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On the Accurate Analysis of Vibroacoustics in Head Insert Gradient Coils

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Abstract

Purpose—To accurately analyze vibroacoustics in MR head gradient coils.

Theory—A detailed theoretical model for gradient coil vibroacoustics, including the first description and modeling of Lorentz damping, is introduced and implemented in a multi-physics software package.

Methods—Numerical finite-element method simulations were used to establish a highly accurate vibroacoustic model in head gradient coils in detail, including the newly introduced Lorentz damping effect. Vibroacoustic coupling was examined through an additional modal analysis. Thorough experimental studies were used to validate simulations.

Results—Average experimental sound pressure levels (SPLs) and accelerations over the 0–3000Hz frequency range were 97.6dB, 98.7dB, and 95.4dB, as well as 20.6g, 8.7g, and 15.6g for the *X*-, *Y*-, and *Z*-gradients, respectively. A reasonable agreement between simulations and measurements was achieved. Vibroacoustic coupling showed a coupled resonance at 2300Hz for the *Z*-gradient that is responsible for a sharp peak and the highest SPL value in the acoustic spectrum.

Conclusions—We have developed and used more realistic multi-physics simulation methods to gain novel insights into the underlying concepts for vibroacoustics in head gradient coils, which will permit improved analyses of existing gradient coils and novel SPL reduction strategies for future gradient coil designs.

Keywords

gradient coil; insertable head gradient; acoustics; vibration; vibroacoustics; sound pressure level; Lorentz damping

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Introduction

The conductors of MR gradient coils are subject to large oscillating Lorentz forces due to currents in the presence of a strong static magnetic field. These forces cause gradient coil conductor vibrations, which in turn generate vibrations of the gradient coil surfaces and then the patient bore wall and other surfaces and structures. The induced pressure variation of the air inside the bore results in an acoustic pressure wave and sound radiation.

Sound pressure levels (SPL) inside gradient coils can reach well over 100 dB and mean that hearing protection is required for patients and sometimes operators (1). Loud sounds represent not only a serious safety risk but also cause other unwanted side effects (2),(3).

Recent research has focused on the development of high performance gradient coils meant for head-only imaging within whole-body MRI systems (4)–(16). These head gradients are designed to produce higher gradient strengths and slew rates than body gradients, and promise major increases in scanner performance in terms of more efficient spatial/diffusion encoding and/or faster data acquisition. These performance enhancements may, however, be accompanied by increases in SPLs and surface vibrations compared to conventional body-size gradient coils, and there is the additional concern that those surfaces are much closer to the patient's ears.

Most early analyses of gradient coil vibroacoustics employed a theoretical modal analysis based on the use of thin shell theory and finite-length cylindrical ducts (17)–(19). The first approach by Taracila et al. addressed the calculation of vibrational shell modes (17). An analytic analysis strategy involving finite length cylindrical ducts with open end termination was used by Shao et al. (18),(19). This acoustic-only work was then extended by Li et al. (20), forming a comprehensive analysis that included both vibrational shell modes as well as acoustic wave propagation in open-ended cylindrical ducts. An extension of the thin shell analysis to intermediate wall thicknesses was presented by Mechefske et al. in (20),(21). This group described, for the first time, the coupling between vibrational and acoustic modes (20).

The first approach to numerical analysis of gradient coil acoustics was carried out using statistical energy analysis (SEA) by Edelstein et al. (1), in which the scanner model was simplified by identifying a set of vibroacoustic sources, subsystems, and interconnecting acoustic pathways. SEA is conventionally applied to systems with a large number of vibroacoustic eigenmodes (i.e. acoustically large or complex systems) and is solved as an energy balance problem. This highly reduces computational cost and makes the SEA methodology attractive for problems such as gradient coil vibroacoustics, compared to more detailed methods such as finite element methods (FEM). A boundary element method (BEM) based numerical analysis was used in an early attempt to verify the open ended cylindrical duct approach in (18). The first FEM analysis of gradient coil acoustic was published by Mechefske et al. (20),(22)–(24), and investigated the vibrational modes of a stand-alone intermediate-walled cylinder with a single gradient winding. This was followed by subsequent works from the same group that separated the vibroacoustic FEM analysis into a vibrational analysis that was then used as an excitation to determine the acoustics of a

stand-alone thick-walled gradient cylinder (22)–(24). A recent study uses FEM analysis for a split MRI-LINAC system (25),(26). More advanced FE simulations are presented by Rausch et al. in (27),(28). Experimental validation of these models was carried out for the theoretical vibroacoustically coupled model in (20), the SEA model in (1), as well as for the FEM analyses in (20),(22)–(24),(26),(27).

