A sidelobe suppressing near-field beamforming approach for ultrasound array imaging

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A method is proposed to suppress sidelobe level for near-field beamforming in ultrasound array imaging. An optimization problem is established, and the second-order cone algorithm is used to solve the problem to obtain the weight vector based on the near-field response vector of a transducer array. The weight vector calculation results show that the proposed method can be used to suppress the sidelobe level of the near-field beam pattern of a transducer array. Ultrasound images following the application of weight vector to the array of a wire phantom are obtained by simulation with the Field II program, and the images of a wire phantom and anechoic sphere phantom are obtained experimentally with a 64-element 26 MHz linear phased array. The experimental and simulation results agree well and show that the proposed method can achieve a much lower sidelobe level than the conventional delay and sum beamforming method. The wire phantom image is demonstrated to focus much better and the contrast of the anechoic sphere phantom image improved by applying the proposed beamforming method.

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I. INTRODUCTION

Beamforming of a transducer array has been used in many applications, such as radar, sonar, seismology, wireless communications, and medical imaging, etc., to improve the signal to noise ratio. In medical ultrasound phased array imaging, an image is formed by transmitting a narrow beam at a number of angles and processing the received signals from all channels of the transducer array. The usual technique in ultrasound imaging is delay-and-sum (DAS) beamforming, which delays the received or transmitted signals by array elements based on their distances from the main target of imaging and then sums the delayed signals to construct the echo signal originating from the main target. In ultrasound imaging, off-axis echoes, originating from off-axis targets, degrade the quality of images by introducing broad image clutter. Therefore it is desirable to eliminate the contributions of the off-axis targets, which in turn results in clearer images with improved contrast. Thus sidelobe levels of the transducer array need to be suppressed as much as possible. The sidelobes of the beam pattern are often high for the DAS beamforming, causing off-axis interference and

greatly degrading the image contrast and quality in ultrasound imaging.

One way to calculate the beamforming weight vector of transducer array and reduce the sidelobe levels in ultrasound imaging is to apply Hanning or Hamming apodization to the transducer array. But this method has some drawbacks. First, the same weighting values are applied to the transducer array independent of depth and imaging line. Second, the weighting values only include amplitude but do not include phase. This is surely not optimized.

Another way to calculate the beamforming weight vector of transducer array in ultrasound imaging is the minimum variance (MV) adaptive beamformers.^{1–5} In this method, the weights are calculated by estimating the data covariance matrix and minimizing the power of the beamformer output subject to the constraint that the beamformer must exhibit the given response in the looking-direction. This method also has some drawbacks. First, the performance of the MV beamformer is dependent on the signal-to-noise ratio (SNR) of the imaging system.⁵ Second, the MV beamformer lacks robustness and is sensitive to the errors in the system.

Recently, some other methods have been developed to calculate the weighting function to reduce the sidelobes of transducer array for ultrasound imaging.^{6–8} Seo and Yen⁶ used dual apodization with cross-correlation technique to suppress the sidelobes of transducer array in ultrasound

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imaging. Guenther and Walker^{7,8} developed optimal apodization functions using constrained least squares theory to reduce the sidelobe levels of transducer array.

The medical ultrasound imaging system can be modeled and simulated with the Field II program.⁹ The Field II program can be used to calculate the acoustic emitted field and scattered field of transducer array for ultrasound imaging. It is based on the concept of spatial impulse response as developed by Topholme and Stepanishen.¹⁰⁻¹² The spatial impulse response method provides a fast, efficient time-domain method to calculate the acoustic field of transducers and arrays.¹³⁻¹⁶

The beamforming focus location of ultrasound transducer array is always in the near-field region of the array for ultrasound imaging, that is, inside a range of $D^2/4\lambda$, D is the aperture width of the array and λ is the wavelength. In the past, the near-field beamforming problem of transducer array has been studied by some researchers.¹⁷⁻²¹ In the near-field region, the sound wave propagating path must be considered. The near-field response vector of the transducer array, that is, the amplitude and phase of the signal received by each transducer in the array can be calculated based on the sound wave propagating path when the focus location, which includes the depth and scan directions, is given. And then we can use the near-field response vector to calculate the optimized beamforming weight vector of the transducer array considering both amplitude and phase.

