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Joint Optimization of Fluence Field Modulation and Regularization in Task-Driven Computed Tomography

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Abstract

Purpose—This work presents a task-driven joint optimization of fluence field modulation (FFM) and regularization in quadratic penalized-likelihood (PL) reconstruction. Conventional FFM strategies proposed for filtered-backprojection (FBP) are evaluated in the context of PL reconstruction for comparison.

Methods—We present a task-driven framework that leverages prior knowledge of the patient anatomy and imaging task to identify FFM and regularization. We adopted a maxi-min objective that ensures a minimum level of detectability index (d') across sample locations in the image volume. The FFM designs were parameterized by 2D Gaussian basis functions to reduce dimensionality of the optimization and basis function coefficients were estimated using the covariance matrix adaptation evolutionary strategy (CMA-ES) algorithm. The FFM was jointly optimized with both space-invariant and spatially-varying regularization strength (β) - the former via an exhaustive search through discrete values and the latter using an alternating optimization where β was exhaustively optimized locally and interpolated to form a spatially-varying map.

Results—The optimal FFM inverts as β increases, demonstrating the importance of a joint optimization. For the task and object investigated, the optimal FFM assigns more fluence through less attenuating views, counter to conventional FFM schemes proposed for FBP. The maxi-min objective homogenizes detectability throughout the image and achieves a higher minimum detectability than conventional FFM strategies.

Conclusions—The task-driven FFM designs found in this work are counter to conventional patterns for FBP and yield better performance in terms of the maxi-min objective, suggesting opportunities for improved image quality and/or dose reduction when model-based reconstructions are applied in conjunction with FFM.

I. INTRODUCTION

Recent years has seen increased interest in integrating fluence field modulation (FFM) devices with diagnostic CT systems.¹⁻⁴ Compared to tube current modulation which is routinely used in modern commercial CT scans, FFM devices allow for greater freedom in shaping the dose distribution in the patient, and, at the same time, can cater to flexible image quality requirements in the reconstruction. Current research efforts have focused largely on hardware development and FFM designs for filtered-backprojection reconstruction using variance as a measure for image quality^{1,5,6}. This work aims to address the following question: assuming we can achieve any FFM pattern, what is the optimal FFM design for

model-based iterative reconstruction (MBIR) that maximizes an objective consistent with task-based definition of image quality?

Towards this end, we adopt a task-driven imaging framework that has been applied in the optimization of a wide range of imaging parameters.^{7,8} In particular, this work extends a previous study⁹ of tube current modulation (TCM) for MBIR that found cases where the optimal TCM for MBIR is drastically different from that proposed for FBP reconstruction. This work investigates the effect of regularization on optimal fluence modulation patterns and presents an algorithm for jointly optimizing fluence field modulation and regularization strength.

II. THEORETICAL AND EXPERIMENTAL METHODS

A. Optimization using a Task-Driven Imaging Framework

Previous work^{7,8} introduced a task-driven imaging framework that takes advantage of previous knowledge of the patient anatomy (via previously existing scans or a low dose 3D scout) and the imaging task (assumed to be known exactly for this initial investigation) to prospectively optimize acquisition and reconstruction parameters of interest. The framework adopts a predictive *system model* to compute spatial resolution and noise in the reconstructed image as a function of the patient anatomy and imaging parameters to be optimized. The *objective function* is based on a task-based image quality metric – detectability index (denoted d'), that quantifies performance of a binary hypotheses task such as detection or discrimination. Detectability index was computed for a signal-known-exactly, background-known-exactly task, and uses a non-prewhitening observer to quantify the upper bound of performance while taking into account of human's inability to prewhiten correlated noise. The mathematical form of d' is:

$$d'^2_j(\Omega_A, \Omega_R) = \frac{\left[\iiint T_j(\Omega_A, \Omega_R) \cdot W_{Task}^2 df_x df_y df_z \right]^2}{\iiint S_j(\Omega_A, \Omega_R) |T_j(\Omega_A, \Omega_R) \cdot W_{Task}|^2 df_x df_y df_z} \quad (1)$$

with dependence on the modulation transfer function, T , and noise power spectrum, S , in the reconstructed image as well as a task function, W_{Task} that describes the detection hypothesis. The subscript j illustrates the *local* nature of the above metrics where within a small neighborhood around voxel j . This local formulation accommodates global nonstationarity but presumes local shift-invariance/stationarity. We note that detectability is a function of the acquisition (Ω_A) and reconstruction parameters (Ω_R) to be optimized due to the dependence of noise and resolution on these same parameters.

