Multichannel myopic deconvolution in underwater acoustic channels *via* low-rank recovery

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This paper presents a technique for solving the multichannel blind deconvolution problem. The authors observe the convolution of a single (unknown) source with K different (unknown) channel responses; from these channel outputs, the authors want to estimate both the source and the channel responses. The authors show how this classical signal processing problem can be viewed as solving a system of bilinear equations, and in turn can be recast as recovering a rank-1 matrix from a set of linear observations. Results of prior studies in the area of low-rank matrix recovery have identified effective convex relaxations for problems of this type and efficient, scalable heuristic solvers that enable these techniques to work with thousands of unknown variables. The authors show how *a priori* information about the channels can be used to build a linear model for the channels, which in turn makes solving these systems of equations well-posed. This study demonstrates the robustness of this methodology to measurement noises and parametrization errors of the channel impulse responses with several stylized and shallow water acoustic channel simulations. The performance of this methodology is also verified experimentally using shipping noise recorded on short bottom-mounted vertical line arrays. © 2017 Acoustical Society of America.

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I. INTRODUCTION

Acoustic remote sensing, such as ocean tomography and geoaoustic inversion, typically relies on a controlled active source to probe the environment.^{1,2} In this case, the channel impulse response (CIR), which is the Green's function for a given ocean environment, can readily be obtained from recorded signals since the broadcast source signal is known. A potential alternative for performing acoustic remote sensing is to use sources of opportunity (e.g., radiating ships) instead of deploying and operating controlled sources. However, using sources of opportunity means that the broadcast source signal is then unknown; thus, estimating both the CIR and the actual source signal using only recorded signals, a process commonly referred to as blind deconvolution, is generally an ill-posed problem without any additional *a priori* information or assumptions, especially for propagating sound through an ocean waveguide as it is distorted by multipath effects. This paper develops a scalable methodology

that solves the multichannel deconvolution problem for a source of opportunity radiating in an ocean waveguide using *a priori* information about CIRs between the source and the array of receiver elements. Because the method is not performed in a totally blind manner, it is referred to as myopic deconvolution.

For decades, the multichannel deconvolution problem has been an active field of research with applications in wireless communication,^{3,4} image restoration,^{5,6} seismic signal processing,⁷ and underwater acoustics.^{8–11} The essence of blind (or myopic) deconvolution rests on the exploitation of either channel structures¹² or input source properties, or both. In the underwater acoustics community, the blind deconvolution problem has been studied by many researchers and several algorithms have been proposed.^{13–16} Traditional multichannel blind deconvolution methods can broadly be categorized into two main approaches: deterministic methods and statistical methods.¹⁷ Deterministic methods, such as subspace methods,¹⁸ usually reduce the blind deconvolution problem to a regularized least squares problem, hence critically depend on an accurate estimate of the duration of the CIR (i.e., channel

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length). These methods also entail high computational cost and sensitivity to noise. Statistical methods^{19,20} typically rely on statistical information about an unknown source signal, which, in practice, require a long recording duration to build up the statistical information.

This paper examines a sensing scenario under the multiple-channel framework [Fig. 1(a)], in which a network of distributed sensors (e.g., a vertical line array) listens to the same source [Fig. 1(b)]. Compared to traditional multichannel blind deconvolution approaches, the proposed myopic deconvolution approach does not rely on accurate channel length estimation or any statistical information about the input source, so it can be used with short recording duration (e.g., a single snapshot). Instead, the proposed approach investigates the multichannel deconvolution problem as a low-rank matrix recovery problem, which significantly differs from the problem formulation of the aforementioned blind deconvolution approaches. Compared to the recently developed single-channel blind deconvolution method using a similar formulation,²¹ the multichannel deconvolution method exploits the spatial diversity of CIRs across channels to solve the blind deconvolution problem.

Under the general deconvolution framework [Fig. 1(a)], the multichannel deconvolution problem is typically posed as a bilinear system of equations that can be recast as a system of linear equations whose solution has a rank-1 structural constraint. This system of linear equations is solved using the low-rank matrix recovery tools developed in recent years.^{22,23} The proposed approach estimates CIRs and source by solving an optimization program. This formulation has the advantage that *a priori* information about CIRs can be incorporated by adding constraints. Furthermore, this also allows one to use standard numerical techniques to solve the problem on a relatively large scale. The numerical experiments in Sec. IV demonstrate recovery of 100 channels with 200 taps each from a source of length 1000. Additionally, when the dimension of the subspace containing the admissible CIRs is consistent with the available *a priori* information, and it is small compared to the length of measurements, the myopic deconvolution can be achieved using a small number of receivers.

The remainder of this paper is organized as follows. Section II presents the theoretical formulation of the proposed myopic deconvolution method based on recasting the multichannel deconvolution problem as a low-rank matrix recovery problem. A general formulation of the method

based on the assumption that CIRs are time-limited is first introduced, and the formulation is then extended to CIRs that can be represented by a general linear model. Section III introduces a linear model for parameterizing multipath CIRs in ocean waveguides in order to incorporate *a priori* knowledge of the arrival-time structure of the CIR in the low-rank matrix recovery problem. Section IV first investigates the performance and the robustness of the proposed deconvolution method using numerical simulations of general stylized channels such as time-limited channels and pulse-like channels. This section then presents numerical simulation results for realistic acoustic channels in an ocean waveguide and deconvolution results using at-sea data recorded in shallow water using bottom-mounted short vertical line arrays. Finally, Sec. V summarizes the findings of this article.

