Frequency shifts in distortion-product otoacoustic emissions evoked by swept tones

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When distortion-product otoacoustic emissions (DPOAEs) are evoked using stimuli whose instantaneous frequencies change rapidly and continuously with time (swept tones), the oscillatory interference pattern known as distortion-product fine structure shifts slightly along the frequency axis in the same direction as the sweep. By analogy with the temporal mechanisms thought to underlie the differing efficacies of up- and down-swept stimuli as perceptual maskers (e.g., Schroeder-phase complexes), fine-structure shifts have been ascribed to the phase distortion associated with dispersive wave propagation in the cochlea. This paper tests an alternative hypothesis and finds that the observed shifts arise predominantly as a methodological side effect of the analysis procedures commonly used to extract delayed emissions from the measured time waveform. Approximate expressions for the frequency shifts of DPOAE distortion and reflection components are derived, validated with computer simulations, and applied to account for DPOAE fine-structure shifts measured in human subjects. Component magnitudes are shown to shift twice as much as component phases. Procedures for compensating swept-tone measurements to obtain estimates of the total DPOAE and its components measured at other sweep rates or in the sinusoidal steady state are presented. © 2016 Acoustical Society of America. [<http://dx.doi.org/10.1121/1.4960592>]

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I. INTRODUCTION

Otoacoustic measurement and analysis methods in which the evoking sound stimuli are rapidly swept across frequency can significantly reduce recording times for both distortion-product and stimulus-frequency otoacoustic emissions (e.g., Long *et al.*, 2008; Choi *et al.*, 2008; Bennett and Özdamar, 2010; Kalluri and Shera, 2013). In contrast to traditional discrete-tone methods, in which different frequency components of the otoacoustic emission (OAE) are measured stepwise in the sinusoidal steady state, swept-tone techniques smoothly traverse the full frequency range of interest, often at rates exceeding 1 octave per second. Although swept-tone methods generally yield estimates of OAE amplitude and phase that closely match those obtained with discrete tones (e.g., Long *et al.*, 2008; Kalluri and Shera, 2013; Abdala *et al.*, 2015), systematic differences are sometimes apparent. For example, distortion-product otoacoustic emission (DPOAE) fine structure—the pattern of spectral ripples that results from alternating constructive and destructive interference between the reflection and distortion components of the total DPOAE—has been found to shift to the right or to the left—that is, either upward or downward in frequency, in the same direction as the sweep—by amounts that vary

somewhat idiosyncratically from subject to subject but appear to depend systematically on the sweep rate (Henin *et al.*, 2011; AlMakadma *et al.*, 2015; Abdala *et al.*, 2015).

To illustrate the frequency shifts observed in emission fine structure, Fig. 1 compares averaged DPOAE levels obtained using primaries swept alternately up and down at a rate of 2 octaves/s. The emissions were extracted from the measured pressure waveform using least-squares fitting (e.g., Long *et al.*, 2008; Kalluri and Shera, 2013). The least-squares procedure estimates emission magnitude and phase versus frequency by fitting a model that includes the frequency components of interest (e.g., f_1 , f_2 , $2f_1 - f_2$) to windowed segments (frames) of the recorded time-domain pressure waveform. As seen most clearly in the figure inset, the two fine-structure patterns appear offset from one another. The offsets evident here are typical of those previously reported—up-sweeps shift measured fine-structure patterns rightward to higher frequencies, down-sweeps shift them leftward to lower. In both cases, the shifts are relative to fine-structure patterns obtained using discrete tones (or at slower sweep rates). Corresponding shifts appear in the microstructure of the DPOAE phase, as well as in derived estimates of the underlying DPOAE components, principally in the reflection component (Henin *et al.*, 2011; AlMakadma *et al.*, 2015).

