \cdot CTDI_c^{\text{adult}} + w_2 \cdot \langle CTDI_p^{\text{adult}} \rangle \\ CTDI_{\text{xsec}}^{\text{ped}} = w_1 \cdot CTDI_c^{\text{ped}} + w_2 \cdot \langle CTDI_p^{\text{ped}} \rangle \end{cases}, \quad (5)$$

where $CTDI_i^{\text{adult}}$ and $CTDI_i^{\text{ped}}$ ($i = \text{xsec}, c, p$) are Monte Carlo simulated $CTDI_{100}$ values for the adult and pediatric phantoms, respectively. The weighting coefficients were solved from Eq. (5) using the *linsolve* command of MATLAB® (R2011b; Mathworks, Natick, Massachusetts).

In addition to Eqs. (4) and (5), we also examined an alternative definition and an alternative method for solving the weighting coefficients. These are discussed in the [Appendix](#).

3 Results and Discussion

3.1 Sample Values of $CTDI_{100}$ and $CTDI_{\text{xsec}}$

As examples of the Monte Carlo simulation results, Tables 2–4 summarized $CTDI_{100}$ and $CTDI_{\text{xsec}}$ for aspect ratios 1, 1.4, and 1.72. Regardless of phantom shape, the average periphery-to-center $CTDI_{100}$ ratios (denoted as $\langle CTDI_p \rangle / CTDI_c$) were around 2 for the adult phantoms and around 1 for the pediatric phantoms. As the aspect ratio increased, $\langle CTDI_{p, \text{minor}} \rangle$ differed more from $\langle CTDI_{p, \text{major}} \rangle$ as one would expect; however, the average of the four peripheral values differed less from the central value as reflected in the decreased $\langle CTDI_p \rangle / CTDI_c$ with an increasing aspect ratio. $\langle CTDI_p \rangle / CTDI_c$ also decreased with increasing kVp, consistent with the fact that a higher energy beam produces more uniform dose distribution.¹²

3.2 Definition of $CTDI_w$ for Elliptical Phantoms

Table 5 summarizes the solutions to Eq. (5) for various kVp and aspect ratios. Although Eq. (5) was solved without the constraint of $w_1 + w_2 = 1$, the two coefficients summed up to be ~ 1 for all combinations of kVp and aspect ratio. This is consistent with the physical meaning of these two weighting coefficients: they approximate to the relative areas represented by the central and peripheral CTDI measurements. In other words, a fraction (w_1)

Table 2 Monte Carlo simulated CTDI (mGy/100 mAs) for circular phantoms (aspect ratio=1).

	70 kVp	80 kVp	100 kVp	120 kVp	140 kVp
Adult phantom					
CTDI _c	0.6	1.0	2.3	4.0	6.3
$\langle \text{CTDI}_p \rangle^a$	1.4	2.3	4.5	7.4	11.2
CTDI _{xsec}	1.1	1.8	3.7	6.2	9.5
$\langle \text{CTDI}_p \rangle / \text{CTDI}_c$	2.5	2.2	2.0	1.9	1.8
Pediatric phantom					
CTDI _c	2.4	4.0	7.9	13.2	19.7
$\langle \text{CTDI}_p \rangle^a$	2.7	4.2	8.2	13.4	19.9
CTDI _{xsec}	2.6	4.2	8.2	13.4	19.9
$\langle \text{CTDI}_p \rangle / \text{CTDI}_c$	1.10	1.06	1.03	1.01	1.01

^aDue to the circular symmetry of the phantoms, all four peripheral locations had nearly identical simulated dose value. Only $\langle \text{CTDI}_p \rangle$ is shown in this table.

of the cross-sectional area receives a dose that can be approximated by the central measurement, whereas the rest (w_2) receives a dose that can be approximated by the average of the four peripheral measurements.

Table 3 Monte Carlo simulated CTDI (mGy/100 mAs) for elliptical cylinders with an aspect ratio of 1.4.

