Dynamic susceptibility of concentric permalloy rings with opposite chirality vortices

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The high frequency dynamic behaviour of concentric permalloy nanorings with vortex domain structures with a thickness of 20 nm, a width in the range of 100 nm–250 nm, and a separation in the range of 10 nm–600 nm is investigated by micromagnetic simulations. The aim is to explore the ferromagnetic resonance of the concentric ring structure as a function of geometric parameters of the system. The dynamic susceptibility spectrum and spatial localization of the ferromagnetic resonance mode are investigated for varying ring widths and separations. The frequency of oscillation is significantly impacted by the presence of the magnetostatic interaction between each ring and can be modulated by a variation in the ring width and separation. The spatial localization of the uniform mode is found to vary as a function of ring separation, which corresponds to a large variation in amplitude of the real and imaginary components of the dynamic susceptibility.

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I. INTRODUCTION

Nanomagnetism has become an important area of research due to recent advances in the synthesis and characterization of magnetic samples comprising micro- and sub-micrometer scales. The dynamic behaviour of nanometer sized magnetic elements has been the subject of considerable research for use in spintronic devices,¹ logic devices,² data storage,^{3,4} and microwave oscillators.⁵ Precise control of the magnetic susceptibility in the high frequency range is necessary for microwave devices which require precise tunability of the resonance frequency of the uniform gyromagnetic mode.⁶ Many studies have focused on the dynamic behaviour of nanometer size magnetic elements such as squares,⁷ disks,⁸ ellipses,⁹ and circular/elliptical rings.^{10–15} The geometry of the magnetic element can significantly impact the dynamics and the hysteretic behaviour, and a variety of geometries have been investigated with the aim of developing the fastest and most reproducible switching mechanisms.¹⁶ In particular, nanorings have been intensively studied for magnetic random access memory (MRAM) applications with most research focusing on ground states,^{17–19} hysteretic behaviour,^{20,21} phase diagrams,^{22–24} and spin dynamics.^{25–28} Miniaturization of nanomagnets is of additional importance when designing novel magneto-electronic devices, such as logic or data storage mechanisms that utilise magnetic vortices. In addition to the geometry, the effect of magnetostatic interactions between rings when placed in close proximity, such as adjacent,²⁸ concentric,^{28,29} or vertically stacked multilayers,³⁰ can also significantly modify the hysteretic properties and lead to considerable changes in their dynamics.

In sufficiently thin magnetic disks, a vortex appears as the ground state, where the in-plane component of the vortex state is characterized by a clockwise or counter-clockwise magnetisation chirality.³¹ The polarity of the vortex is determined by an out-of-plane component of magnetisation at

the central core of the vortex with two possible orientations $p = \pm 1$ which can be controlled by an external magnetic field aligned perpendicular to the disk plane. If a central hole is introduced into the disk, thus making it a nanoring, a core is no longer formed, which leads to an increase in the stability of the vortex at even smaller scales.³² Recently, concentric ring structures have been proposed as multi-bit storage devices for MRAM technology where the vortex chirality (clockwise and counter-clockwise) of each ring can represent 0 or 1 in binary format.³³ Concentric rings have also been considered as contacts in spin injection devices for semiconductors which can utilise the properties of magnetic nanorings, such as low stray fields and high stability.³⁴

For a single thin-ring structure at remanence, a stable bi-domain state called the onion state is observed with head-to-head and tail-to-tail domain walls.^{20,35,36} When a magnetic field is applied parallel to the plane of the ring, one of the domain walls de-pins and moves toward the other domain until the onion state is annihilated and forms a vortex domain structure. As the magnetic field that is applied parallel to the plane of the ring is increased, the vortex state annihilates and forms an onion state of opposite polarity. A sudden drop in the hysteresis loop represents a cascade in magnetisation as the onion state is destroyed and converted to a vortex domain structure, reducing the overall magnetisation of the system. Onion-vortex and vortex-to-reversed-onion transitions correspond to irreversible jumps in the ferromagnetic resonance (FMR) spectra of thin ferromagnetic rings.³⁷ In a concentric ring configuration, the magnetostatic interactions between the rings determine the alignment of the head-to-head and tail-to-tail domain walls.³³ For a sufficiently large saturation field, all the head-to-head domain walls are aligned in one half of the structure, while the tail-to-tail domains are aligned in the other half. When the external saturation field is decreased, the domain walls of each ring propagate in either clockwise or anticlockwise directions in order to minimise the overall energy of the system. The annihilation of

the clockwise and anticlockwise propagating domain walls then results in the formation of vortex domains of opposite chirality in each ring.

