Predicting the Effects of Interventions: A Tutorial on the Disequilibrium Model

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Abstract The disequilibrium approach to reinforcement and punishment, derived from the probability-differential hypothesis and response deprivation hypothesis, provides a number of potentially useful mathematical models for practitioners. The disequilibrium approach and its accompanying models have proven effective in the prediction and control of behavior, yet they have not been fully espoused and integrated into clinical practice. The purpose of this tutorial is to detail the disequilibrium approach and adapt its mathematical models for use as a tool in applied settings. The disequilibrium models specify how to arrange contingencies and predict the effects of those contingencies. We aggregate these models, and provide them as a single tool, in the form of a Microsoft Excel® spreadsheet that calculates the direction and magnitude of behavior change based on baseline measures and a practitioner's choice of intervention parameters. How practitioners take baseline measures and select intervention parameters in accordance with disequilibrium models is explicated. The proposed tool can be accessed and downloaded for use at <https://osf.io/knf7x/>.

Keywords Contingent activity · Contingency management · Disequilibrium model · Instrumental activity · Premack principle · Response deprivation

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The purpose of this tutorial is to provide practitioners with a tool that will predict the direction and magnitude of behavior change that follows an intervention. This tool is a direct derivation of Timberlake and Allison's (1974) response deprivation hypothesis which culminated in Timberlake's (1980) molar equilibrium theory, also known as the disequilibrium approach (Timberlake & Farmer-Dougan, 1991). This approach has a long history beginning with Premack's (1959) probability-differential hypothesis (i.e., "the Premack principle"), which stated that high-probability behavior reinforces low-probability behavior. We do not intend this tutorial to be an exhaustive review of the response deprivation and disequilibrium literature, so we refer readers to Adams (2000) for a brief history, Timberlake (1980) for an extended history and summary of basic laboratory findings, and Timberlake and Farmer-Dougan (1991) for a history as it relates to application. Suffice it to say that Timberlake's disequilibrium approach to reinforcement and punishment not only explains Premack's probability-differential hypothesis but also expands its precision and scope (Klatt & Morris, 2001).

The proposed tool comes in the form of a mathematical model that can be conceived as a set of rules. If applied with fidelity, these rules will allow behavior analysts to predict the direction and magnitude of behavior change that follows an intervention. Before explicating these rules of operation, however, we must first specify the basic assumptions of the disequilibrium approach. As specified by both Premack (1959) and Timberlake (1980), reinforcement is defined as a relation, where one responds for contingent access to an activity, not contingent access to a stimulus. Food, for example, does not reinforce behavior. Instead, the *action of eating* functions as a reinforcer (Killeen & Jacobs, 2016). It is for this reason that the disequilibrium literature talks of *instrumental* and *contingent actions* instead of stimulus–response or response–reinforcer relations.

An instrumental activity is a response that produces an opportunity to engage in another activity (Pierce & Cheney, 2013). Self-injurious behavior, for example, might be instrumental in obtaining caregiver attention, where attention is considered a form of interaction. A contingent activity is one the organism gains access to by engaging in the instrumental activity (Pierce & Cheney, 2013). In this example, the social interaction we call “attention” qualifies as the contingent activity. For simplicity, we can construe instrumental activity as any behavior that occurs because it is effective in producing reinforcers (Domjan, 2015). However, those reinforcers are contingent activities, not stimuli devoid of actions. In Killeen’s (2014) words,

“Most of the things we work for are the opportunities to expend time and energy in doing things that we enjoy: cooking, driving a car, listening to music, reading a book, playing golf ... As is often true of travel, it is the journey that is the destination, the destination an excuse for the journey” (p. 545).

Although unorthodox, the disequilibrium approach to reinforcement is not incompatible with already established standards of practice in applied behavior analysis (see Timberlake & Farmer-Dougan, 1991). It is simply a different lens through which practitioners might solve problems of social significance. To exemplify this point, we defer to the gold standard of applied behavior analysis—the “functional analysis” (Iwata, Dorsey, Slifer, Bauman, & Richman, 1994; Schlinger & Normand, 2013). Functional analysis is a pretreatment assessment method by which practitioners systematically alter potential antecedents and potential reinforcers so as to determine their control over a target behavior (Sarafino, 2012; Schlinger & Normand, 2013). Whereas the standard functional analysis test conditions—escape, attention, tangible, and alone—focus on a target response in relation to a discrete reinforcer, the disequilibrium approach focuses on a relation between any two activities. Table 1 defines the functional analysis test conditions in terms of the disequilibrium approach. As one can see, disequilibrium definitions are not incompatible with existing practice and provide a different orientation by which practitioners might assess and intervene. For example, attention does not terminate in the presentation of a vocal verbal stimulus such as “good job.” Rather, attention is defined as a two-person interaction in which clients are not passive when receiving attention. According to the disequilibrium approach, attention is an interactive activity between parent and child, teacher and student, or practitioner and client.

Following from the view that reinforcers are stimuli, the Iwata et al. (1994) functional analysis method primarily focuses on the various dimensions of instrumental activity in any of the conditions. For instance, the rate of escape attempts, rate of

Table 1 Functional analysis test conditions in terms of instrumental and contingent activity

