Published in final edited form as:

IEEE Trans Biomed Eng. 2017 April ; 64(4): 904–916. doi:10.1109/TBME.2016.2582643.

Separation of physiological Signals using Minimum Norm Projection Operators

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Abstract

Objective—This paper presents the development of a fast, robust method which can be applied to multi-channel physiologic signals for the purpose of either removing a selected interfering signal or separating signals that arise from temporally correlated and spatially distributed signals such as maternal or fetal cardiac waveform recordings.

Methods—Projection operators based upon both the weighted and un-weighted minimum norm equations are presented. The weighted formulation uses models based on signal covariance and the un-weighted formulation requires that a statistical model be built using time-locked averaging.

Results—We present examples that demonstrate the utility of our projection operators when applied to maternal and fetal magneto-cardiograms. In addition, we demonstrate the ability to separate fetal breathing signals from both maternal and fetal cardiac signals.

Conclusion—The method is effective, robust, fast, and does not require significant input from a user.

Significance—Although we demonstrate the utility of our projection operators applied to biomagnetic signals, the method can easily be adapted to other applications where the goal is to either separate or suppress selected signal components.

Index Terms

ICA; minimum norm; MEG; MCG; ECG

I. Introduction

Active current sources in the body produce measurable magnetic fields and electric potentials. Such recordings may be the result of several spatially distinct sources arising from the various organs such as the heart, brain, and muscle. It is common that a number of spatially distinct sources associated with an organ are temporally correlated. The adult human heart is the most obvious example in which the P wave and QRS complex arise from tissue depolarization in spatially distinct regions while the T wave arises from repolarization,

yet all are temporally correlated. It is often desirable to select one signal to study while discarding others, or it may be desirable to suppress a single large interfering signal to reveal weaker signals that are of interest. In practice, a range of methods may be applied, sometimes in sequence, to achieve the desired result [1–20]. Our aim here is to separate the maternal magnetocardiogram (mMCG), the fetal magnetocardiogram (fMCG), and the fetal breathing signal. Our methods are based upon the minimum norm approach that is commonly used to estimate the strength of spatially distributed current dipole [21–23].

We first summarize the minimum norm method as presented in [22]. Then, we introduce our new approach, which uses the minimum norm idea and applies it to time series at the sensor level. The foundational assumption is that the magnetic fields produced by a large number of current dipoles can be represented by a set of lead fields associated with a set of surrogate current sources as presented in the following matrix formulation:

$$\mathbf{X} = \mathbf{L}\mathbf{Q}. \quad (1)$$

After including system noise, the observed data is given by

$$\mathbf{Y} = \mathbf{X} + \mathbf{n}. \quad (2)$$

$\mathbf{Y}$ is the observed data and each row of $\mathbf{Y}$ is the time series from one sensor, $\mathbf{X}$ is the modeled signals arising from current dipoles within the body, and $\mathbf{n}$ describes the system noise. $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{n}$ are size N_c (number of channels) by N_t (number of time samples). $\mathbf{Q}$ has dimension N_q (number of sources) by N_t and $\mathbf{L}$ has dimension N_c by N_q . The matrix $\mathbf{Q}$ describes the time series of the biologically active sources that give rise to the observed signals and each column of the matrix $\mathbf{L}$ is the lead field associated with each current source. We only consider the case where the set of equations defined by (1) and (2) is under-determined.

We take the covariance to be estimated by the expectation operator. Then,

$$\mathbf{C}_x = E\{\mathbf{X}\mathbf{X}^T\} = \mathbf{L}(E\{\mathbf{Q}\mathbf{Q}^T\})\mathbf{L}^T = \mathbf{L}\mathbf{C}_q\mathbf{L}^T, \quad (3)$$

and assuming independence between noise and currents gives

$$\mathbf{C}_y = \mathbf{L}\mathbf{C}_q\mathbf{L}^T + \mathbf{C}_n. \quad (4)$$

Continuing with the formulation as shown in [22], the weighted minimum norm estimate for the current sources is

$$\begin{bmatrix} \hat{\mathbf{Q}} \\ \hat{\mathbf{n}} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_q \mathbf{L}^T \\ \mathbf{C}_n \end{bmatrix} (\mathbf{L} \mathbf{C}_q \mathbf{L}^T + \mathbf{C}_n)^{-1} \mathbf{Y}. \quad (5)$$

If no estimate of the source covariance is available, it is common practice to use zero order Tikhonov regularization [22, 23] so that (5) reduces to

$$\hat{\mathbf{Q}} = \mathbf{L}^T (\mathbf{L} \mathbf{L}^T + \lambda \mathbf{I})^{-1} \mathbf{Y}. \quad (6)$$

We now outline the basic concept of our new approach. As an alternative to the model of (1), one can think of using time-averaged observations at the sensor level, or templates, that arise from the activity of a group of physiologically linked current sources like those found in the heart to describe the signals themselves. We refer to the averaged data as the statistical model produced by a group of coordinated sources. We will derive our projection operators to work on sensor level data only. Neither the number of sources, nor their spatial separation, nor the lead fields are considered. We propose replacing (1) with a model based system of equations such that

$$\mathbf{X} = \mathbf{M} \mathbf{S} \text{ where} \quad (7)$$

$\mathbf{M}$ is a model that is derived from the time locked averages of various signals of interest and $\mathbf{S}$ is the set of explanatory signals associated with each component of the model. $\mathbf{M}$ is analogous to $\mathbf{L}$ in that $\mathbf{M}$ is an embodiment of the spatial information in the signals, and $\mathbf{S}$ is analogous to $\mathbf{Q}$ in that $\mathbf{S}$ is the temporal embodiment of the signals. We note that both (1) and (7) are assumed approximations to a more complex reality. Starting with (7), we then apply the minimum norm solutions from (5) or (6) to derive our projection operators.

We develop projection operators that either pass a selected signal, for example the fetal heart, or suppress a selected signal, for example the maternal heart. While cardiac signals are excellent signals for this method, the method is by no means limited to cardiac signals.

II. Theory

A. Model approach

Equations (8) through (14) set up the basis for our method. We also neglect noise during the development of the projection operators. Let the noiseless signal $\mathbf{X}$ be made up of the summation signals arising from currents $\mathbf{Q}$ distributed throughout the body. Further, there will be grouped current sources, $\mathbf{Q}_i$ (with associated $\mathbf{L}_i$), that are both spatially and temporally associated. They are best thought of as a collection of sources with various orientations distributed throughout a limited volume, but all working in a semi-synchronous pattern. Then, $\mathbf{X}$ may be written as the sum of the signals from all signal groups so that

$$\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3 + \mathbf{X}_4 + \dots \quad (8)$$

Using (7) we assume that a good estimate of $\mathbf{X}$ is $\hat{\mathbf{X}}$ such that

$$\hat{\mathbf{X}} = \mathbf{M}\hat{\mathbf{S}} \quad \text{where} \quad (9)$$

$\hat{\mathbf{S}}$ is computed using the unbiased minimum norm estimate from (6). In our example application, typically $\hat{\mathbf{X}}$ is size $[150 \times 10^5]$, $\mathbf{M}$ is size $[150 \times 700]$, and $\hat{\mathbf{S}}$ is size $[700 \times 10^5]$. The meaning of $\hat{\mathbf{S}}$ will be made clear with the introduction of (12) below.

