Denoising techniques in adaptive multi-resolution domains with applications to biomedical images

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Variational mode decomposition (VMD) is a new adaptive multi-resolution technique suitable for signal denoising purpose. The main focus of this work has been to study the feasibility of several image denoising techniques in empirical mode decomposition (EMD) and VMD domains. A comparative study is made using 11 techniques widely used in the literature, including Wiener filter, first-order local statistics, fourth partial differential equation, nonlinear complex diffusion process, linear complex diffusion process (LCDP), probabilistic non-local means, non-local Euclidean medians, non-local means, non-local patch regression, discrete wavelet transform and wavelet packet transform. On the basis of comparison of 396 denoising based on peak signal-to-noise ratio, it is found that the best performances are obtained in VMD domain when appropriate denoising techniques are applied. Particularly, it is found that LCDP in combination with VMD performs the best and that VMD is faster than EMD.

1. Introduction: Biomedical image processing and denoising is of great interest in different engineering applications [1–6]. For instance, empirical mode decomposition (EMD) is one of the techniques gaining interest in biomedical images denoising [7–9]. This technique was introduced by Huang *et al.* [10] as fully adaptive multi-resolution decomposition technique used to carry out a joint space-spatial frequency representation of the original signal in terms of intrinsic mode functions. Recently, Dragomiretskiy and Zosso [11] introduced the variational mode decomposition (VMD) used to represent the original signal in terms of variational modes which are concurrently extracted from it. Specifically, the VMD seeks a number of modes and their respective centre frequencies to accurately represent the original image (signal) in least-squares sense [11]. Contrary to EMD, the VMD is mathematically derived, does not require a specific interpolation algorithm, and is less sensitive to noise [11].

The aim of this work is to compare the performance of several denoising techniques in EMD and VMD domains. Contrary to previous works where biomedical signals denoising was performed by discrete wavelet transform (DWT) in EMD domain [7–9] and VMD domain [12, 13], in this comparative study 11 different denoising techniques are used to perform denoising task. Indeed, we seek to investigate how each denoising technique performs in each type of adaptive decomposition domain. The included denoising techniques are Wiener filter [14], first-order local statistics (FOLS) [15, 16], fourth partial differential equation (FOPDE) [17], nonlinear complex diffusion process (NLCDP) [18], linear complex diffusion process (LCDP) [18], probabilistic non-local means (PNLM) [19], non-local Euclidean medians (NLEM) [20], non-local means (NLM) [21], non-local patch regression (NLPR) [22], DWT [23] and wavelet packet transform (WPT) [24]. All experiments will be applied to six different biomedical images.

The rest of the Letter as follows: Section 2 presents EMD, VMD and denoising techniques. Section 3 provides experimental results and comparisons. Finally, we conclude our work in Section 4.

2. Methods: The experiments are conducted as follows. First, the biomedical image is processed by means of EMD/VMD for decomposition purpose to obtain its k IMFs and k variational mode functions (VMFs) when EMD and VMD are employed, respectively. Second, the obtained IMFs/VMFs are processed by a specific denoising technique; for instance, Wiener filter, FOLS,

FOPDE, NLCDP, LCDP, PNLM, NLEM, NLM, NLPR, DWT and WPT. In the third step, the denoised biomedical image is reconstructed by adding up the denoised IMFs/VMFs. Finally, quality of the obtained denoised biomedical image is evaluated by using the prominent peak signal-to-noise ratio (PSNR) statistic. All methods are described next.

2.1. Empirical mode decomposition: The EMD [10] breaks a signal down into a sum of IMFs located at different scales, going from fine to coarse, by using the sifting process which is an iterative algorithm. Consider an image $f(x,y)$, then, the sifting process yields to a representation of the image in this manner

$$f(x, y) = \sum_{l=1}^L \text{imf}_l(x, y) + \text{res}(x, y) \quad (1)$$

2.2. Variational mode decomposition: The goal of the VMD is to separate an input signal into k modes each one is characterised by a limited bandwidth in spectral domain [11]. In this framework, each mode k is mostly compact around a centre pulsation ω_k calculated during the decomposition process [11]. The constrained variational problem follows [11]

$$\min_{u_k, \omega_k} = \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2 \right\} \quad (2)$$

subject to

$$\sum_k u_k = f \quad (3)$$

where f is the signal, u is its mode, ω is the frequency, δ is the Dirac distribution, t is the time script, k is the number of modes and $*$ denotes the convolution. For a two-dimensional signal, the analytic signal $\hat{u}_{AS,k}(\omega)$ is defined in the frequency domain as follows [25]

$$\hat{u}_{AS,k}(\omega) = (1 + \text{sgn}(\omega \cdot \omega_k)) \hat{u}_k(\omega) \quad (4)$$