To overcome the shortcoming of the MV beamformer, some sidelobe pattern control is required. Several array optimization approaches to sidelobe control have been proposed,^{22,23} such as iterative approach to adaptive beamforming with sidelobe control and quadratic beampattern optimization method, etc. Other well-known approaches are the diagonal loading method and the penalty function technique.^{1,5,22} The shortcoming of these techniques is that they cannot guarantee that the sidelobes of the beam pattern are exactly below the required level, and the algorithm of these approaches are indirect and complex as they need to use the data covariance matrix of the array.

In this paper, we propose a new array optimization method to calculate the weight vector using the near-field response vector to reduce the sidelobes of transducer array for ultrasound imaging. An optimization routine is developed whereby the mainlobe response of the transducer array will be kept to be unity and the norm of the weight vector made to be minimum and at the same time multiple additional quadratic inequality constraints outside the mainlobe beampattern area used. These constraints guarantee that the beam pattern sidelobe level remains lower than a certain prescribed value. This optimization problem can be solved efficiently using the second-order cone (SOC) optimization algorithm.²³⁻²⁶ In Refs. 23-26, the far-field beam pattern optimization problem of the transducer array is considered, while in this paper, the near-field beam pattern optimization problem is solved.

This new approach is assessed by simulation with the Field II program and experimentally with a 64-element 26 MHz phased array on a wire phantom and an anechoic sphere phantom.

II. OPTIMIZATION METHOD OF NEAR-FIELD BEAM PATTERN OF TRANSDUCER ARRAY

Suppose that the transducer array is composed of N elements, an optimized weight vector to reduce the sidelobes of the near-field beam pattern of a transducer array for ultrasound imaging needs to be determined. All of the elements of the array are assumed to be identical and have an omnidirectional response. The n th element of the array is located at the position (x_n, y_n, z_n) . The field point within the near-field region of the array is located at the position (x_p, y_p, z_p) . The distance from the field point to the n th array element r_n is given by

$$r_n = \sqrt{(x_p - x_n)^2 + (y_p - y_n)^2 + (z_p - z_n)^2}. \quad (1)$$

In the beamforming weight vector calculation process, when the array receives signals, we assume that an acoustic point source is located at the field point (x_p, y_p, z_p) , and it has a spherical wavefront. Its wavelength is denoted by λ and the corresponding wave number is $k = 2\pi/\lambda$. Each element in the array receives the signal irradiated by the point source. Suppose that the initial phase of the acoustic point source is zero. Then according to the acoustic propagation principle in a lossless medium, the signal received by the n th array element from the acoustic point source at the field point (x_p, y_p, z_p) is given by

$$S_n(x_p, y_p, z_p) = \frac{A}{r_n} \exp(-jkr_n), \quad (2)$$

where A is a constant related to the sound radiation pressure intensity of the acoustic point source. r_n is the distance from the field point to the n th array element obtained from Eq. (1). A/r_n is the amplitude of the received signal. It can be seen that the distance r_n affects not only the phase of the signal but also its amplitude in the near-field region of the array.

Then the near-field response vector of the transducer array composed of N transducers from the acoustic point source at the field point (x_p, y_p, z_p) can be represented as

$$\mathbf{S}(x_p, y_p, z_p) = [S_1 \ S_2 \ \cdots \ S_N]^T, \quad (3)$$

where S_n is the signal received by the n th array element from the acoustic point source at the field point (x_p, y_p, z_p) obtained by Eq. (2).