The matrix $\mathbf{M}$ is the statistical model of the time-varying observations and is obtained by averaging the original data based on an easily found repetitive event such as the maximum point of the maternal or fetal R wave (explained in more detail later). The matrix $\mathbf{M}$ is written as

$$\mathbf{M} = [\mathbf{m}_1 \dots \mathbf{m}_{j-1} \mathbf{m}_j \dots \mathbf{m}_{k-1} \mathbf{m}_k \dots \mathbf{m}_m] \quad (10)$$

where each $\mathbf{m}$ is a column vector $[N_c \times 1]$ that is found by computing the time locked average of the signal $\mathbf{X}$ at the same point in a repeating waveform. For example, the column vectors $[\mathbf{m}_1 \dots \mathbf{m}_{j-1}]$ might describe the averaged maternal cardiac cycle in all channels spanning a time from just before the maternal P wave until just after the maternal T wave. Likewise, $[\mathbf{m}_j \dots \mathbf{m}_{k-1}]$ could describe the fetal cardiac cycle and $[\mathbf{m}_k \dots \mathbf{m}_m]$ might be used to describe fetal breathing. We call each subgroup of $\mathbf{M}$ a statistical model for that physiological signal and $\mathbf{M}$ can be thought of as an array of concatenated templates. There are N_m columns in $\mathbf{M}$, and N_m is typically much larger than the number of channels in the system so that (7) is an under-determined set of equations. In our data, $\mathbf{M}\mathbf{M}^T$ is not full rank and requires regularization to invert. See Fig. 14 for a three-component model as described in (10). Given $\mathbf{M}$ and the assumed form of (7), the explanatory matrix $\hat{\mathbf{S}}$ is computed using the minimum norm solution defined by (6).

$$\hat{\mathbf{S}} = (\mathbf{M}^T)(\mathbf{M}\mathbf{M}^T + \lambda\mathbf{I})^{-1}\mathbf{Y}. \quad (11)$$

To get some sense of the meaning of $\mathbf{S}$, we ignore noise and then use (1) to give

$$\hat{\mathbf{S}} = \mathbf{M}^T(\mathbf{M}\mathbf{M}^T + \lambda\mathbf{I})^{-1}\mathbf{L}\mathbf{Q}. \quad (12)$$

The explanatory matrix, $\mathbf{S}$, is a scaled, linear mixture of the source activity $\mathbf{Q}$. In general, any one source will contribute to the signal in multiple rows of $\mathbf{S}$ and at any instant there are probably multiple sources active. However, a group of spatial-temporal current sources, such as the cardiac sources, will primarily give rise to activity in a corresponding group of explanatory signals. Partitioning the matrix $\mathbf{M}$ into three sub-matrices as suggested by (10) gives the maternal cardiac cycle $\mathbf{M}_M = [\mathbf{m}_1 \dots \mathbf{m}_{j-1}]$, the fetal cardiac cycle $\mathbf{M}_F = [\mathbf{m}_j \dots \mathbf{m}_{k-1}]$, and fetal breathing signal $\mathbf{M}_B = [\mathbf{m}_k \dots \mathbf{m}_m]$. Given $\mathbf{M}$ that is partitioned accordingly, there are three corresponding sub-matrices of $\hat{\mathbf{S}}$; $\hat{\mathbf{S}}_M$ is the first 'j-1' rows of $\hat{\mathbf{S}}$, $\hat{\mathbf{S}}_F$ is row 'j' through 'k-1', and $\hat{\mathbf{S}}_B$ is row 'k' through 'm'. Then

$$\hat{\mathbf{X}} = \mathbf{M}_M \hat{\mathbf{S}}_M + \mathbf{M}_F \hat{\mathbf{S}}_F + \mathbf{M}_B \hat{\mathbf{S}}_B. \quad (13)$$

Equation (13) is key to group level signal separation. As seen in (13), the sub-matrices of $\hat{\mathbf{S}}$ align with the grouped columns of $\mathbf{M}$ that create them through (11). The sub-matrix $\hat{\mathbf{S}}_M$ is the subgroup of explanatory signals associated with maternal cardiac activity. Likewise for $\hat{\mathbf{S}}_F$ and $\hat{\mathbf{S}}_B$.

However, we can eliminate $\hat{\mathbf{S}}$ altogether. Using (7), (11), and (13) we can write

$$\hat{\mathbf{X}}_i = \mathbf{M}_i \mathbf{M}_i^T (\mathbf{M} \mathbf{M}^T + \lambda \mathbf{I})^{-1} \mathbf{Y} \quad (14)$$

where $\hat{\mathbf{X}}_i$ is the estimate of the 'ith' subgroup signal corresponding to the 'ith' sub-model of $\mathbf{M}$ from (13). Although the minimum norm solution of (11) is the minimum norm fit for all signals, we find that in practice subgroups are well separated if the signals are adequately modeled. Equation (14) defines the projection operator for the ith signal subgroup. Thus

$$\mathbf{P}_i = \mathbf{M}_i \mathbf{M}_i^T (\mathbf{M} \mathbf{M}^T + \lambda \mathbf{I})^{-1}. \quad (15)$$

Ideally, the projection operator $\mathbf{P}_i$ passes the $\mathbf{X}_i$ signal sub-group with near unity gain and attenuates all other signals. We also note that only the noise in $\mathbf{Y}$ that is aligned with the vectors of $\hat{\mathbf{X}}_i$ can pass through the operator without attenuation. Because there are several variants to consider, we write our projector in general terms as shown below:

$$\mathbf{P} = \mathbf{B}(\mathbf{A} + \lambda \mathbf{I})^{-1}. \quad (16)$$

$\mathbf{P}$ is identified as needed to distinguish between the variants and the makeup of $\mathbf{A}$ and $\mathbf{B}$ define the variant of the operator.

B. Projection Operators

1) Null projection operator—If $\mathbf{PY}$ isolates one or more signals from all others with good fidelity and unity gain, then a null projector can be computed that suppresses the isolated signal. The null projector, $\mathbf{P}_n$, is defined by

$$\mathbf{P}_n = \mathbf{I} - \mathbf{P} \quad (17)$$

where $\mathbf{I}$ is the identity matrix. We point out that the signal rejection may not be perfectly complete and in some cases, the rejection is deliberately limited so that the spatial redistribution of the remaining signals is reduced.

In data such as MCG recordings, the maternal signal dominates all fetal signals, so that initially the only signal available for forming a model is the mMCG. For suppression of the mMCG, $\mathbf{B} = \mathbf{M}_M \mathbf{M}_M^T$ and $\mathbf{A} = \mathbf{B}$. Using (15) and (17) gives

$$\mathbf{P}_n = \mathbf{I} - (\mathbf{M}_M \mathbf{M}_M^T) (\mathbf{M}_M \mathbf{M}_M^T + \lambda \mathbf{I})^{-1}. \quad (18)$$

Using our data, $\mathbf{M}_M \mathbf{M}_M^T$, is singular and requires regularization. However, the principal consideration for the choice of λ is to suppress the action of undesired eigenvectors. For a practical explanation using actual data, see Section IV.