II. MULTICHANNEL DECONVOLUTION VIA LOW-RANK RECOVERY

A. The multichannel deconvolution problem scenario

The multichannel deconvolution problem is illustrated in Fig. 1(a). A common source signal $s(t)$ drives K different channels with the CIRs $h_1(t), h_2(t), \dots, h_K(t)$. Over a period of time, we observe samples of convolution outputs $y_1(t), \dots, y_K(t)$, where $y_k(t) = s(t) * h_k(t)$. From these observations alone, we wish to estimate both $h_k(t)$ ($k = 1, \dots, K$) and $s(t)$. This problem has direct application in underwater acoustic passive sensing as shown in Fig. 1(b): for instance, assuming sound from a shipping source that propagates toward each hydrophone based on each CIR $h_k(t)$. So under this model, each hydrophone from a vertical line array (forming multiple channels) listens to the same ship noise. The goal is to deconvolve the CIRs and the source by measuring only the outputs of a hydrophone array. This problem does not have a unique solution if we know nothing about the source signal $s(t)$ or the CIR $h_k(t)$. However, if we make some structural assumptions about the channels, namely that they live in some known subspace with a low dimension, then the problem, which becomes well posed, can be solved by a fast and scalable algorithm.

If we treat both the source $s(t)$ and the CIR $h_k(t)$ as unknowns, and our observations consist of linear combinations of entries of $s(t)$ and the $h_k(t)$ multiplied by one another, multichannel blind deconvolution is challenging since it is equivalent to solving a system of bilinear equations. To precisely state the problem, an inverse problem is

(a) Problem illustration.

(b) Underwater acoustic channel model.

FIG. 1. (Color online) Multichannel deconvolution problem: (a) Problem illustration. (b) Underwater acoustic channel model.

written down using the language of linear algebra. We assume that both the $s(t)$ and $h_k(t)$ are band limited, so recovering discretized samples $h_k[n]$ of $h_k(t)$ and $s[n]$ of $s(t)$, spaced at the corresponding Nyquist rate or closer, is the same as recovering the CIRs themselves and the source (limited to this frequency band). We define the general convolution model as

$$y_k[\ell] = \sum_{n=-\infty}^{\infty} h_k[n]s[\ell - n], \quad \text{for } \ell = 0, 1, \dots, L - 1. \quad (1)$$

This problem does not have a unique solution if we assume $s[n]$ and $h_k[n]$ have no structure, because many signals and CIRs can produce the same observations $y_k[\ell]$. Even if we were to observe an infinite number of samples $\{y_k[\ell], \ell \in \mathbb{Z}\}$ over an infinite amount of time and treat the channels jointly, the $h_k[n]$ and $s[n]$ would still not be uniquely determined.

B. FIR channels

The story does improve, however, if we make structural assumptions about the CIR $h_k[n]$. For example, we might assume that the $h_k[n]$'s are time-limited [i.e., finite impulse response (FIR) channels] such that $h_k[n]$ can be nonzero only for $0 \leq n \leq N - 1$. Then we can limit the index summation in the convolution model Eq. (1)

$$y_k[\ell] = \sum_{n=0}^{N-1} h_k[n]s[\ell - n], \quad \text{for } \ell = 0, 1, \dots, L - 1. \quad (2)$$

Equation (2) has $N + (L + N - 1) = L + 2N - 1$ unknowns, and L bilinear equations that combine these unknowns in different ways: each sample of y_k involves a sum over products of various combinations of $\{h_k[0], \dots, h_k[N - 1]\}$ and $\{s[-N + 1], \dots, s[0], \dots, s[L - 1]\}$. Treating the channels jointly adds more equations and more unknowns. However,

since the channels are all being driven by a common source, the number of unknowns is smaller than the number of equations. The system of equations corresponding to the observations $\{y_k[\ell], \ell = 0, \dots, L - 1; k = 1, \dots, K\}$ has KL equations and $KN + (L + N - 1) = (K + 1)N + L - 1$ unknowns. Thus, when the number of channels K is large and $L > N$, the number of equations in fact is much larger than the number of unknowns.

Only comparing the number of equations to the number of unknowns does not tell the whole story, especially since the equations are non-linear (bilinear in the unknowns of $\mathbf{s}, \mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K$). The observations above reveal in some loose sense that as the number of observations grows, so does the ratio of the information we have (equations) versus the information we do not (variables). We take another step toward formalizing this trade-off by recasting the bilinear equations in Eq. (2) as a system of linear equations with a rank constraint. The common source $\mathbf{s} \in \mathbb{R}^{L+N-1}$ used in Eq. (2) and the CIR $\mathbf{h}_k$ can be defined as a vector as below

$$\mathbf{s} = \begin{bmatrix} s[-N+1] \\ \vdots \\ s[-1] \\ s[0] \\ s[1] \\ \vdots \\ s[L-1] \end{bmatrix}, \quad \mathbf{h}_k = \begin{bmatrix} h_k[0] \\ h_k[1] \\ \vdots \\ h_k[N-1] \end{bmatrix}. \quad (3)$$

Then we can arrange all pairs of variables appearing in the sum in Eq. (2) in a large $(L + N - 1) \times NK$ matrix $\mathbf{X}_0$

$$\mathbf{X}_0 = [\mathbf{s}][\mathbf{h}_1^T \quad \mathbf{h}_2^T \quad \dots \quad \mathbf{h}_K^T] = [\mathbf{sh}_1^T \quad \mathbf{sh}_2^T \quad \dots \quad \mathbf{sh}_K^T]. \quad (4)$$

Each observation $y_k[\ell]$, an example of which is shown graphically in Fig. 2, is now a linear combination of entries in $\mathbf{X}_0$. Concatenating the observations from each channel

FIG. 2. (Color online) To illustrate the proposed approach, the matrix $\mathbf{X}_0$ defined in Eq. (4) is explicitly written with a number of measurements per channel of $L = 9$, channel length $N = 3$, and a number of channels $K = 3$. Each observation $y_k[\ell]$ is a sum along one of the skew diagonals of the a submatrix of $\mathbf{X}_0$, as illustrated by the red lines above. Recovering an $L + N - 1 \times NK$ matrix from measurements of this type is virtually impossible, but explicitly incorporating the fact that $\mathbf{X}_0$ has rank-1 structure into the recovery makes the recovery possible for appropriate values of L, N, K .