These analyses have laid the foundations for the field of MR acoustics, and have introduced important theoretical underpinnings. Given the limited computational power available at the time of some of these early studies, prior numerical results simplified the calculations by neglecting certain physical effects, and therefore do not fully predict the observed acoustic and vibrations responses. In essence, the SEA analysis neglected a detailed geometrical and spectral analysis, while many of the FEM based models mostly used stand-alone cylinders exclusively without any additional scanner features or outside air geometry, and without realistic mechanical fixation of the gradient coil. Moreover, the wire patterns were not embedded, but a pre-calculated Lorentz force distribution was applied as the source term for the vibrational analysis. An exception to some of these simplifications is found in the more advanced simulations found in literature as part of the magnetomechanical-acoustic modeling by Rausch et al. (27),(28) and Wang et al. (25),(26).

In the present work, we have found that a number of these previously neglected effects turn out to be important for accurate prediction of the vibroacoustic phenomena occurring in MR systems. Thanks to the rapid increase of computational performance over the past decade, and the efficient parallelization of modern-day multi-physics numerical methods, we were able to analyze in detail the vibroacoustic responses of modern MR systems. Our simulation results were compared directly against experimental measurement of both surface vibrations (accelerations) and acoustic SPLs, using a high-performance head gradient coil (5) operating inside a 3T MR scanner. The reasonable agreement between simulation and experimental results validated our model assumptions.

Theory

Lorentz Forces

The Lorentz force $d\vec{F}_L(S, \omega)$ (in N) acting on an infinitesimally small section $d\vec{l}(s) = \hat{t}(s) l ds$ of a conductor can be modeled as:

$$d\vec{F}_L(s, \omega) = I(\omega) \left(d\vec{l}(s) \times \vec{B}_0(s) \right) = I(\omega) \left(\hat{t}(s) \times \vec{B}_0(s) \right) l ds \quad [1]$$

where $I(\omega) = Ie^{j\omega t}$ is the frequency-dependent current waveform (A) at an angular frequency ω in phasor notation flowing through the conductor, and $\vec{B}_0(s)$ is the static magnetic field (T) of the magnet. ds is the infinitesimally small conductor element with $s \in [0, 1]$ as the unitless relative position parameter running along the length of the conductor. l is the total length of the conductor, whereas $\hat{t}(s)$ is a unit vector denoting the tangential direction to ds in Cartesian coordinates (x, y, z) .

Note that with a perfectly homogeneous static B_0 -field oriented in the z -direction over the region of the gradient windings – a very good approximation in the case of our insertable head gradients – only x - and y -components of the Lorentz force are generated.

Lorentz Damping

The conductive wires vibrate due to the oscillatory current $I(\omega)$ which induces an oscillatory Lorentz force $d\vec{F}_L(s, \omega)$. The motion of the conductor in the static magnetic field induces an opposed eddy current, which therefore decreases the total force, leading to what we refer to as “Lorentz damping”, described mathematically as follows:

If the local displacements $\vec{u}(s, \omega)$ of the incremental wire section $d\vec{l}(s)$ are small, each moving point in the solid conductor sees a constant magnetic field, and the induced eddy current density in the wire in A/m² is:

$$d\vec{J}_{eddy}(s, \omega) = \sigma \left(\dot{\vec{u}}(s, \omega) \times \vec{B}_0(s) \right) ds \quad [2]$$

where σ is the conductivity of the wire (S/m) and $\dot{\vec{u}}(s, \omega)$ is the velocity of the incremental wire section (m/s), then the locally induced eddy current at a certain position in the wire in A is:

$$d\vec{I}_{eddy}(s, \omega) = d\vec{J}_{eddy}(s, \omega) * A_{eff} \quad [3]$$

where A_{eff} is the cross section of the wire (m²).

These locally induced incremental eddy currents $d\vec{I}_{eddy}(s, \omega)$, integrated over the length of the entire conductor of one gradient axis, will oppose and counteract the externally applied current $I(\omega)$.

The local eddy current will induce a secondary (or counter-) Lorentz force $d\vec{F}_{CL}(s, \omega)$ at the location of the local section of moving wire, equal to:

$$\begin{aligned} d\vec{F}_{CL}(s, \omega) &= l A_{eff} \left(d\vec{I}_{eddy}(s, \omega) \times \vec{B}_0(s) \right) \\ &= l \sigma A_{eff} \left[\left(\dot{\vec{u}}(s, \omega) \times \vec{B}_0(s) \right) \times \vec{B}_0(s) \right] ds \end{aligned} \quad [4]$$

The total force acting on the wire is then the sum of both force contributions:

$$d\vec{F}(s, \omega) = d\vec{F}_L(s, \omega) + d\vec{F}_{CL}(s, \omega) \quad [5]$$

These Lorentz force waveforms can then be applied to the wire patterns in order to obtain a spatio-temporal displacement pattern.