While there is some understanding of the static magnetic behaviour of concentric rings, the dynamic behaviour, resulting from application of high frequency excitations is unexplored. This understanding is important for the development of high speed magnetic devices and components. Here, we investigate the dynamic behaviour of concentric nanorings to explore the susceptibility spectrum as a function of geometric parameters of the system. The ferromagnetic resonance of the coupled ring structure is analysed and compared for different values of ring separation and their width. The micromagnetic investigation is carried out with the recently developed graphics processing unit (GPU) based solver MuMax.³⁸

II. METHODOLOGY

The high frequency dynamic susceptibility of permalloy concentric rings with a thickness of 20 nm, a width in the range of 100 nm–250 nm, and a separation in the range of 10 nm–600 nm with vortex domain structures is investigated by micromagnetic simulations. The Landau-Lifshitz equation is solved numerically to obtain the magnetisation for each cell in the structure,

$$\frac{\partial \mathbf{M}}{\partial t} = -\mu_0 \gamma ((\mathbf{M} \times \mathbf{H}_{\text{eff}}) - \alpha (\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}))), \quad (1)$$

where $\mathbf{M}$ is the magnetisation, $\mathbf{H}_{\text{eff}}$ is the effective magnetic field, γ is the gyromagnetic ratio, α is the damping constant, and μ_0 is the permeability of free space. Here, the effective field is a combination of the external magnetic field, the demagnetising field, and exchange interactions. The standard values of permalloy were chosen where the saturation magnetisation is given by $M_s = 8.6 \times 10^5$ A/m, exchange constant $A = 1.3 \times 10^{-11}$ J/m³, damping factor $\alpha = 0.01$, and negligible anisotropy with constant $K_1 = 0$. The cell-size was chosen to be $x = y = 4$ nm, which is less than the exchange length of permalloy $\sqrt{2A/\mu_0 M_s^2} \sim 5$ nm. The cell-size in the z-direction was chosen to be 20 nm as the rings are too thin compared to the other dimensions in the ring structure to support a significant out-of-plane magnetisation component.

The system was excited by applying an in-plane pulse in the y-direction with a Gaussian profile, and the evolution of the y-component of the magnetisation $M(t)$ was recorded with respect to time. A weak pulse was chosen ($H_y = 5$ Oe) to ensure that excitation occurs within the linear region of the system. The average susceptibility $\chi(\omega)$ was determined from the ratio of the Fourier transform of the y-component of the average magnetisation $M(t)$ and Fourier transform of the applied field,

$$\chi(\omega) = M(\omega)/h(\omega), \quad (2)$$

where $M(\omega)$ and $h(\omega)$ are the Fourier transforms of the average magnetisation and applied field, respectively.

III. RESULTS AND DISCUSSION

A. Hysteresis

The effect of magnetostatic coupling on the static behaviour of concentric rings was investigated for an inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm with varying outer ring parameters. The rings were initially saturated with an external biasing field applied in the y-direction. The magnetisation M_y was recorded as the field was swept in the region of $H_y = 1000$ and $H_y = -1000$ Oe in steps of 10 Oe.

The hysteresis curves for the inner and outer rings of a concentric system are shown individually in Fig. 1(a). Here, the average normalized magnetisation M_y/M_s for the entire concentric system, which includes both the inner and outer rings, is represented as the dashed black line. As the field is decreased in strength, switching occurs at larger field strengths for the inner ring due to the smaller width and increased shape anisotropy when compared with the outer ring. The point “a” in Fig. 1(a) corresponds to a mixed onion-vortex state where each of the inner and outer rings possesses vortex and onion domain states, respectively. Here, the onion state of the inner ring is destabilized by the magnetostatic interaction between the concentric rings, resulting in the formation of the vortex state for non-zero biasing fields. This is in contrast to single nanorings for which the onion state exists as a meta-stable domain structure and onion-vortex transitions require a non-zero external