into a single vector $\mathbf{y} \in \mathbb{R}^{KL}$, we form a linear system of equations

$$\mathbf{y} = \mathcal{A}(\mathbf{X}_0), \quad (5)$$

where $\mathcal{A}$ takes sums over skew diagonals of submatrices $\mathbf{s}\mathbf{h}_k^T$. We can write out this linear operator $\mathcal{A}$ in the form of inner products of measurement matrix and the rank-1 matrix $\mathbf{X}_0 = \mathbf{s}\mathbf{h}^T$, where $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_1^T, \dots, \mathbf{h}_K^T]^T$. Measurement matrix $\mathbf{A}_{k\ell}$ defines the skew-diagonal sum operation yielding observation $y_k[\ell]$, and we can write $y_k[\ell]$ as below.

$$y_k[\ell] = \sum_{i,j} A_{k\ell}[i,j] X_0[i,j] = \text{trace}(\mathbf{X}_0^T \mathbf{A}_{k\ell}) = \langle \mathbf{A}_{k\ell}, \mathbf{X}_0 \rangle. \quad (6)$$

An example of $\mathbf{A}_{k\ell}$ is given in Eq. (11). Then for all measurements across channels, we form the following expression:

$$\mathbf{y} = \begin{bmatrix} y_1[0] \\ y_1[1] \\ \vdots \\ y_K[L-1] \end{bmatrix} = \mathcal{A}(\mathbf{X}_0) = \begin{bmatrix} \langle \mathbf{A}_{10}, \mathbf{X}_0 \rangle \\ \langle \mathbf{A}_{11}, \mathbf{X}_0 \rangle \\ \vdots \\ \langle \mathbf{A}_{K(L-1)}, \mathbf{X}_0 \rangle \end{bmatrix}. \quad (7)$$

We can now treat the recovery of the CIR $\mathbf{h}_k$ and the source $\mathbf{s}$ as a *matrix recovery* problem. The vector $\mathbf{y}$ contains KL different linear combinations of entries of a $(L+N-1) \times NK$ matrix. Regardless of the values of L , N , and K , the number of equations will always be less than the number of entries in the unknown matrix, indicating that the system of linear equations in Eq. (5) is underdetermined. However, by definition in Eq. (4), $\mathbf{X}_0$ is a rank-1 matrix. To recover $\mathbf{X}_0$ from $\mathbf{y}$, we might search for a $(L+N-1) \times NK$ matrix that satisfies

$$\mathcal{A}(\mathbf{X}) = \mathbf{y}, \quad \text{rank}(\mathbf{X}) = 1. \quad (8)$$

For an arbitrary $\mathbf{y}$, a solution $\hat{\mathbf{X}}$ that satisfies the constraints above exactly may or may not exist. Given $\mathbf{y}$, we might search instead for such a feasible solution $\hat{\mathbf{X}}$ by solving

$$\min_{\mathbf{X}} \text{rank}(\mathbf{X}) \quad \text{subject to} \quad \mathcal{A}(\mathbf{X}) = \mathbf{y}. \quad (9)$$

The deconvolution problem is now recast as a low-rank matrix recovery problem, which can then be solved using low-rank matrix recovery tools developed in recent years, such as nuclear norm minimization, low-rank approximation, iterative hard thresholding, and alternating methods.²⁴ Studies in this area of low-rank matrix recovery have identified effective convex relaxations^{25,26} for problems of this type and efficient, scalable heuristic solvers that enable these techniques to work with thousands of unknown variables. A description of the heuristic solver used in this paper is given in the [Appendix](#). The Burer-Monteiro heuristic solver is based on the low-rank factorization method and notably efficient in memory. Numerical simulations using this solver are presented in Sec. IV.

C. A general linear model for the CIR

The linear operator $\mathcal{A}$ in Sec. II B was derived from the assumption that the CIR is time-limited, so the length of each CIR is N . More generally, we can incorporate *a priori* information about the CIR (e.g., expected arrival-time structure in a multipath environment) into a general linear model, that is, we can assume each CIR $\mathbf{h}_k$ can be written as

$$\mathbf{h}_k = \mathbf{C}_k \mathbf{u}_k, \quad k = 1, \dots, K, \quad (10)$$

where $\mathbf{C}_k \in \mathbb{R}^{N \times D}$ will be called the channel subspace matrix, $\mathbf{u}_k \in \mathbb{R}^D$ is the coefficient vector, N is the length of the vector $\mathbf{h}_k$, and D is the number of columns of the matrix $\mathbf{C}_k$. Therefore, the CIR $\mathbf{h}_k$ lives in a subspace that is the linear span of the columns of the matrix $\mathbf{C}_k$. The time-limited CIR example in Sec. II B is a special case, which corresponds to $\mathbf{C}_k = \mathbf{I}^{N \times N}$. We shall note here the length of the channel N no longer needs to satisfy the constraint $L > N$, but we require that the length of the channel coefficient vector D satisfies $L > D$, since the number of unknowns of the CIR per channel is now D instead of N .

With the subspace model in place, the goal is now to recover the source $\mathbf{s}$ and the channel subspace coefficients $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K$. As before, we recast this problem as recovering the rank-1 matrix $\mathbf{X}_0 = [\mathbf{s}\mathbf{u}_1^T \quad \mathbf{s}\mathbf{u}_2^T \quad \dots \quad \mathbf{s}\mathbf{u}_K^T]$. Each observation $y_k[\ell]$ is again a linear combination of the entries in $\mathbf{X}_0$. Here the new linear operator $\mathcal{A}$ is implemented with the channel model embedded. For example, we can write out explicitly how to calculate $y_k[0]$ using linear algebra with $N=4$

$$y_k[0] = [\mathbf{s}^T] \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} [\mathbf{h}_k] = \langle \mathbf{A}_{k0}, \mathbf{s}\mathbf{u}_k^T \rangle, \quad (11)$$

$$\mathbf{A}_{k0} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{C}_k.$$

In this case, each measurement matrix is the corresponding skew diagonal sum matrix times the channel subspace matrix. We still keep the general notation $\mathbf{y} = \mathcal{A}(\mathbf{X}_0)$ the same.