For the case of MR gradient coils, where the static B_0 -field can be approximated as having no transverse component, the counter-Lorentz force $d\vec{F}_{CL}(s, \omega)$ in [5] can be simplified as follows:

$$\begin{pmatrix} dF_{CL,x}(s, \omega) \\ dF_{CL,y}(s, \omega) \\ dF_{CL,z}(s, \omega) \end{pmatrix} = -\sigma A_{eff} B_0(s)^2 \begin{pmatrix} \dot{u}_x(s) \\ \dot{u}_y(s) \\ 0 \end{pmatrix} l ds \quad [6]$$

By analogy, the Lorentz force $d\vec{F}_L(\vec{r}, \omega)$ in [1] can be simplified as:

$$\begin{pmatrix} dF_{L,x}(s, \omega) \\ dF_{L,y}(s, \omega) \\ dF_{L,z}(s, \omega) \end{pmatrix} = I(\omega) B_0(s) \begin{pmatrix} t_y(s) \\ -t_x(s) \\ 0 \end{pmatrix} l ds \quad [7]$$

The counter-electromotive force $dF_{CL}(s, \omega)$ is proportional to the velocity and therefore becomes a damping term to the equation of motion of each point on the conducting wire.

It is important to distinguish these motion-induced eddy currents from conventionally known eddy currents as follows. Eddy currents stem from Faraday's law of induction, where a time-varying magnetic flux creates an electric field and as such a flow of electric current in any conductive structure. The time-varying magnetic flux in conventionally known eddy currents is the alternating magnetic field generated either by the RF coil or gradient switching. In Lorentz damping, the time-varying magnetic flux is a result of the conductor moving/vibrating within a magnetic field. This effect can be compared to the mechanical resistance that has to be overcome when rotating an aluminum plate within the magnetic field of the MRI scanner.

Equation of motion

The epoxy material into which all modern day gradient coils are potted can be treated as a linear elastic material. The equation of motion for an infinitesimally small section of a conductor can then be written as:

$$\begin{aligned} m \begin{pmatrix} \ddot{u}_x(\vec{r}, s) \\ \ddot{u}_y(\vec{r}, s) \\ \ddot{u}_z(\vec{r}, s) \end{pmatrix} ds + (\sigma l A_{eff} B_0(s)^2 + \eta) \begin{pmatrix} \dot{u}_x(s) \\ \dot{u}_y(s) \\ 0 \end{pmatrix} ds + k_i \begin{pmatrix} u_x(s, \omega) \\ u_y(s, \omega) \\ u_z(s, \omega) \end{pmatrix} ds \\ = I(\omega) B_0(s) \begin{pmatrix} t_y(s) \\ -t_x(s) \\ 0 \end{pmatrix} l ds \end{aligned} \quad [8]$$

with m as the mass of the conductor section $d\vec{r}(s)$, η as the damping coefficient of the epoxy material, and k as the elasticity of the surrounding material, related to the material's Young's modulus and Poisson's ratio. The index i denotes the i^{th} degree of freedom of the system. The equations of motion for points along the conductor are then coupled with the equations

of motion for each point in the surrounding material. It is clear from [8] that the damping term depends on the squared static field strength, while the excitation Lorentz force term depends only linearly on B_0 .

The vibrational response is then arrived at by means of a harmonic analysis by solving [8], assuming sinusoidal currents.

Methods

A novel insertable head gradient coil (5) was analyzed in the present work, using the following modeling and experimental setup:

General modeling

We chose to conduct numerical modeling using the finite-element multi-physics package COMSOL (COMSOL 5.2, Inc., Burlington, MA, USA). Simulations were carried out using harmonic excitation with an alternating current (AC) current of amplitude 50 A in order to be able to match to experimental results obtained with the same excitation (this current was constrained to be less than 10% of the hardware maximum current output of the gradient drivers, to prevent any possibility of damage to the gradient coil or MR system due to excitation of mechanical resonances). This harmonic study was carried out over a frequency range of from 0–3000 Hz, which spans the frequency content of most pulse sequences.

Description of specific novel model features

- I. *General analysis:* The head gradient coil structure was modeled as a thick-walled short epoxy cylinder (inner/outer diameter: 338/490 mm, length: 330 mm to cover the folded wire patterns plus an additional 120 mm length acting as a rear extension to embed cable and cooling fluid connections). The epoxy cylinder was modeled as a linear elastic material (Young's modulus E : 10 GPa, density ρ : 1600 kg/m³, Poisson's ratio ν : 0.4, damping coefficient η : 0.018), whereas the air inside the bore was modeled as a pressure acoustic fluid domain governed by the acoustic wave propagation equation (speed of sound c : 343 m/s, density ρ : 1.2 kg/m³). The coil was modeled as sitting on a rigid patient bridge using a boundary condition that fixed the contact point in the vertical (y -) direction.
- II. *Additional patient bore:* A cylindrically shaped bore duct of 60 cm diameter, with added boundary to represent the patient bridge (Fig. 1a), was modeled with hard sound boundary wall conditions (vanishing normal air particle velocity at the boundary). For the head gradient simulations, all bore surfaces were modeled as acoustic-structural boundaries, as the head gradient coil is fully surrounded by air.
- III. *Air volume outside patient bore:* A hemispherical air volume of radius 1 m was added to the analysis, flush with the patient bore end (Fig. 1b). A simulation domain of infinite size was mimicked using perfectly matched layers of 20 cm thickness in order to allow sound waves radiated from the patient bore to propagate freely toward the outside without reflections.

