Selection of internal wave beam directions by a geometric constraint provided by topography

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Direct numerical simulations are performed to investigate the generation of internal waves in a linearly stratified fluid by oscillating barotropic flows over a model continental shelf-slope topography. The presence of a third wave-beam emitted from an abrupt shelf break and transverse to the topography, which has not been adequately interpreted, is now explained in terms of a geometric constraint provided by the topography. This explanation applies to wave beam selection at any abrupt topographic junction point, no matter whether it is convex or concave, or its nearby slope is subcritical or supercritical. One exception is that, at an abrupt concave point with a nearby supercritical slope, the blocking effect leads to the presence of “dead water” (i.e., no flow) and thus no wave beam is emitted. On a critical slope, two beams with opposite directions are emitted from an amphidromic point that has a distinct distance from the shelf break. In addition to the internal wave dispersion relation that restricts possible wave beam directions to form an X-pattern, the geometric constraint proposed in the present work serves as a second selection mechanism that further restricts wave beam directions. The reflective direction of a wave beam incident onto a slope can also be explained by this geometric constraint. The present work provides an updated explanation of internal wave beams emitted at abrupt topographic junction points and unifies the explanation of the wave beam direction for both wave generation and reflection processes. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4984245>]

I. INTRODUCTION

Internal waves of frequency σ in a fluid of constant buoyancy frequency N and Coriolis frequency f have a characteristic slope given by the dispersion relation

$$\alpha = \sqrt{(\sigma^2 - f^2)/(N^2 - \sigma^2)}. \quad (1)$$

Interesting wave beam patterns may appear when barotropic tide flows over ridges, continental shelf slopes, and other rough bottom topography (e.g., [Lamb, 2004](#)). In the present work, we focus on internal wave generation over a continental shelf-slope topography.

Numerical simulations of [Gerkema et al. \(2004\)](#) and laboratory experiments of [Gostiaux and Dauxois \(2007\)](#) showed that, for a continental shelf with a supercritical slope, two internal wave beams were generated which radiated away from the shelf break in both upward and downward directions. There was no wave beam radiated transversely to the topography in these studies, in contrast to the theory of [Baines \(1982\)](#) and the numerical simulations of [St. Laurent et al. \(2003\)](#). In these latter two studies, such a third beam was present and radiated upward from the shelf break. [Gostiaux and Dauxois \(2007\)](#) conjectured that this disagreement might come from the fact that the shelf break in [Baines \(1982\)](#) and [St. Laurent et al. \(2003\)](#) was a sharp corner, while it was a gentle one

in [Gerkema et al. \(2004\)](#) and their own work. Recently, [Wang et al. \(2012\)](#) designed laboratory experiments using both abrupt and gentle shelf breaks to verify this conjecture. Their results showed that, indeed, the presence of an abrupt point in the shelf break leads to the appearance of the third beam. These authors continued to explain the presence of this additional beam by the non-differentiability of the bottom profile $h(x)$ at the abrupt junction point using the body force (driving internal wave motion) expression of [Baines \(1982\)](#),

$$\mathbf{F} = -\frac{QN^2}{\sigma} \frac{zh'(x)}{h^2} \sin \sigma t \hat{\mathbf{z}}, \quad (2)$$

in which $h'(x) = dh/dx$ and Q is the volume flux that is constant. [Wang et al. \(2012\)](#) explained that “the body force at the junction, which is a small region, is large and decreases rapidly as it moves away from the junction,” and “as a result, the generated internal waves are sensitive to the variations in stratification in this small region.” However, these latter two basic facts are always true whether $h(x)$ is differentiable or not at the junction ([Baines, 1982](#)). That $h(x)$ is non-differentiable right at the junction point means that $h'(x)$ are not equal on the two sides of the point. But this does not give rise to an unusually large body force or has any apparent reason in exciting the third beam. Thus, the appearance of the third beam has not been adequately explained, and this provides direct motivation of the present work.

We perform direct numerical simulations to reproduce the experiments of [Wang et al. \(2012\)](#) with a supercritical shelf slope. Critical and subcritical slopes are also studied

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and the wave energetics are briefly discussed. It is shown that, in addition to the dispersion relation [Eq. (1)], a geometric constraint provided by the topography serves as a second selection mechanism for internal wave beam directions.