III. LINEAR MODELS FOR MULTIPATH CIR IN OCEAN WAVEGUIDES

In Sec. II, the multichannel deconvolution problem has been recast as a low-rank matrix recovery problem, and we

require the ambient dimension of the measurements to be bigger than the dimension of the CIR to solve the problem. Because *a priori* knowledge of the CIR is usually available, dimension reduction of the CIR using a linear model is possible. For instance, in an ocean waveguide, an accurate linear model for CIRs can be fit using *a priori* knowledge of the environmental parameters (expected variations of the sound-speed profile) and the source-receiver configuration.²⁷ Incorporating a linear model for CIRs reduces the dimension of the subspace spanned by the admissible CIRs such that the deconvolution becomes more tractable numerically. Two types of linear models for the CIR in ocean waveguides are discussed below.

A. The multipath CIR with known arrival-times and unknown amplitudes

Our first CIR model, based on the ray approximation, assumes that the CIR consists of multiple pulses that arrive at different time delays with different amplitudes. This model can be interpreted as the same pulse emitted from a common source but taking multiple paths to reach the receivers, as illustrated in Fig. 1(b). The CIR $h_k(t)$ can be written as a sum of pulse function $p(t)$ with different delays $t_1, t_2, \dots, t_D$ and different amplitudes $u_k[1], u_k[2], \dots, u_k[D]$

$$h_k(t) = u_k[1]p(t - t_1) + u_k[2]p(t - t_2) + \dots + u_k[D]p(t - t_D). \quad (12)$$

In this CIR linear model, we know the pulse profile $p(t)$ [as an example, we assume the pulse is a band limited Gaussian-windowed sinusoid $p(t) = u \cdot \sin(2\pi f_c t) e^{-(t/\tau)^2}$, and the decay parameter τ is determined by the effective bandwidth of the pulse] and pulse time delays $t_1, t_2, \dots, t_D$ as the *a priori* information, and the unknowns are the amplitude coefficients $u_k[1], u_k[2], \dots, u_k[D]$ of each ray-like arrivals. Discretizing $h_k(t)$ and $p(t - t_d)$ generate vectors $\mathbf{h}_k \in \mathbb{R}^N$ and $\mathbf{p}_{t_d} \in \mathbb{R}^N$. In terms of linear algebra, Eq. (12) implies that the CIR $\mathbf{h}_k$ lies in D -dimension subspace spanned by vectors $\mathbf{p}_{t_1}, \mathbf{p}_{t_2}, \dots, \mathbf{p}_{t_D}$ instead of the N -dimension vector length ($D < N$). The subspace matrix $\mathbf{C}_k \in \mathbb{R}^{N \times D}$ for the CIR $\mathbf{h}_k$, which is explained in Eq. (10), is simply constructed by stacking D time-domain pulse basis vectors column-wise. Therefore, $\mathbf{C}_k$ can be written as

$$\mathbf{C}_k = [\mathbf{p}_{t_1} \quad \mathbf{p}_{t_2} \quad \dots \quad \mathbf{p}_{t_D}]. \quad (13)$$

B. The multipath CIR with uncertain arrival-times and unknown amplitudes

In practice, the arrival-time structure of the CIR in ocean waveguides is never known exactly as a result of environmental fluctuations (e.g., sound speed variations). However, assuming that the range of expected sound-speed variations can be estimated, using for instance an oceanographic model, we can assume that the arrival-times t_i ($i = 1 \dots D$) for each ray arrival lies within a known time-interval Δt_i , that is, each ray arrival might occur within the time window $t_i \pm \Delta t_i$. Examining all possible pulses that

arrive within one time interval $t_i \pm \Delta t_i$ shows that they form a collection of shifted pulses that lie on a certain region of a manifold. Depending on the range of the time interval, the region of the manifold differs. Now the model for pulses on the manifold in terms of the arrival-time is nonlinear. The manifold needs to be embedded into a linear model that can approximate the CIR by a linear subspace with preferably a low dimension so that the deconvolution method can be implemented. Since all the pulses that lie on the manifold are highly correlated, the principal component analysis (PCA) is performed here to build a low dimension linear subspace for all shifted pulses.

A systematic way to construct the lower dimension subspace based on PCA is introduced in this section. An over-complete subspace matrix, denoted by $\mathbf{B}_{1o}$, is first formed by column-wise stacking vectors that represent all pulses arriving within the time interval $t_i \pm \Delta t_i$. Using the same notation as in Sec. III A for shifted pulse basis vectors, we form the matrix $\mathbf{B}_{1o}$ as

$$\mathbf{B}_{1o} = [\mathbf{p}_{t_1+\Delta t_i} \quad \mathbf{p}_{t_1+\Delta t_i-\delta t} \quad \dots \quad \mathbf{p}_{t_1-\Delta t_i+\delta t} \quad \mathbf{p}_{t_1-\Delta t_i}],$$