Afterwards, Lagrange multipliers are employed to find the optimal u_k in Fourier domain [25]:

$$\hat{u}_k^{n+1} = \arg \min_{\hat{u}_k} \left\{ \alpha \left\| j(\boldsymbol{\omega} - \boldsymbol{\omega}_k) \left[(1 + \text{sgn}(\boldsymbol{\omega} \cdot \boldsymbol{\omega}_k)) \hat{u}_k(\boldsymbol{\omega}) \right] \right\|_2^2 + \left\| \hat{f}(\boldsymbol{\omega}) - \sum_k \hat{u}_k(\boldsymbol{\omega}) + \frac{\hat{\lambda}(\boldsymbol{\omega})}{2} \right\|_2^2 \right\} \quad (5)$$

Consequently, the subsequent Wiener filter is achieved [25]

$$\hat{u}_k^{n+1}(\boldsymbol{\omega}) = \left(\hat{f}(\boldsymbol{\omega}) - \sum_{i \neq k} \hat{u}_i(\boldsymbol{\omega}) + \frac{\hat{\lambda}(\boldsymbol{\omega})}{2} \right) \frac{1}{1 + 2\alpha \|\boldsymbol{\omega} - \boldsymbol{\omega}_k\|^2} \quad (6)$$

$\forall \boldsymbol{\omega} \in \Omega_k: \Omega_k = \{\boldsymbol{\omega} \cdot \boldsymbol{\omega}_k \geq 0\}$

Hence, Wiener filtering updates modes in Fourier domain [11, 25]. In this study, k is simply set to four.

2.3. Discrete and wavelet packet transform: The EMD is employed to decompose the noisy image to get its associated IMFs and a thresholding technique based on DWT is performed to each one of them [23]. In this regard, a nonlinear thresholding is applied on each DWT [26] subband coefficient. For instance, a subband coefficient is set to zero when it is smaller than the threshold. If not, it is retained or changed. To finish, an inverse DWT [26] is applied to recuperate the denoised signal. In the wavelet domain, coefficients thresholding technique is simple to separate information from noise [23]. In this Letter, the optimal threshold value is obtained by minimising Stein's unbiased risk estimator [23] which is more accurate as more data are available [23]. The WPT denoising approach [24] is similar to the DWT; where WPT [27] performs a complete analysis of the image at all subbands; including both approximation and detail coefficients. For both DWT and WPT, Daubechies-4 and Symlet-4 are employed and second and third levels of decomposition are used.

2.4. Wiener filter: Wiener filter [14] is an adaptive technique based on local mean (μ) and variance (σ^2) in the region of each pixel of a noisy image $f(n_1, n_2)$. In particular, Wiener filter creates a pixel wise filtering using estimated mean and variance to obtain an estimated or denoised image $f(n_1, n_2)$ given by

$$f(n_1, n_2) = \mu + \frac{\sigma^2 - v^2}{\sigma^2} (f(n_1, n_2) - \mu) \quad (7)$$

where v is the noise variance that could be estimated as average of all local estimated variances.

