Technical Note: On the accuracy of parametric two-parameter photon cross-section models in dual-energy CT applications

Dong Han^{a)} and Mariela A. Porras-Chaverri

Medical Physics Graduate Program, Department of Radiation Oncology, Virginia Commonwealth University, Richmond, VA 23298, USA

Joseph A. O'Sullivan

Department of Electrical and Systems Engineering, Washington University, St. Louis, MO 63130, USA

David G. Politte

Electronic Radiology Laboratory, Mallinckrodt Institute of Radiology, Washington University School of Medicine, St. Louis, MO 63110, USA

Jeffrey F. Williamson^{a)}

Medical Physics Graduate Program, Department of Radiation Oncology, Virginia Commonwealth University, Richmond, VA 23298, USA

(Received 10 October 2016; revised 28 February 2017; accepted for publication 28 February 2017; published 25 April 2017)

Purpose: To evaluate and compare the theoretically achievable accuracy of two families of two-parameter photon cross-section models: basis vector model (BVM) and modified parametric fit model (mPFM).

Method: The modified PFM assumes that photoelectric absorption and scattering cross-sections can be accurately represented by power functions in effective atomic number and/or energy plus the Klein-Nishina cross-section, along with empirical corrections that enforce exact prediction of elemental cross-sections. Two mPFM variants were investigated: the widely used Torikoshi model (tPFM) and a more complex “VCU” variant (vPFM). For 43 standard soft and bony tissues and phantom materials, all consisting of elements with atomic number less than 20 (except iodine), we evaluated the theoretically achievable accuracy of tPFM and vPFM for predicting linear attenuation, photoelectric absorption, and energy-absorption coefficients, and we compared it to a previously investigated separable, linear two-parameter model, BVM.

Results: For an idealized dual-energy computed tomography (DECT) imaging scenario, the cross-section mapping process demonstrates that BVM more accurately predicts photon cross-sections of biological mixtures than either tPFM or vPFM. Maximum linear attenuation coefficient prediction errors were 15% and 5% for tPFM and BVM, respectively. The root-mean-square (RMS) prediction errors of total linear attenuation over the 20 keV to 1000 keV energy range of tPFM and BVM were 0.93% (tPFM) and 0.1% (BVM) for adipose tissue, 0.8% (tPFM) and 0.2% (BVM) for muscle tissue, and 1.6% (tPFM) and 0.2% (BVM) for cortical bone tissue. With exception of the thyroid and Teflon, the RMS error for photoelectric absorption and scattering coefficient was within 4% for the tPFM and 2% for the BVM. Neither model predicts the photon cross-sections of thyroid tissue accurately, exhibiting relative errors as large as 20%. For the energy-absorption coefficients prediction error, RMS errors for the BVM were less than 1.5%, while for the tPFM, the RMS errors were as large as 16%.

Conclusion: Compared to modified PFMs, BVM shows superior potential to support dual-energy CT cross-section mapping. In addition, the linear, separable BVM can be more efficiently deployed by iterative model-based DECT image-reconstruction algorithms. © 2017 American Association of Physicists in Medicine [https://doi.org/10.1002/mp.12220]

Key words: computed tomography, dual-energy, photon cross-section

1. INTRODUCTION

Two-parameter photon cross-section models are used in quantitative dual-energy computed tomography (QDECT) to describe monoenergetic or spectrally averaged linear attenuation coefficients and other radiological quantities as functions of two independent parameters. Two commonly used two-parameter representations are effective atomic number (Z^*) and electron density (ρ_e^*) or alternatively, the weights of two dissimilar basis materials with known linear attenuation

coefficients. The utility of a given two-parameter model in QDECT depends on the accuracy with which it predicts photon cross-sections. The domain within which high prediction accuracy is required depends on the application. For example, the DECT stoichiometric technique¹ for postreconstruction mapping of electron density and mean excitation energy for proton therapy requires only that the model predict spectrally averaged linear attenuation coefficients in the diagnostic CT energy range. On the other hand, model-based CT image-reconstruction algorithms^{2–6} that incorporate higher order

beam-hardening corrections require that the model accurately predict monoenergetic linear attenuation coefficients across the entire diagnostic X-ray energy range, while using QDECT to support Monte Carlo-based treatment planning for low-energy brachytherapy requires high accurate monoenergetic predictions down to 20 keV for partial cross-sections, differential cross-sections, and energy-absorption coefficients as well as total linear attenuation coefficients.^{7,8}