where δt is a tiny shifting grid for each pulse. However, all those time-shifted pulses [i.e., Fig. 3(a) is an example of those shifted pulses] are highly correlated. We then perform a PCA using a singular value decomposition of this collection of shifted pulse basis vectors to reduce its dimension, resulting in a low-dimension linear model for approximating any pulse arriving in this time region. The matrix $\mathbf{B}_{1o}$ can be written as $\mathbf{B}_{1o} = \mathbf{U}\mathbf{S}\mathbf{V}^T$. Because the singular value spectrum of this matrix decays to near zero quickly as shown in Fig. 3(b), we only need the first R (R is a small number in this typical example) singular values of $\mathbf{B}_{1o}$ to capture nearly all energy (more than 99.999% in simulation) of the singular values. Then the R corresponding singular vectors [i.e., Fig. 3(c) is an example of the first $R=6$ singular vectors] form the basis of the low dimension subspace. Column-wise stacking of these R singular vectors generates a matrix $\mathbf{B}_1$ that can accurately represent any pulse that arrives within the time region $t_1 \pm \Delta t$. The matrix $\mathbf{B}_1$ is written as

$$\mathbf{B}_1 = [\mathbf{U}(:, 1), \dots, \mathbf{U}(:, R)]. \quad (14)$$

The same procedure is performed for each path of the CIR and generates corresponding $\mathbf{B}_2, \dots, \mathbf{B}_D$ for each time region. Finally, the subspace matrix $\mathbf{C}_k$, introduced in Eq. (10) for the CIR $\mathbf{h}_k$ is then formed by

$$\mathbf{C}_k = [\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_D]. \quad (15)$$

Because of the uncertainty of the paths' arrival-time, the CIR $\mathbf{h}_k$ is approximated by a subspace with the dimension $R \times D$ compared to a subspace of the dimension D in Eq. (13).

We need to point out that PCA is not the only method of finding the low dimension subspace for pulses that arrive within a known time region. Based on the *a priori* information of the CIR structure, other efficient subspaces that can approximate $\mathbf{h}_k$ with a linear model also exist. For example,

FIG. 3. (Color online) (a) An illustration of the small time shifts of a band limited basis pulse (400–600 Hz). All shifted pulses form an over-complete subspace matrix $\mathbf{B}_{1o}$ for any ray-like pulse that arrives within an uncertainty window of duration $\Delta t = 3$ ms. (b) The singular value spectrum of the corresponding over-complete matrix $\mathbf{B}_{1o}$. The plot contains the first 20 of 600 singular values. The first $R = 6$ singular values capture more than 99% of the total energy. (c) The pulse subspace matrix $\mathbf{B}_1$, consisting of the $R = 6$ principal components of $\mathbf{B}_{1o}$.

the discrete prolate spheroidal sequences²⁸ can form a highly efficient basis that represents band limited and time concentrated pulses, which can also be an adequate linear model for the current application.

IV. NUMERICAL RESULTS

In this section, the multichannel myopic deconvolution method is implemented on various types of channel structures and the effectiveness and robustness of the deconvolution algorithm are demonstrated for both stylized and realistic CIRs. In simulations, the common driving source signal, $\mathbf{s} \in \mathbb{R}^{L+N-1}$, where $L + N - 1 = 1000$, is Gaussian white noise filtered in an arbitrary bandwidth representative of shipping noise spectra (400–600 Hz) with a sampling frequency $f_s = 2000$ Hz.

A. Stylized simulations

Stylized channel structures introduced in Secs. II and III are simulated in this section as direct implementations of the proposed deconvolution method. These general CIR assumptions do not directly represent any specific underwater acoustic channels, but are to illustrate the method. Realistic underwater acoustic channel simulations will be discussed later in Sec. IV C.

1. Time-limited channels

The first simulation investigates the time-limited channel model that corresponds to the scenario illustrated in Fig. 2. For time-limited channels, CIR vectors $\mathbf{h}_1, \dots, \mathbf{h}_K \in \mathbb{R}^N$ have finite length, which means that elements of the CIR vector $h_k[n]$ are nonzero only for $0 \leq n \leq N - 1$ and they are normally distributed random variables (i.e., FIR channels). Figure 4(b) illustrates a typical recovery result for parameters $N = 200$ (the CIR vector length) and $K = 100$ (number of recording channels). In this case, both the source signal and all of the CIRs (21 000 variables) are recovered with relatively small errors from 100 observed outputs (100 000 total samples). The myopic deconvolution method here benefits from having totally unstructured CIRs, which efficiently increases the mixing of the source signal and CIRs and thus the diversity of observations in the $K = 100$ recording signals.

2. Channels with known arrival-times and unknown amplitudes

To examine the method proposed in Sec. III A, we implement a general linear model for CIRs with a finite number of discrete arrivals with *a priori* known arrival-times but unknown amplitudes. This linear model is realized for multipath channels

FIG. 4. (Color online) Examples of recovery for finite-duration CIRs ($N = 200$ taps) using $K = 100$ channels. (a) Comparison of the original source signal (solid line) and the recovered source signal (dashed line). The relative error is 3×10^{-5} (only the first 300 samples out of 1000 samples are plotted). (b) Comparison of the true underlying CIRs (plotting only the first 50 taps out of $N = 200$ for the first 3 channels) and the recovered CIRs. The average relative error across all 100 channels is 10^{-4} .

FIG. 5. (Color online) (a) Comparison of the actual CIR and recovered CIR for five channels assuming known arrival-times and unknown amplitudes for the three pulse-like arrivals of each CIR. The average relative error across all $K = 50$ channels is 2×10^{-10} . (b) Same as (a) but assuming uncertain arrival-times ($\Delta t = 1.5$ ms) and unknown amplitudes. The average relative error across all $K = 50$ channels is 3×10^{-5} .

in an ocean waveguide as introduced in Sec. III. In this simulation scenario, each CIR consisted of three different individual pulse arrivals that correspond to $D = 3$, defined in Sec. III A. The basis pulse $p(t)$ throughout the rest of simulations is defined as a Gaussian window pulse in the frequency band of 400–600 Hz. The length of the CIR vector is $N = 500$, and the number of channels is $K = 50$. Because accurate time delays t_{1k} , t_{2k} , t_{3k} for each pulse are assumed to be known, we only need to estimate the amplitudes of each pulse, which are $u_k[1]$, $u_k[2]$, $u_k[3]$.