2.5. FOLS filter: FOLS [15, 16] filter estimates a pixel characteristic by calculating subregion statistics estimated over a neighbourhood W . The filter is expressed as follows [16]

$$f_{ij} = \bar{g}_{ij} + k_{ij} (g_{ij} - \bar{g}_{ij}) \quad (8)$$

where f_{ij} is the estimated pixel at location (i, j) on the image with an original value of g_{ij} , $\bar{g}_{ij}$ is grey value local mean of an $m \times n$ neighbourhood around and including g_{ij} ; k_{ij} is the weighing factor with $k \in [0, 1]$. The factor k_{ij} is expressed as follows [16]

$$k_{ij} = \frac{1 - \bar{g}_{ij}^2 \sigma_{ij}^2}{\sigma_{ij}^2 (1 + \sigma^2)}, \quad \sigma^2 = \sum_{ij} \frac{\sigma_{ij}^2}{\bar{g}_{ij}} \quad (9)$$

where σ_{ij}^2 and σ^2 are, respectively, local noise variance in the moving window W and noise variance in the whole image. In this study, a local neighbourhood window W of size 5×5 is considered.

2.6. Fourth-order partial differential equation: Let $f(x, y)$ be an image and $f_0(x, y)$ be the same image corrupted with random

noise $\varepsilon(x, y)$. Therefore, the goal is to get a new image by minimising the cost function $E(f)$ as follows [17]

$$E(f) = \int_{\Omega} \psi(|\nabla^2 f|) dx dy \quad (10)$$

where ∇^2 is the Laplacian operator and $\psi(\cdot) \geq 0$ and also $\psi'(\cdot) > 0$. Since the noisy image represents the initial condition, the solution is provided by the following partial differential equation as the time script tends to infinity

$$\frac{\partial f}{\partial t} = -\nabla^2 \left[\psi'(\nabla^2 f) \frac{\nabla^2 f}{|\nabla^2 f|} \right] \quad (11)$$

The minimisation of $E(f)$ yields to a smoothed image.

2.7. Linear and NLCDP: Let the initial value problem for a one-dimensional signal is as follows

$$I_t = c I_{xx}, \quad t > 0, \quad x, c \in \mathbb{R} \\ I(x; 0) = I_0 \in \mathbb{R}, \quad (12)$$

where c is the diffusion coefficient and $\mathbb{R}$ is a real number. The complex fundamental solution $h(x; t)$ must respect the following equation [18]

$$I(x; t) = I_0 * h(x; t) \quad (13)$$

where $*$ denotes the convolution operator. When the complex diffusion coefficient is represented as $c = re^{i\theta}$, the fundamental solution follows [18]

$$h(x; t) = g_{\sigma}(x; t) e^{i\alpha(x; t)} \quad (14)$$

where $g(\cdot)$, σ , α and θ are, respectively, the Gaussian function, its standard deviation, the exponent function and the angle. They are expressed as follows

$$\alpha(x; t) = \frac{x^2 \sin \theta}{4tr}, \quad \sigma(t) = \sqrt{\frac{2tr}{\cos \theta}} \\ g_{\sigma}(x; t) = \frac{1}{\sqrt{2\pi\sigma(t)}} e^{(-x^2/2\sigma^2(t))} \quad (15)$$

The NLCDP equation is given by [18]

$$I_t = \frac{\partial}{\partial x} (c(\cdot) I_x), \quad c(s) = c(|I_{xx}|) \quad (16)$$

Subsequently, the general solution is expressed as follows [18]

$$I_t = \frac{\partial}{\partial x} \left(\frac{I_x}{1 + I_{xx}^2} \right) = \frac{1 + I_{xx}^2 - 2I_x I_{xxx}}{(1 + I_{xx}^2)^2} I_{xx} \quad (17)$$

2.8. Standard and probabilistic NLM: The PNLM [19] is used to introduce probabilistic weights to the standard NLM algorithm. For instance, consider the noisy image pixel y_L to be expressed as $y_L = f_L + n_L$ where f_L and n_L are, respectively, the clean image pixel and the pixel noise that follows a Gaussian distribution with mean zero and variance σ^2 . Then PNLM estimated clean pixel is given by [19]

$$\hat{f}_L = \sum_{k \in S} \frac{w_{L,k} y_k}{W_L} \quad (18)$$

where y_k is the k th noisy pixel, W_L is the summation of all weights

Fig. 1 Original biomedical images

and $w_{L,k}$ is the probabilistic weight given by [19]

$$w_{L,k} = \text{PDF}_{L,k} \left(\frac{\hat{D}_{L,k}}{\rho^2} \right) \quad (19)$$

where $\text{PDF}_{L,k}(\cdot)$ is the theoretical probability density function, $\hat{D}_{L,k}$ is the estimated distance between the patches around y_L and y_k , and ρ is a tuning parameter. For the standard NLM, the weights are calculated by quantifying the similarity between two local patches (P) around noisy pixels y_L and y_k using a smoothing parameter h as follows

$$w_{L,k} = \exp \left(- \sum_{j \in P} (y_{L+j} - y_{k+j})^2 h^{-1} \right) \quad (20)$$

