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Abstract. We present a fast, noise-efficient, and accurate estimator for material separation using photon-counting x-ray detectors (PCXD) with multiple energy bin capability. The proposed targeted least squares estimator (TLSE) is an improvement of a previously described A-table method by incorporating dynamic weighting that allows the variance to be closer to the Cramér–Rao lower bound (CRLB) throughout the operating range. We explore Cartesian and average-energy segmentation of the basis material space for TLSE and show that, compared with Cartesian segmentation, the average-energy method requires fewer segments to achieve similar performance. We compare the average-energy TLSE to other proposed estimators—including the gold standard maximum likelihood estimator (MLE) and the A-table—in terms of variance, bias, and computational efficiency. The variance and bias were simulated in the range of 0 to 6 cm of aluminum and 0 to 50 cm of water with Monte Carlo methods. The Average-energy TLSE achieves an average variance within 2% of the CRLB and mean absolute error of $3.68 \pm 0.06 \times 10^{-6}$ cm. Using the same protocol, the MLE showed variance within 1.9% of the CRLB ratio and average absolute error of $3.10 \pm 0.06 \times 10^{-6}$ cm but was 50 times slower in our implementations. Compared with the A-table method, TLSE gives a more homogeneously optimal variance-to-CRLB ratio in the operating region. We show that variance in basis material estimates for TLSE is lower than that of the A-table method by as much as ~36% in the peripheral region of operating range (thin or thick objects). The TLSE is a computationally efficient and fast method for material separation with PCXD, with accuracy and precision comparable to the MLE. © 2017 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.JMI.4.2.023503]

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1 Introduction

Conventional computed tomography (CT) images display the linear attenuation coefficient (often represented in Hounsfield units) of the material at a single effective energy.¹ This value represents the tissue property in the voxel but is affected by the x-ray spectrum used. Conventional CT images can also suffer from beam hardening artifacts. In contrast, x-ray spectral information can be used to reconstruct images solely of object properties (i.e., independent of energy) and can, in principle, be free of beam hardening artifacts. Macovski and Alvarez² introduced a theoretical framework for material decomposition. Away from absorption edges, the energy-dependent linear attenuation coefficient $\mu(E)$ can be approximated as a weighted sum $\mu(E) = a_1 f_1(E) + a_2 f_2(E)$ of two basis functions $f_1(E)$ and $f_2(E)$. The two basis functions can be the energy dependence of the two major physical interaction phenomena—photoelectric effect and Compton scattering—or two basis materials. In the current work, we use water and aluminum as the two basis materials and write the attenuation line integral at energy E as

$$\int \mu(E) dl = t_{\text{H}_2\text{O}} \mu_{\text{H}_2\text{O}}(E) + t_{\text{Al}} \mu_{\text{Al}}(E), \quad (1)$$

where $t_{\text{H}_2\text{O}}$ and t_{Al} are the basis material lengths for water and aluminum, respectively. Equation (1) shows that away from K-edges, a transmission through any material can be effectively represented as attenuation by a combination of the two basis materials. An extension to three or more basis functions requires additional use of K-edge materials, or constraints on volume or mass conservation, which have been previously studied,^{3–5} but are beyond the scope of this paper.

For two material decompositions, estimation of $\tilde{\mathbf{t}} = (\tilde{t}_{\text{H}_2\text{O}}, \tilde{t}_{\text{Al}})$ requires measurements with at least two different energies. Clinical CT systems offer spectral imaging using two measurements. Photon-counting x-ray detectors, currently under active development, offer the possibility of separating the polychromatic x-ray spectrum into more than two energy bands. More than two measurements are needed for K-edge techniques, but even when estimating two basis materials, it has been shown that using more than two measurements can be advantageous.⁶

Dual-basis-material decomposition estimators for two spectral measurements have been well studied, and these estimators are well behaved in terms of accuracy, noise, and speed.^{7,8} However, when PCXD are used to measure more than two energy bins material estimation becomes overdetermined,^{6,9,10} and developing an estimator that is fast, accurate, and noise efficient is challenging. This overdetermined case for material decomposition application is studied in this paper.