Our work has previously demonstrated that one particular two-parameter model, the linear basis vector model (BVM),^{9,10} is able to recover monoenergetic photon cross-sections⁹ and charged particle stopping powers¹¹ with mean absolute errors less than 0.5%. However, the most widely used model in the QDECT cross-section imaging literature is the parametric fit model (PFM), first introduced by Spiers¹² and Mayneord.¹³ PFM postulates that cross-sections can be accurately represented by nonlinear, nonseparable functions of Z^* and ρ_e^* . The variant used in many QDECT applications^{14,15} is a modified version (tPFM), introduced by Torikoshi et al.,¹⁶ tPFM simplifies the Spiers¹² model and adds table-based corrections that enforce exact prediction of elemental monoenergetic linear attenuation coefficients.¹⁴ However, to date, no one has assessed the accuracy with which the tPFM is able to predict monoenergetic photon cross-sections for mixtures and compounds representing the range of tissue compositions encountered in normal mammalian anatomy. The purpose of this note is to compare the theoretical accuracy of the modified PFM and BVM for predicting linear attenuation coefficients, partial cross-sections, and energy-absorption coefficients in the 20–1000 keV energy range for such biological tissues or tissue substitutes.

2. METHODS AND MATERIALS

We evaluated photon cross-section prediction accuracy of BVM and tPFM in the energy range from 20 keV to 1000 keV, along with a more complex modified PFM that we call the “VCU modified parametric fit model” (vPFM). The photon cross-sections in this study included the total linear attenuation, photoelectric effect, scattering, and energy-absorption coefficients, all as function of energy E . The model accuracy was evaluated for 43 tissues and tissue substitutes with elemental compositions recommended by ICRU,^{17,18} ICRP,¹⁹ or other references,²⁰ in terms of mean percent relative error in the lower energy range (20 keV to 50 keV), and higher (50 keV to 1000 keV) energy ranges. The reference photon cross-sections data were calculated applying the mixture rule to elemental cross-sections from the NIST XCOM library.²¹ All of the computational methods utilized in this work were implemented in MATLAB environment (version 12.0, The Math Works Inc., Natick, MA, USA) on a Windows 7 PC.

2.A. Description of two-parameter models

The BVM model assumes that the photon cross-sections of both mixtures and elemental substances can be approximated by a linear combination of two dissimilar basis

materials. For example, the total linear attenuation of an unknown tissue at energy E ($20 \text{ keV} \leq E \leq 1000 \text{ keV}$), position $\mathbf{x}$ can be written in the following form

$$\mu(\mathbf{x}, E) = c_1(\mathbf{x})\mu_1(E) + c_2(\mathbf{x})\mu_2(E) \quad (1)$$

where $\mathbf{x}$ refers to the image voxel location and $\mu_i(E)$ ($i = 1, 2$) denotes the linear attenuation coefficient of the i -th basis material in the pure form. In this work, a water-polystyrene pair was chosen for soft tissues, while a water- CaCl_2 solution (23% concentration) pair was used for bony tissues.⁹ The details, including criteria for basis pair choice of tissues, are discussed elsewhere.⁹ $c_i(\mathbf{x})$ images from DECT imaging were computed from low (90 kVp) and high (140 kVp + tin filter) energy scanning spectra approximated by effective energies 45 keV and 90 keV.¹¹ The photoelectric and scattering cross-sections were also computed by using the $c_i(\mathbf{x})$ images.⁹