- IV. *Accurate wire patterns:* Accurate conductor wire patterns were available from the in-house head gradient coil design, and were embedded in the epoxy to ensure a correct representation of the spatial excitation distribution for the analysis (Fig. 2).
- V. *Coupled vibroacoustic analysis:* The analysis included full coupling between acoustics and structural vibration. The acoustic module of COMSOL provides an interface that uses two coupled partial differential equations, interlinked by their respective sound pressure and displacements, as the acoustic-structure boundary. In this study, for all gradient axes, the inner and outer gradient tube walls, as well the end faces, were defined in this way as acoustic-structure boundaries.
- VI. *Motion-induced Lorentz damping:* The effect of displacement damping due to vibration-induced eddy currents in the conductors (Lorentz damping) according to [4] was modeled using solid round copper wire of diameter 3.26 mm for the *X*- and *Y*-gradients, and 5.08 mm for the *Z*-gradient. These dimensions were chosen to match the cross-sectional area of the $2 \times 7 \text{ mm}^2$ Litz wire that was used for the transverse axes and $5 \text{ mm} \times 10 \text{ mm}$ (outer dimensions, with wall thickness of 0.75 mm) hollow copper tube that was used for the *Z*-gradient.

Simulations

- A. Realistic wire patterns were embedded in the epoxy cylinder for the head gradient, and a current excitation yielded the Lorentz force patterns varying along the conductor paths. These force patterns then served as the spatial excitation for steps C and D outlined below.
- B. A modal analysis was performed to reveal the acoustic and structural eigenfrequencies of the stand-alone head coil geometry, from which potential structural-acoustic coupling modes and frequencies could be identified.
- C. A stand-alone analysis of the head gradient coil was then carried out to provide some intuitive understanding of the underlying vibroacoustics. We applied the spatio-temporal Lorentz force excitation of step A to a vibroacoustic analysis using harmonic currents over the frequency range of 0–3000 Hz. Each of the three gradient axes was analyzed separately. The structural mechanics were modeled with simply-supported boundary conditions at the bore faces. In this stand-alone analysis, the acoustic analysis did not include an external patient bore or bridge, nor did it include an outside air volume; rather, the acoustics were modeled with soft sound boundaries at both bore ends.

A simpler version of such a stand-alone analysis was used in (22). The stand-alone analysis presented here is more sophisticated, including Lorentz damping and structural-acoustic coupling. This step was used to compare prior vibroacoustic gradient coil analyses to our newer, more realistic analysis in step D. In addition, the resulting spectral resonances and vibroacoustic coupling could be traced back and compared to the eigenmodes in step B.

- D.** A realistic analysis was then performed which used the setup in step C with added features, including the presence of the patient bore/bridge as well as the outside air volume. The simply supported boundary condition was replaced by an acoustic-structural boundary, and the fixation to the rigid patient bridge was modeled using a hard constraint on vertical displacement.

Experiments

Experiments using an in-house developed head gradient coil were carried out to validate the accuracy of our vibroacoustic modeling of realistic gradient coils. The simulations in step D are a realistic emulation of the experiment and are therefore expected to show agreement to experiment.

To this aim, the head coil was driven (one gradient axis at a time) with a frequency-swept sinusoid (50–3000 Hz) of 60 s sweep duration and 50 A current amplitude. The MR system was a GE Discovery MR750 (3T) system. SPLs were measured using a Behringer ECM800 condenser microphone (Long & McQuade, Toronto, ON, Canada) positioned at 2 cm increments along x and z midlines. Reproducibility was assessed by repeating the experimental measurements.

Vibration levels were characterized by acceleration measurement using a single-axis Analog Devices EVAL-ADX001-70Z accelerometer (Analog Devices, Norwood, MA), sensitive to acceleration amplitudes of $\pm 70g$. The signals from the accelerometers were digitized using a 12-bit, Measurement Computing USB-201 analog to digital converter (Measurement Computing, Cleveland, OH), capable of digitizing up to eight signals at 100k total samples/second (12.5k samples/second for each channel when using all eight inputs). The accelerometers were positioned on the inner bore wall at $x = 16.9$ cm, $y = 0$ cm, and $z = +16$ cm (i.e. at the intersection of the central coronal plane with the inner bore wall, near the front of the coil). Acquired data was processed using a sliding window Fast Fourier Transform with window width of 92 ms.

Results

In the following sections, simulated and experimental results are shown for the head gradient coil described above.

Lorentz forces (step A)

Fig. 3 shows the simulated Lorentz force distributions that act on the conductors of the X -, Y -, and Z -gradients. The maximum force $\frac{dF}{dt}L$ per unit conductor length is 150 N/m. For all gradients it is observed that the forces are radially oriented. X - and Y -gradients for the head coil show that the circumferential variation in Lorentz forces should lead to odd-order acoustic circumferential mode excitation due to the single variation around the coil circumference (Fig. 3a,b). The Z -gradient in the head coil has a circumferentially uniform force distribution (Fig. 3c) with radially opposite Lorentz forces acting on the primary windings as compared to those acting on the shield windings, as is expected. This likely

excites structural modes with an acoustic circumferential mode number of zero and radial modes.