Simulation results presented in Fig. 5(a) demonstrate the exact deconvolution of the first five CIRs [the common source signal is also exactly recovered as in Fig. 4(a), but not plotted in Fig. 5]. The method proposed in this article yields an accurate performance if the CIR linear channel model is precise and the dimension of the subspace representing the model is small. The multipath CIR model is only one special example commonly applied in the ocean waveguide. Another typical channel model in other applications is *sparse* channel, that is, each CIR has D non-zero components at known sparse locations, but we do not know the corresponding amplitudes at each location.²⁹

3. Channels with uncertain arrival-times and unknown amplitudes

The channel model introduced in Sec. III B assuming uncertain arrival-times and unknown amplitudes is examined hereafter. Parameters of the simulation are similar to those used in Sec. IV A 2 except that the arrival-times of each pulse-like arrival of each CIR can lie anywhere within the time interval $t_i \pm \Delta t$ ($i = 1 \dots D$), assuming the same uncertainty window $\Delta t = 1.5$ ms for all arrivals for the sake of simplicity. The uncertainty of the arrival-time increases the dimension of the subspace that represents the CIR, and yields a small model error in representing CIRs because of truncating insignificant singular values. The results in Fig. 5(b) demonstrate that the method still accurately estimated the CIR.

B. Robustness analysis

1. The effect of additive measurement noise

The recovery performance in the presence of additive measurement noises is examined in this section. Gaussian

measurement noise $e_k(t)$ is added on each channel's convolution measurements $y_1(t), \dots, y_K(t)$. In this simulation, the CIR model is the same as in Sec. IV A 2. The recovery error and measurement noise are plotted in a log-log scale in Fig. 6. The measurement signal-to-noise ratio (SNR) is defined as $\text{SNR} = 20 \log_{10}(\|\mathbf{y}_k\|_2 / \|\mathbf{e}_k\|_2)$, and the CIR recovery relative error is defined as $20 \log_{10}(\|\mathbf{h}_k\|_2 / \|\mathbf{h}_k - \mathbf{h}_{k,\text{Approx}}\|_2)$. The result shows that our algorithm is robust to additive noise: as the noise level increases, no catastrophic failure occurs, and the recovery error grows linearly. Hence, the proposed deconvolution method recovers the CIR and the source accurately, and the performance is robust to noise when a linear model represented by Eq. (10) exists.

2. The effect of model errors

In the real environment, there is not only the measurement noise, but also model errors that are caused by lack of accurate *a priori* information to build an exact linear model for the CIR. The model error incurred in representing the CIR by the subspace matrix can be defined as $\mathbf{h}_k = \mathbf{C}_k \mathbf{u}_k + \mathbf{e}_k$. In the simulation, the channel model error is generated under the assumption that we do not have information about the later and

FIG. 6. (Color online) Variations of the recovery error (in logarithmic scale) for the deconvolution method (averaged over 100 independent realizations) vs SNR of the recorded measurements.

FIG. 7. (Color online) (a) Example of channel model error caused by lack of accurate *a priori* information on the arrival-time structure of the actual CIR (upper plot) containing five arrivals. Lower plot shows the approximated CIR assuming only three arrivals are present. (b) Variations of the recovery error (in logarithmic scale) for the deconvolution method (averaged over 100 independent realizations) vs channel model error.

weaker arrival pulses: in typical shallow water environments, those later arrivals have significantly weaker amplitudes and more randomly distributed arrival-times, when compared to the earlier arrivals, as they correspond to ray paths cumulating more reflections from the absorbing ocean bottom and the fluctuating ocean surface. For example, the true CIR shown in Fig. 7(a) consists of five pulses that arrive at time t_1, t_2, t_3, t_4, t_5 , but the *a priori* information only provides the knowledge of the first three pulses that arrive at time t_1, t_2, t_3 . The linear model that is generated by including stronger arrivals only at time t_1, t_2, t_3 would introduce model error in approximating the true CIR. The best approximation for the CIR is also depicted in Fig. 7(a). Channel model error is defined as the normalized Euclidean distance between the true CIR and CIR approximation using the linear model, which is $20 \log_{10}(\|\mathbf{h}_k\|_2 / \|\mathbf{e}_k\|_2)$. Figure 7(b) illustrates the method is also robust to this type of channel model error if the model error (averaged here over 100

independent realizations) is within an upper bound of ≈ 10 dB. It is worth noting that the model error $\mathbf{e}_k$ can be generated and measured by other variables as well. For example, the model error can be caused by a tilted array rather than a perfect vertical array; therefore, the accuracy of *a priori* information will depend on a variable that measures the overall array tilt or individual receiver locations more generally.

C. Numerical simulations in an ocean waveguide

In this section numerical simulations are conducted in a generic shallow water environment to assess the performance of the myopic deconvolution method for the case of a surface source arbitrarily set at depth of 5 m, which represents a shipping source. This source is assumed to broadcast Gaussian white noise filtered in the frequency band 400–600 Hz as a simple model for shipping noise.³⁰ Figure 8(a) describes the

FIG. 8. (Color online) (a) Computational shallow water waveguide. (b) Experimentally-measured sound speed profile. (c) *A priori* information of the ray arrival-time structure of the CIRs for $K = 16$ channels. For each ray arrival, the width of the black shaded area indicates the duration of the corresponding uncertainty time-window $2\Delta t$ for the shallow source shown in (a). (d) Comparison of the actual CIRs (blue solid line) obtained from normal-mode simulations and the recovered CIRs (red dashed line) across all $K = 16$ channels using the deconvolution method for the shallow source shown in (a). The average relative error across all $K = 16$ channels is $\approx 10^{-1}$.