II. MODEL SETUP AND METHODS

The two-dimensional model extends from $x = -2.5$ m to 2.5 m, with an upper surface at $z = 0$ m and a water depth of $H = 0.272$ m. The abrupt shelf break is located at $(x, z) = (0, -0.122)$ m and the slope is $s = 1$ [Fig. 1(a)]. In Fig. 1(b), the abrupt shelf break is replaced by an arc of radius 0.15 m to represent the gentle case. The water is linearly stratified (surface and bottom density being 1003 kg/m^3 and 1025 kg/m^3 , respectively) with a constant buoyancy frequency $N = 0.88/\text{s}$. The barotropic velocity at the left open boundary ($x = -2.5$ m)

is $u = u_0 \sin(\sigma t)$, in which $u_0 = 0.00126 \text{ m/s}$ and $\sigma = 2\pi/T$ is the frequency (period $T = 15 \text{ s}$). The rotational effect is neglected and the internal wave characteristic slope from Eq. (1) is $\alpha = 0.54$. This gives a steepness parameter of $\gamma = s/\alpha = 1.84$ and the continental slope in Fig. 1 is thus supercritical. The tidal excursion parameter is $\varepsilon = u_0 \sigma L = 0.02$, in which $L = 0.15 \text{ m}$ is the horizontal length of the slope. Since $\varepsilon \ll 1$, the dynamics is in the linear internal tide regime. The above model and setup are exactly the same as the laboratory experiments of Wang *et al.* (2012), except for the following two aspects. First, the experiments of Wang *et al.* (2012) were performed in a closed-end tank and a block (the topography) of relatively short length (in the x -direction) was oscillated back and forth to model the barotropic tide (see Fig. 1). In other words, the block does not extend to connect the right vertical tank wall, due to difficulty in the experimental realization. Internal waves will thus inevitably be generated at the right margin of the block that can propagate to blur the wave field near the left margin, which is of our concern. This difficulty can be easily avoided in simulations by extending the continental shelf to the right domain boundary, which is an open boundary. Second, the gentle shelf break in Wang *et al.* (2012) was modelled by one quarter of a circle with radius 0.15 m , while in Fig. 1(b) this is modelled by an arc of the same radius and tangent to both the supercritical slope and the shelf. Simulations for both setups were performed, which showed no noticeable difference in the flow field, but the latter setup facilitates the discussion of the gentle shelf break with different curvature radius and the same shelf height and slope [Fig. 1(c)].

We performed direct numerical simulations using an in-house numerical code (Chen *et al.*, 2010a; 2010b; and 2014) that solves the full Navier-Stokes equations. The kinematic viscosity and mass diffusivity are set to be molecular values, which are $\nu = 10^{-6} \text{ m}^2/\text{s}$ and $\kappa = 10^{-7} \text{ m}^2/\text{s}$, respectively. The rigid-lid approximation is adopted at the upper surface, where the horizontal velocity is stress-free. A no-slip condition is applied at the bottom boundary. Near each of the two lateral open boundaries, a small sponge layer is added to avoid the reflection of outward travelling waves. The horizontal direction is discretized using 4000 grid cells, with $\Delta x = 0.8 \text{ mm}$ between $x = -0.3 \text{ m}$ and $x = 0.2 \text{ m}$, and then it is gradually stretched to $\Delta x \approx 2.2 \text{ mm}$ near the two open boundaries. The vertical direction is discretized using 250 grid cells, with $\Delta z = 0.54 \text{ mm}$ near the bottom on the deep water side and $\Delta z = 0.25 \text{ mm}$ on the shelf. Δz is then stretched upwards by a factor of 1.005. The boundary layer thickness is estimated to be $\sqrt{\nu/\sigma} = 1.54 \text{ mm}$, in which there are at least 3 grid cells. The time step is $\Delta t = 7.5 \times 10^{-3} \text{ s}$. Simulations started from a motionless state lasted for five oscillation periods, and the results from the last period were selected for analysis. Companion runs with homogeneous fluid were also performed to obtain the barotropic flow fields, as were done in the experiments of Wang *et al.* (2012). For the abrupt supercritical case [Fig. 1(a)], convergence tests with spatial and temporal resolutions doubled changed the computed velocities by less than 1%, and halving the resolutions changed the velocities by less than 3%.