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The maximum likelihood estimator (MLE) is the gold standard for this material decomposition problem as it is an asymptotically unbiased and noise-efficient estimator.^{4,10,11} However, it has several drawbacks. MLE is an iterative likelihood maximization algorithm and is computationally inefficient.^{4,6} It requires an accurate and realistic forward model, and modeling of PCXD is complicated by pixel variations and nonidealities, such as pulse pileup and spectral tailing.^{12,13} Errors could also arise from using an inaccurate model for the incident spectrum. Aging of x-ray source components can change the incident spectrum over time. Inherence in iterative methods are also possible problems of lack of convergence or convergence to a local minimum. The A-table method, proposed by Alvarez,⁶ attempts to overcome these limitations. Using measurements through a calibration phantom, this method allows estimation of material decomposition noniteratively and without any forward model. In this paper, we show that the A-table method is not always noise optimal. Our proposed estimator—targeted linearized least squares estimator (TLSE)—circumvents the noise degradation of the A-table method using locally optimized parameters. In this paper, we compare the performance of our proposed method with the MLE and A-table methods.

2 Methods

Spekt¹⁴ was used to generate a 120-kVp spectrum discretized at 1-keV intervals and filtered with 4 mm of aluminum. The beam was transmitted through 0 to 50 cm of water and 0 to 6 cm of aluminum with 200 steps of each material using tabulated attenuation coefficients.¹⁵ The performance of material decomposition estimators, especially those that take the logarithm of the measurements, in regions of very low counts can be problematic.¹⁶ To avoid this, the intensity of the beam before the Al filter was adjusted to 10^8 photons, so that photon counts after transmission through the thickest materials (and after binning) were in the order of 10^2 and the estimators would not run into photon starvation. Poisson noise was added for each ray to create 2000 realizations of each case.

In the initial simulations, the PCXD was modeled as having no spectral distortions (e.g., from pulse pileup, spectral tailing, and other effects). The detector had five abutting two-sided energy bins. The energy bin thresholds for the i 'th bin, l_i and u_i , were selected such that the number of photons in each of

the five energy bins is roughly equal at the center of the operating range of basis material thickness (25 cm for water and 3 cm for aluminum) (Fig. 1).

We used the Cramér–Rao lower bound (CRLB) to assess the noise performance of the algorithms. CRLB gives the estimate of minimum variance an unbiased estimator can achieve. The Fisher information matrix $\mathbf{F}$ for a multibin photon-counting detector system has previously been derived for the case that is uncorrelated between energy bins.¹⁷ As the CRLB gives the lower bound on the variance that can be achieved, we define the noise efficiency parameter R as

$$R = \frac{\text{Var}(\tilde{t})}{C}, \quad (2)$$

where $\text{Var}(\tilde{t})$ is the observed variance in a basis material with an estimator at that operating point, and C is the corresponding CRLB estimate of the basis material. R_{water} is shown for this study and will be referred to below as simply R . For an unbiased estimator, $R \geq 1$ as the CRLB is a lower bound, and the estimator is noise efficient if R is asymptotically close to 1.

2.1 Maximum Likelihood Estimator

Use of the MLE for material decomposition with multibin PCXD has been previously proposed,^{4,10,11} and is posed as

$$(\tilde{t}_{\text{H}_2\text{O}}, \tilde{t}_{\text{Al}}) = \arg\max \mathcal{L}^*(\mathbf{I} | t_{\text{H}_2\text{O}}, t_{\text{Al}}), \quad (3)$$

where $\tilde{t}_{\text{H}_2\text{O}}$ and $\tilde{t}_{\text{Al}}$ are the most likely estimates of the water and aluminum with $\mathbf{I}$ as the spectral measurement and is solved as maximization of the log-likelihood function $\mathcal{L}^*$ of computed transmission through different water and aluminum lengths, $t_{\text{H}_2\text{O}}$ and t_{Al} , respectively. $\mathcal{L}^*$ is obtained from the likelihood function based on the uncorrelated Poisson model. We used `fmincon`, a greedy, gradient-based optimization function in the MATLAB[®] Optimization Toolbox¹⁸ with the interior-point algorithm and boundary constraints set to -1.2 to 7.2 cm of Al and -10 to 60 cm of water.

The MLE is a noise-efficient estimator, and we find that R is close to 1 with a maximum deviation of $\sim 2\%$ for both basis materials across the operating range (Fig. 2 shows the R value for estimates of the water basis material; the aluminum basis

Fig. 1 (a) 120-kVp spectrum transmitted through combinations of materials showing energy bin thresholds. (b) Photon count histogram of detected signal at transmission through different thicknesses of water and aluminum.