FA conditions	Instrumental activity	Contingent activity
Escape	Any activity that functions to remove aversive activities	Opportunity to engage in activities other than that which is aversive
Attention	Any activity that produces an opportunity for peer or caregiver interaction	Interaction with peer, parent, teacher, practitioner, etc.
Tangible	Any activity that produces an opportunity to access tangibles	Object manipulation, imaginative play, peer-peer interaction, practitioner–client interaction, etc.
Alone	Any activity that occurs by itself without producing access to other behavior	—

responses for attention, rate of responses for tangibles, and rate of responses while alone are obtained for comparison. The occurrences of reinforcers—as stimuli that follow these responses—are measured, but the activities those stimuli afford are not. When using the disequilibrium approach, one need also focus on contingent activities: duration or rate of activities during escape from demands, duration or rate of attention activity, and duration or rate of client interaction with a tangible. Notice that no contingent activity is identified for automatically maintained behavior (i.e., automatically reinforced activity occurring in the alone condition) in Table 1. This is because reinforcement in the disequilibrium approach is defined as a contingent relation between two activities. Automatically maintained behavior does not function, per this definition, to produce access to a contingent activity. This is not to say that automatically maintained behaviors like self-injury do not have effects (see, e.g., Thompson, Symons, Delaney, & England, 1995, for the endogenous effects of self-injury). Like the feel of the ground underfoot when walking, automatically maintained self-injury is all at once an activity and an effect that occurs while alone. This perspective does not preclude automatically maintained behavior from intervention, for practitioners can train alternative instrumental activities that have similar sensory consequences (e.g., Piazza, Adelinis, Hanley, Goh, & Delia, 2000).

A central tenet of the disequilibrium approach is that reinforcement works when the reinforcing contingent activity is restricted below its normal baseline level of occurrence (Timberlake & Allison, 1974). Restricting contingent activities below their baseline level of occurrence is termed “response deficit” (Timberlake, 1980). Generally, restricting access to a reinforcing contingent activity is a part of any successful intervention that increases behavior; it is the rule rather than the exception (Domjan, 2015; Timberlake & Farmer-Dougan,

1991). When shaping client behavior with little or no probability of occurrence, access to a preferred activity is restricted until the client obtains it by engaging in a prespecified instrumental response. If the terminal response criterion is to utter the word “supercalifragilisticexpialidocious,” then a prespecified instrumental response might be to utter “sup” followed by “super,” “supercali,” and so on. For every prespecified instrumental response, access to the preferred activity is restricted so as to establish it as a reinforcing contingent activity. Token economies, described as one of the most successful behavioral interventions (Hackenberg, 2009), also restrict access to contingent activities until a client engages in a predetermined instrumental response that meets a prespecified criterion (e.g., every third instrumental response earns you a point, and a total of 10 points will gain you access to a contingent activity of your choosing). In the cases of both shaping and token economies, access to a contingent activity is restricted and an increase in instrumental responding for that activity ensues.

In contrast to reinforcement, decreases in behavior, or punishment effects, are observed when clients are required to engage in a contingent activity at a level greater than what occurred during baseline (Heth & Warren, 1978). This procedure is termed “response excess” (Timberlake, 1980). Examples of behavioral reduction strategies that fall within the purview of response excess are overcorrection (Timberlake & Farmer-Dougan, 1991) and noncontingent reinforcement (NCR). Activities like property destruction, for example, are reduced when clients are required to “cleanup” at a level well above their baseline levels of cleaning. When an intervention transforms property destruction into an instrumental response for cleaning, requiring the client to engage in cleaning well above baseline levels, property destruction declines. In the case of NCR, or using response-independent schedules to reduce problem behavior (see Richman et al., 2015), access to a contingent activity is provided at levels greater than they occur during baseline. When attention maintains self-injury, for instance, a response excess of attention will reduce that self-injury.

Impose a contingency that cuts into a child’s playtime (response deficit) and that child will increase his or her efforts to obtain that activity; impose a contingency that requires a child to play more than usual (response excess) and that same child will decrease his or her efforts to obtain that activity. These are contingency arrangements at the basis of most, if not all, interventions (Killeen, 2014). In the following sections, we present “the disequilibrium model,” a mathematical formulation of the aforementioned contingency arrangements. Such a mathematical model can be used as a tool to guide clinical decision-making, as it predicts the general direction of behavior change (increase or decrease) and helps ensure that interventions will have their desired effect.

The Disequilibrium Model

The disequilibrium model is an attempt to quantify and formalize reinforcement and punishment. In order to use this model, practitioners must acquire baseline measures of both the instrumental and contingent activities. Once baseline measures of those activities are acquired, practitioners can begin arranging interventions and making predictions based on the rules of the model. Below we describe how to model both reinforcement and punishment, later we describe the means by which these models can be put to use.

Modeling Reinforcement

The rules for increasing behavior were first established by Eisenberger, Karpman, and Trattner (1967). These rules were later elaborated by Timberlake and Allison (1974) and can be read as follows:

$$\frac{I}{C} > \frac{O_i}{O_c}, \quad (1)$$

where I stands instrumental activity, C stands for contingent activity, and O_i and O_c stand for baseline measures of instrumental and contingent activities, respectively. Both I and C are intervention parameters, together comprising the contingency arranged by the practitioner. Therefore, I/C represents the relative proportion of instrumental activity to contingent activity. A client must do I amount of the instrumental activity in order to gain access to the contingent activity of amount C . O_i/O_c represents the client’s free operant baseline levels of instrumental (O_i) and contingent (O_c) responding. These are measures of how often the client would engage in either response with relatively few restrictions. The ratio O_i/O_c represents the relative proportions of baseline instrumental activity to contingent activity. Table 2 provides a summary of definitions for each of these terms.

In narrative form, Eq. 1 can be read as follows:

If the instrumental to contingent requirement (I/C) is *greater than* what a client usually does during his or her free time (O_i/O_c), then you will observe an *increase* in behavior.

This is an appreciable rule because it has been verified across species and contexts (Timberlake, 1980; Timberlake & Farmer-Dougan, 1991). It is also falsifiable, which is important for the future development of this and other models that spawn from it.