Assume that $\mathbf{w}(x_f, y_f, z_f)$ is the beamforming weight vector of the transducer array steering to the focus point (x_f, y_f, z_f) , which is

$$\mathbf{w}(x_f, y_f, z_f) = [w_1 \ w_2 \ \cdots \ w_N]^T. \quad (4)$$

Then the beamformed output response of the transducer array from the acoustic point source at the field point (x_p, y_p, z_p) with the mainlobe of the beam pattern steering to the focus point (x_f, y_f, z_f) becomes

$$B(x_p, y_p, z_p) = \mathbf{w}^H(x_f, y_f, z_f) \mathbf{S}(x_p, y_p, z_p), \quad (5)$$

where $\mathbf{w}(x_f, y_f, z_f)$ is the weight vector steering to the focus point (x_f, y_f, z_f) , which is a complex value including both amplitude and phase weighting, and $\mathbf{S}(x_p, y_p, z_p)$ is the near-field response vector of the transducer array from the acoustic point source at the field point (x_p, y_p, z_p) , which can be obtained from Eqs. (3) and (2). For the same weight vector, when we change the field point position (x_p, y_p, z_p) in Eq. (2), we can obtain the beamforming responses of the transducer array from acoustic point sources at different field points, which are equivalent to different directions, from Eq. (5). To steer an ultrasound beam to a different focal point, we need to design a different weight vector.

It is necessary to carry out the beam control for the transducer array to obtain a low-sidelobe near-field beam pattern for ultrasound imaging. We establish an optimization problem as follows: we keep the mainlobe response of the transducer array, which is steering to the focus point (x_f, y_f, z_f) , to be unity; make the norm of the weight vector be minimum, which can increase the robustness of the beamformer for the norm of the weight vector is equal to the white noise gain of the array; and at the same time, use multiple additional quadratic inequality constraints outside the mainlobe beam pattern area, which guarantees that the beam pattern sidelobe level remains lower than a certain prescribed value. It leads to the following optimization problem:

$$\begin{cases} \min_{\mathbf{w}} \mu \\ \text{subject to } \mathbf{w}^H \mathbf{S}(x_f, y_f, z_f) = 1 \\ \|\mathbf{w}\| \leq \mu \\ |\mathbf{w}^H \mathbf{S}(x_p, y_p, z_p)| \leq \delta_s, \forall (x_p, y_p, z_p) \in \Theta_{SL}, \end{cases} \quad (6)$$

where $\mathbf{w}$ is the weight vector of the transducer array, μ is a nonnegative real variable, (x_f, y_f, z_f) is the focus point, (x_p, y_p, z_p) is the field point in the sidelobe region, Θ_{SL} is the sidelobe region, which includes a number of directions and δ_s controls the sidelobe level of the beam pattern. $\mathbf{S}(x_f, y_f, z_f)$ and $\mathbf{S}(x_p, y_p, z_p)$ are, respectively, the near-field response vectors of the transducer array from acoustic point sources at the focus point (x_f, y_f, z_f) and field points (x_p, y_p, z_p) . We uniformly divide the circular arc in the process of beam scanning at a certain distance from the center of the transducer array in the acoustic near field of the array into M field points. The lines connecting these field points and the array center denote M directions of the acoustic near field. The line connecting the focus point (x_f, y_f, z_f) and the array center denotes the mainlobe direction and that connecting the field point (x_p, y_p, z_p) and the array center denotes the sidelobe direction. We can obtain a number of sidelobe directions in the sidelobe region Θ_{SL} when we change the field point positions.

The optimization problem [Eq. (6)] can be solved by means of the optimization algorithm of the second-order cone programming.²³ Thus the optimized weight vector $\mathbf{w}$ of the transducer array is calculated to achieve a low-sidelobe near-field beam pattern. This optimization method of calculating the weight vector can be applied to transducer arrays

of arbitrary shape including linear array, convex array, circular array, etc.

To assess the performance of this new approach, ultrasonic images of a wire phantom are obtained by simulation with Field II program, and the images of a wire phantom and an anechoic sphere phantom are obtained experimentally with a 64-element phased array.