Fig. 2 Noise performance of MLE, depicted as the R value across the operating range.

component behaves similarly), and it shows a negligible mean of absolute bias of $(3.10 \pm 0.06) \times 10^{-6}$ cm.

2.2 A-Table Estimator

Alvarez proposed a two-step method to address some shortcomings of the maximum likelihood (ML) estimator, with an initial estimate that is based on a linearized ML optimized at one operating point and the second step that corrects for residual bias away from the optimization point using a lookup table. Because the ML is linearized, the first step involves just matrix multiplication. The A-table therefore operates noniteratively, as opposed to ML, and also eliminates the need of an accurate forward model. The method is based on measurements of a step-wedge calibration phantom [Fig. 3(a)] so it does not require accurate knowledge of the spectrum or of the detector's energy response.

The linearized ML used in the first step of the A-table uses an $N \times N$ weighting matrix $\mathbf{W}$ that determines the statistical contribution from each of the N energy bins. It is crucial for noise performance. Alvarez used a $\mathbf{W}$ that is optimal for the center of the operating range to compute the linearized ML estimate for all measurements. As expected, the noise is optimal ($R \sim 1$) at the center of the operating range but degrades if the transmission is significantly higher or lower because that changes the measurement statistics, as shown at the top-right and bottom-left of Fig. 3(b). To assess the degradation of R , we divided the operating region into 8-by-8 rectangular segments and found the

maximum of the average R -values over these 64 segments was as high as 1.36 in our simulation.

2.3 Targeted Least Squares Estimator

TLSE is based on the A-table⁶ but uses a local weighting to improve the noise performance, accounting for the dependence of the measurement statistics on the object that is attenuating the beam. TLSE incorporates local statistics to yield noise-efficient performance. We divide the operating range (i.e., range of thicknesses of the two basis materials) into multiple smaller regions and choose $\mathbf{W}$ to optimize the noise performance in each region. In the first step, we begin with a linearized forward model

$$\begin{bmatrix} \ln\left(\frac{I_o[n=1]}{I[n=1]}\right) \\ \ln\left(\frac{I_o[n=2]}{I[n=2]}\right) \\ \vdots \\ \ln\left(\frac{I_o[n=N]}{I[n=N]}\right) \end{bmatrix} = \begin{bmatrix} \mu_{\text{H}_2\text{O}}[n=1] & \mu_{\text{Al}}[n=1] \\ \mu_{\text{H}_2\text{O}}[n=2] & \mu_{\text{Al}}[n=2] \\ \vdots & \vdots \\ \mu_{\text{H}_2\text{O}}[n=N] & \mu_{\text{Al}}[n=N] \end{bmatrix} \times \begin{bmatrix} t_{\text{H}_2\text{O}} \\ t_{\text{Al}} \end{bmatrix}. \quad (4)$$

Equation (4) is abridged into $\mathbf{P} = \mathbf{M}\mathbf{t}$. Data from the calibration phantom [Fig. 3(a)] measurements can be used to compute a matrix $\mathbf{M}$ for each local region using least squares. Once the matrix $\mathbf{M}$ is determined, the weighted least squares basis material estimates are given by

$$\hat{\mathbf{t}}^{\text{linear}} = \begin{bmatrix} \hat{t}_{\text{H}_2\text{O}}^{\text{linear}} \\ \hat{t}_{\text{Al}}^{\text{linear}} \end{bmatrix} = (\mathbf{M}'\mathbf{W}\mathbf{M})^{-1}\mathbf{M}'\mathbf{P}. \quad (5)$$

Equation (5) maps the negative log of normalized counts $\mathbf{P}$ into the two basis material estimates. We want to use the matrix $\mathbf{W}$ that minimizes noise in the estimates. The covariance of the material estimate $\Sigma_{\hat{\mathbf{t}}^{\text{linear}}}$ is given by

$$\Sigma_{\hat{\mathbf{t}}^{\text{linear}}} = (\mathbf{M}'\mathbf{W}\mathbf{M})^{-1}\mathbf{M}'\Sigma_{\mathbf{P}}\mathbf{M}(\mathbf{M}'\mathbf{W}\mathbf{M})^{-1}, \quad (6)$$

where $\Sigma_{\mathbf{P}}$, the covariance of $\mathbf{P}$ for measurements following Poisson distribution reduces to