To exemplify the utility of the disequilibrium reinforcement model in action, we defer to Dougher (1983). In the course of working with a patient diagnosed with schizophrenia in a hospital setting, Dougher noticed that this patient’s long bouts of

Table 2 Terms and definitions in the disequilibrium model

Terms	Definitions
O_i	Baseline level of instrumental activity
O_c	Baseline level of contingent activity
O_i/O_c	Free operant baseline proportion of instrumental and contingent activity
I	Intervention level of instrumental activity
C	Intervention level of contingent activity
I/C	Intervention proportion of instrumental and contingent activity

disruptive coughing were occasionally followed by a free cup of coffee. Coffee consumption was one of the few activities, besides coughing, with some frequency of occurrence. Based on these baseline observations, Dougher (1983) used coffee consumption (i.e., the contingent activity) as leverage to increase socially appropriate responses (i.e., the instrumental activity). Figure 1a presents the mean levels of responding during baseline and intervention. The closed circle represents the baseline proportion of instrumental to contingent activity, where approximately one socially appropriate instrumental response (O_i) was made for every 13 oz. of coffee consumed (O_c). The open circle represents the results of the intervention Dougher implemented to increase socially appropriate instrumental activity (O_i) from a rate of approximately 1 to 27 responses. Dougher increased socially appropriate responding by arranging his I/C such that the patient received less than an ounce of coffee (C) for every socially appropriate instrumental response (I). In other words, the client was required to work to access

coffee. Notice, in Fig. 1a, that approximately 13 oz. of coffee was consumed during baseline, but only about 10 oz. was consumed during intervention. Therefore, the I/C schedule produced a response deficit in coffee consumption relative to baseline (O_i/O_c) and required the client to engage in *more* instrumental behavior to gain access to that coffee.

Important to note is the flexibility of the disequilibrium model, for Dougher (1983) used rate as a measure for instrumental activity (i.e., O_i and I) and ounces of coffee consumed as a measure for contingent activity (i.e., O_c and C). The disequilibrium model allows practitioners to utilize any unit of measurement so long as those units are consistent across baseline and intervention (Timberlake & Farmer-Dougan, 1991). Previous studies have utilized rate of responding, duration of responding (Konarski, 1987), and time sampling procedures (Farmer-Dougan, 1998). Derivative measures such as percent correct have also been used (Konarski, Crowell, & Duggan, 1985), but more research, in this regard, is needed. For this reason, our recommendation is to utilize rate, duration, and time sampling until future research has more thoroughly shown the compatibility of derivative measures.

Modeling Punishment

In regards to punishment effects, Heth and Warren (1978) showed that the inverse of the above Eq. 1 specified the conditions for decreasing behavior. The rule can be read as follows:

$$\frac{I}{C} < \frac{O_i}{O_c}, \quad (2)$$

Fig. 1 a, b Re-plotted data from Dougher (1983) showing reinforcement (left panel) and punishment contingency (right panel). Instrumental and contingent activities are expressed along the x- and y-axes, respectively.

Closed circles denote baseline means, and the open circles indicate the mean results of intervention. The dashed lines denote the coordinates of the data points

where all variables are the same, but the intervention ratio (I/C) should be *less than* the baseline ratio. In narrative form, the Eq. 2 can be read as follows:

If the instrumental to contingent requirement (I/C) is *less than* what a client usually does during his or her free time (O_i/O_c), then you will observe a *decrease* in behavior.

Figure 1b is an example in which Dougher (1983) decreased the previously mentioned patient's disruptive coughing. The closed circle represents the baseline proportion of instrumental to contingent activity, where an average of 49 instrumental coughing responses (O_i) were made for every 13 oz. of coffee consumed (O_c). The open circle represents the results of the intervention Dougher implemented to decrease coughing from a mean rate of 49 responses to a mean rate of approximately 26 responses. Coughing decreased by providing more coffee than the patient consumed in baseline. That is to say, for every coughing response (I), the client was required to consume a prespecified amount of coffee (C) that would result in an excess of coffee consumption if the client were to continue engaging in the instrumental coughing activity. Notably, coffee consumption increased from approximately 13 to 20 oz.—an excess in consumption that resulted from I/C being less than the free operant baseline ratio (O_i/O_c).

Summary

Figure 1a, b schematizes the utility of the disequilibrium model: So long as there is a discrepancy between what a client usually does (O_i/O_c) and what that client is required to do (I/C), you will observe a change in behavior. Equation 1 specifies reinforcement and results in a deficit in contingent activity relative to baseline such that instrumental activity increases (Timberlake, 1980). As shown in Fig. 1a, the client engages in more socially appropriate behavior such that he approaches his baseline levels of coffee drinking. In doing so, the client ends up engaging in instrumental activity at a rate greater than baseline (O_i). Equation 2 specifies punishment and results in an excess in contingent activity relative to baseline such that instrumental activity decreases (Timberlake, 1980). As shown in Fig. 1b, the client consumes more coffee during intervention than baseline such that the rate of disruptive coughing decreases.

The disequilibrium model (Eqs. 1 and 2) quantifies what practitioners already know: The arrangement of discrepancies between what a client usually does (O_i/O_c) and what a client is required to do during intervention (I/C) will result in a change in behavior. Although intuitive in its narrative form, the disequilibrium model goes further in that it formalizes that intuition by quantifying it and providing a means to predict a likely result (see Critchfield & Reed, 2009 and Mazur, 2006,

for the benefits of quantification over narration). In formalizing reinforcement and punishment, the disequilibrium model specifies some of the necessary conditions that factor into the success of a particular intervention: A contingency that produces a response deficit will increase behavior whereas a contingency that produces a response excess will decrease behavior.

Provided our description of the disequilibrium approach and its models, our next step is to describe how to use those models. First, we will explain how to get O_i and O_c using a paired baseline. Next, we will explain how to select I and C for your intervention. Finally, we will put both of these together and review a quantitative model that reliably predicts not only the direction of behavior change (increase or decrease) but also how much behavior change to expect.