III. SIMULATION RESULTS

The transducer array simulated is a linear phased array composed of 64 transducers. The center frequency of the array is 26 MHz. The center-to-center element spacing is 30 μm , which is nearly half-wavelength. The height of the transducers is 2 mm. The sound speed in the medium is assumed to be 1480 m/s. Two methods are used to calculate the weight vector of the transducer array for comparison. The first method is the conventional DAS beamforming method, which delays the received signals to the array elements based on their distances to the focus point and then sums the delayed signals to construct the beamformed output.^{20,21} The second method is the proposed method, which uses Eq. (6) to calculate the optimized weight vector of the transducer array. Then the near-field directivity of the transducer array is calculated from Eq. (5) with the obtained weight vector and the near-field response vector. The field points used to calculate the near-field response vector are on the circular arc at a distance of 8 mm from the center of the transducer array. Figures 1 and 2 are, respectively, the near-field directivity of the transducer array when the beam scan angles are 10° and 20°. The x axis represents the scan angle of the directivity which is from -60° to 60°. The y axis represents the near-field beam pattern of the transducer array denoted in decibels where the maximum value is normalized to 0 dB. The solid line represents the results with the weight vector calculated by the proposed method, which constrains the sidelobe level to below -20 dB. The dotted line represents the results from the conventional DAS beamforming method. It is found that the proposed method is capable of reducing the sidelobe level of the near-field beam pattern of

FIG. 1. The near-field directivity of the array when the beam scan angle is 10°.

FIG. 2. The near-field directivity of the array when the beam scan angle is 20° .

the transducer array. The sidelobe level of the proposed method is much lower than that of the DAS beamforming method while the mainlobe beamwidths of the two methods are similar. Here only the directivities in the beam scan directions of 10° and 20° are shown; the beam pattern can also be calculated in other beam scan directions simply by changing the beam scan angle.

The Field II program is used to simulate the ultrasound images of the above transducer array. The weight vectors calculated by the two methods are applied to the array. The weight vectors of the proposed beamforming method are complex values including both amplitude and phase weighting. A wire phantom image is simulated. The phantom is composed of five wire targets. In the Field II program, the wire targets are represented by the points of scatters. The positions of the scattering points are, respectively, $(1.3 \ 0 \ 3.5)$, $(0.65 \ 0 \ 5)$, $(0 \ 0 \ 6.5)$, $(-0.65 \ 0 \ 8)$, and $(-1.3 \ 0 \ 9.5)$, with a unit mm. In the simulation process, the transmitting beam is generated by the conventional DAS beamforming method, which gives the transmitting signal of each transducer of the array a different time delay based on its distance from the target of imaging with the same amplitude. The transducer array transmitted signals at a series of beam scan angles by the DAS beamforming method, which are from -45° to 45° with an increment step 0.5° , focused at 5 mm. The excitation pulse is modeled as a two-cycle sinusoid at the central frequency of the array. The transmitted signals are scattered back by the targets. The transducer array receives the scattered signals. The images are then formed by processing the received signals from all channels of the transducer array applying the calculated beamforming weight vectors at the corresponding beam scan angles. The received signals of the transducer array are converted from real data to complex data using the Hilbert transform, which can be multiplied by the complex weight vectors, and at the same time the amplitude of the beamformed output signal can be obtained. The beam scan angles are also from -45° to 45° with an increment step of 0.5° . The depths of targets are calculated using the propagation time of the acoustic signals and the sound

FIG. 3. (Color online) The simulated wire phantom ultrasound imaging result with the DAS beamforming method.

speed in the medium. Through scan conversion, the beam-formed output, which is in polar coordinates, can be converted into Cartesian coordinates, and the wire phantom images can be formed and displayed. Figures 3 and 4 are, respectively, the wire phantom ultrasound images obtained by the transducer array using the DAS beamforming method and the proposed beamforming method. The images are displayed with a 45 dB dynamic range. The actual positions of the wire phantoms are denoted by the sign of star in the figures. The x axis represents the lateral distance of the transducer array, where the unit is millimeters. The y axis represents the axial distance of the transducer array with a unit of millimeters. The origin is the center of the transducer array. It is clear from these figures that the sidelobe level of the proposed method is much lower than that of the DAS beamforming method. The white spots that reveal the wire images become tighter in Fig. 4. The sizes of the images of the five targets in Fig. 4 are, respectively, reduced to 73.3%, 60%, 71.4%, 60%, and 70.6% of those in Fig. 3 in the lateral direction. The sizes of the images in the axial direction do

FIG. 4. (Color online) The simulated wire phantom ultrasound imaging result with the proposed beamforming method.

not change much. The criteria that we use to quantify the reduction are 45 dB down from the maximum value. The wire phantom image using the proposed method focuses much better and the width of the wire image becomes narrower than that using the DAS beamforming method.