FIG. 1. (a) Hysteresis loop for the outer ring (green), inner ring (red), and total concentric system (black). The outer ring has an outer diameter of 1000 nm, a width of 200 nm, and a thickness of 20 nm. The inner ring has an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm. (b) Hysteresis loop for a ring of an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm interacting with an outer ring with a width of 100 nm and a thickness of 20 nm for ring separations of 50 nm (green), 25 nm (blue), and 12.5 nm (red). The dashed black line represents a single ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm for which no magnetostatic coupling is present.

biasing field.^{36,39} As the external field is reversed, the outer ring undergoes a transition from the onion to vortex domain [see point “b” in Fig. 1(a)]. In addition, small drops and jumps in the hysteresis loop of the inner ring are found to occur at the point of magnetisation switching for the outer ring [see, for example, point “c” in Fig. 1(a)]. The effect of different ring separations on the hysteretic behavior is shown in Fig. 1(b). Here, the vortex-to-onion transitions can be seen to vary as a function of the ring separation. A decrease in the separation between each ring results in a decrease in the threshold field strength required to induce switching of the domains.

B. Ring width

The impact of magnetostatic coupling on the high frequency dynamic susceptibility of concentric rings was investigated as a function of the ring width. The concentric rings were relaxed into vortex domains of opposite chirality [see Fig. 2(a)] for an inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm that was interacting with an outer ring with an inner diameter of 800 nm, a thickness of 20 nm, and the width varied in the region of 100 nm–250 nm. The stable state of the total system was excited with a Gaussian pulse of strength $H_y = 5$ Oe. The dynamic susceptibility was calculated from expression (2), where $M(\omega)$ is given by the Fourier transform of the time varying magnetisation component $M_y(t)$ corresponding to the inner ring. The influence of varying the outer ring width on the dynamic susceptibility of the inner ring is shown in Figs. 3(a) and 3(b). The width of the outer ring is varied in the range of 100 nm–250 nm, while all geometric parameters of the inner ring are kept fixed. Two modes are pronounced in the susceptibility spectrum for the inner ring: a lower frequency mode corresponding to the uniform resonance of the outer ring which stems from magnetostatic interactions between the concentric rings and the second at ~ 11.5 GHz corresponding to the uniform resonance of the inner ring. This can be confirmed analytically by approximating each arm of the inner/outer ring by an elliptic cylinder saturated in the x direction,³⁹ in which case the uniform resonance frequency can be readily shown to be

$$\omega_r = \mu_0 \gamma M_s \sqrt{N_y N_z}, \quad (3)$$

where $N_y = T/(W + T)$ and $N_z = W/(W + T)$ are the demagnetising factors of the ring arms with width W and

FIG. 2. (a) Schematic of the domain structure of concentric nanorings with vortex domains of opposite chirality. (b) Schematic of the perturbed state of interacting rings following pulse excitation.

FIG. 3. (a) Real and (b) imaginary components of the susceptibility for an inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm interacting with an outer ring with an inner diameter of 800 nm and a thickness of 20 nm. The inner ring is kept fixed while the width of the outer ring is varied in the region of 100 nm–250 nm. (c) The frequency of the ferromagnetic resonance of the outer ring with an inner diameter of 800 nm and a thickness of 20 nm plotted against the width. (d) The amplitude of the imaginary component of the susceptibility for the inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm plotted against the width of the outer ring. The dashed line represents the amplitude of the susceptibility corresponding to the ferromagnetic resonance of the inner ring when no magnetostatic coupling is present.

thickness T . Substituting the dimensions of the inner ring yields a resonance frequency of 11.3 GHz, in agreement with micromagnetic simulations.

Increasing the width of the outer ring leads to a decrease in the frequency of the uniform resonance of the outer ring [see Fig. 3(c)], which in turn leads to a decrease in frequency for the corresponding mode in the susceptibility spectrum of the inner ring [see Figs. 3(a) and 3(b)]. The frequency of the uniform resonance mode of the inner ring at ~ 11.5 GHz remains largely unchanged with varying outer ring widths at this separation. However, a large variation in the maximum amplitude of the susceptibility can be observed with varying outer ring widths [see Fig. 3(d)]. For an outer ring width of ~ 100 nm, where the eigenfrequencies of each individual ring are similar, there is enhancement of the amplitude of oscillation corresponding to the uniform resonance of the inner ring [see Fig. 3(d)]. When the system is perturbed by pulse excitation, the demagnetising field acts to align the magnetisation anti-parallel in each adjacent ring, resulting in frequency beating and modulation [see Figs. 4(a) and 4(b)].