Obtaining O_i and O_c : The Paired Baseline

Values for O_i and O_c are obtained using the free operant paired baseline method. The paired baseline method entails taking measures on two or more simultaneously available activities (Timberlake & Allison, 1974). This baseline method is termed “paired” because at least two activities are simultaneously available to a client. Additionally, and most importantly, these activities are freely available in the sense that no restrictions are placed on the client's behavior while collecting baseline data. While other free operant methods assess for preference (e.g., Roane, Vollmer, Ringdahl, & Marcus, 1998), the free operant paired baseline method assesses the probability of activities in a given context. Instead of rank-ordering stimuli according to relative preference, activities following a paired baseline are designated as either instrumental (O_i) or contingent (O_c).

Conducting a paired baseline requires providing clients with access to simultaneously available activities that might include anything from academic tasks to play with tangibles or attention from peers or caregivers. Konarski, Crowell, Johnson, and Whitman (1982), for example, provided developmentally disabled clients access to either reading or math tasks. Time spent on both reading and math tasks was measured during 20 min sessions within a classroom setting. Some clients spent more time doing reading than math, while other clients spent more time doing math than reading. Traditionally, tasks on which clients spend more time are labeled “high-probability” and tasks on which clients spend less time are labeled “low-probability.”

Tradition in accordance with the Premack principle asserts that high-probability behavior should be designated as the contingent activity (O_c) while low-probability behavior should be designated as the instrumental activity (O_i). This is so because high-probability behavior is said to reinforce low-probability behavior (Premack, 1959). The purpose of

the Konarski et al. (1982) study, though, was to show how low-probability behavior could function to reinforce high-probability behavior. Contrary to the Premack principle, low-probability behavior can function to increase high-probability behavior so long as your I/C schedule ratio is *greater than* the O_i/O_c baseline ratio (see Timberlake & Wozny, 1979, for a laboratory exposition). This is to say that, following a paired baseline assessment, either activity can be designated as O_i or O_c regardless of whether or not that activity is high-probability or low-probability. The only caveat is that your I/C ratio has to be *greater than* the O_i/O_c baseline ratio if you want to increase behavior and *less than* the O_i/O_c baseline ratio if you want to decrease behavior.

Table 3 presents data from the Konarski et al. (1982) study in which high-probability reading behavior (I) was increased by providing contingent access to low-probability math behavior (C). These data exemplify how I/C is always *greater than* O_i/O_c even when O_c is a low-probability behavior. The finding that low-probability behavior can reinforce high-probability behavior is important because it widens the class of activities that can function as a reinforcer. Therefore, if behavior has some probability of occurrence, it can be used as a reinforcer. This finding might prove beneficial in the case of undifferentiated functional analysis data, for practitioners could use any of the traditional behavior functions as O_i and O_c (except, perhaps, automatically maintained behavior) provided the behavior has a nonzero level of occurrence.¹

Given that almost any activity can function as a reinforcer or punisher, Farmer-Dougan (1998) stated, “From an applied perspective, this eliminates the need for complex reinforcer assessment procedures. All that is necessary is a very quick assessment of baseline rates” (p. 81). All that is needed, then, is a measure of O_i and O_c . Preference assessments identify activities that are preferred but not which activities function as reinforcers (Pace et al., 1985). In contrast, the disequilibrium model ensures that one of the activities assessed during a paired baseline *will* function as a reinforcer or punisher.

Choosing I and C

Ascertaining your O_i and O_c values is an important first step because it has bearing on your selection of I/C values. To restate our basic assumptions:

1. To increase (reinforce) behavior, the I/C ratio should be *greater than* the baseline ratio (O_i/O_c)

¹ By “undifferentiated,” we are referring to the case in which there is no readily apparent differentiation between rates of responding across functional analysis conditions. This is to say that the target behavior is either multiply controlled or automatically maintained (see Roane, Fisher, Kelley, Mevers, & Bouxsein, 2013, for visual inspection criteria that aid in the discernment of conditions controlling behavior).

Table 3 When O_c is a low-probability behavior (Konarski et al., 1982)

Participant	Activities	I/C	< or >	O_i/O_c
Timmy	Read/Math	3.5/0.5	>	13.4/5.1
	Ratio	7		2.6
Billy	Read/Math	3.0/1	>	9.4/9.3
	Ratio	3		1

Data in minutes

2. To decrease (punish) behavior, the I/C ratio should be *less than* the baseline ratio (O_i/O_c).

These assumptions provide a rough guide regarding what I/C ratios will work, but they do not specify whether those ratios are practical given the situation. In the case of reinforcement, one must be careful not to set the I/C ratio too high. For example, one of the clients Konarski et al. (1980) assessed, Sam, spent approximately 1.4 min reading (O_i) and 16.3 min doing math (O_c). Konarski et al. (1980) arranged their I/C intervention such that Sam had to do 3 min of reading (I) to access 2 min of math (C)—2 min of math being a response deficit compared to baseline. The resulting equation looked as follows:

$$\frac{3}{2} > \frac{1.4}{16.3}$$

This ratio correctly predicted the resulting increase in instrumental reading.

Konarski et al. (1980) chose the 3:2 ratio not only because it would result in an increase in behavior but also because it was practical. Three minutes of reading is more than what Sam did during baseline, but is feasible and would result in Sam accessing the reinforcing contingent activity up to four times within each 20 min session. Konarski et al. (1980) could have arranged the I/C schedule such that Sam had to do 8 min of reading for every 2 min of math, resulting in the equation looking as follows:

$$\frac{8}{2} > \frac{1.4}{16.3}$$

According to the disequilibrium model, using this I/C ratio, we should still observe an increase in instrumental reading; but practically speaking, Sam might quit reading before ever accessing the reinforcing contingent activity. This example brings up two noteworthy issues that require empirical investigation: ratio strain and maintenance. First, requiring Sam to engage in 8 min of reading activity is almost six times higher than his 1.4 min bout of reading during baseline, which may result in Sam’s behavior (the instrumental activity) coming to a stop just prior to accessing the contingent activity. This is ratio strain. In this case, the instrumental activity may come to

function as an aversive demand that Sam attempts to avoid or escape. Second, if Sam does meet the 8:2 requirement, he would gain access to the reinforcing contingent activity only twice during the 20-min session. As such, Sam would have only done math for a total of 4 min, which is much less than his usual 16.3 min doing math. Making the reinforcing contingent activity available only twice within a session may or may not result in a performance that maintains under the same or similar circumstances (generalization). Both ratio strain and matters of maintenance and generalization have not, to the best of our knowledge, been thoroughly examined in the context of the disequilibrium model in applied settings. Additionally, there are no formal rules regarding the most optimal I/C schedule that results in maintenance and generalization without straining the client. Until further research is established, I/C should be selected on the basis of the model's predictions *and* on the basis of practical considerations (e.g., session length) surrounding any intervention.