2.9. NLEM and NLPR: The Euclidean mean is the minimiser of $\sum_j w_j \|f - f_j\|^2$ overall image patches f of size $k \times k$ centred at pixel j . The NLEM [16] seeks to minimise $\sum_j w_j \|f - f_j\|$ over all f ; as the median is more robust to outliers than the mean; by using the iteratively reweighted least-squares algorithm [16]. Finally, the purpose of the NLPR algorithm [22] is to minimise $\sum_j w_j \|f - f_j\|^d$ where $0 < d < 1$ to better suppress the residuals $\|f - f_j\|$ induced by the outliers [22]. In this study, the parameter d is set to 0.5.

3. Experimental results: All image denoising models are tested on human brain MRI (176×208), brain sagittal MRI (225×225), brain coronal MRI (168×168), computed tomography (CT)-based pancreas image (200×134), prostate digital image (275×183) and DNA microarray image (225×225), all shown in Fig. 1. Each image is corrupted by different additive Gaussian noise levels. The noise has a normal distribution with zero mean and standard deviations $\sigma_1 = 1$, $\sigma_2 = 1.5$ and $\sigma_3 = 2$. The experimental results for brain MRI, brain sagittal MRI, brain coronal MRI, CT-based pancreas image, prostate tissue image and DNA microarray image are, respectively, provided in Tables 1–6. The Daubechies (db4) and symlet (sym4) wavelets at levels 3 and 4 are applied for DWT and WPT. The number k of IMFs/VMFs is set to 4 to speed up processing and for simplicity of computations. In total, 396 experiments are conducted. As can be seen from all tables, the experimental results indicate that most of denoising techniques used in this study performed better in EMD domain. However, the best performances were achieved in VMD domain. The relative superiority of VMD over EMD for image denoising can be explained by its capability to find modes and their respective centre frequencies in least-squares sense and its ability to be less sensitive to noise. In particular, LCDP in combination with VMD provides the best results contrarily to the other techniques applied in VMD domain. The explanation follows. In one hand, VMD is based on Wiener filtering process that automatically makes the decomposition process robust to noise. In the other hand, linear complex diffusion process provides a stable flow of the diffusion process and preserves perceptual properties such as edge detection thanks to the imaginary part. Besides, it is interesting to notice that the results for the denoised microarray image show a remarkable

Table 1 PSNR values for denoised brain MRI

	σ_1		σ_2		σ_3	
	EMD	VMD	EMD	VMD	EMD	VMD
Wiener	11.83	11.38	11.75	8.94	11.61	9.15
FOLS	11.34	9.63	11.10	7.90	10.84	7.04
FODPE	11.43	12.56	11.35	10.56	10.98	9.66
NLCDP	11.41	12.03	11.26	10.06	10.85	9.08
LCDP	11.67	12.69	11.48	12.27	11.12	10.22
PNLM, NLEM, NLM, NLPR, DWT, WPT	11.33	9.60	11.09	7.88	10.82	7.02

Table 2 PSNR values for denoised brain sagittal MRI

	σ_1		σ_2		σ_3	
	EMD	VMD	EMD	VMD	EMD	VMD
Wiener	9.42	8.45	9.39	8.27	9.38	8.77
FOLS	9.09	6.89	8.93	6.24	8.84	6.52
FODPE	8.93	9.91	8.84	9.87	8.93	10.02
NLCDP	8.83	8.99	8.71	8.77	8.76	8.86
LCDP	9.55	10.23	9.35	9.63	9.43	10.36
PNLM, NLEM, NLM, NLPR, DWT, WPT	9.05	6.87	8.92	6.22	8.82	6.50

improvement when compared with EMD or other images under test. This could be explained by the fact that microarray images contain less edges and geometric features than the other images. In this

