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Interior Tomography from Differential Phase Contrast Data via Hilbert Transform Based on Spline Functions

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Abstract

X-ray phase contrast imaging is an important mode due to its sensitivity to subtle features of soft biological tissues. Grating-based differential phase contrast (DPC) imaging is one of the most promising phase imaging techniques because it works with a normal x-ray tube of a large focal spot at a high flux rate. However, a main obstacle before this paradigm shift is the fabrication of large-area gratings of a small period and a high aspect ratio. Imaging large objects with a size-limited grating results in data truncation which is a new type of the interior problem. While the interior problem was solved for conventional x-ray CT through analytic extension, compressed sensing and iterative reconstruction, the difficulty for interior reconstruction from DPC data lies in that the implementation of the system matrix requires the differential operation on the detector array, which is often inaccurate and unstable in the case of noisy data. Here, we propose an iterative method based on spline functions. The differential data are first back-projected to the image space. Then, a system matrix is calculated whose components are the Hilbert transforms of the spline bases. The system matrix takes the whole image as an input and outputs the back-projected interior data. Prior information normally assumed for compressed sensing is enforced to iteratively solve this inverse problem. Our results demonstrate that the proposed algorithm can successfully reconstruct an interior region of interest (ROI) from the differential phase data through the ROI.

Keywords

Interior Tomography; Differential Phase Contrast Imaging; Hilbert Transform

1. INTRODUCTION

X-ray phase contrast tomography (PCT) has attracted a significant interest because it utilizes phase shift signals and could provide better contrast for soft biological tissues and low atomic number samples¹⁻³. The grating interferometer for differential phase contrast (DPC) imaging is among the most promising methods that can stably extract phase signals^{2, 4-6}. A typical grating based differential phase contrast tomography (DPCT) system usually consists of three gratings: a source grating (G0) to produce a periodic array of micro-sources, a phase grating (G1) to produce interference fringes at the so-called Talbot distances, and an

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analyzer grating (G2) to detect the Talbot fringes. The first DPCT system was implemented at the x-ray synchrotron radiation facility⁷⁻¹¹ and later on modified to work with an ordinary x-ray tube¹². Preclinical computed tomography (CT) equipped grating interferometry has been prototyped. Then, several pre-clinical experiments have demonstrated an enhanced soft-tissue contrast in the phase images¹³⁻¹⁶. Despite the promising future of DPCT, one of the main practical obstacles is the fabrication of large-area gratings of a small period and a high aspect ratio. Imaging large objects with a size-limited grating setup results in data truncation representing a new type of the interior problem.

Data truncation has been extensively studied in the field of CT image reconstruction. The differential back-projection (DBP) reconstruction techniques have been proved to handle many data truncation problems^{17, 18}. These methods utilize the relationship between the DBP image and the 1-D Hilbert transform along certain lines. The data completeness condition states that as long as there exists one set of parallel PI-lines that covers the local ROI and are not transversely truncated, the local ROI can be stably and exactly reconstructed¹⁹. However, when the field-of-view (FOV) is totally inside the object, this condition cannot be satisfied that the so-called interior problem has no unique solution in an unconstrained space. To make the solution unique and stable, prior information of an underlying object is needed. One example of such prior information is a known sub-region within the ROI^{20, 21}. Another method is the compressed sensing (CS) technology assuming that the object is piecewise constant²² or piecewise polynomial²³.

The DPCT reconstruction is similar to the CT reconstruction except that the data acquired is the first-order derivative along the detection line. Since the first step of most DBP-like algorithms is to take the first-order derivative of the projection data, it is natural to apply these methods for DPCT ROI reconstruction. In this paper, we follow this idea and propose an iterative algorithm combining CS prior to solve the interior DPCT problem. In the following section, we first recall the link between the DBP image and the Hilbert transform of the object, and then describe our method to solve the inverse Hilbert transform in terms of B-spline basis functions. In the third section, we report a numerical simulation results. In the last section, we discuss relevant issues and conclude the paper.