IV. EXPERIMENTAL RESULTS

The experimental data of a wire phantom composed of five wire targets are collected with a 64-element prototype linear phased array in a water bath. The parameters of the array are the same as the preceding simulated array. The center frequency of the array is 26 MHz. The center-to-center element spacing is 30 μm . A wire phantom composed of five wire targets is placed in a water bath. The length and diameter of the wire targets are, respectively, 2 cm and 20 μm . The positions of the targets are, respectively, (1.3 0 3.5), (0.65 0 5), (0 0 6.5), (−0.65 0 8), and (−1.3 0 9.5) with a unit millimeter, which are also the same as those used in the simulation. The coordinates provided refer to the centers of the objects. The transducer array transmitted pulse signals at a series of beam scan angles by the DAS beamforming method that are from -45° to 45° with an increment step 0.5° , focused at 5 mm. The pulse-echo response signals scattered by the wire targets are measured and collected at the corresponding beam scan angles. Images are acquired using the two beamforming methods. The wire phantom images are displayed in Figs. 5 and 6, respectively, for the DAS beamforming and the proposed beamforming approach. The images are displayed with a 45 dB dynamic range. The x axis and y axis, respectively, represent the lateral and axial distance of the transducer array, where the unit is mm. The origin is the center of the transducer array. Figures 5 and 6 demonstrate that the experimental results are consistent with the simulation results in Sec. III. The five wire targets are clearly displayed in the images. The white spots that reveal the images of wire targets also become tighter in Fig. 6. The sizes of the images of the five targets in Fig. 6 are, respectively, reduced to 80%, 83.3%, 80%, 81.8%, and 78.9% of those in Fig. 5 in the lateral direction. The sizes of the

FIG. 5. The experimental wire phantom ultrasound imaging result with the DAS beamforming method.

FIG. 6. The experimental wire phantom ultrasound imaging result with the proposed beamforming method.

images in the axial direction do not change much. The criteria that we use to quantify the reduction are 45 dB down from the maximum value. The proposed method is shown capable of achieving a much lower sidelobe level and focusing much better than the DAS beamforming method.

The approach is also tested on an anechoic sphere phantom obtained from the University of Wisconsin²⁷ with the 64-element linear phased array in the water bath. In this anechoic sphere phantom, anechoic spheres of different diameters are randomly distributed spatially in a uniformly echogenic tissue-mimicking background material appropriate for high frequency ultrasound imaging. The mean diameter of the anechoic spheres we test in the experiment is 530 μm . Images are obtained with the two beamforming methods. The anechoic sphere phantom images are displayed in Figs. 7 and 8, respectively, for the DAS beamforming and the proposed beamforming approach. The images are displayed with a 45 dB dynamic range. The x axis and y axis, respectively, represent the lateral and axial distance of the

FIG. 7. The experimental anechoic sphere phantom ultrasound imaging result with the DAS beamforming method.

FIG. 8. The experimental anechoic sphere phantom ultrasound imaging result with the proposed beamforming method.

transducer array, where the unit is millimeter. The origin is the center of the transducer array. Figures 7 and 8 show that the anechoic spheres and background of the anechoic sphere phantom are clearly displayed in the images for both beamforming methods and the contrast of the image is improved using the proposed beamforming approach because of the lower sidelobes of the beam patterns.

V. CONCLUSIONS

In this paper, a new method is proposed to suppress sidelobe level of a transducer array for ultrasound imaging. An optimization problem is posed to calculate the optimized weight vector of transducer array, which keeps the mainlobe response of the transducer array to be unity and makes the norm of the weight vector be minimum and at the same time uses multiple additional quadratic inequality constraints outside the mainlobe beam pattern area to guarantee that the beam pattern sidelobe level remains lower than a certain prescribed value. The second-order cone algorithm is used to solve the optimization problem and obtain the optimized weight vector. The results from simulation and experiments show that the proposed method is capable of controlling the sidelobe level, thus yielding improved image quality than that of the conventional DAS beamforming method.

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