Eigenmode analysis (step B)

Table 1 shows the simulated/theoretical acoustic and structural eigenmodes of the stand-alone head gradient coil (i.e. without the added patient bore and outside air volume) from 0 to 3000 Hz with the three indices indicating radial, circumferential, and axial mode numbers, from left to right. The force distribution on a particular wire pattern will then selectively excite only a subset of these structural modes – an analysis showing the modes that are excited by each of the X -, Y -, and Z -gradient wire patterns is shown in the next section (step C). The vibrations of the inner bore wall will then excite a subset of all the acoustic eigenmodes.

In addition, certain acoustic and structural eigenmodes can *couple* (20), which can lead to strongly increased SPLs at that frequency. Such modes exhibit the same resonant frequency and mode shape for both structural and acoustic resonances. Using Table 1, we can identify potentially coupled modes. Fig. 4 shows a graphical visualization that plots mode frequency vs mode number, which can be used as an alternative tool for identifying coupled modes. Coupled modes correspond to “crossings” between the acoustic and structural mode relationships, wherever modes with the same mode code are sufficiently close in frequency.

For our head gradient geometry, the following acoustic modes are candidates for vibroacoustic coupling: (0,2,1) at 1057 Hz, (0,3,0) at 1357 Hz, (1,2,2) at 2300 Hz, (2,0,2) at 2393 Hz, and (1,3,2) at 2702 Hz. These are shown in bold print in Table 1. These correspond to the structural modes (0,2,1) at 1042 Hz, (0,3,0) at 1437 Hz, (0,2,2) at 2238 Hz, (0,0,2) at 2362 Hz, and (0,3,2) at 2689 Hz. If these coupled modes are excited by a given wire pattern and gradient waveform, then we can expect strongly increased sound pressure levels at these frequencies.

Sound pressure levels: stand-alone analysis (step C)

Fig. 5 shows acoustic and vibration analyses for the X -, Y -, and Z -axes of the head gradient coil, using stand-alone boundary conditions (i.e. without the added patient bore and using a soft sound boundary condition for the gradient bore ends instead of the outside air volume). Resonant peaks are labeled according to their corresponding acoustic eigenmode.

We observe a relatively small number of resonant peaks for the head gradient coil, with their frequencies coinciding for the X - and Y -gradients. Each of the resonant peaks can be traced back to its corresponding acoustic eigenfrequency and mode shape in Table 1. It must be noted that only a subset of all eigenmodes is excited by the wire pattern of the particular gradient axis. For the X - and Y -gradients (Fig. 5a, red), all resonances stem from modes with non-zero odd-order circumferential variation (as predicted by the qualitative Lorentz force patterns shown in Fig. 3) and non-zero axial variation. The Z -gradient (Fig. 5b, cyan) excites a different set of modes, namely the modes with no circumferential variation. This can again be explained by the circumferentially uniform distribution of Lorentz forces, as previously seen in Fig. 3. Additionally, a small number of higher-order circumferential modes are excited for the Z -gradient, which is likely attributable to higher order

vibroacoustic coupling effects. For all three gradient axes, only nonzero axial modes are excited.

Prominent resonant frequencies (i.e. resonances with peak SPL levels of >20 dB as compared to surrounding SPL levels) can be extracted from Fig. 5a,b. For the X and Y axes these prominent peaks are at 967 Hz (mode 0,1,2) and 1997 Hz (mode 0,1,5), while for the Z -gradient these are at 381 Hz (mode 0,0,1), 763 Hz (mode 0,0,2), 1907 Hz (mode 0,0,5), 2300 Hz (mainly coupled mode 1,2,2, together with a number of other eigenmodes falling at this frequency), 2540 Hz (mode 2,0,3), and 2671 Hz (mode 0,0,7). The resonance at 2300 Hz exhibits the largest SPL of >130 dB, which is an example of a coupled vibroacoustic mode being excited. This same mode also falls into the same frequency range with many other eigenmodes, which may further contribute to the increased SPL at this frequency.

Accelerations: stand-alone analysis (step C)

Fig. 5d shows the undamped acceleration spectra for all three gradient axes. We observe resonant peaks for the structural bending modes (0,0,1) at 1119 Hz, (0,1,2) at 2222 Hz, and (1,1,0) at 2907 Hz in the X/Y -gradient acceleration spectra and the breathing modes (0,0,1) at 1845 Hz and (0,0,2) at 2362 Hz in the Z -gradient acceleration spectrum. For the X - and Y -gradients, bending modes are excited, whereas for the Z -gradient, radial and breathing modes are found. Moreover, we can observe the previously mentioned coupled structural-acoustic resonance at 2300 Hz for the Z -gradient (shown by the small spike at this frequency, circled in Fig. 5d), denoting the importance of being able to accurately represent mode coupling in our model. The respective coupled modes are (0,2,2) for the structural resonance and (1,2,2) for the acoustic resonance. Fig. 5c shows the fully damped acceleration spectra for all three axes. For X/Y -axes the highest peak occurs at 2222 Hz, directly corresponding to the eigenmode observed in the undamped case. For the Z -gradient the undamped resonances vanish and a maximum acceleration at 2080 Hz is found.