B. Penalized Likelihood Reconstruction with Quadratic Penalty

The detectability in Eq.(1) depends on noise and resolution properties of the reconstructed image. In this work we concentrate on a penalized-likelihood objective with a quadratic penalty, where the reconstruction, $\hat{\mu}$, is given by:

$$\hat{\mu} = \arg \max_{\mu} [\log L(\mu; y) - \beta R(\mu)] \quad (2)$$

Here, $L(\mu; y)$ denotes a likelihood term, $R(\mu)$ is a quadratic roughness penalty, and the regularization parameter, β , controls the relative strength of $R(\mu)$ to $L(\mu; y)$. Typically, a single regularization strength is applied globally; however, we will also consider space-variant regularization β_j that is applied locally. The mean measurement, $\bar{y}$, follows Beer's law attenuation and assumes independent and Poisson distributed noise:

$$\bar{y} = I_0(\theta, u) e^{-\mathbf{A}\mu} \quad (3)$$

where $\mathbf{A}$ is the forward projection operator. Fluence field modulation is accounted for through I_0 - the number of unattenuated photons per detector pixel as a function of projection angle, θ , and horizontal detector location, u . The quadratic roughness penalty term was implemented to penalize first-order neighbor differences.

C. Prediction of Local Noise and Resolution Properties

The predictors for spatial resolution and noise were based on derivations using the implicit function theorem and Fourier approximation for fast computation^{10,11}:

$$T_j \approx \frac{\mathcal{F} \left\{ \mathbf{A}^T D \{ \bar{y}(\mu) \} \mathbf{A} e_j \right\}}{\mathcal{F} \left\{ \mathbf{A}^T D \{ \bar{y}(\mu) \} \mathbf{A} e_j + \beta \mathbf{R} e_j \right\}} \quad 4(a)$$

and

$$S_j \approx \frac{\mathcal{F} \left\{ \mathbf{A}^T D \{ \bar{y}(\mu) \} \mathbf{A} e_j \right\}}{|\mathcal{F} \left\{ \mathbf{A}^T D \{ \bar{y}(\mu) \} \mathbf{A} e_j + \beta \mathbf{R} e_j \right\}|^2} \quad 4(b)$$

The e_j term denotes the j -th unit vector, $D\{\cdot\}$ forms a diagonal matrix with its vector argument along the diagonal, and the matrix $\mathbf{R}$ represents the Hessian of the quadratic penalty which is a constant and independent of y or μ .

D. Joint optimization of fluence field modulation and regularization

We sought a low dimensional parameterization for FFM using 2D Gaussian basis functions, B^i , spanning the (u, θ) domain, assuming views 180° apart to have the same fluence. The fluence field is therefore given by:

$$I_0(u, \theta) = \sum_i^n \omega B^i(u, \theta) \quad (5)$$

where coefficient vector $\vec{\Omega} = [\omega^1, \omega^2, \dots, \omega^n]^T$ can be optimized instead. An example illustration of basis functions with coarse sampling along both directions is shown in Fig. 1. In this work, we used more samples with 17 evenly spaced locations along u and 6 along θ , giving a total of $17 \times 6 = 102$ coefficients.

One can consider FFM to be a more general case of tube current modulation where performance at multiple locations in the image can be optimized simultaneously. There are multiple ways of formulating a task-based objective (e.g., mean d'). In this work, we chose to maximize the minimum d' at a number of sample locations (denoted as the location vector $\vec{v}$ in the image volume so as to ensure a minimum level of d' throughout, i.e.:

$$\vec{\Omega} = \arg \max_{\vec{\Omega}} \min_{\vec{v}} d'(\vec{v}; \vec{\Omega}), \quad (6a)$$

We seek the optimal imaging parameter subject to a total dose constraint such that the total bare beam fluence behind the object (denoted as u^*) is kept constant across different FFM strategies, i.e.,

$$\sum_{(u^*, \theta)} I_0(u^*, \theta) = I_{tot} \quad (6b)$$

FFM Design with Global Regularization—For fixed β values, we adopted the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) algorithm¹² to optimize the coefficients for FFM. To find the optimal constant/space-invariant β , the FFM optimization was performed for discrete values of β spanning orders of magnitude from 1 to 10^7 and the β yielding the highest minimum d' is identified.

FFM Design with Local Regularization—As mentioned above β is typically a scalar, but may also be designed to vary spatially to exert more control over the local spatial resolution, noise and detectability. Towards this end, a joint optimization of fluence field modulation and spatially varying β_j was performed. Contrary to FFM design which couples d' across multiple locations, the effect of β_j is fairly local. A maximin objective may thus be unsuitable for optimization. Instead, a local exhaustive search method¹³ was adopted. The β_j at each discrete location was optimized by an exhaustive search through discrete values and

the β_j corresponding to the maximum local β_j' was identified, i.e., $\arg \max_{\beta_j'} d'(\beta_j)$

The resulting local β_j values were interpolated to form a fully sampled final β_j map assuming the locally optimal β_j is smoothly varying. The fluence field and β_j optimization alternates until convergence indicated by a less than 5% average change in d' at the discrete locations of optimization from the previous iteration. The flowchart of the algorithm is illustrated in Fig. 2.