FIG. 1. Vorticity (ω) near the continental margin with a supercritical slope at maximum leftward barotropic tide: (a) abrupt shelf break; (b) gentle shelf break with an arc radius of 0.15 m . (c) Maximum vorticity on lines a, b, and c indicated in (a) during a full period versus arc radius. Vertical dashed lines in (a) indicate the positions for flux calculation in Fig. 5.

III. RESULTS AND DISCUSSION

A. Shelf break curvature

Figures 1(a) and 1(b) are, respectively, the abrupt and gentle cases studied in Wang *et al.* (2012). In Fig. 1(a), there are three primary beams, beam-a, -b, and -c (with cross sections a, b, and c perpendicular to the corresponding beam, spanning slightly larger than the beam width, and located two beam widths away from the shelf break), emitted from the abrupt shelf break. Also apparent is a second harmonic beam. A constant buoyancy frequency of $N = 0.88/\text{s}$ and a primary wave frequency of $\sigma = 0.42/\text{s}$ allow the presence of up to the second higher harmonic. When the abrupt shelf break is replaced by a gentle one with a curvature radius of 0.15 m [Fig. 1(b)], beam-c disappears. This verifies the experiment of Wang *et al.* (2012), and it is indeed true that the presence of an abrupt junction point leads to the appearance of the third beam (i.e., beam-c). However, as pointed out in the Introduction, the differentiability of $h(x)$ cannot be used to provide an explanation. Actually, in Fig. 1(b), there is a weak signal of beam-c, as also can be discerned from the experiment of Wang *et al.* (2012) (see Figs. 3 and 4). This third beam with very low intensity can also be discerned in the model results of Griffiths and Grimshaw (2007) for a supercritical shelf slope with a gentle shelf break. We performed extra simulations changing the curvature radius of the shelf break [Fig. 1(c)]. The abrupt case corresponds to a curvature radius of zero. It can be seen that, as the curvature radius increases, beam-a and -b get stronger while beam-c becomes weaker. The disappearing process of beam-c with increasing curvature radius is a continuous and slow one. There is no abrupt change in the behavior of beam-c when the curvature radius changes from zero (nondifferentiable) to nonzero (differentiable).

B. Geometric constraint provided by topography

We aim to explain the presence of beam-c in Fig. 1(a) in this subsection. We propose that, in oscillating stratified flows, abrupt points of topography will serve as energy sources from which wave beams in the four possible directions determined by Eq. (1) are emitted, and then the actual presence of beams is determined by a geometric constraint provided by the topography, i.e., each of the four possible beams can exist as long as it lies in the fluid. In order to test this hypothesis, we designed a series of simulations (Fig. 2), in which all topography have the same height as that in Fig. 1 and all have abrupt junction points. The other conditions remain unchanged. It can be seen from Fig. 2(a) that four primary beams emit from the abrupt ridge crest and form an X-pattern, which has also been obtained by Dettner *et al.* (2013) for tidal flows over a similar symmetric supercritical ridge. When compared to Fig. 1(a), one more beam with the right-downward direction appears, since the geometry of a supercritical ridge allows its presence. In Fig. 2(b) with a critical slope, the geometry allows two primary beams to be connected at the abrupt junction point at the base of the slope, which is locally concave (in contrast to the shelf break that is locally convex). One beam lies exactly on the slope, indicating resonant generation, which has been investigated by the laboratory experiments of Zhang *et al.*

FIG. 2. The same as Fig. 1(a) except that the topography is changed to (a) a symmetric ridge with a supercritical slope ($\gamma = 1.84$) identical to that in Fig. 1(a); (b) a shelf with a critical slope ($\gamma = 1$); (c) a shelf with a subcritical slope ($\gamma = 0.71$). The vertical dashed lines in (b) and (c) indicate the positions for energy flux calculation in Fig. 5 and the six white lines on the critical slope indicate the positions for time evolutions of vorticity shown in Fig. 3.