Fig. 5d also shows a comparison of structural resonances with and without adding Lorentz damping in simulation, leading us to the observation that Lorentz damping leads to greatly decreased and smoothed peaks in the acceleration spectrum. For the X - and Y -gradients we observe that the resonant peaks in both damped and undamped spectra occur at the same frequencies, while for the Z -gradient this observation is not reflected. This unexpected result is subject to further investigation.

It should be noted that the structural (mechanical) resonances shown in Table 1 have only a minor influence on the acceleration spectrum (leading to just one major peak in that spectrum) and furthermore, structural resonances do not appear to have any significant impact on the stand-alone acoustic spectrum as none of the sharp peaks in the acoustic spectrum (Fig. 5a,b) correspond to structural resonances (Table 1). We believe this is due to vibrational peak smoothing and broadening caused by Lorentz damping, leading to the suppression of mechanical resonances that therefore do not create resonances in the acoustic spectrum. All sharp resonances that appear in the acoustic spectra in Fig. 5a,b can therefore be attributed to acoustic eigenmodes.

Sound pressure levels: realistic analysis (Step D)

Fig. 6a–c show the SPL spectra for the *X*-, *Y*-, and *Z*-head gradient coil in simulation and experiment, extracted at isocenter. Simulations in this case utilize the realistic analysis as defined by Step D in Methods. We observe a generally satisfactory agreement between simulation and experiment. Table 2 shows the averaged values for SPLs at isocenter for the graphs in Fig. 6, confirming this agreement. Average experimental SPLs over the 0–3000 Hz frequency range are 97.6 dB, 98.7 dB, and 95.4 dB for the *X*-, *Y*-, and *Z*-gradients, with the respective simulated average SPLs deviating by 0.7 dB, 0.5 dB, and 8.9 dB from these experimental values. The full spectra are also matched reasonably well, deviating by mean absolute differences of 9.7 dB, 9.0 dB and 14 dB (spectrally averaged absolute differences between simulation and experimental) for the *X*-, *Y*-, and *Z*-gradients, respectively. Prominent resonances in simulation (more than 20 dB above the immediate baseline near a resonance) occur at 640 Hz, 1280 Hz, and 2340 Hz, for the *X*-gradient and at 640 Hz, 1280 Hz, 2340 Hz, and 2580 Hz for the *Y*-gradient. For the *Z*-gradient, important resonances are found at 640 Hz, 760 Hz, 1280 Hz, 2300 Hz, and 2460 Hz. The resonance at 640 Hz is not an acoustic eigenmode of the head gradient coil per se but can be attributed to the geometries of the added patient bore and bridge. A comparison with the stand-alone analysis in step C reveals similar resonances with a large number of added peaks in the realistic analysis that stem from additional scanner components. The assignment of individual resonances in the step D simulations is difficult due to the highly complex spectrum. In addition, the more complex model required a coarser frequency resolution (20 Hz) due to model size as compared to the stand-alone model that was smaller and was solved in increments of 10 Hz to accurately identify distinct resonances. The goal here is not to compare single resonances between step C and D but rather to gain insights into the fundamentals of gradient vibroacoustics using step C and then to recognize the closer match to experimental measurements provided by the more realistic analysis in step D.

Accelerations: realistic analysis (Step D)

Fig. 7-c shows spectra of radial accelerations as characterized by simulation vs experiment for the *X*-, *Y*-, and *Z*-axes. For all three axes, simulations predict the measured spectrally-averaged acceleration values reasonably well: 26.8 g vs 20.6 g, 5.0 g vs 8.7 g, and 12.4 g vs 15.6 g (overall average deviations of 6.2 g, 3.7 g, and 3.2 g) for the *X*-, *Y*-, and *Z*-gradients, respectively (see Table 2). The mean absolute differences are 9.5 g, 3.8 g, and 9.5 g, for *X*-, *Y*-, and *Z*-gradients, respectively. Note that the accelerations differ between *X*- and *Y*-gradients as a result of the gradient coil being placed onto the patient bridge which constrains the coil in the vertical direction. Additional resonances appear in the experimental data and are attributed to mechanical resonances stemming from unknown and not modeled clamping and mechanical characteristics of the main magnet and other vibrating structures of the scanner, as well as possibly from over-simplifying the modeling of the copper conductors and the epoxy resin.

Discussion

This work has focused on the development of an accurate simulation methodology for the prediction of vibrations and acoustics in MR gradient coils. These new methods were

applied to a new type of insertable head gradient coil (5). Our goal was to show the validity of this new modeling tool which is distinguished by greatly increased accuracy of prediction as compared to existing literature. We used the new capability to more accurately predict SPLs and acceleration levels for our head gradient coil.