Torikoshi et al.¹⁶ proposed a nonseparable parametric fit model (tPFM) by assuming that the total linear attenuation coefficient is a function of electron density ρ_e^* and effective atomic number Z^*

$$\mu(\mathbf{x}, E, \rho_e, Z^*)_{tPFM} = \rho_e(\mathbf{x})(Z^{*4}(\mathbf{x})F(E, Z^*) + G(E, Z^*)) \quad (2)$$

where the first and second right-hand terms represent photoelectric absorption and scattering, respectively. The functions $F(E, Z^*)$ and $G(E, Z^*)$ are precalculated correction terms that force Eq. (2) to exactly recover the elemental linear attenuation coefficients for the elements ($Z = 2$ to 20) on the discrete logarithmic energy grid specified in the XCOM database. For mixtures of elemental materials, Z^* may assume noninteger values. The nonlinear Eq. (2) can be solved iteratively for the idealized DECT scenario where there is exact knowledge of total linear attenuation coefficient at two discrete effective energies (45 keV and 90 keV) for each voxel.

Our vPFM approach combines the more detailed Spiers¹² PFM with elemental correction tables introduced by Torikoshi.¹⁶ We hypothesized that the more complete parametric model would support more accurate interpolation for intermediate E and Z^* values, thereby improving the prediction accuracy for mixtures and compounds.

$$\mu(E, Z^*, \rho_e)_{vPFM} = \rho_e \cdot \left[a \cdot \frac{Z^{*b}}{E^c} \cdot F'(E, Z^*) + \left(\sigma_{KN}^e(E) + d \cdot \frac{Z^{*e}}{E^f} \right) \cdot G'(E, Z^*) \right] \quad (3)$$

where we have dropped the spatial coordinate, $\mathbf{x}$, for simplicity. The energy- and position independent a - f are the best-fit parameters which maximize the accuracy with which PFM predicts the elemental photoelectric effect and total scattering cross-sections (NIST XCOM) for $2 \leq Z \leq 20$ and $20 \leq E \leq 1000 \text{ keV}$ without the empirical correction factors. $F'(E, Z^*)$ and $G'(E, Z^*)$ are derived by comparing best-fit PFM predictions to the known XCOM cross-sections. Since

the prediction accuracy of tPFM and vPFM were so similar, detailed results only for the former are presented.

Monte Carlo simulations often require the linear energy-absorption coefficient, $\mu_{en}(\mathbf{x}, E)$. This quantity also serves as a useful surrogate for absorbed dose⁹ in the approximation where energy impartation is dominated by first-order photon collisions. For elements, $\mu_{en}(E, Z)$ is given by

$$\mu_{en}(E, Z) = \rho_e \left[\sigma_{KN} \left(1 - \frac{\bar{E}_{sca,KN}}{E} \right) + \sigma_{PE} \left(1 - \frac{E - P_K \cdot E_K \cdot \omega_K}{E} \right) \right] \cdot (1 - g_K) \quad (4)$$

where σ_{KN} and σ_{PE} denote Klein-Nishina and photoelectric cross-sections, respectively, per electrons; $\bar{E}_{sca,KN}$ is the average energy scattered per collision; and P_K , E_K and ω_K denote the K-shell vacancy probability, binding energy and fluorescent yield, respectively. The quantities of $P_K \cdot E_K$ are functions of effective atomic numbers; linear interpolation between their elemental values is used to evaluate $\mu_{en}(E, Z^*)$ at noninteger Z^* values. The average fraction of the secondary charged particle energy that is subsequently lost in radiative process is denoted by g_K . Photon energy of interest in this note is assumed greater than the shell binding energy. The dose calculation error introduced by tPFM can be approximated by Eq. (4)

Once $c_i(\mathbf{x})$ ($c_1(\mathbf{x}), c_2(\mathbf{x})$) and $(Z^*(\mathbf{x}), \rho_e(\mathbf{x}))$ pairs have been determined, the mean relative error of photon cross-

sections prediction error for the j -th tissue can be written as

$$\text{Root-mean-square Error}(\%) = 100\% \times \left[\sum_{i=1}^N \left(1 - \frac{\mu_{i,\Delta,DECT}(E)}{\mu_{i,\Delta,ref}(E)} \right)^2 / N \right]^{1/2}_{\Delta=Tot,PE,EN} \quad (5)$$