(2008) and direct numerical simulations of Gayen and Sarkar (2010). According to Gostiaux and Dauxois (2007), the critical point on a gentle shelf break (with a supercritical slope) is an amphidromic point from which internal wave beams are emitted. The phase of beams rotates about this amphidromic point. In Fig. 1(a) or Fig. 2(a), the abrupt shelf break or the sill crest serves as this amphidromic point. However, for the case of a critical slope in Fig. 2(b), is there still an amphidromic point? Are the three primary beams near the abrupt shelf break in Fig. 2(b) still emitted from the shelf break, as in the situation of Fig. 1(a)? Since the whole slope in Fig. 2(b) is now critical, people might expect that internal waves are resonantly generated over the whole slope and the amphidromic point is elongated to be a line on the slope. By probing at four different points above the critical slope, Zhang *et al.* (2008) showed in Fig. 3 that the phase propagation of the wave beam is upward along the critical slope. These authors also commented that there is “a very small phase difference for displacements perpendicular to the slope.” In other words, the phase velocity perpendicular to the slope is very small. These authors did not determine the direction of this across-slope phase velocity. However, this propagation direction is crucial, since it determines the energy propagation direction of the beam, given that phase and group velocities are orthogonal with opposite vertical components. We determine this phase propagation direction along six equally spaced white lines perpendicular to the critical slope [see Fig. 2(b)]. The results show that, for lines 1 to 4, the phase propagation is upward

FIG. 3. Time evolutions of vorticity on lines 4 (a), 5 (b), and 6 (c) indicated in Fig. 2(b) for 2 forcing periods. η measures the distance perpendicular to the bottom.

perpendicular to the slope. For example, the time evolution of vorticity along line 4 for two forcing periods is shown in Fig. 3(a). The much stronger intensity of the cells at the bottom row is due to the viscous boundary layer on the slope. It is clear that the phase is travelling upward as time evolves, indicating a downward energy propagation along the slope. In contrast, the phase on line 6 is travelling downward [Fig. 3(c)], indicating an upward energy propagation along the slope. On line 5, the phase evolution shows an almost standing-wave pattern [Fig. 3(b)], indicating that line 5 is close to the position where internal wave beams are emitted in opposite directions along the slope. Thus, in Fig. 2(b), even though the whole slope is critical, wave beams over the slope are still emitted from a single amphidromic point, which has a distinct distance from the abrupt shelf break. However, beam-c (the third beam) is still emitted from the abrupt shelf break, as in Fig. 1(a). Now, it is clear that the upward phase propagation along the critical slope determined in Fig. 3 of Zhang *et al.* (2008) is the along-slope component of the phase velocity, since the phase rotates around the amphidromic point where internal wave beams are emitted (Gostiaux and Dauxois, 2007).

In Fig. 2(c) with a subcritical slope, only two upward-going beams are present at each of the abrupt convex and concave junction points, which are the only two possible directions allowed by the geometry of the topography. For tidal flows over a subcritical triangular ridge, Petrelis *et al.* (2006) and Dettner *et al.* (2013) showed that two similar upward-going beams were also emitted from the ridge crest (convex) and ridge base (concave).

Since geometric constraints apply at both convex and concave points, one may wonder why it does not apply at the concave points in Figs. 1(a), 1(b), or 2(a). At these points, the geometry of the topography should allow one left-upward going beam, even though the other three directions lie

outside the fluid. However, there are no such left-upward going beams emitted at these points. This is because in these cases the slope is supercritical, and the fluid near the bottom of the slope is blocked to form “dead water.” In other words, there is no relative motion between the fluid and the topography near the concave junction point, and thus no internal waves are generated. The laboratory experiment of Zhang *et al.* (2007) showed that this blocking effect is generic to any supercritical topography under the condition of small tidal excursion parameter ($\varepsilon = 0.02$). The nondimensional combination $Nh_0/u_0 = 105 \gg 1$ (h_0 is the topography height) implies that the blocking effect is indeed very strong (Baines, 1995). Zhang *et al.* (2007) further commented that this blocking effect hydrodynamically screens the steep part of the topography, increases the horizontal length scale, and makes the topography less supercritical. They also conjectured that “the properties of the internal gravity waves are mainly determined by the *local* topographic properties such as the *radius of curvature* at the critical locations.” This conjecture has now been verified by our direct simulations in Fig. 1(c).