	70 kVp	80 kVp	100 kVp	120 kVp	140 kVp
Adult phantom					
CTDI _c	0.6	1.1	2.4	4.2	6.5
$\langle \text{CTDI}_{p,\text{minor}} \rangle^a$	1.6	2.5	5.0	8.2	12.2
$\langle \text{CTDI}_{p,\text{major}} \rangle^a$	1.2	2.0	4.0	6.7	10.1
$\langle \text{CTDI}_p \rangle$	1.4	2.3	4.5	7.4	11.1
CTDI _{xsec}	1.1	1.8	3.7	6.2	9.4
$\langle \text{CTDI}_p \rangle / \text{CTDI}_c$	2.3	2.1	1.9	1.8	1.7
Pediatric phantom					
CTDI _c	2.5	4.1	8.1	13.4	20.0
$\langle \text{CTDI}_{p,\text{minor}} \rangle$	2.8	4.5	8.6	14.0	20.7
$\langle \text{CTDI}_{p,\text{major}} \rangle$	2.5	3.9	7.6	12.5	18.6
$\langle \text{CTDI}_p \rangle$	2.7	4.2	8.1	13.2	19.6
CTDI _{xsec}	2.6	4.2	8.1	13.3	19.8
$\langle \text{CTDI}_p \rangle / \text{CTDI}_c$	1.06	1.02	1.00	0.99	0.98

^aDue to the elliptical symmetry of the phantoms, the 12 and 6 o'clock hole locations had nearly identical simulated dose value. They are represented by $\langle \text{CTDI}_{p,\text{minor}} \rangle$. Similarly, $\langle \text{CTDI}_{p,\text{major}} \rangle$ represents the dose received by the 3 and 9 o'clock hole locations.

Table 4 Monte Carlo simulated CTDI (mGy/100 mAs) for elliptical cylinders with an aspect ratio of 1.72.

	70 kVp	80 kVp	100 kVp	120 kVp	140 kVp
Adult phantom					
CTDI _c	0.7	1.2	2.6	4.6	7.1
$\langle \text{CTDI}_{p,\text{minor}} \rangle$	1.7	2.7	5.3	8.6	12.9
$\langle \text{CTDI}_{p,\text{major}} \rangle$	1.2	1.9	3.7	6.3	9.5
$\langle \text{CTDI}_p \rangle$	1.4	2.3	4.5	7.4	11.2
CTDI _{xsec}	1.1	1.8	3.7	6.2	9.4
$\langle \text{CTDI}_p \rangle / \text{CTDI}_c$	2.1	1.9	1.7	1.6	1.6
Pediatric phantom					
CTDI _c	2.6	4.2	8.2	13.5	20.2
$\langle \text{CTDI}_{p,\text{minor}} \rangle$	2.9	4.6	8.8	14.2	21.1
$\langle \text{CTDI}_{p,\text{major}} \rangle$	2.3	3.7	7.1	11.7	17.4
$\langle \text{CTDI}_p \rangle$	2.6	4.1	7.9	12.9	19.2
CTDI _{xsec}	2.6	4.1	8.1	13.2	19.7
$\langle \text{CTDI}_p \rangle / \text{CTDI}_c$	1.02	0.99	0.97	0.96	0.95

At a given kVp, w_1 increased with an increasing aspect ratio. Correspondingly, w_2 decreased with an increasing aspect ratio. This indicated that as the phantom became more elliptical, a larger percentage of the area received a dose that could be approximated by the central CTDI₁₀₀.

For a given aspect ratio, w_1 decreased with increasing kVp. Correspondingly, w_2 increased with increasing kVp. In general, the variability across kVp settings was small (coefficient of variation: 2% to 5%). Figure 5 shows the kVp-averaged w_1 and w_2 versus the aspect ratio. Both relationships fitted well to a straight line. At an aspect ratio of one, the kVp-averaged w_1 and w_2 were 0.37 and 0.63, respectively. They agreed reasonably well with the conventional weighting coefficients of 1/3 and 2/3 for circular phantoms. As the phantom became more elliptical, the weighting for the central hole (w_1) increased linearly from 0.37 to 0.46, whereas the weighting for peripheral holes (w_2) decreased linearly from 0.63 to 0.54. These results suggest that Eq. (4) can serve as the definition of CTDI_w for both circular and elliptical phantoms as long as the weighting coefficients are adjusted based on the aspect ratio.