2. METHODOLOGY

2.1 Hilbert Transform and Back-projection Image

Let $f(x, y)$ denote a 2-D object and $p(s, \theta)$ denote its 2-D Radon transform acquired in a parallel projection geometry (Figure 1(a)). The classical filtered back-projection algorithm reconstructs the object from its Radon transform by the following equation,

$$f(x, y) = \int_0^\pi \int_{-\infty}^{\infty} P(\omega, \theta) |\omega| e^{2\pi i \omega (x \cos \theta + y \sin \theta)} d\omega d\theta \quad (1)$$

where $P(\omega, \theta)$ is the Fourier transform of the projection data,

$$P(\omega, \theta) = \int_{-\infty}^{\infty} p(s, \theta) e^{-2\pi i \omega s} ds \quad (2)$$

In differential phase contrast tomography, the data denoted by $g(s, \theta)$ are the first-order derivative of $p(s, \theta)$ along s ,

$$g(s, \theta) = \frac{\partial p(s, \theta)}{\partial s} \quad (3)$$

To reconstruct the image from the derivative data, we decompose the filter kernel as

$$|\omega| = 2\pi i \omega \left[\frac{1}{2\pi} (-i \operatorname{sgn} \omega) \right] \quad (4)$$

where the inverse Fourier transform of $2\pi i \omega$ is the derivative operation, and $i \operatorname{sgn} \omega$ is the Hilbert transform kernel. Thus, we have

$$f(x, y) = \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} 2\pi i \omega P(\omega, \theta) (-i \operatorname{sgn} \omega) e^{2\pi i \omega (x \cos \theta + y \sin \theta)} d\omega d\theta \quad (5)$$

Now, applying the 1-D Hilbert transform along the y direction on $f(x, y)$, we have

$$\begin{aligned} H_y f(x, y) &= \int_{-\infty}^{\infty} \frac{f(x, y-y')}{\pi y'} dy' \\ &= \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} 2\pi i \omega P(\omega, \theta) (-i \operatorname{sgn} \omega) e^{2\pi i \omega (x \cos \theta + (y-y') \sin \theta)} \frac{1}{\pi y'} dy' d\omega d\theta \\ &= \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} 2\pi i \omega P(\omega, \theta) (-i \operatorname{sgn} \omega) e^{2\pi i \omega (x \cos \theta + y \sin \theta)} d\omega d\theta \int_{-\infty}^{\infty} \frac{1}{\pi y'} e^{-2\pi i \omega y' \sin \theta} dy' \\ &= \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} 2\pi i \omega P(\omega, \theta) (-i \operatorname{sgn} \omega) e^{2\pi i \omega (x \cos \theta + y \sin \theta)} (-i \operatorname{sgn}(\omega \sin \theta)) d\omega d\theta \\ &= \frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} 2\pi i \omega P(\omega, \theta) (-i \operatorname{sgn} \omega) e^{2\pi i \omega (x \cos \theta + y \sin \theta)} (-i \operatorname{sgn} \omega (\operatorname{sgn}(\sin \theta))) d\omega d\theta \\ &= -\frac{1}{2\pi} \int_0^\pi \int_{-\infty}^{\infty} 2\pi i \omega P(\omega, \theta) (-i \operatorname{sgn} \omega) e^{2\pi i \omega (x \cos \theta + y \sin \theta)} d\omega d\theta \\ &= -\frac{1}{2\pi} \int_0^\pi g(x \cos \theta + y \sin \theta, \theta) d\theta \end{aligned} \quad (6)$$

Similarly, if the back-projection starts from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, we have

$$H_x f(x, y) = -\frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} g(x \cos \theta + y \sin \theta, \theta) d\theta \quad (7)$$

The above two equations show the relationship between the horizontal and vertical Hilbert transforms of an image and its differential back-projections. A generalization of this result relative to an arbitrary starting angle can be found in Noo's paper in 2002¹⁸.

For fan-beam projection data on a flat panel detector array (Figure 1(b)), Fu provided a back-projection filtration (BPF) algorithm to reconstruct the 2-D image²⁴. This BPF algorithm is expressed as

$$f(x, y) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{B_x(x, y)}{x-x'} dx' \quad (8)$$

where $B(x, y)$ is a weighted back-projection image and defined by

$$B_x(x, y) = \int_{\gamma_1}^{\gamma_2} \frac{1}{L(\gamma)} \frac{DR}{\sqrt{s^2 + D^2}} g(s, \gamma) d\gamma \quad (9)$$

with

$$L(\gamma) = \sqrt{(x + R \sin \gamma)^2 + (y - R \cos \gamma)^2}, \quad (10)$$

and the angles γ_1 and γ_2 are determined by intersections between the horizontal PI-line through the point (x, y) and the x-ray source trajectory.