Fig. 3 Noise performance of A-table depicted using the value of water.

$$\Sigma_{\mathbf{p}} = \begin{bmatrix} \frac{1}{I[n=1]} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{I[n=N]} \end{bmatrix}. \quad (7)$$

By choosing the weighting matrix

$$\mathbf{W} = \Sigma_{\mathbf{p}}^{-1} = \begin{bmatrix} I[n=1] & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I[n=N] \end{bmatrix}, \quad (8)$$

we obtain the noise-optimal estimate.⁶ The diagonal components of the weighting matrix $\mathbf{W}$ are the detected binned counts $\mathbf{I}$.

The term $(\mathbf{M}'\mathbf{W}\mathbf{M})^{-1}\mathbf{M}'$ in Eq. (6) can be precomputed, and the linearized ML estimate just involves simple matrix multiplication. The estimate is noise optimal only to the degree that $\mathbf{W}$ matches the measurement statistics. Therefore, $(\mathbf{M}'\mathbf{W}\mathbf{M})^{-1}\mathbf{M}'$ is computed separately for each region with $\mathbf{W}$ selected at the centroid of each region. Note that the noise optimal matrix is invariant to changes in overall intensity. It depends of the relative, not absolute, counts in the energy bins.

The only aspect left is selection of the proper region for each data measurement. We can do this with a first estimate using a global matrix that is used as a segment localizer. The second step is a localized targeted least squares estimate giving $\tilde{t}_{\text{H}_2\text{O}}^{\text{linear}}$ and $\tilde{t}_{\text{Al}}^{\text{linear}}$. The bias correction is also specific to the localized region and uses a two-dimensional (2-D) bicubic spline interpolation,

Fig. 4 Schematic for TLSE algorithm using Cartesian segmentation.

(a)

(b)

Fig. 5 (a) Normalized spectrum at the center of different regions in basis material space showing similar trend and (b) average energy of basis material space showing 3, 4, and 5 piece segments based on average energy.

yielding the final estimates $\tilde{t}_{\text{H}_2\text{O}}$ and $\tilde{t}_{\text{Al}}$. Figure 4 shows the TLSE algorithm with a Cartesian segmentation in which the operating range is segmented into a rectangular grid.

The optimality of the estimate depends on the matching of the relative counts in the energy bins to those in $\mathbf{W}$. The noise performance of the A-table method, Fig. 3(b), showed a trend from thin to thick objects (bottom left to upper right) and less dependence on the substitution of water for aluminum (top left to lower right), suggesting that perhaps segmentation based on the average energy of the measured spectrum may be efficient. To demonstrate this further, Fig. 5(a) shows a plot of normalized spectrum (or normalized diagonal components of $\mathbf{W}$) for several operating points. Optimal weights for segments that fall along the upper-left to lower-right diagonals are relatively close to each other. Thus, segmentation based on spectral shape may minimize the number of segments required for noise optimization.

We used the average energy of the spectrum,

$$E_{\text{ave}} = \frac{\sum_{i=1}^N I[i] \times E[i]}{\sum_{i=1}^N I[i]}, \quad (9)$$

where $I[i]$ is the counts in the i 'th energy bin and $E[i]$ is the midpoint of the i 'th energy bin used as a scalar surrogate of spectral shape for segmentation. Specifically, for our operating range, E_{ave} varied from 51 to 91 keV, and we divided this into uniformly spaced values of E_{ave} . Some sample of average segmentations are shown in Fig. 5(b).

In average-energy TLSE (Fig. 6), segment localization is performed using E_{ave} rather than with a global linearized ML. The steps following segment selection are common to both variants of the TLSE method and comprise a locally linearized estimate followed by a local bias correction.

2.4 Evaluation

In addition to using the noise efficiency parameter R and the mean bias, we calculated the L2-norm of the deviation of the weighting matrix from the optimal weighting matrix

$$W_{\text{error}} = \left\| \left\{ \frac{\text{diag}(\mathbf{W}_{\text{optimal}})}{\text{trace}(\mathbf{W}_{\text{optimal}})} - \frac{\text{diag}(\mathbf{W}_{\text{segment}})}{\text{trace}(\mathbf{W}_{\text{segment}})} \right\} \right\|, \quad (10)$$

Fig. 6 Block diagram for the TLSE with average-energy segmentation.

where $\mathbf{W}_{\text{optimal}}$ is the optimal weighting matrix at any point within a region, and $\mathbf{W}_{\text{segment}}$ is the weighting matrix used for the segment. For each segment, we use the 95th percentile error $W_{\text{error}}^{95\%}$ to compare the deviation of the weighting compared with the optimal weighting.