$\Delta = Tot, PE$, and EN represent total linear, photoelectric absorption, and energy-absorption coefficients, respectively. The subscript “ref” refers to the ground truth values derived from the XCOM library.²¹

3. RESULTS

3.A. Estimation error by the BVM and tPFM

Figure 1 shows the BVM and tPFM root-mean-square (RMS) prediction errors of total cross-section estimation as functions of effective atomic number derived from tPFM model for the low $20 \leq E \leq 50$ keV and high $50 \leq E \leq 1000$ keV energy ranges. The BVM model outperforms the tPFM, achieving RMS errors of less about 0.5% and 0.1% for the low- and high-energy ranges, respectively, except for Teflon and inflated lung tissues. At low energies, tPFM predictions exhibit RMS errors over 1% for most tissues and up to 2% for tissue substitutes (phantom materials) with RMS errors are less than 0.6% at higher energies. The

FIG. 1. Percent RMS error of the total linear attenuation coefficient predicted by BVM or tPFM for 43 tissues and phantom substitutes as functions of tPFM effective atomic number for the (a) $20 \leq E \leq 50$ keV and (b) $50 \leq E \leq 1000$ keV energy ranges. The thyroid tissue prediction error is outside the plotting range of the low-energy plot. [Color figure can be viewed at wileyonlinelibrary.com]

tPFM prediction errors are generally larger for bony tissues. Due to its high iodine content (0.1% by weight with $Z = 53$), both BVM and tPFM fail to accurately model the thyroid tissue cross-sections, with errors as large as 7.9% and 7.7%, respectively.

Figure 2 compares the tPFM and BVM estimation errors of photoelectric and scattering cross-sections. BVM predicts soft and bony tissues low-energy photoionization cross-sections with accuracies 0.5% to 2% except for a 4.5% error for Teflon. In contrast, tPFM models the soft tissue photoionization cross-section with comparable accuracy but exhibits much larger errors (2–4%) for bony tissues. A similar pattern obtains for higher energy scattering cross-sections: BVM prediction accuracy is generally better than 0.25%, while tPFM exhibits errors in excess of 2% for bony tissues. Neither BVM nor tPFM accurately predicts the thyroid gland photoionization cross-sections with RMS errors 17.0% and 18.7%, respectively, while scattering cross-sections exhibit RMS error less than 1%. More extensive data can be found in Table S1 of our supplementary materials.

Figure 3 shows that BVM estimates of $\mu_{en}(E)$ exhibits errors less than 1% (never more than 1.6%, except for Teflon and thyroid) for all soft and bony tissues. For tPFM, soft tissue estimation errors are mostly larger than 1% and, 2% to 3.5% for bony tissues. Iodine-based tissues have $\mu_{en}(E)$ estimation errors of 14.7% and 16.2% for BVM and tPFM across the two energy ranges, respectively.

Despite the fact that the Spiers formula¹² yields a much better fit to the elemental total cross-section data than the

uncorrected Torikoshi formula, tPFM and vPFM modeled total, photoelectric, scattering and energy-absorption coefficient data with equivalent accuracies. In general, prediction errors rarely deviated by more than 0.5% to 1.0% between the two models. As an example, Fig. 4(b) shows that in the range of 20–1000 keV, RMS errors for prediction $\mu_{en}(E)$ are virtually identical for the vPFM and tPFM models. A more complete data set is shown in Table S2 of our supplementary materials.

We also investigated the possible correlations between the tissue composition surrogate quantities [i.e., $c_1(x)/(c_1(x) + c_2(x))$ ratio (c-ratio) for BVM and effective atomic number (Z^*) for tPFM and vPFM in Fig. 4] and $\mu_{en}(E)$ prediction errors. BVM c-ratios of soft tissues are clustered about unity (Fig. 4(a)), exhibiting prediction errors in 0.2–1.2%, while for bony tissues, the corresponding c-ratios assume a broad distribution centered around 0. Figure 4(b) shows that no clear correlation between RMS error and Z^* for soft tissues; however for bony tissues with Z^* greater than or less than 10, different linear correlations appear. Figure 4(b) illustrates another important finding: prediction errors for the vPFM and tPFM are nearly identical.