Vorticity profiles on sections a, b, and c indicated in Figs. 1(a), 2(b), and 2(c) are shown in Fig. 4. Since beam-a (propagating left-downward) does not exist in the subcritical case, and for the critical case it is emitted from a point [near line-5 in Fig. 2(b)] that is quite different from the shelf break, only the supercritical case is shown in Fig. 4(a). At this instant (maximum tide from right to left), beam-a is transiting from a bimodal (with one positive and one negative peak in the vorticity band) to a unimodal (with only one peak) state. Beams-b [Fig. 4(b)] in the supercritical and critical cases are clearly in a bimodal state, while in the subcritical case it is in a

FIG. 4. Vorticity profiles on sections (a) a, (b) b, and (c) c indicated in Fig. 1(a) (supercritical), Fig. 2(b) (critical), and Fig. 2(c) (subcritical). The plot starts from the upper end point of section-a and the lower end points of both section-b and -c.

unimodal state, due to interference of the two right-upward going beams emitted at the abrupt convex and concave points, respectively. In the critical case, due to resonant generation, the intensity of beam-b is the strongest. This is consistent with the result of [Dettner *et al.* \(2013\)](#) that, under critical conditions, intense boundary currents are generated with large kinetic energy densities. Beams-c [Fig. 4(c)] in all three cases are in a unimodal state, with the intensity increasing with the slope. Fig. 2.19 in [Vlasenko *et al.* \(2005\)](#) also exhibits a similar feature. Resonant generation in the critical case does not increase the intensity of beam-c. It is the “sharpness” (i.e., the nearby slope) at the abrupt junction point that determines the intensity of beam-c. This is also similar to the situation in Fig. 1(c), in which the curvature radius represents “sharpness.”

Geometric constraints apply not only in internal wave beam generation but also in beam reflection from a boundary. It is well known that an incident internal wave beam propagating onto a slope will be transmissive (forward) if the slope is subcritical and reflective (backward) if it is supercritical. This is because, except for the incident beam, the transmissive or reflective direction is the only one (among the three remaining directions) that is allowed by the geometry of the topography. When the slope is critical, i.e., critical reflection, geometric constraint allows the internal wave energy to propagate both upwards and downwards along the slope, creating intense boundary flows [see Fig. 1 in [Cacchione *et al.* \(2002\)](#)]. The experiment of [Zhang *et al.* \(2008\)](#) showed that, in addition to critical reflection ([Cacchione *et al.*, \(2002\)](#)), resonant generation of internal waves on a critical slope can also provide an explanation for the intense boundary flows [as simulated in Fig. 2(b)]. The present work shows that, in addition to remotely generated internal waves incident onto a slope, geometric constraint also applies to the selection of wave beams that are locally generated.

C. Energetics

Horizontal baroclinic energy flux is calculated as $F' = p'u'$, in which p' and u' are pressure and horizontal velocity disturbances obtained by subtracting the corresponding barotropic values (from companion barotropic runs) from the full ones. The depth-integrated and time-averaged fluxes across the vertical dashed lines (located half beam width outside the beams emitted from the junction points) indicated in Figs. 1(a), 2(b), and 2(c) are shown in Fig. 5(a). In the subcritical cases, the off- and on-shelf fluxes are of comparable magnitude. As γ increases, the on-shelf fluxes decrease slowly while the off-shelf ones increase dramatically. The fraction of on-shelf relative to the total flux (summing absolute values of both off- and on-shelf fluxes) is shown in the inset, which is very similar to the behavior for the increasing ratio $\delta = h_0/H$ shown in Fig. 8 of [St. Laurent *et al.* \(2003\)](#) for the situation of an abrupt step ($\gamma = \infty$). The fraction in the inset seems to saturate for a supercritical slope and it is 5.0% for the supercritical abrupt case. This value agrees very well with that shown in Fig. 8 of [St. Laurent *et al.* \(2003\)](#) (with $\delta = h_0/H = 0.55$) by linear inviscid calculation. Also in Fig. 5(a), for critical and supercritical slopes, the on-shelf flux is slightly larger in

FIG. 5. (a) Energy fluxes propagating off- (negative) and on-shelf (positive) for both abrupt and gentle (replacing both the convex and concave points by an arc of radius 0.2 m) cases. The inset shows the fraction of on-shelf relative to the total flux. (b) Total flux and conversion rate compared to the linear theories of Llewellyn Smith and Young [2002 (JPO02); 2003 (JFM03)].

the gentle than in the abrupt cases. This is because the on-shelf propagating beam-b is stronger in the gentle than in the abrupt case [see Fig. 1(c)]. However, for a subcritical slope, the reverse is true, i.e., it is slightly larger in the abrupt than in the gentle case. This is because in the abrupt case, one more on-shelf propagating beam is emitted from the concave point at the bottom of the slope [Fig. 2(c)]. The off-shelf flux is always larger in the abrupt than in the gentle case, due to the left-upward going beams emitted from the abrupt convex and concave points. It is clear that additional beams in the abrupt cases have a more significant influence on the off-shelf than on the on-shelf flux.