2) Orthogonal projector—The orthogonal projector, $\mathbf{OP}$, is a type of null projector and is described in [5]. It is presented here because of similarities with $\mathbf{P}_n$ and because we will compare the results obtained with both projectors. To compute $\mathbf{OP}$, one starts with the time-averaged signal of the maternal cardiac signal that we have already defined as $\mathbf{M}_M$. The time point where $\mathbf{M}_M$ has the largest amplitude is found and the data at that time point are taken as a column vector, $\mathbf{v}$. A null projector is computed using $(\mathbf{I} - \mathbf{v}(\mathbf{v}^T \mathbf{v})^{-1} \mathbf{v}^T)$ and is then applied to the signal to annihilate the largest component in $\mathbf{M}_M$. $\mathbf{M}_M$ is recomputed using the projected data and the process is repeated until the maternal signal is suppressed to an acceptable level. The sequential process was shown in [5] to be equivalent to forming a single projection operator using a matrix, $\mathbf{V}$, such that the columns of $\mathbf{V}$ are the individual column vectors, $\mathbf{v}$, found at each step as described above. Then, a single projection vector, $\mathbf{OP}$, may be computed as follows:

$$\mathbf{OP} = \mathbf{I} - \mathbf{V}(\mathbf{V}^T \mathbf{V})^{-1} \mathbf{V}^T. \quad (19)$$

Applying the $\mathbf{OP}$ operator to the signal $\mathbf{Y}$ will suppress the major components of the maternal cardiac signal, leaving a small residual mixed with fetal signals. $\mathbf{OP}$ uses a set of linear independent vectors, typically 5 to 10 in number, to build the projector and there is no need to regularize the inverse in (19). On the other hand, $\mathbf{P}_n$ of (18), is built with several hundred vectors that are linearly dependent by nature. Although regularization of the inverse is

required, the primary considerations in selecting λ are redistribution and the chosen level of mMCG suppression. The level of mMCG suppression for **OP** depends upon the number of iterations (the number columns vectors in $\mathbf{V}$). In contrast, we will show that the level of mMCG suppression for $\mathbf{P}_n$ is continuously varied by the parameter λ .

3) Model projectors—The aim of these projectors is the extraction of the fetal heart signal or the fetal breathing signal. To find a model for the fetal heart or fetal breathing, one must first remove the maternal heart signals using either $\mathbf{P}_n$ or **OP**. Once the maternal cardiac signal is projected out and the fetal R waves are found, $\mathbf{M}_F$ is computed and added to the model. If the fetus presents a significant fetal breathing signal and if it is possible to find the time locked average of the fetal breathing signal over some window, then $\mathbf{M}_B$ can be added to the model. Then let $\mathbf{A} = \mathbf{M}\mathbf{M}^T$ where $\mathbf{M}$ is the concatenation of $\mathbf{M}_M$, $\mathbf{M}_F$, and, depending on the data, $\mathbf{M}_B$. Inclusion of $\mathbf{M}_B$ is desirable since that will generally improve the performance of model-based projectors.

To form a projector that will extract the fMCG signal, let $\mathbf{B} = \mathbf{M}_F\mathbf{M}_F^T$. Using (15), we define the fetal cardiac projector, $\mathbf{P}_F$ as

$$\mathbf{P}_F = (\mathbf{M}_F\mathbf{M}_F^T)(\mathbf{M}\mathbf{M}^T + \lambda\mathbf{I})^{-1}. \quad (20)$$

Likewise, we define the fetal breathing projector by letting $\mathbf{B} = \mathbf{M}_B\mathbf{M}_B^T$.

$$\mathbf{P}_B = (\mathbf{M}_B\mathbf{M}_B^T)(\mathbf{M}\mathbf{M}^T + \lambda\mathbf{I})^{-1}. \quad (21)$$

Again, the choice of λ used in (20) and (21) is data dependent and is discussed in Section IV. It must be noted that both $\mathbf{P}_n$ and **OP** cause spatial redistribution of signals that can corrupt the fetal models and a scheme for reducing this effect is also presented in Section IV.

4) Covariance projectors—Combining (1) and (5) gives

$$\hat{\mathbf{X}} = \mathbf{L}\hat{\mathbf{Q}} = \mathbf{L}\mathbf{C}_Q\mathbf{L}^T(\mathbf{L}\mathbf{C}_Q\mathbf{L}^T + \mathbf{C}_n)^{-1}\mathbf{Y}. \quad (22)$$

Using the definitions of (3) and (4),

$$\hat{\mathbf{X}} = \mathbf{C}_x\mathbf{C}_y^{-1}\mathbf{Y} \quad (23)$$

Since the subgroup signals in (2) are expected to be uncorrelated, the cross terms in the calculation of $\mathbf{C}_x$ can be ignored. Then

$$\mathbf{C}_x = \mathbf{C}_1 + \mathbf{C}_2 + \mathbf{C}_3 + \mathbf{C}_4 + \dots \quad (24)$$

so that in general we can write

$$\hat{\mathbf{X}}_i = \mathbf{C}_i \mathbf{C}_Y^{-1} \mathbf{Y} \quad (25)$$

Since $\mathbf{C}_Y$ is just the covariance of the data, $E\{\mathbf{Y}\mathbf{Y}^T\}$, we only need to estimate $\mathbf{C}_i$, the covariance of the signal of interest in order to compute a projection operator. We assume that the maternal and fetal MCG signals are sufficiently stationary so that we use the statistical model as a basis to estimate $\mathbf{C}_i$ so that

$$\mathbf{C}_i = E\{\mathbf{M}_i \mathbf{M}_i^T\}. \quad (26)$$

Alternatively, one may be able to isolate a signal in the frequency domain, and if so, $\mathbf{C}_i$ can be computed using the bandwidth limited signal. Fetal breathing is a candidate signal for this technique. However $\mathbf{C}_i$ is estimated, the projection operator defined in (27) is not bandwidth limited. Please consider that equation (26) may overestimate the amplitude of the covariance of discontinuous signals compared to the actual signal covariance in $\mathbf{C}_Y$. Fetal breathing is an intermittent signal as opposed to the continuous fetal or maternal MCG. For example, the fetal breathing model shown in Fig. 14 below, when used in (26), must be scaled down to reflect the percentage of time the fetus spent breathing. The signal specific projector is then written as

$$\mathbf{P}_i = \mathbf{C}_i \mathbf{C}_Y^{-1}. \quad (27)$$

In our data, regularization of $\mathbf{C}_Y$ is not required because the system noise, $\mathbf{C}_n$ of (3) and (4), is sufficiently large so that the inverse is non-singular. Examples will be presented in Section IV.

C. Redistribution Estimation

Any projector other than the identity matrix has the potential to redistribute signals within signal space. Since both $\mathbf{P}_n$ and $\mathbf{OP}$ are derived from the averaged mMCG, which varies from patient to patient. The degree of signal redistribution cannot be known beforehand. However, the off-diagonal elements of the projector cause redistribution. To estimate the potential for any null projector to redistribute a signal, we define a redistribution estimate, R_e , as the mean of the magnitudes of the off-diagonal matrix elements of $\mathbf{P}_n$ (or $\mathbf{OP}$):

$$R_e = \text{mean}(|\mathbf{P}_n^*|), \quad (28)$$

where the asterisk indicates the removal of the diagonal elements. Clearly, R_e goes to zero as $\mathbf{P}_n$ becomes more like the identity matrix, which causes no redistribution at all.