4. DISCUSSION

For a wide range of tissues and tissue substitutes, our study demonstrates that BVM models linear attenuation, photoelectric effect and energy-absorption coefficients with significantly better accuracy than tPFM. This confirms our

FIG. 2. tPFM and BVM percent RMS predictions for 43 tissues and phantom substitutes as functions of tPFM effective atomic number for (a) photoelectric effect cross-sections in the $20 \leq E \leq 50$ keV range and (b) photon scattering cross-sections in the $50 \leq E \leq 1000$ keV range. The thyroid tissue prediction error is outside the plotting range of the low-energy plot. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 3. Percent RMS error of energy-absorption coefficients estimate for 43 tissues and phantom materials as functions of tPFM effective atomic number for the (a) $20 \leq E \leq 50$ keV and (b) $50 \leq E \leq 1000$ keV energy ranges. The thyroid tissue prediction error is outside the plotting range of the low-energy plot. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 4. $\mu_{en}(E)$ of RMS estimation errors as functions of (a) c-ratio for BVM and (b) effective atomic number for tPFM and vPFM in the energy range of 20 keV to 1000 keV. Thyroid tissue is excluded. [Color figure can be viewed at wileyonlinelibrary.com]

earlier study,⁹ which showed that the simple, linear two-parameter BVM implementation can accurately represent (within 2%) monoenergetic photoelectric cross-sections and other radiological quantities needed to implement model-based brachytherapy dose calculations for energies as low as 20 keV, just below the mean energy of photons emitted during ¹⁰³Pd decay. While tPFM and vPFM definitely outperform the uncorrected PFM e.g., reducing photoionization cross-section modeling errors from 22.4% to 4.5%, modified PFM errors for bony tissues (3.2 to 6.4%) exceed the 3% maximum marginal DECT cross-section uncertainty that we believe is necessary to support meaningful DECT-based low-energy brachytherapy dose calculations.²² In contrast, the corresponding range of errors for BVM is 0.3 to 1.5%. Our study suggests that the modified PFM family is unsuitable for mapping low-energy photon cross-sections in brachytherapy permanent seed applications. However, we note that tPFM or other variants of PFM have been successfully applied (experimentally and theoretically) to DECT mapping of proton stopping power ratios,^{1,15,23} because they depend more heavily on electron density than effective atomic number. This suggests that while PFM models may be more suitable for heavy charged particle stopping power mapping in DECT than for mapping photon cross-sections.

The prediction errors of vPFM and tPFM are nearly identical (see Fig. 5 for three typical tissues and Fig. 4(b) for RMS mean errors). This indicates that more elaborate modeling of cross-section atomic number dependence does not lead to more accurate predictions for noninteger Z^* values, corresponding to compounds and mixtures of biological interest.

This finding was unexpected, since the energy- and atomic number- dependent corrections of the Spiers formula¹² are significantly smaller than those of the simple Z^4 correction assumed by the tPFM model. This underscores the fact that effective atomic number can only serve as an energy-independent surrogate for tissue composition only for elemental substances. However, the tPFM error vs. energy plot does exhibit a small “scallop” effect which is significantly reduced in the vPFM error profile. This indicates that vPFM does in fact, support more physically plausible interpolation between tabulated elemental correction factors $F'(E, Z^*)$ and $G'(E, Z^*)$.

Our purpose was to assess the theoretical accuracy of two competing families of two-parameter cross-section models independently of the real-world uncertainties associated with commercial CT scanners which can undermine the one-to-one correspondence between CT voxel intensities and associated radiological quantities. To this end, our validation scheme utilized an idealized scanner model which ignored all measurement uncertainties, including statistical fluctuations of sinogram signal intensities, photon scatter, polychromaticity of the scanning spectra, reconstruction artifacts, and overlap of the low- and high-energy spectra. Although a comprehensive analysis of the impact of such uncertainties on cross-section estimation error is outside of the scope of this Technical Note, we have made a comparative uncertainty propagation analysis.²⁴ Three levels of image-intensity uncertainties (specified as spectrally averaged $(\mu(\mathbf{x})/\mu_{\text{wat}})$ ratios) (0.2%, 0.1%), (0.6%, 0.3%), and (1.5%, 1.0%), where each pair corresponds to low- and high-energy CT images, respectively. The largest uncertainty pair, (1.5%, 1.0%), represents