The origin and significance of the frequency shifts remain unclear. Henin *et al.* (2011) suggest that the shifts arise due to dispersive wave propagation in the cochlea (see

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FIG. 1. Frequency shifts in DPOAE fine structure. The figure shows the DPOAE levels at $2f_1 - f_2$ evoked from a human young adult using swept primary tones. Emissions were extracted from the measured time waveform using narrowband least-squares fitting (LSF). Line thickness and color code the direction of the sweep: thick black shows the DPOAE level obtained when the primaries were swept upward, from low frequencies to high; thin gray shows the response to a down-sweep. The inset zooms to show the 1.5–2.3 kHz region in more detail. The measurement noise floor, which averages around -20 dB SPL, is shown at the bottom of the figure. Stimulus parameters: primary frequency ratio $f_2/f_1 = 1.22$; primary levels $\{L_1, L_2\} = \{65, 65\}$ dB SPL; sweep rates $b = \pm 2$ octave/s.

also AlMakadma *et al.*, 2015). Cochlear dispersion means that the speed of the traveling wave changes as it propagates (or, equivalently, that the group delay of the motion at any location depends on frequency). Dispersion implies that complex, multi-component stimuli such as chirps or so-called Schroeder-phase complexes (Schroeder, 1970), when swept across frequency in opposite directions, can produce basilar-membrane (BM) vibration patterns with strikingly different temporal envelopes (Recio and Rhode, 2000; Summers *et al.*, 2003). Thus, as outlined by AlMakadma *et al.* (2015), the different BM vibration patterns produced by up- versus down-swept stimuli may induce corresponding phase or frequency shifts in the emissions they evoke. For example, as a consequence of cochlear dispersion, up- and down-swept stimuli are known to have different efficacies as perceptual maskers (e.g., Smith *et al.*, 1986; Kohlrausch and Sanders, 1995; Wojtczak and Oxenham, 2009). By extension, the different BM vibration patterns produced by these two stimuli may also yield different amounts of suppression or other nonlinear interactions that affect the process of emission generation.

A simpler albeit less mechanistically interesting hypothesis is that fine-structure shifts arise via an artifact inherent in the swept-tone procedure and the analysis methods used to extract the emission from the microphone recording. The intuition underlying this alternative suggestion is that when the analysis procedure operates on a time waveform containing an emission whose instantaneous frequency is changing, apparent frequency shifts can arise simply because the emission components are delayed relative to the stimulus. For example, if the total emission pressure consists of two components with different latencies, then the two interfering components recorded by the microphone at any given instant have slightly different frequencies. Because the system is

not in the sinusoidal steady state, neither one of these instantaneous frequencies necessarily matches the emission frequency assumed and assigned by the analysis procedure. In effect, sweep rate, analysis methodology, and emission delay combine to produce systematic mislabelings of the frequency axis, mislabelings that generally differ for each emission component as well as for their sum. The frequency shifts predicted by this scenario thus depend both on the component latencies, which generally vary with frequency and across subjects, and on the sweep rate and direction.

Since neither of these explanations for sweep-induced fine-structure shifts precludes the other, our goal here is to help assess their relative contribution to the measured shifts. Because the evaluation of human cochlear dispersion remains challenging—although not impossible, e.g., via determination of the phase curvature of the auditory filters (Oxenham and Dau, 2001)—we focus on quantifying and testing the predictions of the second, more methodological hypothesis. After deriving approximate theoretical expressions for the shifts and validating them in simple models, we apply our expressions to measured data to assess whether they can account for the frequency shifts seen in real ears. Finally, in the Discussion, we outline a procedure that compensates for the frequency shifts arising from the use of swept tones, enabling one to reconstruct an estimate of the steady-state DPOAE and its components.

II. THEORETICAL FREQUENCY SHIFTS

A. Shifts in component magnitude

Acoustic distortion products arise when cochlear nonlinearities driven at the stimulus frequencies (e.g., f_1 and f_2) produce mechanical motion at combination-tone frequencies (e.g., $2f_1 - f_2$). To estimate the frequency shifts expected from the swept-tone paradigm due solely to emission latency, we first consider the idealized case in which the emission consists of a single component (e.g., reflection or distortion) that emerges with a well-defined, but possibly frequency-dependent phase-gradient delay, τ . (To give it a name, we will call it component “X.”) In swept-tone measurements, the stimulus frequencies are varied continuously so that the *nominal* emission frequency at any instant [e.g., $f_{dp}(t) = 2f_1(t) - f_2(t)$] varies at some known rate, b , given, for example, in Hz/s. Because of the delays involved in emission generation, including round-trip propagation through the middle ear and along the cochlea, the *actual* emission frequency being measured by the microphone at time t is not $f_{dp}(t)$ but $f_{dp}(t - \tau)$. During the elapsed time τ that it took the emission to arise and emerge from the cochlea, the nominal distortion-product (DP) frequency has changed by an amount