shallow water environment used for the simulations. All environmental parameters including the sound speed profile [Fig. 8(b)] were selected to be representative of the experimental scenario and the actual environment introduced in Sec. IV D. Additionally, as implemented in the actual at-sea experiment, only a very short bottom-mounted vertical linear array (VLA) with 16 elements and 1-m spacing located from 128 to 143 m deep were simulated here. The range between the source and the VLA elements was arbitrarily set to 2 km and the corresponding CIR waveforms were computed using the normal-mode formulation implemented with the software KRAKEN [Fig. 8(c), blue line].³¹

A priori information of the ray arrival-time structure of the CIRs was obtained independently from a simple ray tracing simulation, using the standard BELLHOP (Ref. 32) model, and used for the CIR parametrization discussed in Sec. III [Fig. 8(b)] (note that no amplitude information for the various paths was used and that the ray simulations do not account for any dispersive effects presented in the actual CIR waveforms computed from normal-mode simulations). A classic shallow-water multipath structure can be observed where for instance the first wavefront corresponds to the direct and first surface-bounced arriving nearly simultaneously for this very shallow source. The uncertainty time window of each simulated ray arrival (of up to $\Delta t = 7$ ms) corresponds to variations of the sound-speed profile of ± 3 m/s which are representative of the expected variations of the environmental parameters at the experimental site described in Sec. IV D. The ray arrival-time intervals are then further combined if their corresponding uncertainty windows overlap. The resulting ray arrival-time uncertainty windows are then used to construct the CIR subspace using the same method described in Sec. III B. Figure 8(c) shows that a small relative error of more than -20 dB is achieved with the CIRs estimated from myopic deconvolution (red dashed line) when compared to the original CIRs computed from normal-mode simulations (blue solid line) even though the *a priori* information available about the CIRs is not very accurate in this shallow-ocean acoustic channel application (i.e., resulting in fairly large uncertainty time window of up to $\Delta t = 7$ ms).

Furthermore, Figs. 9(a) and 9(b) demonstrate that similar performance can be achieved even when we reduce the number of recording channels to only eight and four channels. These simulation results indicate that the proposed deconvolution

method can be used with a very limited number of receivers, provided that fairly accurate *a priori* information of the arrival-time structure of the CIR is available. This means that the myopic deconvolution method does not require a dense array to perform well, which is advantageous in comparison to some traditional deconvolution methods such as the ray-based deconvolution (RBD) method⁹ which not only it needs a long aperture, but the array should also be relatively dense with appropriate element spacing for beamforming to work.

D. Deconvolution of experimentally measured shipping noise recordings

The myopic deconvolution method is applied here to shipping noise recorded on a short bottom-mounted vertical line array which was moored in a shallow and nearly range-independent section of the Coronado Bank (water depth ≈ 150 m), approximately 20 km offshore of San Diego, CA. Based on prior studies, the environmental parameters for the test site were estimated to be similar to those shown in Fig. 8(a). The VLA had 16 elements uniformly spaced by 1 m; with the first element approximately 7 m above the sea-floor. Other technical features of the hydrophone array deployment and the electronic system have been described previously.^{33–35} The research vessel (R/V) New Horizon was used as a surface source of opportunity to test the myopic deconvolution method using the recorded shipping noise data on the VLA. Since no ground truth was available for the actual CIR between the R/V and the VLA, estimated CIRs from the myopic deconvolution were compared to the estimated CIRs independently obtained using a previously described RBD method.

To do so, $K = 16$ CIRs between the R/V and the VLA were first estimated using RBD for an arbitrarily selected location of the R/V ≈ 600 m away from the VLA, referred to hereafter as the “library” location. Based on visual inspection of spectrograms of the recorded shipping noise^{34,35} from the R/V at this distance, its dominant frequency band was found to be 300–800 Hz and the 10-s long snapshot of recorded data was filtered in this frequency band. These $K = 16$ estimated CIRs with RBD [see Fig. 10(a)] were then used to infer *a priori* arrival-time structure of the CIRs (in the vicinity of this selected library location) as input to the myopic deconvolution method. In order to estimate the arrival-time uncertainty for each ray-path of the CIRs, we

FIG. 9. (Color online) Same as Fig. 8(c) but using only the simulated-received waveforms on (a) $K = 8$ channels or (b) $K = 4$ channels for the myopic deconvolution method. The relative error across all channels for both cases is $\approx 10^{-1}$.

FIG. 10. (Color online) Experimental results using shipping noise. (a) Estimated CIRs using the library data set from the RBD method when the R/V was ≈ 600 m from the VLA. (b) Same as (a) but using a different data set (the event set) when the R/V was within the same location on a different day. (c) Comparison of the estimated CIRs from either the RBD (red line) or the myopic deconvolution method (green line) using the same event data set. (d) Comparison of the estimated CIRs with RBD using either the event data set [red line, same as shown in (b)] or the library data set [blue line, same as shown in (a)].

used historical sound-speed profiles measurements from CTD casts collected in the area and representative of the ocean sound speed fluctuations (≈ 3 m/s) occurring during the time period of the experiments. By running multiple ray

simulations using the software BELLHOP for a source-array separation distance of 2 km, through this collection of sound-speed profiles we obtained a maximum uncertainty window of $\Delta t = 3$ ms for all considered ray arrivals. An

FIG. 11. (Color online) (a) and (b): Same as Figs. 10(c) and 10(d) but using only $K = 8$ channels for the myopic deconvolution method. (c) and (d): Same as Figs. 10(c) and 10(d) but using only $K = 4$ channels for the myopic deconvolution method.

arrival-time window of $\Delta t = 3$ ms was used here for each ray-like arrival.