FIG. 5. Relative estimation error of energy-absorption coefficients for adipose, muscle and cortical bone by (a) BVM and (b) vPVM and tPFM models. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 6. Percent unexpanded uncertainty (coverage factor $k = 1$) of the linear energy-absorption coefficient for PMMA as a function of energy for the (a) tPFM and (b) BVM models. The low, medium, and high curves denoted the uncertainties corresponding to the (low, high) energy image-intensity uncertainties (0.2%, 0.1%), (0.6%, 0.3%), and (1.5%, 1.0%), respectively. [Color figure can be viewed at wileyonlinelibrary.com]

FIG. 7. RMS error of total cross-section estimates predicted by dBVM (double-basis pair) and sBVM (single-basis pair) for 43 tissues and phantom substitutes as functions of effective atomic number for the (a) low ($20 \leq E \leq 50$ keV) and (b) high ($50 \leq E \leq 1000$ keV) energy ranges. The thyroid tissue prediction error is outside the plotting range of the low-energy plot. [Color figure can be viewed at wileyonlinelibrary.com]

the lowest combined random and systematic uncertainties achievable with current commercially available scanner hardware and reconstruction engines. Using the error propagation formulas derived in our earlier work,^{9,11} the unexpanded uncertainties of $\mu_{en}(E)$ were evaluated for three typical tissues: adipose, muscle, and cortical bone as well as polymethyl methacrylate (PMMA). Figure 6 shows the result for PMMA (See Supplemental Materials for the complete analysis). In summary, vPFM and tPFM were found to have nearly identical error propagation characteristics, while BVM cross-section estimation uncertainties were slightly better than the modified PFM methods. Similar to our prior work,^{9,22} Figure 6 shows that image-intensity uncertainties must be limited to 0.2–0.5% in order to achieve cross-section mapping uncertainties of less than 3% at the lowest clinically relevant energy (20 keV). Our prior experimental evaluations of BVM-based postreconstruction QDECT conducted on a commercial CT scanner, have experimentally demonstrated that when these conditions are satisfied (only under highly controlled conditions), systematic cross-section mapping errors range from 0.8–1.5%.²² More comparisons of $\mu_{tot}(E)$, $\mu_{pe}(E)$, and $\mu_{en}(E)$ for standard tissues are shown in supplementary materials.

To ensure a fair comparison between BVM and modified PFM, we evaluated BVM accuracy for double-basis pair (dBVM) and single-basis pair BVM (sBVM) implementations. The sBVM implementation deploys a polystyrene and CaCl₂ (23% concentration) solution pair. The comparisons between dBVM with and sBVM for all three groups of tissues are shown in Fig. 7 of supplemental materials. For soft and tissue groups, the dBVM prediction accuracy of total linear attenuation outperforms the sBVM. Both of the BVM models show similar prediction accuracy for bony tissue group at low- and high-energy ranges. The sBVM model also shows better accuracy than the family of PFM models. For the low-energy range $20 \leq E \leq 50$ keV, RMS errors of both sBVM and dBVM models for prediction of total cross-section are well within 1.0% across three tissue groups except for inflated lung tissue and Teflon. However, the dBVM (error averaged over all tissue of 0.25%) outperforms the sBVM (average error of 0.6%). In the higher energy range $E \geq 50$ keV, both models show similar prediction accuracy, achieving RMS errors less than 0.1% except for Teflon and inflated lung ti.