The optimization was demonstrated in a digital abdomen phantom (Fig. 3a) based on a CT scan of a cadaver at 120kV and reconstructed at ~0.7mm voxel size. The imaging task was a mid-frequency task (Fig. 3b) usually associated with a discrimination task. The discrete locations over which both fluence field and β_j were optimized were sampled along the circumference of concentric ellipses resembling the shape of the phantom and marked as yellow crosses on Fig. 3(a).

E. Evaluation of conventional FFM strategies for MBIR

We evaluate conventional FFM strategies proposed for FBP in the context of MBIR, and compare the detectability performance to designs from the task-driven framework detailed in Sec.D. We extended the scalar parameterization of tube current proposed by Gies et al. to fluence field as follows:

$$I_0(u^*, \theta) = \frac{e^{\alpha l(u^*, \theta)}}{\sum_{\theta} e^{\alpha l(u^*, \theta)}} I_{tot}, \quad (7)$$

and investigated FFM when 1) $\alpha = 1.0$ which yields a flat fluence field behind the object, and 2) $\alpha = 0.5$ which produces a modulation pattern that minimizes the mean variance in FBP reconstruction.⁶

III. RESULTS AND BREAKTHROUGH WORK

The FFM (in terms of a scale factor to a nominal fluence) commonly proposed for FBP reconstruction (Sec.II.E) and their corresponding detectability map, d'_j , for the mid-frequency task (Fig. 3b) is shown in Fig. 4. Both the $\alpha = 1.0$ and $\alpha = 0.5$ cases distribute more fluence through more attenuating views. The unmodulated acquisition yields d'_j that is highly location dependent whereas the $\alpha = 1.0$ case yields a flat d'_j map. The $\alpha = 0.5$ case falls in between the previous two.

The optimal fluence field modulation as a function of β is plotted in Fig. 5(a). An interesting inversion in the modulation pattern is observed as β increases. For small β , the FFM resembles conventional FFM proposed for FBP where the peak of modulation coincides with more attenuation lateral views. This is perhaps not surprising since PL reconstruction behaves like a weighted FBP reconstruction when the regularization strength is negligible. As β increases, the pattern is flipped with more fluence assigned to the less attenuating anterior-posterior (AP) and PA views. These results are reminiscent of previous findings⁹ for tube current modulation in PL reconstruction. The presence of regularization intrinsically applies greater smoothing to noisier data. Assigning more fluence to noisier views in PL reconstruction therefore does not result in a big noise advantage as one would find in (unweighted) FBP. Instead, it is more beneficial for d' to make less noisy views even better by way of increasing fluence and therefore spatial resolution along radial directions corresponding to those views.

Detectability map tend to be flat across all β values, consistent with a maxi-min objective which attempts to increase fluence to low d' area (center of the phantom in the unmodulated

acquisition) while decrease fluence to high d' area (edge of the phantom). The minimum d' as a function of β is plotted in Fig. 5(b), with the optimal β equal to 10^5 .

Results from the alternating optimization of fluence field and spatially varying β_j is shown in Fig. 6. The fluence field pattern is very similar to the optimal modulation pattern ($\beta = 10^5$) in Fig. 5. The β_j map is more or less constant, with small local variations that results in a more uniform detectability map compared to results shown in Fig. 5. The minimum detectability is 9.2, outperforming all three conventional strategies shown in Fig. 4.

IV. DISCUSSION and CONCLUSION

We presented a joint optimization of fluence field modulation and regularization strength in penalized-likelihood reconstruction with a quadratic penalty. The regularization strength parameter can have dramatic impact on the shape of fluence field modulation patterns that are optimal for detectability index. For jointly optimized regularization and fluence field, the optimization results suggest more fluence through the less noisy views, contrary to what has been historically proposed for FBP reconstruction. This suggests that we should reconsider standard acquisition protocols when model-based reconstruction methods are being used. Future work will incorporate more sophisticated performance models include those that model signal uncertainty in the detectability index model and extend the optimization to a multi-task, multi-location scenario.

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Fig. 1.

Example 2D Gaussian basis functions in the (u, θ) domain for a low dimensional parameterization of fluence field modulation. A more densely sampled basis functions were employed in the optimization with 17 locations along u and 6 along θ .

Fig. 2.

Flowchart alternating optimization of fluence field modulation and spatially varying β_j

Fig. 3.

a) Abdomen phantom with locations (yellow crosses) over which fluence field and β_j are optimized. b) The imaging task centers on mid-frequencies.

Fig. 4. Fluence field modulation commonly proposed for FBP reconstruction and the corresponding detectability map for the mid frequency task in Fig. 3b when PL reconstruction is applied.

Fig. 5.

(a) For fixed, space-invariant β , the optimal fluence field modulation and detectability map for the abdomen phantom and mid-frequency task are shown. The location with the minimum d' is indicated by a black marker on the detectability map. The optimal β is indicated by the green box. (b) Minimum d' as a function of β , suggesting an optimum at $\beta = 10^5$.

Fig. 6. Fluence field modulation, spatially varying β map, and detectability map from the alternating optimization scheme shown in Fig. 4.