$$\Delta f_{dp} = b\tau. \quad (1)$$

Consequently, the instantaneous emission amplitude measured at time t is not $|X|[f_{dp}(t)]$ but $|X|[f_{dp}(t) - \Delta f_{dp}]$. Thus, when $|X|$ is plotted versus its nominal rather than its actual frequency, it appears shifted along the frequency axis by an amount Δf_{dp} proportional to the sweep rate.¹

B. Shifts in component phase

Remarkably, frequency shifts for component phase turn out to differ from shifts for component magnitude. The complication arises because phases are expressed relative to a reference. Because the stimulus itself contains no distortion products to serve as phase reference, DPOAE phases are conventionally referenced to the corresponding DPs produced by an assumed instantaneous nonlinearity located at the microphone port. Thus, DPOAE phase at $2f_1 - f_2$ is referenced to $2\phi_1 - \phi_2$, where ϕ_1 and ϕ_2 are the measured phases of the primary tones in the ear canal (e.g., [Dhar and Abdala, 2010](#)). Because the stimulus frequencies are varied continuously, we consider the phase as a function of time, t . Let $\varphi(t)$ be the phase, in cycles, of the DP component produced by the imagined reference nonlinearity. Because of the assumed delay, τ , we expect the emission phase measured at time t to be $\varphi(t - \tau)$. Expressed relative to the reference, the measured phase is thus $\varphi_{dp}(t) = \varphi(t - \tau) - \varphi(t)$. To approximate this difference, we expand $\varphi(t - \tau)$ in a Taylor series,

$$\varphi(t - \tau) = \varphi(t) - \dot{\varphi}\tau + \frac{1}{2}\ddot{\varphi}\tau^2 + \dots, \quad (2)$$

where the diacritical dots represent time derivatives. Thus,

$$\varphi_{dp}(t) \cong -f_{dp}\tau + \frac{1}{2}b\tau^2, \quad (3)$$

where we have substituted the values $f_{dp} = \dot{\varphi}$ for the instantaneous DP frequency and $b = \ddot{\varphi}$ for the sweep rate.

Equation (3) indicates that there are two contributions to the measured component phase. The first term ($-f_{dp}\tau$) is the phase shift due to the delay τ itself; the second term arises because the nominal DP frequency, and thus the reference phase it provides, changes over time. Note that only the second term depends on the sweep rate, b . The first term contributes even when the DP frequency is constant ($b = 0$) and the measurements are made in the sinusoidal steady state using the discrete-tone paradigm. The second term, proportional to b , represents the additional phase shift arising from the use of swept tones. Our analysis predicts that the phase difference between swept- and discrete-tone measurements is

$$\Delta\varphi \cong \frac{1}{2}b\tau^2. \quad (4)$$

The phase shift $\Delta\varphi$ corresponds to an equivalent shift, Δf , along the frequency axis, given by

$$\Delta f \cong \frac{\Delta\varphi}{\tau} \cong \frac{1}{2}b\tau. \quad (5)$$

Note that the predicted frequency shift for component phase is *half* of that obtained in Eq. (1) for component magnitude ($\Delta f \cong \frac{1}{2}\Delta f_{dp}$). We return to and test this prediction for the relative frequency shifts in Sec. V.

C. Shifts in fine structure

The frequency shifts derived above have consequences for measurements of DPOAE fine structure, such as those

illustrated in Fig. 1. Naively, swept tones might be expected to induce a shift in magnitude fine structure equal to Δf_{dp} , the shift derived for component magnitude in Eq. (1). However, the spectral ripples characteristic of DPOAE fine structure are not so much the consequence of frequency variations in component magnitude as they are of the rotation of component phase. Furthermore, the origin of fine structure involves not just a single component (e.g., component X), but two (distortion and reflection). These complications all affect the expected fine-structure shift.