Clearly, Eq. (8) states that the original image is a 1-D Hilbert transform of $B_x(x, y)$, i.e.

$$H_x f(x, y) = -\frac{1}{2} B_x(x, y) \quad (11)$$

Similarly, if we select the vertical PI-line and choose γ_1 and γ_2 accordingly, we have

$$H_y f(x, y) = -\frac{1}{2} B_y(x, y). \quad (12)$$

2.2 Spline Basis based Interior Reconstruction

To recover an image from its Hilbert transform. We represent the image in terms of B-splines

$$f(x, y) = \sum_i \sum_j q_{i,j} \beta_m(x-i) \beta_m(y-j) = B_m(x, i) Q(i, j) B_m(y, j) \quad (13)$$

where $\beta_m(\cdot)$ is a B-spline function of degree m and

$$\begin{aligned} B_m(y, j) &= \begin{bmatrix} \beta_m(y_1 - j_1) & \cdots & \beta_m(y_N - j_1) \\ \vdots & \ddots & \vdots \\ \beta_m(y_1 - j_N) & \cdots & \beta_m(y_N - j_N) \end{bmatrix} \\ Q(i, j) &= \begin{bmatrix} q(i_1, j_1) & \cdots & q(i_N, j_N) \\ \vdots & \ddots & \vdots \\ q(i_N, j_1) & \cdots & q(i_N, j_N) \end{bmatrix} \\ B_m(x, i) &= \begin{bmatrix} \beta_m(x_1 - i_1) & \cdots & \beta_m(x_N - i_N) \\ \vdots & \ddots & \vdots \\ \beta_m(x_N - i_1) & \cdots & \beta_m(x_N - i_N) \end{bmatrix} \end{aligned} \quad (14)$$

From the above equations, we can see that $B_m(x, i) = B_m(y, j)^T = B_m^T$, so $f(x, y) = B_m^T Q B_m$.

The benefit of choosing B-spline basis functions is that its Hilbert transform has an analytical expression, which satisfies the following recursion relations

$$H\beta_{m,k}(t) = \frac{t - \tau_k}{\tau_{k+m-1} - \tau_k} H\beta_{m-1,k}(t) + \frac{\tau_{k+m} - t}{\tau_{k+m} - \tau_{k+1}} H\beta_{m-1,k+1}(t-1) \quad (15)$$

with

$$H\beta_{1,k} = \frac{1}{\pi} \ln \left| \frac{t - \tau_k}{t - \tau_{k+1}} \right| \quad (16)$$

where $\tau_k (k = 1, 2, \dots)$ denotes the B-spline nodes. The Hilbert transform of $f(x, y)$ can be represented as a matrix production form

$$\begin{aligned} H_x f(x, y) &= (H B_m^T) Q B_m = (H B_m^T) (B_m^T)^{-1} B_m^T Q B_m = (H B_m^T) (B_m^T)^{-1} f(x, y) \\ H_y f(x, y) &= B_m^T Q (H B_m) = B_m^T Q B_m (H B_m) = f(x, y) (B_m)^{-1} (H B_m) \end{aligned} \quad (17)$$

By re-ordering the 2-D image into a 1-D vector, we can rewrite the above equations in the matrix by vector format. Furthermore, combining Eqs. (6) (7) (11) (12) and (17), we obtain two sets of linear equations,

$$\begin{aligned} A_x f &= B_x \\ A_y f &= B_y \end{aligned} \quad (18)$$

Then, the object can be recovered by solving the two systems.

When an object is fully covered by the detector array, the back-projection images B_x and B_y can be accurately calculated, and the matrices A_x and A_y are full ranked. Solving the full rank equations can easily recover the images. However, if only a region-of-interest (ROI) inside the object is covered by a detector module, the back-projection images are only accurate over the ROI. In this case, we should only use the equations that have the correct values on the right hand side of Eq. (18). This would always result in an underdetermined problem. To solve this problem, a priori knowledge on the object is required. Based on the previous results^{22, 25}, if the object is piecewise constant, the interior reconstruction can be performed via total variation (TV) minimization:

$$\begin{cases} \min TV(f) \\ s.t. A_x f = B_x, A_y f = B_y \end{cases} \quad (19)$$

This is a convex problem and can be solved using the split-Bregman approach²⁶.