D. Regularization

We first consider the inverse term in (16): $(\mathbf{A} + \lambda \mathbf{I})^{-1}$. $\mathbf{A}$ is symmetric by definition because $\mathbf{A} = \mathbf{M}\mathbf{M}^T$, and with minimal regularization we have found that $\mathbf{A}$ is positive definite. Under that condition, we can decompose $\mathbf{A}$ into its eigenvectors, $\mathbf{V}$, and associated eigenvalues that lie on the diagonal of matrix $\mathbf{D}$, so that

$$\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^T. \quad (29)$$

It is straightforward to show that

$$(\mathbf{A} + \lambda \mathbf{I})^{-1} = \mathbf{V}(\mathbf{D} + \lambda \mathbf{I})^{-1}\mathbf{V}^T. \quad (30)$$

Since the null projector defines $\mathbf{B} = \mathbf{A}$, we can write a general expression for the null projector in terms of eigenvectors and eigenvalues by combining (18), (29), and (30). After some basic simplification we have

$$\mathbf{P}_n = \mathbf{V}\lambda(\mathbf{D} + \lambda \mathbf{I})^{-1}\mathbf{V}^T. \quad (31)$$

The null projector can be expressed in terms of the eigenvectors of $\mathbf{A}$, which is derived from the model, but has modified eigenvalues given by the diagonal matrix $\lambda(\mathbf{D} + \lambda \mathbf{I})^{-1}$. Let d_i be an eigenvalue of the diagonal matrix $\mathbf{D}$ and let μ_i be the corresponding eigenvalue of $\mathbf{P}_n$, then

$$\mu_i = \lambda / (d_i + \lambda). \quad (32)$$

Since each eigenvector is normalized, each μ_i is the gain associated with that eigenvector. Consequently, μ_i is the gain of the signal component that gave rise to that eigenvector. Eq. (32) shows that for the larger maternal signals where $d_i \gg \lambda$, then $\mu_i \approx \lambda/d_i$ so that the maternal signals that align with these eigenvectors are attenuated by the amount λ/d_i . In practice, we choose λ so that most non-mMCG eigenvalues fall below λ which implies $d_i \ll \lambda$, making $\mu_i \approx 1$. As suggested by (17), $\mathbf{P}_n$ is similar to the identity matrix except that it is designed to suppress the mMCG eigenvectors while passing other signals.

In our data, the trace of $\mathbf{A}$ is typically $\sim 10\%$ larger than the largest eigenvalue associated with the mMCG R wave. We can utilize t_A as an easily computed approximate value for the largest eigenvalue, $d_{\max}$. We now define a dimensionless scalar, sf , as follows:

$$\lambda = sf \cdot t_A, \text{ where } (33)$$

t_A is the trace of matrix $\mathbf{A}$. From (33), we see that $sf \approx \lambda/d_{\max}$ so that sf is the gain (attenuation) of the dominant mMCG signal. However, sf is not the attenuation of the whole mMCG waveform since each eigenvector has a different gain according to (32).

Since the eigenvalues of $\mathbf{P}_n$ are derived from the mMCG model, we assume, as a first approximation, that each eigenvector has an associated signal $x_i = \sigma_i \cdot u_i(t)$ where $u_i(t)$ is the normalized signal and σ_i is the RMS amplitude of the signal. It then follows that the power of that component gives rise to the eigenvalue so that $d_i = \sigma_i^2$. To estimate the RMS output for each component, we substitute $d_i = \sigma_i^2$ into (32) to estimate that gain of that signal and then multiply by the input x_i to get the estimated output signal y_i .

$$y_i = \lambda x_i / (\sigma_i^2 + \lambda) = (\lambda \sigma_i / (\sigma_i^2 + \lambda)) \cdot u_i(t) \quad (34)$$

Setting $\sigma_i = \lambda^{1/2}$ maximizes (34), so that the RMS of all residual components should be less than or equal to $\frac{1}{2}\lambda^{1/2}$. If we let $\sigma_i = \lambda^{1/2}$, and combine (33) and (34) we have the maximum mMCG residual component given by

$$y_{\max} = (t_A \cdot sf / 4)^{1/2}. \quad (35)$$

The total residual mMCG is the sum of all residual components, but (34) and (35) suggests that the residual mMCG $\propto sf^{1/2}$. The proper selection of sf is application dependent and details will be presented in section IV.

We now consider the general projector described by (16) when $\mathbf{B} = \mathbf{A}$. First, we note that the most general model $\mathbf{M}$ can have more than one component, for instance fMCG and fetal breathing models may be added. If so, then there should be some subset of the eigenvectors of $\mathbf{A}$ associated with those models. Similar to (29) we write $\mathbf{B}$ in terms of an eigenvalue matrix $\mathbf{G}$ and eigenvectors $\mathbf{W}$. Then

$$\mathbf{B} = \mathbf{W} \mathbf{G} \mathbf{W}^T. \quad (36)$$

Using (16), (29), (30), and (36) the projection operator is expressed in terms of eigenvectors and eigenvalues

$$\mathbf{P}=(\mathbf{W}\mathbf{G}\mathbf{W}^T)(\mathbf{V}(\mathbf{D}+\lambda\mathbf{I})^{-1}\mathbf{V}^T). \quad (37)$$

We define g_i as the eigenvalues of $\mathbf{G}$. Using (30) we write an expression for the eigenvalues of the second term shown in parentheses. We define θ_i as the eigenvalue of the inverse term. Then,

$$\theta_i=1/(d_i+\lambda). \quad (38)$$

For a projector designed to pass the fMCG, we would typically set λ well below the eigenvalues associated with the major fMCG components in (38). Given that the model $\mathbf{M}_F$ is the basis of $\mathbf{B}$ and that $\mathbf{M}_F$ is included as part of $\mathbf{A}$, we assume that some eigenvectors and eigenvalues will be common to both (or at least similar). Given the condition that $d_i \gg \lambda$ and there are joint eigenvectors, then the projector's gain is $g_i/d_i \approx 1$ for each eigenvector common to $\mathbf{A}$ and $\mathbf{B}$. All other signals are suppressed by the selectivity of $\mathbf{B}$ (lack of a matching eigenvector) and the scaling from (38). The values of sf used for $\mathbf{P}_F$, and $\mathbf{P}_B$ range from 10^{-4} to 10^{-6} and must be determined by trial and error for each type of application.

The covariance formulation of (27) has two factors that are similar to the two factors in (37). $\mathbf{C}_i$ is functionally associated with $\mathbf{B}$ and the inverse of $\mathbf{C}_Y$ is functionally associated with $(\mathbf{A} + \lambda\mathbf{I})^{-1}$. As we will show in section IV, our system noise essentially sets the minimum value for λ . Since the fetal signals are larger than the system noise, there is no need to regularize the inverse of $\mathbf{C}_Y$.

III. Data Collection

Data presented in this paper was collected using the SARA system, a 151 channel MEG system based on SQUID technology [24, 25]. In the SARA system, the magnetic sensors are radial gradiometers with a 2 cm diameter coil and 8 cm coil separation (base line) [3, 6]. The instrument is installed in a magnetically shielded room (Vakuumschmelze Hanau, Germany), to reduce the effects of environmental noise. The gradiometers cover the whole maternal abdomen and capture maternal and fetal MCG, fetal breathing, and fetal brain activity. The patients sat in upright position during the recording. All signals were bandpass filtered with an eighth order, zero-phase, filter having a passband of 1 Hz to 50 Hz unless stated otherwise. The sample rate was 312.5 samples per second. A total of 113 recordings of 10 minutes duration were taken from normal fetuses ranging in gestation age from 26 weeks to 38 weeks. Data collection protocols were approved by the IRB at University of Arkansas for Medical Sciences.

IV. Results and Discussion

A. Performance of OP and $\mathbf{P}_n$

To calculate either **OP** or $\mathbf{P}_n$, $\mathbf{M}_M$ must first be computed. Since the mMCG dominates all other signals, it is easy to find the maternal R markers [27]. The median R-to-R time interval is taken as the duration of $\mathbf{M}_M$ (or window). The raw signal is time averaged using the maternal R markers so that 40% of the time window precedes each R marker and 60% follows. This basic procedure captures information about the P and T waves and the QRS complex and is used to compute either mMCG or fMCG cardiac models. The averaged mMCG, $\mathbf{M}_M$, is then used to compute either the $\mathbf{P}_n$ or **OP** null projector as specified by (18) or (19), respectively.