Table 3 PSNR values for denoised brain coronal MRI

	σ_1		σ_2		σ_3	
	EMD	VMD	EMD	VMD	EMD	VMD
Wiener	8.87	6.90	8.88	6.22	8.82	6.57
FOLS	8.45	6.12	8.38	4.84	8.24	4.85
FODPE	8.50	8.51	8.44	8.42	8.32	7.64
NLCDP	8.36	7.84	8.27	7.21	8.11	6.92
LCDP	8.70	9.74	8.53	9.78	8.49	8.89
PNLM, NLEM, NLM, NLPR, DWT, WPT	8.42	6.09	8.35	4.81	8.21	4.82

Table 4 PSNR values for denoised pancreas CT

	σ_1		σ_2		σ_3	
	EMD	VMD	EMD	VMD	EMD	VMD
Wiener	5.60	4.82	5.59	4.92	5.60	4.30
FOLS	5.45	4.36	5.40	4.19	5.38	3.32
FODPE	5.14	5.19	5.06	5.23	5.11	4.65
NLCDP	5.13	5.00	5.02	4.71	5.05	4.24
LCDP	5.46	6.06	5.31	5.72	5.33	5.71
PNLM, NLEM, NLM, NLPR, DWT, WPT	5.42	4.30	5.37	4.14	5.33	3.27

Table 5 PSNR values for denoised prostate

	σ_1		σ_2		σ_3	
	EMD	VMD	EMD	VMD	EMD	VMD
Wiener	3.58	3.28	3.59	2.82	3.61	2.99
FOLS	3.46	2.82	3.42	2.10	3.41	2.12
FODPE	3.27	3.42	3.33	3.50	3.33	2.89
NLCDP	3.19	3.06	3.22	2.75	3.21	2.36
LCDP	3.35	3.49	3.43	3.57	3.42	3.42
PNLM, NLEM, NLM, NLPR, DWT, WPT	3.41	2.76	3.39	2.04	3.37	2.08

Table 6 PSNR values for denoised microarray

	σ_1		σ_2		σ_3	
	EMD	VMD	EMD	VMD	EMD	VMD
Wiener	10.08	62.31	10.03	9.52	9.97	3.54
FOLS	9.52	57.38	9.38	7.42	9.23	2.45
FODPE	9.25	57.98	9.26	11.01	9.03	5.92
NLCDP	9.02	57.89	8.99	8.27	8.76	5.59
LCDP	9.58	57.83	9.61	9.36	9.22	11.06
PNLM	9.51	57.46	9.38	6.66	9.21	2.44
NLEM	9.51	59.25	9.38	6.66	9.21	2.44
NLM	9.51	57.47	9.38	6.66	9.21	2.44
NLPR	9.51	58.49	9.38	6.66	9.21	2.44
DWT	9.51	57.43	9.38	6.66	9.21	2.44
WPT db4-L4	9.51	63.69	9.39	6.71	9.21	2.45
WPT sym4-L4	9.51	63.60	9.39	6.71	9.22	2.45
WPT L3	9.51	64.94	9.39	6.71	9.21	2.45

regard, denoising complexity is simpler and results are more effective in comparison with other type of images.

Finally, the processing time of the EMD (VMD) was 625.9902 s (293.7004) for brain MRI, 1470.2432 s (443.6261) for brain sagittal MRI, 400.0473 s (215.7050 s) for brain coronal MRI, 247.4103 s (229.3780 s) for pancreas CT, 782.6120 s (480.4362 s) for prostate image and 1082.7018 s (444.4205 s) for DNA microarray image. Therefore, the VMD is highly faster than EMD in image decomposition.

4. Conclusion: The central goal of this study was to use and compare a wide range of image denoising techniques when applied to six biomedical images in EMD and VMD domains. Results from 396 experiments have shown that several denoising techniques performed better in EMD domain. However, the best performances are obtained in VMD domain when an appropriate denoising technique is applied. Above all, we found that LCDP in combination with VMD performs the best unlike the other techniques applied in VMD domain. In addition, VMD was found to be faster than EMD.

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