3.3 CTDI_{xsec}

Another interesting finding of this study was in regard to the planar average CTDI₁₀₀ (denoted as CTDI_{xsec}). For both the adult and the pediatric phantoms, CTDI_{xsec} was independent of the aspect ratio (Fig. 6). In other words, when the cross-sectional area (and hence volume) was kept the same, all the phantoms had the same cross-sectional average dose regardless of how elliptical they were. This result indicates that for the purpose of assessing the planar average dose, which is what the commonly used concepts of volume-weighted CTDI (CTDI_{vol}) and size-specific dose estimate (SSDE)⁶ represent, circular phantoms are

Table 5 Weighting coefficients in the definition, $CTDI_w = w_1 \cdot CTDI_c + w_2 \cdot \langle CTDI_p \rangle$, for various kVp and aspect ratios.

Aspect ratio	70 kVp	80 kVp	100 kVp	120 kVp	140 kVp	Mean	COV (%) ^a
w_1							
1	0.4004	0.3862	0.3749	0.3648	0.3634	0.3779	4
1.1	0.4099	0.3868	0.3779	0.3689	0.3675	0.3822	5
1.2	0.4169	0.3987	0.3844	0.3765	0.3772	0.3907	4
1.3	0.4281	0.4040	0.3934	0.3859	0.3878	0.3998	4
1.4	0.4367	0.4180	0.4044	0.3972	0.3954	0.4103	4
1.5	0.4494	0.4316	0.4188	0.4093	0.4076	0.4233	4
1.6	0.4621	0.4469	0.4376	0.4309	0.4287	0.4412	3
1.72	0.4815	0.4634	0.4558	0.4476	0.4488	0.4594	3
w_2							
1	0.6098	0.6235	0.6331	0.6409	0.6415	0.6298	2
1.1	0.6025	0.6213	0.6296	0.6366	0.6381	0.6256	2
1.2	0.5959	0.6118	0.6240	0.6292	0.6296	0.6181	2
1.3	0.5854	0.6047	0.6137	0.6192	0.6184	0.6083	2
1.4	0.5748	0.5905	0.6027	0.6082	0.6093	0.5971	2
1.5	0.5616	0.5769	0.5879	0.5961	0.5968	0.5838	3
1.6	0.5464	0.5603	0.5682	0.5737	0.5755	0.5648	2
1.72	0.5260	0.5418	0.5486	0.5548	0.5529	0.5448	2

^aCoefficient of variation = (standard deviation/mean) $\times$ 100%.

sufficient despite the fact that they do not represent the shapes of the human body well.

3.4 Implications for CT Dosimetry

The results of our study have two implications for CT dosimetry:

1. For the sole purpose of accessing the cross-sectional average dose, the traditional circular CTDI phantoms are sufficient and elliptical CTDI phantoms are not necessary. However, the latter is advantageous in that it also permits the evaluation of tube current modulation systems. When using elliptical phantoms, one can still obtain $CTDI_w$, in its traditional meaning of a cross-sectional average dose, by combining central and peripheral measurements using shape appropriate weighting coefficients.
2. Because SSDE represents the dose received by a water cylinder having the same diameter as the diameter of the patient's body,⁶ SSDE is a reasonable representation of the cross-sectional average dose received by the patient in the midplane ($z = 0$) of the scan volume, even if the patient has a very elliptical body shape.

Fig. 5 Weighting coefficients in the definition, $CTDI_w = w_1 \cdot CTDI_c + w_2 \cdot \langle CTDI_p \rangle$, as a function of aspect ratio. Each data point represents an average over five kVp settings. The error bar reflects one standard deviation across the five kVp settings.