Distortion-product fine structure results from interference between multiple DPOAE components with different phase-gradient delays (e.g., [Kim, 1980](#); [Kemp and Brown, 1983](#); [Brown et al., 1996](#); [Heitmann et al., 1998](#); [Talmadge et al., 1998](#); [Mauermann et al., 1999](#); [Kalluri and Shera, 2001](#)). To a good first approximation, we can ignore variations in component magnitudes. Assume there are two components (labeled “d” and “r”) with phases $\varphi_d(f)$ and $\varphi_r(f)$. Fine-structure peaks occur at frequencies where the phases φ_d and φ_r are equal, so that the components add constructively. Let f_0 be the frequency of a fine-structure peak measured using discrete tones. If τ_d and τ_r are the respective delays of the two components, then near f_0 the phases have the form $\varphi_d = -(f - f_0)\tau_d + \varphi_0$ and $\varphi_r = -(f - f_0)\tau_r + \varphi_0$, where φ_0 is a common constant that specifies the value at the fine-structure peak: $\varphi_d(f_0) = \varphi_r(f_0) = \varphi_0$. When measured using the swept-tone paradigm, the phases of the two components shift by amounts, $\Delta\varphi_d$ and $\Delta\varphi_r$, that depend on their delays, as outlined above [Eq. (4)]. The phases become $\varphi_d = -(f - f_0)\tau_d + \Delta\varphi_d + \varphi_0$ and $\varphi_r = -(f - f_0)\tau_r + \Delta\varphi_r + \varphi_0$. The phase shifts produce a change, Δf_{fs} , in the frequency of the fine-structure peak. To find that frequency shift, we set $f = f_0 + \Delta f_{fs}$, require that the two phases again be equal (constructive interference), and solve for Δf_{fs} . The result is

$$\Delta f_{fs} = \frac{\Delta\varphi_r - \Delta\varphi_d}{\tau_r - \tau_d}. \quad (6)$$

When $\Delta\varphi_{d,r} = \frac{1}{2}b\tau_{d,r}^2$, the frequency shift reduces to

$$\Delta f_{fs} = \frac{1}{2}b(\tau_r + \tau_d) = \Delta f_r + \Delta f_d. \quad (7)$$

The predicted swept-tone fine-structure shift is simply the sum of the frequency shifts of the phases of the individual DPOAE components. As expected, the frequency shift is symmetric with respect to interchange of the two component delays.

Consistent with the patterns observed experimentally, the direction of the predicted fine-structure shift depends on the direction of the sweep (i.e., on the sign of b). Because the delays are generally positive, frequency up-sweeps ($b > 0$) shift the fine-structure pattern to higher frequencies ($\Delta f_{fs} > 0$); down-sweeps shift the pattern in the opposite direction. Equation (7) implies that the total shift between an up-sweep and a down-sweep measured at the same sweep rate is twice the unidirectional result:

$$\Delta f_{\uparrow} = |b|(\tau_r + \tau_d). \quad (8)$$

The same formula holds for logarithmic sweeps (with b in octaves/s and the frequency shift $\Delta f_{\uparrow}$ then measured in octaves) when $|b\tau| \ll 1$ octave. Typical sweep rates ($|b| \sim 1$ oct/s) and emission delays ($\tau_r + \tau_d \sim 10$ ms) yield expected fine-structure shifts $\Delta f_{\uparrow} \sim 0.01$ oct (or, equivalently, 12 cents).² The size of the predicted shift varies in direct proportion to the rate. Conveniently, quantitative tests of Eq. (8) can be performed without making discrete-tone measurements.