Finally, we would like to emphasize that tPFM and related PFM-based DECT stoichiometric methods have been shown to support sufficiently accurate proton stopping power and higher energy (> 100 keV) photon cross-section mapping. However, all of these studies have been limited to the post reconstruction scenario where only spectrally averaged linear attenuation coefficients need to be accurately estimated. Our prior²² and current work⁸ demonstrate the potential of iterative statistical image-reconstruction (SIR) algorithms, based on physically accurate spectral and scatter distribution measurements, to limit input errors to the $< 0.5\%$ level required to address the poorly conditioned problem of DECT low-energy

cross-section mapping. For example, by using a polyenergetic alternating minimization (AM) reconstruction process^{4,6,7} was able to limit image-nonuniformity errors to 0.3%, independently of phantom size and location therein using a commercial 16-row CT scanner. Our subsequent SIR extensions reconstruct the $(c_1(\mathbf{x}), c_2(\mathbf{x}))$ image directly by operating jointly on unprocessed low- and high-energy experimentally acquired sinograms.⁸ Such innovations would not be possible without the BVM model, which accurately and efficiently (due to its linearity and separability¹¹) supports estimation of monoenergetic linear attenuation coefficients.

5. CONCLUSION

The theoretical accuracy of two families of two-parameter cross-section models for DECT photon cross-section mapping was evaluated for a broad range of human tissue compositions and tissue substitutes. The BVM model consistently supports superior photon cross-section modeling accuracy compared to the tPFM model. For tissues containing elements with effective atomic number less than 20, the achievable accuracy of the BVM model was found to be within 3% or better down to 20 keV photon energies. Thus, the linear, separable, two-parameter BVM is a reasonable model for extrapolating DECT cross-section maps to the low-energy permanent seed brachytherapy energy range. Neither variant of the PFM model (tPFM or vPFM) is able to predict the relevant radiological quantities with the requisite 3% uncertainty or less.

ACKNOWLEDGMENTS

This study was supported by NIH R01 CA149305 and Varian Medical Systems.

CONFLICT OF INTEREST

The authors have no COI to report.

^{a)}Author to whom correspondence should be addressed. Electronic mails: radon.han@gmail.com; jeffrey.williamson@vcuhealth.org.

REFERENCES

1. Bourque AE, Carrier J-F, Bouchard H. A stoichiometric calibration method for dual energy computed tomography. *Phys Med Biol*. 2014;59:2059–2088.
2. De Man B, Nuyts J, Dupont P, Marchal G, Suetens P. An iterative maximum-likelihood polychromatic algorithm for CT. *IEEE Trans Med Imaging*. 2001;20:999–1008.
3. Elbakri IA, Fessler JA. Segmentation-free statistical image reconstruction for polyenergetic x-ray computed tomography with experimental validation. *Phys Med Biol*. 2003;48:2453–2477.
4. O'Sullivan JA, Benac J. Alternating minimization algorithms for transmission tomography. *IEEE Trans Med Imaging*. 2007;26:283–297.
5. Beister M, Kolditz D, Kalender WA. Iterative reconstruction methods in X-ray CT. *Phys Medica*. 2012;28:94–108.