Another guide for selecting I/C values is based on the assumption that the magnitude of the change in instrumental responding is directly related to the magnitude of the initial discrepancy between baseline and intervention (Timberlake, 1980, p. 14). Said another way, the size of the discrepancy between I/C and O_i/O_c will be related to the change in instrumental responding, assuming no barriers to responding such as ratio strain. This assumption works as a general rule of thumb in that it approximates how much instrumental activity will change given your selection of I/C values. Clinical decision-making, though, calls for more than a general rule of thumb as “best guess” (Iwata et al., 1994, p. 208). For this reason we provide another, better than best guess, model that predicts how much behavior will change provided you input some I/C intervention parameters. Such a model will help in discerning the most optimal I/C values prior to implementing an intervention.

Heth and Warren's (1978) Predictive Model

Predicting how much behavior will change is a difficult endeavor given the nuances of circumstances surrounding any intervention. An effective model is one with a generality that cuts across most nuanced circumstances and still makes predictions with precision. The disequilibrium approach provides opportunities for successful prediction across such circumstances, as its literature is replete with potentially useful mathematical models beyond the disequilibrium model already described (see Timberlake & Wozny, 1979).

For the purposes of this tutorial, we demonstrate the predictive utility of quantitative models using one described by Heth and Warren (1978). We choose this model over others because of its relative simplicity and apparent predictive utility. The model is simple in that it utilizes variables with which

we are already familiar: If values for O_i/O_c and I/C have been discerned in the context of the disequilibrium model, then these values are readily applicable to Heth and Warren's predictive model. Heth and Warren's predictive model can be read as follows:

$$X_i = \frac{I(O_i + O_c)}{(I + C)}, \quad (3)$$

where X_i is predicted instrumental responding and I , C , O_i , and O_c are the same as in Eqs. 1 and 2. X_i is the predicted measure of instrumental activity based on baseline measures of behavior (O_i and O_c) and the parameters of the intervention (I and C) arranged by the practitioner.

In the laboratory study where Heth and Warren (1978) put their X_i model to the test, they gave undergraduates a choice between listening to audio and watching a video. The choice between audio and video was a paired baseline assessment that informed Heth and Warren's choice of I/C values. In one condition, participants' video watching (O_i) was increased through the production of a response deficit in listening to audio (O_c). In a second condition, different participants' video watching (O_i) was decreased through the production of a response excess in listening to audio (O_c). As predicted by the disequilibrium models of reinforcement and punishment, video watching increased for those in the response deficit condition and decreased for those in the response excess condition. The X_i model was also very accurate in its predictions of how much time participants would spend watching video. Figure 2 is a plotted version of Heth and Warren's (1978) actual versus predicted data across participants. We implemented a linear

Fig. 2 Results from Heth and Warren (1978) in which the predicted values (X_i) are plotted according to the actual data. Linear equation and coefficient of determination (R^2) are noted above the line

regression to determine the accuracy of the model. This regression has an R^2 value of 0.98, meaning that X_i accounts for 98 % of the variance between the predicted and actual measures of instrumental activity. According to Reed (2009), high R^2 values, ranging from 85 to 100 %, are expected when conditions are highly controlled in a laboratory. Thus, Heth and Warren's X_i model qualifies as a very good predictive model of resultant instrumental activity in their study.

In order to test the predictive utility of the X_i model across circumstances and populations, we plotted and compared the model's predictions to already acquired data from another laboratory study (Konarski, 1987) and an applied study (Konarski et al., 1982). In the laboratory study, Konarski (1987) provided developmentally disabled participants with an option between looking at pictures via a slide show and listening to music (O_i/O_c). Once paired baseline levels of responding were acquired, Konarski (1987) selected I/C values such that he would observe an increase in one of the behaviors ($I/C > O_i/O_c$) or no change in behavior ($I/C = O_i/O_c$). We tested the X_i model's predictions against the actual behavior Konarski (1987) observed by inputting his I , C , O_i , and O_c values into the X_i model, and running a linear regression to determine how well the X_i model predicts actual responding. In turn, the X_i model very closely predicted the instrumental activity that Konarski (1987) actually observed. Figure 3a shows the predicted versus actual, across groups, with an R^2 value of 0.94. The X_i model, then, accounts for 94 % of variance in the actual data observed.

In the applied study, Konarski et al. (1982) assessed and changed the reading and math behavior of developmentally disabled children in a special education classroom. Like Konarski (1987), clients were exposed to conditions in which

a discrepancy between baseline and intervention was either present ($I/C > O_i/O_c$) or absent ($I/C = O_i/O_c$). As before, we ran a linear regression between the actual and predicted instrumental behavior. Not only did the X_i model correctly predict an increase in behavior when a discrepancy between baseline and intervention was present, but X_i also very closely predicted the duration of instrumental activity actually observed. Figure 3b shows the actual versus predicted durations across both participants and conditions with an R^2 of 0.90. Konarski et al. (1982) is an applied study, so an R^2 according to which approximately 90 % of variance is accounted for by the X_i model is impressive. Reed (2009) states that R^2 values derived from applied studies are usually much lower, with 50 % being acceptable because the conditions in applied settings are not as controllable as conditions in the laboratory.