III. SIMULATED FINE-STRUCTURE SHIFTS

To test the predictions of Eq. (8) for the fine-structure shifts $\Delta f_{\uparrow}$, we first apply them to simulated DPOAE measurements. Perhaps the simplest meaningful simulation is one in which the model DPOAEs consist of two components, each with constant magnitude and delay. Thus,

$$\hat{P}_{dp}(f) = |P_d|e^{-2\pi i f \tau_d} + |P_r|e^{-2\pi i f \tau_r}. \quad (9)$$

The diacritical hat indicates that $\hat{P}_{dp}(f)$ given by Eq. (9) represents the composite DPOAE spectrum as it appears in the steady state (i.e., when measured using discrete tones). To simulate a measurement performed using swept tones, $\hat{P}_{dp}(f)$ was converted into a time-domain, swept-frequency waveform using Fourier analysis. More specifically,

$$p_{dp}(t) = \mathcal{F}^{-1}\{\hat{P}_{dp}(f)\} * s(t), \quad (10)$$

where $\mathcal{F}^{-1}\{\cdot\}$ is the inverse Fourier transform, the asterisk (*) represents convolution, and $s(t)$ is a swept-tone waveform, defined by

$$s(t) = O(t) \cos[2\pi\phi(t)]. \quad (11)$$

In this equation, $O(t)$ is an overall ramping window used to taper the tone onset and offset, and $\phi(t)$ is the ongoing phase (in cycles). We used logarithmic sweeps whose instantaneous frequency spanned the range 0.5–8 kHz, varying as $f_{dp}(t) = \dot{\phi}(t) = f_0 2^{b(t-t_0)}$ in a direction determined by the sign of b . For the numerical computations, signals were sampled at 48 kHz. Emissions were extracted from the swept-frequency signals using least-squares fitting (e.g., Long *et al.*, 2008; Kalluri and Shera, 2013; Abdala *et al.*, 2015).

Figure 2 shows the resulting DPOAE levels in a narrow frequency region centered on 2 kHz, the location of a fine-structure peak in the steady-state spectrum. Line types indicate the three sweep rates: Dotted lines show the fastest sweeps (2 oct/s) and solid lines the slowest ($\frac{1}{2}$ oct/s); dashed lines are intermediate (1 oct/s). Emission levels at each sweep rate are shown both for up-sweeps (black lines) and for down-sweeps (gray lines). Comparison with the scale bars, whose lengths give values of $\Delta f_{\uparrow}$ computed from Eq. (8), demonstrates that the fine-structure shifts observed in the simulations match theoretical predictions based on the sweep rates and DPOAE component delays used in the model.

Although sufficient to illustrate the mechanisms at play, Eq. (9) provides only an idealized representation of the composite DPOAE and its components. Fine-structure shifts

FIG. 2. DPOAE fine-structure patterns obtained near 2 kHz from simulated swept-tone measurements with different sweep rates and directions. The three different line types (dotted, dashed, and solid) show results for the three sweep rates (2, 1, and $\frac{1}{2}$ oct/s, respectively). Up-sweeps are shown using thick black lines and down-sweeps using thin gray lines. The length of each horizontal scale bar matches the corresponding value of $\Delta f_{\uparrow}$, the magnitude of the theoretical frequency shift computed from Eq. (8). Arrowheads on the scale bars indicate the sweep direction. DPOAE spectra were simulated using Eq. (9) and then transformed into frequency sweeps [Eq. (10)] for LSF analysis. Parameter values: $\tau_d = 0.5$ ms; $\tau_r = 5.5$ ms; $|P_r/P_d| = 0.4$.

obtained from simulations performed using a phenomenological model that produces more realistic DPOAE spectra are shown in Fig. 3. Unlike the simple representation of the DPOAE spectrum of Eq. (9), in which the two components of $\hat{P}_{dp}(f)$ have constant magnitude and delay, the improved model's distortion- and reflection-source components capture more of the complexity found in actual DPOAE measurements. In particular, both components have phase gradients that vary with frequency based on empirical trends, including a change in phase slope at the apical-basal transition near 1.3 kHz (Shera and Guinan, 2003; Dhar *et al.*, 2011). In addition, both components have fine structure in their magnitude and phase, producing variations in emission delay that correlate with undulations and notches in emission level (Sisto *et al.*, 2007). Although the fine-structure of the model's distortion-source component arises through an entirely ad-hoc procedure—low-pass filtering a sequence of