After projecting out the mMCG using either **OP** or $\mathbf{P}_n$, some residual mMCG will be leftover. Time averaging null projected data based on the maternal R markers will produce the averaged residual mMCG. We use the largest magnitude found in the averaged residual mMCG to judge the performance of a null projector. The same measure is normally used to either stop the **OP** nulling algorithm of (19) or to choose λ (or sf) in (18).

To demonstrate the general properties of the null projectors, a ten-minute recording taken from a 36-week-old fetus was processed using the **OP** and $\mathbf{P}_n$ projectors and the results are shown in Fig. 1. Using the **OP** method of (19), null vectors were sequentially placed in the signal space until the magnitude of the residual mMCG fell below 0.1 pT (our typical stopping threshold). Seven null vectors were needed to satisfy the stopping threshold for the **OP** method. As presented in the theory section, the value of sf sets the degree of mMCG suppression of a $\mathbf{P}_n$ projector. We adjusted sf iteratively until the $\mathbf{P}_n$ projector produced to within one percent the same averaged residual mMCG as produced by **OP**. We hereafter refer to such a projector as a matched $\mathbf{P}_n$. Fig. 1A shows a two second window of the recorded data, $\mathbf{Y}$. Both mMCG and fMCG signals are visible in 1A. The $\mathbf{M}_M$, **OP**, and matched $\mathbf{P}_n$ were derived from averaged mMCG data shown in Fig. 1B. Figs. 1C and 1E show the output of the **OP** and matched $\mathbf{P}_n$ applied to the data shown in 1A. Note that the mMCG is effectively suppressed in both 1C and 1E. Figs. 1D and 1F show the resulting averaged residual mMCG plots for **OP** and matched $\mathbf{P}_n$.

In order to study the performance of **OP** and matched $\mathbf{P}_n$, a total of 113 datasets were processed and various measures were extracted. First the maternal R markers for each dataset were found. Then the fetal R markers were found after applying $\mathbf{P}_n$ with $sf = 10^{-6}$. Without regard to a stopping threshold, **OP** projectors with 1 through 15 column vectors in $\mathbf{V}$ were computed for each dataset using (19). A corresponding matched $\mathbf{P}_n$ was computed for every **OP** projector. The peak magnitude of the averaged residual mMCG, the peak magnitude of the averaged fMCG (using fetal R markers), and sf were found for each of the 1,695 combinations of data and projectors. Fig. 2 shows how $\mathbf{P}_n$ affected the magnitude of the residual mMCG and of the averaged fMCG as a function of sf . The results presented in Fig. 2 show that the averaged residual mMCG is proportional to $(sf)^{1/2}$ over six decades as (35) suggests. Over the range $10^{-1} < sf < 10^{-4}$, the amplitude of the fMCG is approximately constant, but begins to drop when sf is smaller than 10^{-4} . There is little point in using $sf < 10^{-6}$ because the fMCG and mMCG are reduced equally.

We now describe a number of measures used to compare and evaluate the performance of the null projectors $\mathbf{P}_n$ and $\mathbf{OP}$. We define windows for the fMCG QRS complex, pre-QRS, and post-QRS as follows: a) the QRS window is 67.2 ms (± 10 samples @ 312.5 Hz) and centered on the fetal R marker, b) the pre-QRS window is variable in length and includes all values that precede the QRS window, and c) the post-QRS window is variable in length and includes all values that follow the QRS window. See Fig. 3A as an example. We define the P-Q window as a 32 ms (10 samples @ 312.5Hz) window that exhibits minimal change in signal level and is located between the end of the P wave and the start of the QRS complex. At each possible window position, the window mean for each channel is subtracted from the data and the L_1 norm is computed. The window position with the lowest norm is selected. The position relative to the R marker and the P-Q window means are saved for later use. Fig. 3B shows an example of the P-Q window position after the means have been subtracted (baseline correction).

To show how the null projectors affected fetal QRS amplitude, the amplitude before and after application of the projector was examined. To estimate the actual fetal QRS amplitude we simply averaged the un-projected data (raw) using the fetal R markers and extracted the amplitude of the largest signal. See Fig. 4A for an example. Some remaining mMCG is present in the fMCG signal. To minimize this influence, we rejected datasets where the RMS value of the QRS window was not at least ten times the RMS value of the concatenated pre-QRS and post-QRS windows. Ninety-five datasets met our criterion. We then determined the largest peak in the time averaged fetal QRS before and after applying $\mathbf{OP}$ or matched $\mathbf{P}_n$. The ratio gave the attenuation associated with each $\mathbf{OP}$ iteration and matched $\mathbf{P}_n$. As an example, the averaged fMCG that results from applying two iterations of $\mathbf{OP}$ is shown in Fig. 4B, and the averaged fMCG that results from applying the matched $\mathbf{P}_n$ is shown in Fig. 4C. We also note that we used the same peak for each paired $\mathbf{OP}$ and matched $\mathbf{P}_n$ calculation so that there is no bias in favor one or the other because of the residual mMCG.

The attenuation at each $\mathbf{OP}$ step is presented in Fig. 5 in a boxplot format. Normally, the $\mathbf{OP}$ algorithm terminates between 5 and 10 iterations. Considering that range, the median attenuation for $\mathbf{OP}$ was less than the matched $\mathbf{P}_n$, but gain variation was more. By these measures, the two methods appear to be roughly equivalent with both methods showing increased attenuation and variation with increased $\mathbf{OP}$ iteration (mMCG attenuation).

In practice, we use the peak magnitude of the averaged mMCG residual as a metric for measuring the suppression of the mMCG waveform, but it can be argued that a better metric would be the RMS of the mMCG residual. To explore that possibility, we computed the RMS of un-averaged data using the previously defined windows: QRS, pre+post QRS, and P-Q (with baseline removed). After applying a $\mathbf{P}_n$ projector, QRS data around each fetal R marker were collected and combined, and then the RMS value was computed. Data not marked as QRS were classified as pre+post QRS data and the RMS value was computed. Using the averaged fMCG, the position of the P-Q window and the channel means were found as described above. The P-Q window means were subtracted from the raw data on a per channel basis. The P-Q data relative to each R marker was then combined and the RMS value was computed. Computing the RMS during the P-Q interval with the mean removed is our best attempt to measure the RMS residual mMCG without fMCG contamination. Even

though the QRS and P-Q windows are short, there were sufficient samples for computing the RMS because each dataset had at least 1000 fetal R markers. Finally, the averaged residual mMCG was computed. All 113 recordings were processed. The median values obtained using the $\mathbf{P}_n$ operator are shown in Fig. 6. The values for iteration “0” were computed on the original data without application of a projector. We do not show the $\mathbf{OP}$ results because they are similar to the $\mathbf{P}_n$ results.

The three RMS values computed using the raw data plotted at position “0” have almost identical median values, indicating that the short window lengths do not bias the results and that the mMCG is the dominant RMS signal in the raw data. Over the first few $\mathbf{OP}$ iterations, the fetal RMS values and the residual mMCG parallel each other. However, by the fifth iteration, the change in gain shown in Fig. 5A fully accounts for the monotonic drop in the fetal RMS median values. In contrast, the median residual mMCG data closely follows (35). Clearly the RMS values of the P-Q window set the upper limit for the RMS of the residual mMCG for all $\mathbf{OP}$ iterations. But the projection operator will pass some system noise so that we are not certain of the makeup of the P-Q signal especially since the median values track the QRS and do not follow (35). On this basis, we adopted the use of the residual mMCG (based on averaging) for our metric of mMCG suppression.