Based on our preliminary investigation, the X_i model appears to predict instrumental activity with a precision similar to that observed in basic laboratories. As mentioned, however, Heth and Warren's (1978) X_i model is not the only mathematical model the disequilibrium literature has to offer. Furthermore, the X_i model requires more empirical investigation across applied settings and populations. Scientist-practitioners might verify the utility of these models under certain conditions and falsify their utility under others. For purposes of using, testing, and improving these models in the field, we have provided a Microsoft Excel platform within which users can input their O_i/O_c baseline values and their choice of I/C values. All one need do is provide the O_i , O_c , I , and C values, and the Excel platform will calculate the predictions of the X_i model and tell you whether or not I/C is greater than or less than baseline (O_i/O_c). We are offering this

Fig. 3 a, b Predicted (X_i) and actual data from Konarski (1987) (left panel) and Konarski et al. (1982) (right panel)

resource through the Open Science Framework (<https://osf.io/>), an online open source platform designed to facilitate collaborations in scientific research. The Microsoft Excel platform and directions regarding its use can be found at <https://osf.io/knf7x/>. We provide this resource for not only its use as a potential tool in clinical practice, but also for purposes of research and development. In other words, this is a call for research from the standpoint that reinforcers are activities. The disequilibrium approach is notoriously difficult to understand (see, e.g., Timberlake, 1984), so it is with this tutorial and Excel platform that basic researchers, practitioners, and scientist-practitioners might explore and expand upon what Premack, Timberlake, and Allison taught us so long ago.

The Disequilibrium Approach in 10 Steps

In this section, we present the general steps a practitioner can take to implement the disequilibrium approach in his or her own practice. These steps are condensed from the information presented above. They are purposefully general and should be implemented flexibly, based on circumstances and future research.

1. Identify and define the target activity you would like to change. This activity will serve as the instrumental activity (I).
2. Identify an activity that can serve as a reinforcer or punisher (almost any behavior will do). This behavior will serve as the contingent activity (C).
3. Select the behavioral dimensions for the instrumental and contingent activities. Most dimensions (e.g., rate, duration, or latency) will work. Recording systems such as time sampling are also feasible (Farmer-Dougan, 1998).
4. Conduct a paired baseline assessment to acquire measures of both instrumental (O_i) and contingent (O_c) activities. In the applied literature, paired baselines have been as short as 10 min (Farmer-Dougan, 1998) and as long as 2 h (Dougher, 1983). However, Bourret (2005)—an unpublished dissertation—used a 5 min baseline, and there is precedent in the literature for free operant preference assessments as short as 5 min.
5. Select a feasible I/C ratio (schedule of reinforcement or punishment). To increase, or reinforce, behavior, I/C should be greater than O_i/O_c . To decrease, or punish, behavior, I/C should be less than O_i/O_c .
6. Predict the magnitude of behavior change of your selected I/C ratio. You will need your intervention and baseline values, designated as I , C , O_i , and O_c . You may use the Microsoft Excel calculator we provide here <https://osf.io/knf7x/>. This calculator relies on the following formula:
7. If necessary, revise your I/C values based on the predicted results. A small predicted increase or decrease in activity may be a good starting point but may not result in sustained change. A large predicted increase in behavior might entail ratio strain.
8. Equate baseline and intervention session durations. If baseline sessions were 20 min in length, then intervention sessions should be 20 min in length. The X_i model operates on the assumption that the client will have the same amount of time to respond during both baseline and intervention.
9. Implement your intervention and increase or decrease the density of the I/C schedule requirements, throughout the course of intervention, on the basis of additional paired baselines taken.
10. Measure the effects of your intervention and compare those results to the results predicted by the X_i model.

In addition to the steps provided here, we encourage readers to provide feedback on the OSF website regarding the utility of the spreadsheet we provided and to publish the result of any controlled studies regarding the use of the disequilibrium model.

An Application of the Disequilibrium Approach

The purpose of this section is to illustrate a hypothetical implementation of the disequilibrium approach in discrete-trial teaching (DTT), a widely applied and highly effective intervention method for individuals diagnosed with autism spectrum disorder (Ghezzi, 2007). In this case example, imagine a practitioner tasked with increasing a socially relevant and desirable instrumental response. Given this task, the practitioner might select a target instrumental response on the basis of a variety of assessments and curricula such as the *Promoting the Emergence of Advanced Knowledge Relational Training System* (PEAK; Dixon, 2014) and the *Verbal Behavior Milestones Assessment and Placement Program* (VB-MAPP; Sundberg, 2008). For the purposes of this case example, suppose the practitioner chooses to target vocal labeling responses in the presence of corresponding picture cards and a vocal instruction (e.g., “Tell me what this is.”).

Following the selection of a target instrumental response, the practitioner is faced with selecting a contingent activity. According to the disequilibrium approach, any activity can serve as the contingent activity so long as it has a nonzero level of occurrence. More specifically, the activity selected must have a nonzero level of occurrence in the DTT context. This is to say that what happens in other contexts may not happen in the DTT context. As such, the practitioner might arrange a paired baseline to include multiple candidate contingent activities in addition to the target instrumental activity.

$$X_i = \frac{I(O_i + O_c)}{(I + C)}$$

The practitioner can do so in a manner similar to studies that have examined response allocation in multi-operant environments (Green & Striefel, 1988; Bourret, 2005).