The myopic deconvolution method was then applied to another set of shipping noise data (referred to as “event” data) recorded from the same R/V when it passed again across the library location a day later to estimate the corresponding CIRs (referred to as recovered CIRs) between the R/V and the VLA [see Fig. 10(c)]. As a validation, the CIRs were also estimated independently using the RBD method for the same event data which closely compared to the recovered CIRs from the myopic deconvolution method [see Figs. 10(b) and 10(c)] with an average correlation coefficient across the 16 channels of 0.85. It can also be noted that the CIRs estimated from RBD using the library data (i.e., used only as *a priori* information for the myopic deconvolution method) and the event data (which matched the CIRs obtained from the myopic deconvolution method) did significantly differ—as expected—due to variations of the environmental parameters across the selected 2 days at this shallow water test site: the average correlation coefficient across the 16 channels was only 0.58. Finally, Fig. 11 shows that the myopic deconvolution method can also be implemented successfully using only a smaller number of receiver elements (respectively, eight and four elements), provided that sufficiently accurate *a priori* information of the arrival-time structure of the CIRs is available (e.g., using simulated data for the selected test site). If such *a priori* information is available, this may be a significant advantage of the myopic deconvolution method over the RBD method which requires an array with sufficiently long aperture to beamform individually the different ray arrivals composing the CIR wavefronts.⁹

V. CONCLUSIONS

This article introduced a method for solving the deconvolution problem using acoustic sources of opportunity and *a priori* information about the CIR via the low-rank recovery problem formulation. For the shallow-water acoustic channel application, we developed a systematic way to build an efficient and accurate linear model incorporating *a priori* information about the expected CIRs’ arrival-time structure so that the low-rank recovery method can be implemented for the proposed myopic deconvolution method. Stylized numerical simulations demonstrated that the proposed method perfectly deconvolved both the noise source signal and CIRs simultaneously. The proposed method was also shown to be robust in the presence of additive measurement noises and model errors in the CIR parameterization. Furthermore the applicability of the proposed myopic deconvolution method for estimated CIRs in shallow water using shipping noise and short bottom-mounted VLAs was demonstrated both numerically and experimentally. This method is likely to be applicable to other environments supporting waveguide-like propagation (e.g., seismic or structural waveguides). The proposed myopic deconvolution method can naturally be extended to the multiple input multiple output deconvolution problem (i.e., the case of multiple sources of opportunities) and receivers.

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APPENDIX

1. The heuristic solver

The multichannel deconvolution problem becomes computationally intractable as the number of channels increases. To recover $\mathbf{X}_0$, we are searching over a matrix of size $(L + N - 1) \times NK$ that minimizes the nuclear norm which satisfies the linear constraints. If we want to recover $K = 100$ channels with each channel of length $N = 500$, and a source of length $L + N - 1 = 1000$, then the rank-1 matrix $\mathbf{X}_0$ is of size $1000 \times 50\,000$, which the storage becomes an issue in computation. However, the nonlinear programming algorithm developed by Burer and Monteiro²³ using low-rank factorization can greatly reduce the number of variables, and in the meanwhile, provides reliable performance adapted in our application.

By construction, we know that the target matrix $\mathbf{X}_0$ is rank-1, so instead of iterating matrix $\mathbf{X}$ of size $(L + N - 1) \times NK$ to minimize its nuclear norm, we can reformulate the problem to optimize over vectors $\mathbf{Z} \in \mathbb{R}^{(L+N-1) \times 1}$ and $\mathbf{H} \in \mathbb{R}^{NK \times 1}$, where $\mathbf{X} = \mathbf{Z}\mathbf{H}^T$ (note that $\mathbf{Z}$ and $\mathbf{H}$ correspond to the source signal vector $\mathbf{s}$ and CIR vectors $\mathbf{h}$ in Eq. (4), respectively). Hence we only need to store two vectors $\mathbf{Z}$ and $\mathbf{H}$ in the memory for each iteration. This reformulation is driven by the fact that the nuclear norm is equal to the minimum Frobenius norm factorization

$$\|\mathbf{X}\|_* = \min_{\mathbf{Z}, \mathbf{H}} \frac{1}{2} (\|\mathbf{Z}\|_F^2 + \|\mathbf{H}\|_F^2) \quad \text{subject to } \mathbf{X} = \mathbf{Z}\mathbf{H}^T. \quad (\text{A1})$$

We can then minimize the following augmented Lagrange term:

$$\|\mathbf{Z}\|_F^2 + \|\mathbf{H}\|_F^2 - 2\langle \lambda, \mathcal{A}(\mathbf{Z}\mathbf{H}^T) - \mathbf{y} \rangle + \sigma \|\mathcal{A}(\mathbf{Z}\mathbf{H}^T) - \mathbf{y}\|_2^2. \quad (\text{A2})$$

This new formulation of the problem is non-convex because $\mathcal{A}(\mathbf{Z}\mathbf{H}^T)$ is a linear combination of the product of two unknowns. But the Burer-Monteiro heuristic shows that under certain conditions, the local minima in Eq. (A2) are also the global minima.³⁶ This Burer-Monteiro heuristic solver is used in state-of-the-art large scale implementations of matrix recovery problems. To minimize the augmented Lagrangian term in Eq. (A2), the inner operation is executed using Limited-memory Broyden-Fletcher-Goldfarb-Shanno algorithm (LBFGS) with MATLAB solver “minfunc” developed by Recht *et al.*²² The parameters of the augmented Lagrangian are updated according to the schedule proposed by Burer and Monteiro.²³ The parameter λ determines the

trade-off between the fidelity of the solution to the measurements $\mathbf{y}$ and its conformance to the model. The term involving σ measures the Euclidean norm of the infeasibility, and σ is a penalty parameter. We update both parameters for each LBFGS minimization. We also use a gradient-based algorithm because the function and the gradient evaluations of the augmented Lagrange term Eq. (A2) with respect to vectors $\mathbf{Z}$, $\mathbf{H}$ can be performed very quickly.

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