FIG. 3. Up-down fine-structure shifts $\delta f_{\uparrow}(f)$ derived from simulated swept-tone DPOAEs obtained using a quasi-realistic phenomenological representation of $\hat{P}_{dp}(f)$. The symbols give frequency shifts for three different sweep rates computed using the moving cross-correlation described in the text. The solid lines give corresponding theoretical frequency shifts $\langle \Delta f_{\uparrow} \rangle$, as computed from Eq. (8) and subsequently smoothed by local averaging. Frequency shifts are given in cents (i.e., 1200 times the shift in octaves).

random numbers—the reflection-source component is computed using equations adapted from coherent-reflection theory. These equations, developed for a previous study of reflection-source phase gradient delay (Shera and Bergevin, 2012), characterize the scattering of incident traveling waves by micromechanical irregularities arrayed along the organ of Corti. When evaluated using standard parameter values, the model reproduces the major features of measured reflection-source emissions, including their irregular magnitude spectra and rapidly rotating phases (e.g., Shera and Guinan, 1999; Siegel *et al.*, 2005). The dual-component model allows the creation of simulated data representing individual subjects by choosing parameter sets that vary randomly about means chosen to match normative human data (Shera *et al.*, 2002; Abdala *et al.*, 2011).

To estimate the empirical fine-structure shift $\delta f_{\uparrow}$ at frequency f , we compute a moving cross-correlation on the fine-structure patterns extracted from the data. [To distinguish theoretical predictions from shifts estimated from data (whether real or simulated), we write the latter as $\delta f_{\uparrow}$, using a lower-case δ ; theoretical shifts are denoted $\Delta f_{\uparrow}$, as in Eq. (8).] Since the oscillatory fine-structure in $|P_{dp}(f)|$ appears superposed on the distortion component, $|P_d(f)|$, we subtract out this slowly varying “background,” and define the up- and down-sweep fine-structure patterns as

$$F_{\uparrow\downarrow}(f) = \log [|P_{dp}^{\uparrow\downarrow}(f)/P_d^{\uparrow\downarrow}(f)|] \\ = \log [1 + P_r^{\uparrow\downarrow}(f)/P_d^{\uparrow\downarrow}(f)], \quad (12)$$

where the arrows indicate the sweep direction. Although defined here using log-magnitude spectra [i.e., the real part of $\log(P_{dp}/P_d)$], the fine structure patterns might just as well have been derived from the phase difference $\angle P_{dp}^{\uparrow\downarrow}(f) - \angle P_d^{\uparrow\downarrow}(f)$ [i.e., the imaginary part]. $P_{dp}^{\uparrow\downarrow}(f)$ and $P_d^{\uparrow\downarrow}(f)$ are obtained from the time-domain sweep waveforms using narrow- and wideband LSF analysis, respectively. Details of the LSF analysis, including the effects of varying the bandwidth, are described elsewhere (Abdala *et al.*, 2015, 2016). To quantify the local similarity between the fine-structure patterns $F_{\uparrow}$ and $F_{\downarrow}$ as a function of frequency shift (or “lag”), Δf , we compute the windowed (or moving) cross-correlation at frequency f :

$$\chi(\Delta f, f) = (F_{\uparrow} \star^w F_{\downarrow})(\Delta f, f), \quad (13)$$

where

$$(X \star^w Y)(\Delta f, f) = \int w(\nu - f) X(\nu) w(\nu + \Delta f - f) Y(\nu + \Delta f) d\nu. \quad (14)$$

Centered about frequency f , the windowing function $w(\nu - f)$ renders the cross-correlation local to the frequency of interest. For $w(\nu)$ we used a tenth-order low-pass Butterworth filter with a halfwidth at half-power of 0.15 octaves. At each frequency, the fine-structure shift $\delta f_{\uparrow}(f)$ is taken to be the lag Δf that maximizes the cross-correlation function, $\chi(\Delta f, f)$.