Suppose the practitioner arranges a 5 min free operant paired baseline period in which multiple activities, including the target instrumental activity, are available to the client without restriction (see Roane et al., 1998, for a comparable application that serves as a preference assessment variant). The available activities include toy cars, a shape sorter toy, construction blocks, books, and picture cards that pertain to the target instrumental response (i.e., vocal labeling). Prior to the start of the paired baseline, the client is instructed to play freely—to engage with the toys, books, or picture cards as he or she pleases. In the case that the client approaches the picture cards, the practitioner is there to randomly present one of the pictures and ask, “What is this?” The behavioral dimensions in this paired baseline are number of correct instrumental responses and duration of engagement in seconds for each of the candidate contingent activities.

In Table 4, we present hypothetical results from the paired baseline. The number of correct responses (O_i) per engagement and the duration of activities per engagement are listed. The bottom row of Table 4 sums the number of correct responses and duration of activities across those engagements. In determining which of the four activities will be used as the contingent activity, the practitioner’s goal of increasing behavior is best served by selecting the activity with the highest duration. Selecting the activity with highest duration means the practitioner can produce a greater response deficit, which yields a larger predicted increase in behavior. Additionally, selecting the activity with the highest duration is consistent with strategies reported in research on free operant preference assessments. Roane et al. (1998), for instance, showed that the activity most often engaged in during assessment was correlated with a reinforcing function when made contingent upon a target response. In this case example, the activity with the highest total duration is play with toy cars at 170 s. Given this duration, in combination with the number of correct responses (O_i), the practitioner’s paired baseline will look as follows:

$$\frac{O_i}{O_c} = \frac{2_{\text{correct responses-labeling}}}{170_{\text{s-toy car}}}$$

It is important to note that the practitioner does not reduce this ratio to 1/85. Even though 1/85 is mathematically equivalent to 2/170, it yields a different prediction when entered into the X_i model.

Provided the O_i/O_c baseline ratio, the practitioner can now select an I/C intervention. By deferring to Eq. 1, the practitioner knows that an increase in behavior is likely to follow from I/C being greater than O_i/O_c . As such, suppose the practitioner chooses an I/C ratio that provides 15 s of access to the

contingent activity for every one instrumental response. The resulting intervention ratio, then, is this:

$$\frac{I}{C} = \frac{1_{\text{correct response-labeling}}}{15_{\text{s-toy car}}}$$

This choice of ratio is practical in that it does not require the client to do more than what was observed in the paired baseline; it is feasible. Additionally, and importantly, this I/C ratio is greater than the O_i/O_c baseline ratio.

Before implementing this intervention, however, suppose the practitioner enters the obtained O_i , O_c , I , and C values into Eq. 3, Heth and Warren’s (1978) predictive model:

$$X_i = \frac{I(O_i + O_c)}{(I + C)}$$

Solving for X_i can be helpful to the practitioner in that it tells you how many instrumental responses to expect. Given the parameters of the current intervention, solving for X_i is as follows:

$$X_i = \frac{1(2 + 170)}{(1 + 15)},$$

where X_i equals 10.75. With this number, the practitioner can expect to observe approximately 11 target instrumental responses, so long as 15 s access to the contingent activity is provided for every instrumental response. This X_i prediction is useful insofar as the amount of trial presentations for the target instrumental response can be prepared before the start of the intervention session. The practitioner can plan to present 11 trials (i.e., “What is this?” and showing the picture) to meet the client’s level of performance based on the X_i prediction. This also pertains to ceiling effects, as the client may not engage in more than 11 instrumental responses. Furthermore, the imposition of additional trials may result in escape-related problem behaviors commonly found in applied situations (Geiger, Carr, & LeBlanc, 2010).

Another way the practitioner may use the X_i model is to determine whether or not the selected I/C ratio is optimal. Imagine the practitioner uses our proposed Microsoft Excel spreadsheet and populates the O_i , O_c , I , and C columns with many different values for C . Doing so would yield a number of different X_i predictions that aid in the selection of an I/C intervention. In Figure 4, a screenshot of the Microsoft Excel spreadsheet, we present multiple X_i results given a range of C values from 5 to 20 s of contingent access to the toy car. The values for O_i and O_c are held constant at 2 and 170 because they were observed during the paired baseline. The values for I are held constant at one because the practitioner selected a feasible instrumental response requirement based on the client’s performance during baseline. Notice that we bolded the values representing the practitioner’s current choice of I/C

Table 4 Hypothetical paired baseline results

Engagement	Number of correct responses (O_i)	Toy cars (s)	Shape sorter (s)	Books (s)	Construction blocks (s)
1	1	20	10	19	40
2	1	30	4	20	20
3		120	5		
4			2		
Sum	2	170	21	39	60

Baseline period = 5 min (300 s)

intervention. On the one hand, C values far less than 15—for example, 5 and 6—result in a larger predicted increase in responding but pose a risk of ratio strain. On the other hand, C values greater than 15—for example, 19 and 20—result in a smaller predicted increase in responding that may not optimize what the client is capable of doing. The I/C ratio of 1:15 falls within the middle, striking an important balance between what the client can and cannot do.

Once the baseline (O_i/O_c) and intervention (I/C) ratios have been established, the practitioner can arrange the DTT environment, determine the session length, and decide on criteria for what does and does not count as a correct instrumental response. First, imagine the practitioner arranges the DTT environment such that the client can engage in both the instrumental and contingent activity on a tabletop. In this case, the practitioner has control over what the client can and cannot access. Second, the practitioner determines the session length based on the length of the paired baseline: 5 min. Third, the practitioner decides what does and does not constitute a correct instrumental response by establishing exclusionary criteria. This is to say that the contingent activity will be withheld on the basis of certain circumstances and particular responses. Suppose the practitioner determines those exclusionary criteria to be response latency longer than 3 s, incorrect responses, previously acquired responses, and prompted responses.² With these criteria, in addition to the tabletop and session length, the practitioner begins the intervention by providing opportunities (i.e., trials) for instrumental responding. That is, the practitioner presents a picture card with the vocal instruction, “Tell me what this is.” A fixed-ratio 1 (FR1) schedule is imposed such that 15 s access to the contingent activity is made available for every correct instrumental response. At the end of the 15 s contingent activity, the client is asked or prompted to return the toy car, which begins a brief intertrial interval (ITI) that is followed by another trial. This sequence continues until the client completes 11 trials, the predicted X_i , or until the session times out at 5 min.