3. NUMERICAL SIMULATIONS

3.1 Experiment Settings

To demonstrate the capability of the proposed algorithm, we performed a numerical experiment with parallel beam projections. The projection data were acquired from 2880 projection angles ranging from 0 to 360 degrees. Figure 2 shows a modified Shepp-Logan phantom defined on a 256×256 pixels grid. This phantom is piecewise constant and includes several ellipses whose parameters are specified in Table 1. The dashed circle in Figure 2 defines an ROI of 84 pixels in radius. The detector module contained 169 bins and only covered the ROI. Outside the ROI, we added two ellipses to interfere the ROI reconstruction. For comparison, we also reconstructed the image using the FBP method (Eq. 5).

3.2 Results

Figure 3 shows intermediate back-projection images, which only yielded correct values within the ROI. Figure 4 shows the reconstruction results using the proposed method and the FBP algorithm respectively. Both the algorithms can reconstruct internal structures in the ROI. However, the profile comparison in Figure 5 demonstrates that the proposed algorithm can exactly reconstruct the ROI, while the FBP algorithm fails to recover the internal contents faithfully. To show the convergence of the proposed algorithm, we plotted a mean square error (MSE) of the ROI versus the number of iterations in Figure 6. Although the MSE oscillated in the early iterations, it eventually converged.

4. DISCUSSIONS AND CONCLUSIONS

Our numerical simulation demonstrated that the proposed algorithm could successfully reconstruct an interior ROI from differential phase data. The convergence curve in Figure 6 shows the stability of this method. It should be noted that the recovering procedure in fan beam geometry should be the same as in a parallel beam case after the back-projection images are obtained. Our phantom is of a commonly used type but it is very simplistic. In the next stage, we will evaluate our approach with more realistic phantoms and physical experimental data.

In conclusion, we have proposed a new iterative scheme using the Hilbert transform and B-spline functions to solve the interior DPCT problem. Our method is based on the link between the back-projection images and the 1-D Hilbert transform of the object. By implementing the Hilbert transform in the B-spline space, the inverse Hilbert transform can be better solved in a matrix production format. Introducing the CS based interior reconstruction framework, the proposed algorithm can stably and exactly reconstruct an interior ROI, as proved by the numerical simulation. Currently, we are acquiring fan beam data from biological samples for pathological imaging applications.

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Figure 1.

Parallel beam geometry (a) and fan beam geometry (b) with equal distant detectors. p is the Radon line integral; and g is the differential projection data. In the fan beam geometry, D is the distance from the x-ray source focal spot to the detector, and R the distance from the source to the rotation center.

Figure 2.
A modified Shepp-Logan phantom used in the numerical simulation. The dashed circle indicates an ROI inside the phantom.

Figure 3. Back-projection images. (a) The result corresponding to the horizontal Hilbert transform and (b) that corresponding to the vertical Hilbert transform.

Figure 4. Interior reconstruction in parallel beam geometry. (a) An image reconstructed using the proposed method and (b) the counterpart obtained using the FBP method. The display window is $[0.2, 0.6]$ for (a) and $[0, 0.6]$ for (b).

Figure 5.
 Profile comparison along the horizontal (a) and vertical lines (b) in Figure 4.

Figure 6.
Plot of MSE versus the number of iterations on the logarithmic scale.

Table 1

Parameters of the 2D Shepp-Logan phantom, where (x_0, y_0) represent the center coordinates of the ellipse, (a, b) the semi-axes, ω the rotation angle, and f the relative intensity.

No	f	a	b	x_0	y_0	ω
1	1	0.92	0.92	0	0	0
2	-0.7	0.89	0.88	0	-0.0184	0
3	-1	0.11	0.31	0.22	0	-18
4	-0.1	0.16	0.41	-0.22	0	18
5	0.1	0.21	0.25	0	0.35	0
6	0.1	0.046	0.046	0	0.1	0
7	0.1	0.046	0.046	0	-0.1	0
8	1.0	0.046	0.023	-0.08	-0.605	0
9	0.1	0.023	0.023	0	-0.606	0
10	0.1	0.023	0.046	0.06	-0.605	0
11	0.3	0.05	0.05	-0.8	0	0
12	0.3	0.05	0.1	0.35	-0.1	10
13	-0.2	0.2	0.04	0.5	-0.52	60.5