The time-averaged barotropic-to-baroclinic energy conversion rate is calculated as in [Khaliwala \(2003\)](#). This conversion rate and the total flux away from the slope for both abrupt and gentle cases are shown in Fig. 5(b). Due to additional beams generated in the abrupt cases, the conversion rates are roughly 10% larger than those in the corresponding gentle cases. This agrees with the results of [Echeverri and Peacock \(2010\)](#) that the conversion rate and the nature

of the radiated internal waves are sensitive to the topographic shape. It also agrees with the results of Di Lorenzo *et al.* (2006) and Zilberman *et al.* (2009) that inadequate resolution and smoothing of topographic features required to run a numerical model may lead to an underestimate of the true conversion rate. As γ increases, both the conversion rate and total flux increase. The difference between them represents that part of energy that is dissipated over the slope. For example, in the subcritical case, roughly one third of the energy is locally dissipated. It is interesting that, even though critical topography generates intense boundary currents [Fig. 4(b)], the conversion rate and radiated total flux do not peak at critical condition ($\gamma = 1$). This is the main point conveyed by Dettner *et al.* (2013), in which it was further inferred that the more easily measured kinetic energy density cannot be used as a proxy to characterize the conversion of tidal energy to radiated internal wave power. Estimations of the conversion rate from two linear theories of Llewellyn Smith and Young [2002 (JPO02); 2003 (JFM03)] are also plotted for comparison. JPO02 is based on the assumptions of $\varepsilon \ll 1$, $\sigma/N \ll 1$, $\gamma \ll 1$, and $\delta \ll 1$. Except the first one, the latter three conditions are not justified in the present work. However, we use it to give a crude estimate, as it was done in Llewellyn Smith and Young (2002). It can be seen that JPO02 gives a more reasonable estimate for the subcritical case, which is more justified for using the theory. The second theory of JFM03 is for a very steep ridge, in which γ has no effect. Since the present shelf-slope topography has only one deep side and the on-shelf flux is only a small fraction of the total flux for the supercritical case [see Fig. 5(a)], the conversion rate from JFM03 is halved to provide an estimate in Fig. 5(b). It can be seen that this agrees surprisingly well with the supercritical abrupt case.

IV. CONCLUSIONS

It is indeed true that the presence of an abrupt shelf break leads to the appearance of a third beam, as conjectured by Gostiaux and Dauxois (2007) and verified by the experiments of Wang *et al.* (2012). The appearance of this third beam is now more adequately explained by a geometric constraint provided by the topography, which applies to wave generation at any abrupt junction point, whether it is convex or concave, or its nearby slope is subcritical or supercritical. One exception is that, at an abrupt concave point with a nearby supercritical slope, a blocking effect leads to the presence of “dead water” and thus no wave beam is emitted. One more interesting point is that, under critical conditions, the beam lying on the slope is actually composed of two beams with opposite directions emitted from an amphidromic point, located somewhere between the top and bottom of the slope. The specific location of this amphidromic point may deserve some further studies. Geometric constraints can also be used to explain the reflection direction (forward or backward) of an incident beam propagating onto a slope. The beam selecting process *might be tentatively* understood in the following way. Abrupt points of topography in stratified flows or the impingement of an incident beam onto a slope will create disturbances in all directions. However, possible wave propagation directions

are restricted by the dispersion relation [Eq. (1)] to form an X-pattern. Then, the actual appearance of a particular wave beam is determined by a geometric constraint, i.e., it can appear as long as it lies in the fluid. Thus, in addition to the dispersion relation, this geometric constraint serves as a second selection mechanism of internal wave beams. The present work provides an updated explanation for the presence of wave beams at abrupt points and unifies the explanations of wave beam directions for both wave generation and reflection processes.

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