² Due to the wide variety of DTT arrangements and characteristics that might complement the disequilibrium approach (e.g., error correction, prompting, prompt fading, etc.), we refer readers to Plaisance, Lerman, Laudont, and Wu (2016) for a recent discussion on DTT algorithms involving error correction and the presentation of previously acquired responses.

Prior to the start of the next intervention session, the practitioner prepares another free operant paired baseline to obtain a new O_i/O_c ratio. This new O_i/O_c would be for the subsequent session that targets either the same or other classes of instrumental activities. Transition periods between sessions are common in treatment programs, so this may be an opportune time to prepare and collect new paired baseline data. Doing so prior to each intervention session permits the identification of those contingent activities that will be most effective within a particular I/C intervention.

Conclusion

The disequilibrium approach to reinforcement and punishment provides a different method of addressing problems of social significance. How practitioners go about solving problems of social significance changes when reinforcers and punishers are construed as activities rather than stimuli. Vocal verbal stimuli such as “good job” or “awesome” are not one-offs that terminate in the completion of a contingency requirement; they are one aspect of a two-way interaction for which a client often strives. Arguably, the disequilibrium approach increases the precision and scope with which practitioners predict and control behavior (Konarski, Johnson, Crowell, & Whitman, 1981). The disequilibrium model increases precision by formalizing contingency arrangements and reducing practitioner guesswork by predicting what will function as a reinforcer or punisher. Increases in scope follow from the clarification that any behavior can function as a reinforcer or punisher, depending on whether I/C is greater than or less than O_i/O_c .

Although the utility of the disequilibrium approach has been demonstrated (e.g., Farmer-Dougan, 1998; Holburn & Dougher, 1986; Realon & Konarski, 1993), its models for reinforcement, punishment, and prediction of instrumental activity (e.g., X_i) require further investigation. First, the behavioral dimension most measured by disequilibrium researchers is duration of activities. The effectiveness of the disequilibrium model and X_i model need to be tested using other popular measures such as rate of behavior. Second, it is unclear how to systematically discern an optimal I/C ratio

O_i	O_c	O_i/O_c	I	C	I/C	X_i Prediction	Gain/Loss
2	170	0.01	1	5	0.20	28.67	+26.7
2	170	0.01	1	6	0.17	24.57	+22.6
2	170	0.01	1	7	0.14	21.50	+19.5
2	170	0.01	1	8	0.13	19.11	+17.1
2	170	0.01	1	9	0.11	17.20	+15.2
2	170	0.01	1	10	0.10	15.64	+13.6
2	170	0.01	1	11	0.09	14.33	+12.3
2	170	0.01	1	12	0.08	13.23	+11.2
2	170	0.01	1	13	0.08	12.29	+10.3
2	170	0.01	1	14	0.07	11.47	+9.5
2	170	0.01	1	15	0.07	10.75	+8.8
2	170	0.01	1	16	0.06	10.12	+8.1
2	170	0.01	1	17	0.06	9.56	+7.6
2	170	0.01	1	18	0.06	9.05	+7.1
2	170	0.01	1	19	0.05	8.60	+6.6
2	170	0.01	1	20	0.05	8.19	+6.2

Fig. 4 X_i results from C population (range: 5 - 20 s). Note that O_i , O_c , and I are constants and the values coordinated with C at 15 s are in bold

that will produce the most behavior change without the effects of ratio strain. The X_i model is a step in the right direction, as it specifies how much behavior will change based on the I/C values inputted, but it does not definitively solve the problem of whether or not we will observe ratio strain. Lastly, research is needed on the maintenance and generalization of behavior following disequilibrium arrangements that produce a response deficit ($I/C > O_i/O_c$) and response excess ($I/C < O_i/O_c$).

Research discerning the conditions under which the disequilibrium approach might be useful has not been exhausted. There is much work to be done, and the disequilibrium approach to reinforcement and punishment might guide future inquiry. Take for instance the X_i model. It is a simple model that has no fitted parameters (i.e., it does not have parameters like sensitivity and bias in the generalized matching law; see Reed & Kaplan, 2011). The X_i model appears to predict instrumental activity with precision but is still subject to refutation, verification, and elaboration. If elaborated through its refutation and verification in practice, the X_i model might open doors to research in areas such as behavioral prognosis. Knowing what will happen based on the X_i model's predictions also entails knowing the likely course of a behavioral problem (i.e., its prognosis): How long the behavioral problem will persist given every iteration of an I/C intervention.

The disequilibrium models are rules for effectively controlling behavior; rules that also have predictive utility. Such rules are especially effective when practitioners can directly assess contingencies and ensure that the

requirements of a new I/C contingency are met (in homes, schools, etc.). When direct observation is not possible, or when resources, time, or space preclude paired baseline assessments, the disequilibrium models might serve as “molar functional relations” (Waltz & Follette, 2009). In their article describing molar functional relations, Waltz and Follette (2009) translate basic research findings that have implications for assessment and treatment. The molar functional relations they describe are matching, discounting, momentum, and variability (Waltz & Follette, 2009). These are “molar” functional relations because they specify “broad patterns of behavior in context...” (Waltz & Follette, 2009, p. 52). To these four molar functional relations, we add disequilibrium: the systematic disruption of activities that results in increases or decreases in behavior relative to baseline. Therefore, if paired baselines and direct access to contingencies are not possible, then the disequilibrium models might, at their very least, serve as key conceptual cornerstones—molar functional relations—that inform practitioners on how to arrange contingencies with particular effects.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

Ethical Approval This article does not contain any studies with human participants or animals performed by any of the authors.

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