Any noise improvements shown by R are directly applicable in projection (radiographic) spectral x-ray imaging. However, in CT applications, the image value at any point is generated from many rays, each of which passed through different object compositions and thicknesses. To study this effect, we simulated CT scans of a head, thorax, medium and large abdomens, and a pelvis. We started with publicly available DICOM images of a head, thorax, medium abdomen, and pelvis. To simulate a large abdomen, low-density water (−150 HU) was inserted on the outside of the medium abdomen. To generate synthetic photon-counting transmission measurements, image pixels were defined to be weighted sums of water and cortical bone. Pixels with values <500 HU were defined as water at a density that would achieve the pixel value, and anything higher was defined as a linear combination of water and cortical bone with their relative density constrained to sum to one. Although this approach does not achieve compositions that exactly match patients, it was a simple and reasonable test of the methods. Polychromatic forward projections using a 120-kVp spectrum of these synthetically segmented objects were computed, and projection domain basis material estimates were generated using the A-table and average-energy TLSE. Variance maps were produced by backprojecting variance estimates for each ray.¹⁹ Noise in the two basis material estimates is anticorrelated

and tends to cancel when equivalent monochromatic images are formed. To explore the variance in equivalent monochromatic images, the basis material projections were reconstructed with filtered backprojection, and the basis images were linearly combined to produce 65-keV equivalent monochromatic images.

All of the above tests were conducted assuming a detector with perfect energy response prior to energy binning. To explore whether the TLSE method would function well with detectors that have imperfect energy response, we simulated transmission through aluminum and water thicknesses for a detector with the expected energy response of a very thick CdTe detector with $1 \times 1 \text{ mm}^2$ elements.²⁰ The simulated detector exhibited charge sharing and escape of secondary photons but not pulse pileup. On average, 79.9% of detected photons contributed photo-peak events; the rest produced pulses at lower than their true energies.

3 Results

The noise efficiency of the two proposed estimators is shown in Fig. 7. The performance of MLE and A-table is shown in Figs. 2 and 3(b), respectively. For Cartesian TLSE with 4×4 segments, R is uniformly within 4% of unity [Fig. 7(a)]. Figure 7(b) shows that the TLSE with average-energy segmentation provides noise performance within 2% of CRLB with just five segments. When tested with the nonideal spectral response a 1-mm² CdTe detector, we found similar noise improvements compared with the A-table, indicating that spectral weighting is still crucial for noise optimal basis material estimation.

The local maximum R particularly highlights noise efficiency of estimators in different operating conditions. As it is evident in Table 1, the local maximum R of average-energy TLSE is significantly improved (with local maximum R value of 1.02) compared with the A-table (with a value of 1.36).

A further segment number analysis to compare Cartesian and average-energy TLSE is shown in Table 2. Average energy segmentation achieves comparable minimum local R using far fewer segments. This is consistent with Fig. 8, which shows that for the same number of regions, average energy-based segmentation offers the least $W_{\text{error}}^{95\%}$.

It is also apparent in Fig. 8 that segmenting the operating range based on the amount of water or aluminum gives relatively bad noise performance. Similarly, the diminishing returns of increasing the number of segments for better noise property are evident.

Fig. 7 Variance-to-CRLB ratio obtained for (a) Cartesian TLSE and (b) average-energy TLSE. Lines show the segmentation.

Table 1 Average R ratio for different methods (uncertainty shown as standard error).

Method	Segments	Average R (all)	Maximum average R of (8×8) tiles	Mean error l (10^{-6} cm)
MLE	N/A	1.0014 ± 0.0002	1.0186 ± 0.0002	3.10 ± 0.06
A-table	N/A	1.0567 ± 0.0002	1.3625 ± 0.0002	2.06 ± 0.06
Cartesian TLSE	4×4 (16)	1.0086 ± 0.0002	1.0386 ± 0.0002	1.22 ± 0.06
Average-energy TLSE	5	1.0068 ± 0.0002	1.0193 ± 0.0002	3.68 ± 0.06

Table 2 Average R and maximum variance (8×8 grid) shown for Cartesian and average-energy TLSE segmentation (uncertainty shown as standard error).