B. Regularization

Understanding the role of the eigenvalues of matrix $\mathbf{A}$ in (16) is key to selecting the proper value of λ (or sf) for regularization. We first consider the regularization of $\mathbf{P}_n$. The model, $\mathbf{M}_M$, for the null projector, $\mathbf{P}_n$, is simply the time-averaged mMCG computed as described previously. Then $\mathbf{A} = \mathbf{B} = \mathbf{M}_M \mathbf{M}_M^T$ in (16). Using the same dataset used in Fig. 1, the eigenvalues of $(\mathbf{A} + \lambda \mathbf{I})$ were computed using four values of λ . Fig. 7A is a plot of the first 30 eigenvalues ranked from largest to smallest. The solid line is for $\lambda = 10^{-35}$ which is far below any useful value but satisfied the requirements of our matrix inversion algorithm. The dashed lines are curves obtained using the indicated values of λ . In Fig. 7B, the 30 smallest eigenvalues of $\mathbf{P}_n$, ranked from smallest to largest, are plotted. Curves for the three values of λ are shown, but the curve for $\lambda = 10^{-35}$ is completely outside the range of the graph. The data shown in Fig. 8 illustrate the effect λ has upon the performance of $\mathbf{P}_n$. Fig. 8A shows a two second window of recorded data $\mathbf{Y}$, and 8B, 8C, and 8D show the effect of the three indicated values of λ on the properties of $\mathbf{P}_n$. Note the scale change in panels B, C, and D. In this example, there are three maternal and four fetal QRS complexes. The fMCG is barely visible in 8A and the mMCG is not visible in 8D. Note also that the fMCG is significantly attenuated going from 8C to 8D. We did not present data corresponding to the case where $\lambda = 10^{-35}$ because no signal remained. According to (32), in the extreme, when $\lambda \rightarrow 0$, then $\mathbf{P}_n \rightarrow \mathbf{O}$.

If λ is significantly less than an eigenvalue found in $\mathbf{A}$, then any signal associated with that eigenvector will be suppressed by $\mathbf{P}_n$ according to (32). Since the eigenvectors of $\mathbf{A}$ come directly from averaged mMCG, the projector will effectively suppress the major signal components of the mMCG. $\mathbf{P}_n$ is similar to the identity matrix, except for the suppressed mMCG eigenvectors, and as expected, most eigenvalues are very close to unity as indicated

by Fig. 7B. As λ is reduced, more and more eigenvalues are attenuated, and consequently, all signals including the fMCG are affected accordingly.

Since the mMCG is dominant in all of our data, plots of the eigenvalues of $(\mathbf{A} + \lambda \mathbf{I})$ are always similar to Fig. 7A in that one eigenvalue dominates the power in the mMCG. We found that the trace of $\mathbf{A}$ is always just larger than the largest eigenvalue. For the data shown in Fig. 7A, the largest eigenvalue is $4.94 \cdot 10^{-20}$ and the trace is $5.33 \cdot 10^{-20}$. Computing the trace is faster and simpler than computing an inverse and then finding the largest eigenvalue. If we use the trace of $\mathbf{A}$ as an approximate value for the largest eigenvalue, according to (32) we have a convenient method to suppress the dominant component ($d_{\max}$) of the mMCG to a known fraction, specifically $\mu_{\max} \cong sf$.

We now examine the effect of λ on model-based projectors. Starting with the maternal and fetal models, $\mathbf{M}_M$ and $\mathbf{M}_F$ respectively, we concatenated them according to (10) to get $\mathbf{M}$ and set $\mathbf{A} = \mathbf{M}\mathbf{M}^T$ and $\mathbf{B} = \mathbf{M}_F\mathbf{M}_F^T$. The eigenvalues of $(\mathbf{A} + \lambda \mathbf{I})$ were computed using four values of λ using the concatenated model. Each set of eigenvalues was ranked in descending order and the first 30 from each set are shown in Fig. 9. The solid curve is the data using $\lambda = 10^{-35}$ and the dashed curves result from using the indicated values of λ . For reference, the eigenvalues from the individual models $\mathbf{A} = \mathbf{M}_M\mathbf{M}_M^T$ (dot-dash, 'mMCG') and $\mathbf{A} = \mathbf{M}_F\mathbf{M}_F^T$ (dot-dash, 'fMCG') are provided for comparison with the full model. The solid line and the two dash-dot lines below the solid line use $\lambda = 10^{-35}$ to satisfied the requirements of our matrix inversion algorithm.

Observe that the first eigenvector of the fMCG model (bottom trace) is approximately $1 \cdot 10^{-22}$. To demonstrate the effect that λ has on performance, two values of λ just above ($2 \cdot 10^{-22}$) and then below ($4 \cdot 10^{-24}$) were selected. A value much less ($2 \cdot 10^{-27}$) was also included in the analysis. The three values of λ produced projectors with markedly different output. See Fig. 10.

The results verify (38). For all eigenvalues smaller than the value of λ , (38) predicts that the signal will be suppressed. Fig. 10A corresponds to a case where λ was deliberately set larger than the expected largest eigenvalue of the fMCG component of the model $\mathbf{M}$ and the fMCG was attenuated. In that regard, the action of λ is opposite to the action observed for $\mathbf{P}_n$. Fig. 10B and 10C show results from projectors with the value of λ chosen well below the principle eigenvalues of the fMCG components and the resulting fMCG waveforms have almost identical QRS amplitudes. Fig. 10C shows more details, corresponding to the inclusion of more eigenvalues. In Fig. 9, the first eigenvalue found in the fMCG model is the largest eigenvalue of (38). The fourth eigenvalue from the combined model of $\mathbf{A}$ is almost identical with the first eigenvalue of $\mathbf{B}$, and the respective eigenvectors are nearly identical. In reference to (38), we predicted unity gain for eigenvectors common to $\mathbf{A}$ and $\mathbf{B}$ because $g_i/d_i \approx 1$ when $d_i \gg \lambda$. By comparing the R wave amplitude of the averaged fMCG (RAW) shown in Fig. 4A to the R wave amplitudes seen in Fig. 10B and 10C, we see that the projector gain is approximately unity. Finally, there is no discernible mMCG in Fig. 10. As stated, (38) will pass the mMCG at a reduced level relative to the fMCG and there is no eigenvector associated with the mMCG in the $\mathbf{B}$ component of the projector.

The behavior of a covariance projector is similar to a model projector. Using the same dataset, we computed the eigenvalues of the inverse terms from the model-based projector of (20) and the covariance based projector of (27). The 50 largest eigenvalues of the inverse terms are plotted in Fig. 11. The solid line is from the combined model (mMCG and fMCG concatenated) and $\lambda = 10^{-35}$. The dot-dash line is from the same model but $sf = 10^{-5}$. The dash line is from the covariance of $\mathbf{C}_Y$ with no regularization. In our datasets, the system noise serves to regularize the inverse of the data covariance matrix $\mathbf{C}_Y$. While one might choose to add a $\lambda \mathbf{I}$ term, we have never found it to be advantageous.

In Fig. 11, we see that the covariance noise floor is roughly five decades below the largest eigenvalue. To compare performance of the covariance projector versus the performance of the model projector we set $sf = 10^{-5}$ so that the regularization floor is also five decades below the largest eigenvalue. Signals produced by the two projectors are presented in Fig. 12. The output of the model projector is shown in 12A while the output of the covariance projector is shown in 12B. Note that the data shown in Fig. 10 correspond to the same time window as the data shown in Fig. 12. Further, the regularization selected for Fig. 10B appears to produce a signal similar to the covariance projector of Fig. 12B. In Fig. 12C, λ was set about two orders of magnitude above the system noise floor. At this level, some fMCG components are being suppressed.