The stand-alone model presented in this work is based on a fully coupled vibroacoustic analysis and includes the effect of Lorentz damping, which is an aspect of gradient coil acoustics that has not been mentioned to date. This improved stand-alone model gives us important insights into the underlying vibroacoustic characteristics of a gradient coil.

In our more realistic simulations, which feature a number of improvements (added patient bore, added bridge geometry, added outside air volume), SPL spectra match experiments reasonably well, with a larger deviation for the *Z*-gradient as opposed to the *X*- and *Y*-gradients. The acceleration spectra also match experiments reasonably well for the *X*- and *Z*-gradients (not as well for the *Y*-gradient). Additional model features will be required, with more detailed analysis, to obtain closer match to the experimentally measured spectra. The added patient bore and the outside air volume, as well as the more accurate fixation boundary condition, change the discrete and “sparse” spectrum of the stand-alone analysis into a much more complex spectrum with a much greater number of smaller resonances, which is better matched to the experimental spectra. The match of both the average SPL levels and the spectral distribution of resonances in simulation demonstrate the necessity of using such an improved model over a stand-alone model for accurate gradient coil vibroacoustic prediction. The agreement between experiment and simulation is better than in previously published reports. Existing literature has claimed a match between experiment and simulation (20)–(24), however, only modal analyses were presented, and no full acceleration or SPL spectra were shown. In fact, only two published reports even demonstrate a rough comparison. Yao et al. (24) show results for velocity profiles, which are however not shown as an overlaid graph and show significant differences. In addition, a comparison of measured and simulated SPLs is shown but limited to a 2D distribution at a single frequency. A quantitative value for the SPL accuracy is not stated. Edelstein et al. (1) show a comparison that is based on simplified SEA analysis. Experiment and simulation qualitatively agree but show deviations on the order of 15–20 dB for lower frequencies (<1000 Hz) and around 5 dB for higher frequencies (1000 Hz – 10000 Hz). One of the limitations in the past has been computational power. Given the ~100-fold increase in CPU and RAM capabilities in today’s computers compared to two decades ago, the kind of realistic modeling and FEM analysis we describe in this paper is now feasible, with simulation times in the range of minutes to hours, making these tools practical for advanced investigations of gradient coil vibroacoustics.

The Lorentz damping term is introduced for the first time in this work; this term is particularly important for bringing simulated acceleration levels and in turn the observed average SPLs closer to the measured levels. Our results suggest that Lorentz damping acts to approximately decouple the effects that determine average SPL values from those that determine peaks in the acoustic spectrum. This is due to the large structural damping resulting from this effect, which suppresses structural resonant peaks to levels that are at most a few dB higher than off-peak baseline accelerations; as a consequence, structural

Conclusion

Numerical finite-element method simulations were used to model the vibroacoustics in head insert gradient coils in detail, including the effect of the patient bore and the outside air volume. The Lorentz damping effect was introduced for the first time and was shown to have a significant impact on structural vibrations and SPL levels. Thorough experimental studies were carried out to validate the presented analysis. Reasonable agreement between simulations and measurements was reached and many new insights about the causes of structural vibrations and acoustic sound pressure levels were gained as a result of these improved simulation capabilities.

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Fig. 1.

(a) A cylindrically shaped bore duct of 60 cm diameter, with added boundary to represent the patient bridge, was modeled with hard sound boundary wall conditions; (b) A hemispherical air volume of radius 1 m was added to the analysis, flush with the patient bore end.

Fig. 2.

Wire patterns for the head gradient: (a) X -, (b) Y -, and (c) Z -gradient axes.

Fig. 3.
Simulated radial Lorentz forces in N/m for X -, Y - and Z -head gradient axes.

Fig. 4.

Graphical strategy to find coupled modes: Find open and closed circles with same mode code and close to same frequency (overlapping or nearly overlapping symbols in this graph). Strongly overlapped: filled arrow. Partially overlapped: striped arrow. Close to overlapped: unfilled arrow.

Fig. 5. Standalone simulation results: (a) Simulated volume-averaged SPLs for the XY axes, (b) simulated volume-averaged SPLs for the Z gradient. (c) Simulated bore surface averaged accelerations for body X - as well as head X -, Y -, and Z -gradients; (d) Comparison of damped vs. undamped accelerations in all three gradients.

Fig. 6. Simulated vs experimental SPL at isocenter for (a) head X -, (b) Y -, and (c) Z -gradient coil at 3T/50 A.

Fig. 7. Simulated vs experimental acceleration spectrum near the bore end ($z = +16$ cm) for (a) head X -, (b) Y -, and (c) Z -gradient coil at 3T/50 A.

Acoustic and structural eigenmodes of the head gradient coil from 0 to 3000 Hz obtained from numerical simulation with the three indices in ascending order indicating radial, circumferential, and axial mode numbers, respectively. The acoustic resonances obtained from numerical analysis coincide with the theoretical values for the acoustic resonant modes a cylindrical air-filled cavity. The structural resonant frequencies can only be predicted with very rough approximation by thin shell theory, as compared to the accurate frequencies that are calculated here by means of numerical analysis. We can identify the structural modes (0,2,1) at 1042 Hz, (0,3,0) at 1437 Hz, (0,2,2) at 2238 Hz, (0,0,2) at 2362 Hz, and the mode (0,3,2) at 2689 Hz as candidates for vibroacoustic coupling.