6. Williamson JF, Whiting BR, Benac J, et al. Prospects for quantitative computed tomography imaging in the presence of foreign metal bodies using statistical image reconstruction. *Med Phys*. 2002;29:2404–2418.
7. Evans JD, Whiting BR, Politte DG, O'Sullivan JA, Klahr PF, Williamson JF. Experimental implementation of a polyenergetic statistical reconstruction algorithm for a commercial fan-beam CT scanner. *Phys Medica*. 2013;29:500–512.
8. Chen Y, O'Sullivan JA, Politte DG, et al. Line integral alternating minimization algorithm for dual-energy X-ray CT image reconstruction. *IEEE Trans Med Imaging*. 2016;35:685–698.
9. Williamson JF, Li S, Devic S, Whiting BR, Lerma FA. On two-parameter models of photon cross sections: application to dual-energy CT imaging. *Med Phys*. 2006;33:4115–4129.
10. Han D, Sampson A, Politte D, O'Sullivan J, Siebers J, Williamson J. Accuracy of dual-energy CT photon cross-section mapping using a non-separable two-parameter cross-section model. *Med Phys*. 2012;39:3989.
11. Han D, Siebers JV, Williamson JF. A linear, separable two-parameter model for dual energy CT imaging of proton stopping power computation. *Med Phys*. 2016;43:600–612.
12. Spiers FW. Effective atomic number and energy absorption in tissues. *Br J Radiol*. 1946;19:52–63.
13. Mayneord WV. The significance of the Roentgen. *Acta Int Union Against Cancer*. 1937;2:271–282.
14. Bazalova M, Carrier J-F, Beaulieu L, Verhaegen F. Dual-energy CT-based material extraction for tissue segmentation in Monte Carlo dose calculations. *Phys Med Biol*. 2008;53:2439–2456.
15. Yang M, Virshup G, Clayton J, Zhu XR, Mohan R, Dong L. Theoretical variance analysis of single- and dual-energy computed tomography methods for calculating proton stopping power ratios of biological tissues. *Phys Med Biol*. 2010;55:1343–1362.
16. Torikoshi M, Tsunoo T, Sasaki M, et al. Electron density measurement with dual-energy x-ray CT using synchrotron radiation. *Phys Med Biol*. 2003;48:673–685.
17. ICRU. *Stopping powers and ranges for protons and alpha-particles*. Report No.49, 1993.
18. ICRU. *Tissue substitutes in radiation dosimetry and measurement*. Report No. 44, 1989.
19. Valentin J. Basic anatomical and physiological data for use in radiological protection references values: ICRP publication 89. *Ann ICRP*. 2002;32:1–277.
20. Woodard HQ, White DR. The composition of body tissues. *Br J Radiol*. 1986;59:1209–1218.
21. Berger MJ, Hubbell JH, Seltzer SM, et al. *National Institute of Standards and Technology, Gaithersburg, "XCOM: Photon Cross Section Database (version 1.5)*, 2010.
22. Evans JD, Whiting BR, O'Sullivan JA, et al. Prospects for in vivo estimation of photon linear attenuation coefficients using postprocessing dual-energy CT imaging on a commercial scanner: comparison of

analytic and polyenergetic statistical reconstruction algorithms. *Med Phys*. 2013;40:121914.

23. Hünemohr N, Krauss B, Dinkel J, et al. Ion range estimation by using dual energy computed tomography. *Z Med Phys*. 2013;23:300–313.
24. Taylor BN, Kuyatt CE. *Guidelines for evaluating and expressing the uncertainty of NIST measurement results*. NIST Technical Note 1297; 1994.

SUPPORTING INFORMATION

Additional Supporting Information may be found online in the supporting information tab for this article.

Figure S1: Percent unexpanded uncertainty of total linear attenuation for (a) tPFM, (b) vPFM, and (c) BVM, respectively as functions of energy in the range $20 \leq E \leq 1000$ keV, at image-intensity uncertainty levels of (0.6%, 0.3%) and (0.2%, 0.1%) for (low, high) energy CT images. Four typical human tissues and tissue substitute were selected for analysis.

Figure S2: Percent unexpanded uncertainty of photoelectric coefficients for (a) tPFM, (b) vPFM, and (c) BVM, respectively as functions of energy in the range $20 \leq E \leq 50$ keV, at image-intensity uncertainty levels (0.6%, 0.3%) and (0.2%, 0.1%) for (low, high) energy CT images. Four typical human tissues and tissue substitute were selected for analysis.

Figure S3: Percent unexpanded uncertainty of energy-absorption coefficients for (a) tPFM, (b) vPFM, and (c) BVM, respectively as functions of energy in the range $20 \leq E \leq 1000$ keV, at image-intensity uncertainty levels (0.6%, 0.3%) and (0.2%, 0.1%) for (low, high) energy CT images. Four typical human tissues and tissue substitute were selected for analysis.

Table S1: RMS estimation errors by the double-basis pair BVM and tPFM models for all investigated tissues and phantom substitutes in photoelectric effect cross-section (20-50 keV) and scattering cross-section (50-1000 keV).

Table S2: Tissue parameters predicted by double-basis BVM and tPFM and RMS estimation errors of energy absorption (20-1000 keV) and errors in total linear attenuation coefficients at 22 keV.