3.5 Limitations and Caveat

First, our study is limited to a single CT system and a single bowtie filter setting. For other CT systems and bowtie filter properties, the x-ray energy spectra reaching the phantoms would differ. However, considering that w_1 and w_2 did not vary substantially across the broad range of kVp settings

Fig. 6 Planar average CTDI_{100} (denoted as $\text{CTDI}_{\text{xsec}}$) for various kVp, aspect ratio, and phantom sizes.

investigated in this study (Table 5, last column), we do not expect their values to differ substantially for other CT systems and bowtie filter properties.

Second, our study is theoretical in nature. Its results may be further corroborated in an experimental study involving physical elliptical phantoms having the same cross-sectional areas as the standard circular CTDI phantoms. Gafchromic films may be used to measure the cross-sectional average dose and to verify if this quantity is independent of the aspect ratio. The aspect ratio-specific weighting coefficients in Table 5 can also be applied to CTDI measurements made in elliptical phantoms to verify the CTDI_w definition [Eq. (4)].

Third, the weighting coefficients published in this study (Table 5) only apply to the concept of CTDI_{100} . These weighting coefficients may not apply to the new dosimetry quantities proposed by AAPM Task Group 111 to replace CTDI_{100} .² A separate study is warranted.

It should be noted that although elliptical phantoms can be used to test the functionality of tube current modulation systems, the concept of CTDI_w and hence Eq. (4) only apply to fixed tube current scanning mode, which is the scanning mode of choice for all CT quality assurance tests required by the American College of Radiology. For tube current modulated scans, alternative dose metrics should be considered.^{16,17}

4 Conclusion

The concept of CTDI_w can be readily extended to elliptical phantoms if the $1/3$ and $2/3$ weighting coefficients are replaced with linear functions of elliptical aspect ratio. For the purpose of assessing the planar average dose, which is what the commonly used concepts of CTDI_{vol} and SSDE represent, circular phantoms are sufficient, despite the fact that they do not represent the shapes of the human body well.

Appendix: Alternative Definition and Method

In addition to the definition in Eq. (4), we also considered an alternative definition that involved three weighting coefficients, w_1 for the central hole, w_2 for the average of the two holes along the minor axis, and w_3 for the average of the two holes along the major axis:

$$\text{CTDI}_w = w_1 \cdot \text{CTDI}_c + w_2 \cdot \langle \text{CTDI}_{\text{p,minor}} \rangle + w_3 \cdot \langle \text{CTDI}_{\text{p,major}} \rangle. \quad (6)$$

At each kVp and for each aspect ratio, w_1 , w_2 , and w_3 were obtained by solving the following set of three linear equations:

$$\begin{cases} \text{CTDI}_{\text{xsec}}^{\text{adult}} = w_1 \cdot \text{CTDI}_c^{\text{adult}} + w_2 \cdot \langle \text{CTDI}_{\text{p,minor}}^{\text{adult}} \rangle + w_3 \cdot \langle \text{CTDI}_{\text{p,major}}^{\text{adult}} \rangle \\ \text{CTDI}_{\text{xsec}}^{\text{ped}} = w_1 \cdot \text{CTDI}_c^{\text{ped}} + w_2 \cdot \langle \text{CTDI}_{\text{p,minor}}^{\text{ped}} \rangle + w_3 \cdot \langle \text{CTDI}_{\text{p,major}}^{\text{ped}} \rangle \\ w_1 + w_2 + w_3 = 1 \end{cases}, \quad (7)$$

where $\text{CTDI}_i^{\text{adult}}$ and $\text{CTDI}_i^{\text{ped}}$ ($i = \text{xsec}, c, p$) are simulated CTDI_{100} values for the adult and pediatric phantoms, respectively.

Table 6 summarizes the solutions to Eq. (7) for various kVp and aspect ratios. At 70 kVp and an aspect ratio of 1.1, we obtained a negative w_3 value. As the phantom became more circular with a reduced aspect ratio, one would expect a closer match between w_2 and w_3 . However, the solutions of the linear equations showed otherwise (Fig. 7). These results indicate that Eq. (7) does not have physically meaningful solutions.