C. Redistribution of Null Projectors

Null projection operators redistribute signals that are not removed from signal space. To compare the redistribution of $\mathbf{OP}$ and $\mathbf{P}_n$ we defined a redistribution estimator (28). Larger values of R_e indicate a departure from the identity matrix and likely increase in redistribution. Using 113 datasets, the value for R_e was computed for 1,695 $\mathbf{OP}$ and matched $\mathbf{P}_n$. See Fig. 13. The results are presented as box plots for the respective projectors. We reiterate that the redistribution estimator, R_e , is only an estimate of the potential for redistribution because the actual redistribution of fetal signals also depends upon the non-mMCG signals. The potential for redistribution is almost always less for the performance matched $\mathbf{P}_n$ projector. Further, R_e increases with increasing number of $\mathbf{OP}$ vectors (and therefore decreasing sf).

Fig. 13 shows that there has to be a tradeoff between suppression of the mMCG and redistribution of fetal signals used to construct our models. To minimize redistribution errors in the fetal models, we use a combination of averaging and a low redistribution null projector ($R_e \sim .004$) to suppress the mMCG. If the fMCG is large with respect to the mMCG and there are many fetal R markers, then only averaging is needed so that no redistribution is introduced into the model. See Fig. 4A for an example of incomplete averaging and 4C after application of $\mathbf{P}_n$.

D. Computation of Models

In contrast to the dataset in the previous subsections that did not have fetal breathing, a dataset thought to include significant periods of fetal breathing was selected for analysis. Characteristics that identify fetal breathing in magnetic recordings have been reported earlier [29]. Fetal breathing occurs about 30% of the time near term [30] and has the following

characteristics: quasi periodic sinusoidal of about 1 Hz, amplitude comparable with the fMCG, in sensor space it presents close to the fMCG signal, and it is commonly intermittent. The signal identified has all of the above characteristics and for the purposes of this paper, we assume that the observed signal is fetal breathing. Fig. 14 shows a three-component statistical model extracted from the selected dataset that includes mMCG ($\mathbf{M}_M$), fMCG ($\mathbf{M}_F$), and a signal that is likely fetal breathing ($\mathbf{M}_B$).

We first extracted the maternal R markers and then used them to compute the averaged mMCG ($\mathbf{M}_M$) as shown in Fig. 14 column numbers 1 to 284. Computing the fetal model was a multi-step procedure. Using $\mathbf{M}_M$, a $\mathbf{P}_n$ projector with $sf = 4.5 \cdot 10^{-5}$, was applied to the data to reveal the fetal R waves. See Fig. 15A for an example that shows fMCG and fetal breathing. Note that the null projectors primarily suppress the mMCG, but other signals remain, although attenuated. The fetal R markers were extracted using the algorithm described in [27]. Referring back to the discussion concerning Fig. 4, most of the mMCG was removed from the fMCG and fetal breathing models by averaging. To complete the suppression of the mMCG, a second $\mathbf{P}_n$ projector ($sf = 0.1$ and $R_e \approx 0.004$) was applied to the signal. Since the operation is linear, it can be done before or after averaging. See Fig. 15B for an example of the signal after application of $\mathbf{P}_n$ ($sf = 0.1$) but before averaging. Despite the mMCG signals, fMCG and breathing signals are all visible in Fig. 15B. The output of this second null projector was averaged ($N \sim 1300$) to produce the fetal model, $\mathbf{M}_F$, shown in Fig. 14, columns 285 to 428.

$\mathbf{M}_B$ was found by manually identifying the zero crossing point of 19 cycles of the breathing signal and averaging 150 time samples (0.48 s) on either side of the zero crossings. The fetal breathing model, $\mathbf{M}_B$, is shown in columns 429 to 729.

E. Model and Covariance Projectors

The models in Fig. 14 were used to construct three different projectors designed to isolate either the fMCG or the fetal breathing signal. The results are shown in Figs. 16 and 17. For reference, the time windows of Figs. 15, 16, and 17 are the same. Fig. 16A shows the output of an fMCG projector, $\mathbf{P}_F$, computed using (20) but with only $\mathbf{M}_M$ and $\mathbf{M}_F$ included in $\mathbf{M}$. Fig. 16B shows the output of $\mathbf{P}_F$, but in this case $\mathbf{M}_M$, $\mathbf{M}_F$, and $\mathbf{M}_B$ are included in $\mathbf{M}$. In Fig. 16A and 16B, $sf = 10^{-5}$. Fig. 16C shows the output of a covariance-based projector $\mathbf{P}_C$ computed using (27). In this case, the covariance, $\mathbf{C}_i$, of the fMCG signal was estimated from (26) using the fMCG model, $\mathbf{M}_F$, shown in Fig. 14. All three projectors passed the fMCG signal and suppressed the mMCG signal. However, the two-model projector of Fig. 16A, also passed some of the fetal breathing signal, while the other two projectors attenuated the fetal breathing signal. The three-model projector produced comparable results to the $\mathbf{P}_C$ projector when $10^{-4} > sf > 10^{-6}$ while the covariance projector did not require regularization.

Fig. 17A shows the output of a three-model fetal breathing projector using (21), $\mathbf{M}_M$, $\mathbf{M}_F$, and $\mathbf{M}_B$, and $sf = 10^{-5}$. Fig. 17B and 17C show the output of two different covariance based projectors computed using (27) but utilizing two different methods to compute $\mathbf{C}_i$. In 17B, $\mathbf{C}_i$ was estimated using $\mathbf{M}_B$ and (26). In this example, the amplitude of the projector's output

was scaled by 0.2 to match the amplitude seen in 17A. The scaling by 0.2 reflects the intermittent nature of the fetal breathing as indicated in the discussion following (26). In 17C, $\mathbf{C}_f$ was estimated as follows: The signal $\mathbf{Y}$ was filtered with an eighth order, Butterworth, zero-phase, digital filter with a pass-band of 0.5 Hz to 1.25 Hz. The narrow bandwidth filter separated the fetal breathing signal from other signals, and the covariance, $\mathbf{C}_f$, was calculated directly from the bandwidth-limited $\mathbf{Y}$. There was no need to rescale the amplitude. The important point to be made by this example is that any appropriate method can be utilized to estimate the covariance of such an isolated signal.

F. Other Considerations

Conceptually, a covariance based mMCG projector could be used to form a null projector, but this approach fails to achieve the hoped for results, usually leaving residual signals that are larger than the null projector defined in (18). As pointed out, a covariance projector may typically affect the amplitude of a projected signal. A mismatch in amplitude of a few percent between the input and output is inconsequential for an isolation projector, but will limit the rejection when that projector is used to form a null projector. Attempts to scale the gain of multi-model and covariance projectors for use as a null projector fell short of the performance of $\mathbf{P}_n$.

The primary scope of this paper is to define and present effective projectors and to demonstrate their utility when applied to a moderate length recording of ten minutes. Longer recordings make it more likely that either the mother or fetus will move, violating the requirement that the signals remain quasi-stationary. To some extent, non-stationarity can be dealt with by finding periods of maternal and fetal inactivity from which to build the desired projector. Since non-stationarity is data dependent, it can only be addressed in a statistical study, which is beyond the scope of this paper.

Finally, equation (12) introduces the model resolution matrix [30,31] for linear inverse problems, which might provide a means to evaluate crosstalk between signal subgroups.

V. Conclusion

Both the fetal and maternal cardiac signals are generated by multiple current sources that are temporally correlated and spread over many centimeters. Even so, we demonstrate that sensor level projection operators can be designed that will effectively suppress the mMCG signal or selectively pass either the fMCG signal or the fetal breathing signal. Our time-averaged model based projectors utilize the un-weighted minimum norm formalism, while our covariance projectors utilized the weighted minimum norm formalism.