Table 1

f_{Head} (Hz)	Acoust. mode no.	Struct. mode no.	Excited by axis	f_{Head} (Hz)	Acoust. mode no.	Struct. mode no.	Excited by axis	f_{Head} (Hz)	Acoust. mode no.	Struct. mode no.	Excited by axis	type
381	(0,0,1)		Z	1907	(0,0,5)		Z	2567	(0,4,5)		Z	
558		(0,2,0)	Ovaling	1965	(1,0,4)		Z	2571	(1,1,5)		X,Y	
595	(0,1,0)			1997	(0,1,5)		X,Y	2575	(0,5,4)			
706	(0,1,1)		X,Y	2042	(0,3,4)			2591	(1,3,0)			
763	(0,0,2)		Z	2065	(0,4,3)		Z	2603	(1,0,6)		Z	
967	(0,1,2)		X,Y	2068	(1,1,3)		X,Y	2619	(1,3,1)		X,Y	
987	(0,2,0)			2072	(0,5,0)			2651	(1,2,4)			
1042		(0,2,1)	Bend+Oval	2108	(0,5,1)			2662	(0,3,6)			
1057	(0,2,1)			2141		(1,0,0)	Radial	2671	(0,0,7)		Z	
1119		(0,0,1)	Bending	2147	(0,2,5)			2681	(0,6,3)			
1144	(0,0,3)		Z	2168	(1,2,0)			2689		(0,3,2)	Bend+Oval	
1238	(1,0,0)			2201	(1,2,1)			2702	(1,3,2)			
1247	(0,2,2)			2210	(0,5,2)		X,Y	2713		(0,4,1)	Bend+Oval	
1290	(0,1,3)		X,Y	2222		(0,1,2)	Bend+Oval	2735	(2,0,4)		Z	
1296	(1,0,1)		Z	2238		(0,2,2)	Bend+Oval	2736	(0,1,7)		X,Y	
1357	(0,3,0)			2268	(2,0,0)			2760	(2,1,0)			
1411	(0,3,1)		X,Y	2273	(1,0,5)		Z	2773	(0,7,0)			
1437		(0,3,0)	Ovaling	2287	(0,0,6)		Z	2787	(2,1,1)			
1455	(1,0,2)		Z	2298	(0,4,4)			2800	(0,7,1)		X,Y	
1511	(0,2,3)			2300	(1,2,2)			2818	(0,5,5)			
1525	(0,0,4)		Z	2300	(2,0,1)		Z	2833	(1,3,3)		X,Y	
1557	(0,3,2)		X,Y	2301	(1,1,4)		X,Y	2847	(0,2,7)			
1637	(0,1,4)		X,Y	2341	(0,3,5)		X,Y	2862	(1,4,0)			
1660		(0,0,1)	Torsional	2327		(0,1,1)	Bend+Tors	2864	(2,1,2)			

f_{head} (Hz)	Acoust. mode no.	Struct. mode no.	type	Excited by axis	f_{head} (Hz)	Acoust. mode no.	Struct. mode no.	type	Excited by axis	f_{head} (Hz)	Acoust. mode no.	Struct. mode no.	type	Excited by axis
1686	(1,0,3)			Z	2364	(0,1,6)			X,Y	2865	(1,6,0)			
1719	(0,6,0)				2362	(0,0,2)	(0,0,2)	Breathing	Z	2866	(1,1,6)			
1722	(1,1,0)				2368	(0,5,3)				2877	(0,7,2)			X,Y
1731		(0,3,1)	Bend+Oval		2393	(2,0,2)			Z	2888	(1,2,5)			
1760	(0,4,1)			Z	2425	(0,6,0)				2907		(1,1,0)	Radial	X,Y
1765	(1,1,1)			X,Y	2452	(1,2,3)				2944	(1,0,7)			Z
1776	(0,3,3)				2455	(0,6,1)				2964	(2,0,5)			Z
1817	(0,2,4)				2493	(0,2,6)				2988	(2,1,3)			X,Y
1845		(0,0,1)	Breathing	Z	2480		(0,4,0)	Ovaling		2997	(0,3,7)			
1881	(0,4,2)			Z	2540	(2,0,3)			Z	3000	(0,7,3)			
1884	(1,1,2)			XY	2543	(0,6,2)				3002	(1,4,1)			

Table 2

Spectrally mean values for SPLs and accelerations in X -, Y -, and Z -gradients for experiment and simulation. We observe a generally satisfactory agreement between simulation and experiment.

Axis	SPL Simulation	SPL Experiment	Accel. Simulation	Accel. Experiment
X	96.9 dB	97.6 dB	26.8 g	20.6 g
Y	99.2 dB	98.7 dB	5.0 g	8.7 g
Z	104.3 dB	95.4 dB	12.4 g	15.6 g