Even for the definition in Eq. (4), we obtained negative weighting coefficients when attempting to solve the following pairs of linear equations:

$$\begin{cases} \text{CTDI}_{\text{xsec}}^{\text{adult}} = w_1 \cdot \text{CTDI}_c^{\text{adult}} + w_2 \cdot \langle \text{CTDI}_p^{\text{adult}} \rangle \\ w_1 + w_2 = 1 \end{cases}, \quad (8)$$

$$\begin{cases} \text{CTDI}_{\text{xsec}}^{\text{ped}} = w_1 \cdot \text{CTDI}_c^{\text{ped}} + w_2 \cdot \langle \text{CTDI}_p^{\text{ped}} \rangle \\ w_1 + w_2 = 1 \end{cases}. \quad (9)$$

In summary, one cannot find physically meaningful weighting coefficients that add up to be exactly 1. Understandably, the traditional $1/3$ and $2/3$ weighting coefficients are only approximations. It is quite possible that if simulation data are generated for another phantom size (e.g., CTDI phantoms with an equivalent diameter of 24 cm), then another equation can be used

Table 6 Weighting coefficients in the alternative definition, $CTDI_w = w_1 \cdot CTDI_c + w_2 \cdot \langle CTDI_p \rangle$, for various kVps and aspect ratios.

Aspect ratio	70 kVp	80 kVp	100 kVp	120 kVp	140 kVp	Mean	COV (%) ^a
w_1							
1.1	0.4352	0.4012	0.3900	0.3772	0.3770	0.3961	6
1.2	0.4422	0.4155	0.3977	0.3856	0.3895	0.4061	6
1.3	0.4540	0.4187	0.4050	0.3946	0.3988	0.4142	6
1.4	0.4553	0.4294	0.4142	0.4048	0.4023	0.4212	5
1.5	0.4658	0.4435	0.4279	0.4168	0.4140	0.4336	5
1.6	0.4747	0.4572	0.4456	0.4375	0.4347	0.4499	4
1.72	0.4914	0.4701	0.4614	0.4508	0.4510	0.4649	4
w_2							
1.1	0.6849	0.5517	0.5431	0.4858	0.4956	0.5522	14
1.2	0.4849	0.4552	0.4320	0.3968	0.4217	0.4381	8
1.3	0.4207	0.3843	0.3736	0.3598	0.3710	0.3819	6
1.4	0.3628	0.3501	0.3490	0.3420	0.3387	0.3485	3
1.5	0.3341	0.3307	0.3273	0.3263	0.3221	0.3281	1
1.6	0.3077	0.3101	0.3089	0.3072	0.3071	0.3082	0.4
1.72	0.2870	0.2882	0.2894	0.2862	0.2826	0.2867	1
w_3							
1.1	-0.1201	0.0471	0.0669	0.1370	0.1273	0.0517	200
1.2	0.0729	0.1292	0.1702	0.2176	0.1887	0.1557	36
1.3	0.1253	0.1971	0.2214	0.2456	0.2302	0.2039	23
1.4	0.1818	0.2205	0.2368	0.2532	0.2590	0.2303	13
1.5	0.2002	0.2258	0.2448	0.2569	0.2639	0.2383	11
1.6	0.2176	0.2327	0.2455	0.2553	0.2582	0.2419	7
1.72	0.2217	0.2417	0.2491	0.2630	0.2664	0.2484	7

^aCoefficient of variation = (standard deviation/mean) × 100%.

Fig. 7 Weighting coefficients in the alternative definition, $CTDI_w = w_1 \cdot CTDI_c + w_2 \cdot \langle CTDI_{p,minor} \rangle + w_3 \cdot \langle CTDI_{p,major} \rangle$, as a function of the aspect ratio. Each data point represents an average over five kVp settings. The error bar reflects one standard deviation across the five kVps settings. The curves are second-order polynomial fits to the data.

together with the first two equations in Eq. (7) to obtain meaningful solutions. Nevertheless, the definition in Eq. (4) would still be preferred given its simplicity and practicality.

Disclosures

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