In one dataset, where we suspected a fetal breathing signal, we build a three-model projector. The suppression of fetal breathing in the fMCG output was noticeably improved. We also demonstrated the feasibility of using either a time-locked averaged model or a bandwidth-limited signal for computation of a covariance projector. Even though the fMCG and the fetal breathing signal overlapped spatially, the two signals were separated with minimal crosstalk.

We developed equations that predict the gain of our projectors as a function of their eigenvalues. The equations provide an improved understanding of the effects of regularization in the minimum-norm formalism when applied to our projectors.

We demonstrate the effectiveness of a null projector, $\mathbf{P}_n$, that may be used to suppress the mMCG signal and that the $\mathbf{P}_n$ projector is very similar in performance to the $\mathbf{OP}$ projector. Further, the degree of suppression may be set by the regularization parameter. The method was automated and used to find null projectors for 113 datasets without any user intervention.

Finally, we address the issue of redistribution in a quantifiable way and describe how to reduce the effects of the redistribution.

Acknowledgments

This work was sponsored in part by NIH grant R01EB007826-04A1 NIBIB/NIH.

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Biographies

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Fig. 1.

(A), (C), and (E) Results from two seconds of data taken from a 10 minute recording. (A) The raw data, Y , displaying both maternal and fetal MCG signals. (C) and (E) The output of the OP projector and the matched P_n projector applied to Y , respectively. (B), (D), and (F) Data that are time-averaged using the maternal R markers. (B) The time-averaged mMCG. (D) and (F) Are the averaged mMCG after applying the OP projector and the matched P_n projector, respectively. Note the scales in each panel.

Fig. 2.

Fifteen performance matched $\mathbf{P}_n$, corresponding to 15 different $\mathbf{OP}$ iterations, were applied to 113 datasets. The $\log_{10}$ of the peak magnitude of fMCG (symbol 'O') and the $\log_{10}$ of the peak magnitude of residual mMCG (symbol '+') is plotted against $\log_{10}(sf)$. The double arrow indicates the optimal value range for sf , where, a clear separation of the maternal and fetal signal is visible.

Fig. 3.

(A) Shows an example of the averaged fMCG after application of $\mathbf{P}_n$. Three windows were defined as QRS, pre-QRS, and post-QRS intervals. (B) Shows a portion the data in (A) with the P-Q interval indicated and with baseline correction.

Fig. 4.

(A) An example of the averaged fMCG using $\mathbf{Y}$ (RAW). (B) The averaged fMCG after applying only two iterations of the $\mathbf{OP}$ projector. (C) The averaged fMCG after applying the performance matched $\mathbf{P}_n$.

Fig. 5.

Boxplots of the fMCG attenuation resulting from applying null projectors to the raw data.

(A) The results after applying the matched $\mathbf{P}_n$. (B) The results from applying $\mathbf{OP}$. The scatter in the measurements is shown in boxplot format with 25–75 quartile boxes. The ‘+’ symbol indicates outliers.

Fig. 6.

The median of the averaged residual mMCG and the medians of the RMS in the fetal QRS, Pre-Post QRS and P-Q interval windows, as illustrated in Fig. 3, are plotted against the number of **OP** iterations for 113 datasets. The curves shown were obtained using the matched **P_n** operator.

Fig. 7.

(A) A plot of the thirty largest eigenvalues from $(\mathbf{A} + \lambda \mathbf{I})$ computed using four values of λ and $\mathbf{M}_M$. The solid line is the case for $\lambda = 10^{-35}$ and the three dashed lines correspond to larger values of λ as indicated. (B) A plot of the thirty smallest eigenvalues from $\mathbf{P}_n$ computed using the same values of λ and $\mathbf{M}_M$. The data for $\lambda = 10^{-35}$ falls outside the range of the plot.

Fig. 8.

(A) Two seconds of recorded raw data from the dataset associated with the projectors described in Fig. 7. There are four fetal heartbeats and three maternal heartbeats in all panels. (B), (C), and (D) Show the same data as (A), after the application of P_n projectors computed using the indicated values of λ . Note that different scales were selected for clarity.

Fig. 9.

Plots of the thirty largest eigenvalues computed using three models and four values of λ .

The three dashed lines and the solid line are values computed using the concatenated model (mMCG + fMCG). The two dot-dash lines are values computed using the individual models mMCG and fMCG. The values of λ used to compute the top three curves are indicated. For the bottom three curves, $\lambda = 10^{-35}$ was used.

Fig. 10.

(A), (B), and (C) Plots of four seconds of processed output from three different $\mathbf{P}_F$ projectors. All three projectors used the same mMCG + fMCG model $\mathbf{M}$, but differed in the value of λ , $2 \cdot 10^{-22}$, $4 \cdot 10^{-24}$, and $2 \cdot 10^{-27}$ respectively.

Fig. 11.

A covariance and a model projector were computed from the same data as shown in Fig.10. The solid line is the plot of the eigenvalues of $(\mathbf{A} + \lambda \mathbf{I})$ from the model (mMCG + fMCG) with $\lambda = 10^{-35}$. The dash-dot line is the plot of the eigenvalues found using the same model but setting $sf=10^{-5}$. The dashed line is the plot of the eigenvalues found from the covariance of the recorded data, $\mathbf{Y}$.

Fig. 12.

(A) A plot of four seconds of output from a model based projector computed with $sf=10^{-5}$. (B) A plot of four seconds of output from a covariance projector ($\lambda=0$) applied to the same data. (C) A plot of four seconds of output from a covariance projector ($\lambda = 3 \cdot 10^{-25}$). This plot demonstrates that a non-zero λ can degrade the performance of a covariance projector.

Fig. 13.

Redistribution estimates, R_e , from 1695 **OP** and matched **P_n** null type projectors were computed. Larger values of R_e imply more redistribution. (A) A boxplot of the R_e values found for the **OP** operator over the **OP** iteration number. (B) The results from the matched **P_n** null projector.

Fig. 14.

A plot of a three component statistical model from a 148 channel recording. The mMCG model occupies columns 1 to 284, the fMCG model occupies columns 285 to 428, and the fetal breathing model occupies columns 429 to 729.

Fig. 15.

A dataset with fetal breathing was selected for analysis. (A) Signal from a three second window produced by the null projector, $\mathbf{P}_n$ ($sf = 4.5 \cdot 10^{-5}$). This signal is free of mMCG and was used to find fetal R markers. It shows significant fetal breathing. (B) The output of a second $\mathbf{P}_n$ ($sf = 10^{-1}$) for the same three second window. The fMCG model and the breathing model shown in Fig. 14 were computed by averaging this signal around the fetal R markers and the zero crossing of the breathing pattern, respectively.

Fig. 16.

Three seconds of output from three different projection operators designed to pass only the fMCG signal and applied to the same data. (A) The output of a two-model fetal mMCG projector. In this case the fetal breathing component was not included in the model. (B) Output of an fMCG projector using the three-component model shown in Fig. 14. (C) Output of a covariance fMCG projector.

Fig. 17.

(A) Output of a fetal breathing projector using the three-component model shown in Fig. 14. (B) Output of a fetal breathing projector using a covariance projector. In this case, the signal covariance was obtained from the fetal breathing model of Fig. 14 and the amplitude was rescaled to match the amplitude of panel (A). (C) Output of a fetal breathing projector using a covariance projector. In this case, the signal covariance was obtained by applying a digital bandpass filter to the data to isolate the fetal breathing signal. The fetal breathing signal covariance was computed directly rather than from the model shown in Fig. 14. The signal was not rescaled.