C. Ring separation

The width of the outer ring is kept fixed at 100 nm to investigate the effect of ring separation for a pair of concentric rings. A thickness of 20 nm is chosen again for each ring, and the separation is varied in the region of 10–600 nm, while all geometric parameters of the inner ring are kept fixed. The initial magnetisation state and dynamic susceptibility were calculated as in Subsec. III B (Ring width). The

FIG. 4. (a) The y-component of the magnetisation following pulse excitation for a single ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm. (b) The y-component of the magnetisation following pulse excitation for an inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm coupled with an outer ring. The outer ring has an outer diameter of 1000 nm, a width of 150 nm, and a thickness of 20 nm. The magnetisation response $M_y(t)$ of the inner (black) and outer (orange) ring is shown in the inset in (b).

amplitude of oscillation corresponding to the uniform resonance of the inner ring is found to depend in a complex way on the ring separation. The imaginary component of the susceptibility increases in amplitude with reducing separation from 400 nm to 100 nm, reaching a local maximum amplitude at ~ 200 nm, before decreasing and reaching a local minimum at ~ 50 nm [see Figs. 5(a), 5(b), and 5(d)]. For separations smaller than ~ 50 nm, the susceptibility again increases, reaching a maximum value at ~ 12.5 nm [see Fig. 5(d)]. The shift in the frequency of the uniform resonance mode of the inner ring is found to decrease with increasing ring separation [see Fig. 5(c)]. The frequency of the ferromagnetic resonance mode tends towards the dashed black line (representing a single nanoring) with increasing separation in Figs. 5(a) and

FIG. 5. (a) Real and (b) imaginary components of the susceptibility for an inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm interacting with an outer ring with a width of 100 nm, a thickness of 20 nm, and a ring separation in the range of 50–600 nm. (c) The frequency of the ferromagnetic resonance of the inner ring with an outer diameter of 500 nm, a width of 100 nm, and a thickness of 20 nm plotted against the ring separation. (d) The amplitude of the imaginary component of the susceptibility corresponding to the ferromagnetic resonance for the inner ring plotted against the ring separation.

FIG. 6. Normalized spatial Fourier transform of the y-component of the magnetisation following pulse excitation for concentric rings with a width of 100 nm, a thickness of 20 nm, and ring separations of (a) 300 nm, (b) 100 nm, (c) 50 nm, and (d) 12.5 nm. The outer diameter and thickness of the inner ring are kept fixed at 500 nm and 20 nm, respectively. The frequencies shown in (a)–(d) represent the ferromagnetic resonance frequency of the inner ring for each ring separation.

5(b), as the magnetostatic interaction between the concentric rings becomes vanishingly small.

The normalized spatial distribution of the ferromagnetic resonance for the inner concentric ring is shown in Figs. 6(a)–6(d) for varying ring separations. Here, the Fourier transform of the y-component of the magnetisation is calculated following pulse excitation for every cell in the simulation grid. The amplitude of the dynamic susceptibility is found to depend in a complex way on the variation of the spatial localization of the uniform mode (see Figs. 5 and 6). In Fig. 6, the frequencies shown in (a)–(d) correspond to the frequency of the ferromagnetic resonance mode of the inner ring, which shifts downwards in frequency with decreasing ring separation [see Fig. 5(c)]. Spatially inhomogeneous resonance modes emerge in the outer ring at these frequencies, resulting from the long-range stray fields which decay across the width of the outer ring.

IV. CONCLUSION

The high frequency dynamic susceptibility of concentric permalloy nanorings with vortex domain structures with a thickness of 20 nm, a width in the range of 100 nm–250 nm, and a separation in the range of 10 nm–600 nm was investigated by micromagnetic simulations. The frequency of oscillation was found to be significantly impacted by the magnetostatic interaction between concentric nanorings and could be modulated by a variation in the ring separation and width. The variation in amplitude of the real and imaginary components of the dynamic susceptibility was found to correspond to a variation in the spatial distribution of the ferromagnetic resonance mode. An increase in the amplitude of the dynamic susceptibility at the ferromagnetic resonance frequency was observed for concentric nanorings with similar eigenfrequencies and dimensions.

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