Cartesian TLSE			Iso-average-energy TLSE		
Number of segments	Average R	Maximum R (8×8) tiles	Number of segments	Average R	Maximum R (8×8) tiles
2×2	1.0201 ± 0.0002	1.1183 ± 0.002	3	1.0137 ± 0.0002	1.0331 ± 0.0002
3×3	1.0109 ± 0.0001	1.0644 ± 0.0002	4	1.0088 ± 0.0002	1.0209 ± 0.0002
4×4	1.0086 ± 0.0002	1.0386 ± 0.0002	5	1.0068 ± 0.0002	1.0183 ± 0.0002
5×5	1.007 ± 0.0002	1.0285 ± 0.0002	6	1.0058 ± 0.0002	1.0193 ± 0.0002

Table 1 also shows the mean bias as measures of the accuracy of different estimators. As expected, this error is similar and negligible for all the methods tested.

Figure 9 shows the simulated anthropomorphic objects (a)–(e). Figures 9(f)–9(j) show 2-D histograms of the region of the basis material space that was used for each scan. Also shown in Fig. 9 are noise map ratios (standard deviation of the A-table divided by that of TLSE) for water basis images (k–o) and for monochromatic images (p–t). As shown by the noise map ratios in Fig. 9, TLSE achieves lower noise than the A-table for the different simulated CT scans for both water basis images and 65-keV monoenergetic images. In water images [Figs. 9(k)–9(o)], the standard deviation is improved by 10% to 20%. Due to the negative correlation of the noise in the basis images, the improvement is $<3\%$ for monoenergetic images [Figs. 9(p)–9(t)].

Fig. 8 95th percentile of W_{error} for different segmentation methods.

We evaluated the computational efficiency of material estimation using the runtime for 2000 trials at the center of the operating region. The runtime of an iterative method such as ML is dependent on many aspects, such as the optimization algorithm and constraints. In our simulation, the average-energy-based TLSE is on average 50 times faster than MLE.

4 Conclusions

Projection-domain material decomposition algorithms are used for applications including virtual noncontrast imaging, iodine and other contrast media imaging, and effective monochromatic images. In an overdetermined task with multibin PCXDs, attaining accurate and noise optimal estimates in a reasonable computation time is a challenge. MLE is noise efficient and can be accurate, but it is computationally burdensome and requires an accurate forward model. The A-table method is more computationally efficient and does not need a forward model but can have significant noise degradation when the operating region (i.e., the range of basis material thicknesses) is large. TLSE provides a good compromise between speed, accuracy, and noise efficiency compared with the previous methods.

In our tests, compared with the A-table, TLSE can provide noise benefit of up to 36% in radiographic imaging and 19% in basis material CT imaging (analogous to virtual noncontrast images). Due to the negative correlation of basis material errors, noise improvements are unlikely to be visible in effective monochromatic images.

In our simulations, we assumed that all measurements after energy binning had large enough counts to avoid photon starvation and bias after the logarithm. The difficulties resulting from low counts are expected to be similar for TLSE and A-table. For systems and applications in which photon starvation may be observed, algorithmic improvements may be needed.

Fig. 9 Noise performance analysis for TLSE versus A-table for virtual noncontrast and effective monochromatic imaging with simulated scans. (a)–(e) Phantoms displayed with fields-of view of 25, 30, 30, 50, and 30 cm, respectively, (f)–(j) material composition 2-D histogram with 200 bins from 0 to 50 cm water and 0 to 6 cm bone with color map as log of the number of times each occurred in the sinogram, (k)–(o) ratio of standard deviations (A-table divided by average-energy TLSE) of water basis estimates, and (p)–(t) 65-keV effective energy images.

Detectors that exhibit flux-dependent spectral distortion (e.g., from pulse pileup) also present challenges. The A-table and TLSE may be able to deal with these nonidealities if the calibration and object scans are obtained with the same incident flux (kVp and mA). Algorithms that can adapt to varying incident intensity would be helpful. These are all interesting subjects for future study.

Disclosures

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