Figure 3 shows the resulting frequency shifts $\delta f_{\uparrow}(f)$, expressed in cents, at three different sweep rates (open

symbols). The solid lines give theoretical predictions obtained from Eq. (8) using the delays $\tau_d(f)$ and $\tau_r(f)$ estimated by computing phase gradients from the unmixed components. To allow better comparisons with the frequency shifts $\delta f_{\uparrow}$ obtained from the fine-structure patterns, theoretical predictions were smoothed by performing a local weighted average using the same window employed in the cross-correlation analysis; thus,

$$\langle \Delta f_{\uparrow}(f) \rangle = \int w(\nu - f) \Delta f_{\uparrow}(\nu) d\nu / \int w(\nu) d\nu. \quad (15)$$

Figure 3 demonstrates excellent overall agreement between theoretical predictions based on DPOAE component delays and fine-structure shifts measured from the simulated swept-tone data. Although shifts decrease at higher frequencies, where model emission delays are shorter, the decrease appears neither monotonic nor entirely smooth, despite the local averaging employed in the estimators. As shown below, correlated variations in both the empirical and predicted fine-structure shifts are also seen in data from real ears.

IV. MEASURED FINE-STRUCTURE SHIFTS

To test our analysis in actual ears, we made swept-tone measurements in human subjects whose DPOAEs showed measurable fine structure. Acoustic signals were delivered to and recorded from the right ears of two normal-hearing young adults while they were comfortably seated in a sound-attenuating chamber. Swept stimulus waveforms were generated and responses acquired and averaged digitally (48 kHz sampling rate) using an RME Babyface Audio Interface (Audio AG, Haimhausen, Germany) and an ER-10C probe system (Etymotic Research, Elk Grove Village, IL). Hardware was controlled using custom software written in MATLAB (The Mathworks, Natick, MA). *In situ* earphone calibrations, compensated for probe insertion depth to reduce the influence of ear-canal standing-waves (Lee *et al.*, 2012), were performed using chirps at intervals throughout a measurement session. DPOAEs were evoked at emission frequencies spanning four octaves (0.5–8 kHz) using primary tones with instantaneous frequencies f_1 and f_2 chosen so that the cubic distortion product at $2f_1 - f_2$ swept upward or downward at rates of $\frac{1}{8}$, $\frac{1}{2}$, 1, or 2 oct/s. Up- and down-sweeps were interleaved and the starting phases of the primary tones rotated systematically between sweeps so that the stimuli at f_1 and f_2 canceled in the average while leaving the response at $2f_1 - f_2$ unchanged (phase-rotation averaging). After highpass filtering (250 Hz), the microphone signal was treated to online artifact rejection, which eliminated sweep segments with obvious glitches in the measured emission pressure. The number of sweeps contributing to an averaged measurement varied in proportion to the sweep rate, with 32 sweeps for $|b| = 2$ oct/s. The stimulus and averaging parameters were $(L_1, L_2) = (65, 65)$ dB sound pressure level (SPL) and $f_2/f_1 = 1.22$.

Figure 4 shows the fine-structure shifts $\delta f_{\uparrow}(f)$ measured in each of the two subjects and compares them with the predictions of Eq. (8) derived from their respective DPOAE

FIG. 4. Up-down fine-structure shifts $\delta f_{\uparrow}(f)$ measured in swept-tone DPOAEs obtained from two normal-hearing young adults (A and B). The top panels have the same format as those in Fig. 3—the symbols give frequency shifts (in cents) for three different sweep rates computed using the moving cross-correlation. The solid lines give corresponding theoretical frequency shifts. The bottom panels show the level of the reflection-source component of the DPOAE (and the noise floor) at the fastest sweep rate ($b = \pm 2$ oct/s). Note that the shifts quantified in the top panels are not those of the reflection-component visible in the bottom panels—fine-structure shifts $\delta f_{\uparrow}(f)$ are computed from the total DPOAE. The data in panel A are from the same subject shown in Fig. 1.

component delays. As in Fig. 3, $\tau_d(f)$ and $\tau_r(f)$ were estimated by computing phase gradients from the unmixed components. Although noisier than the simulations, the human data show the same trends across frequency and a similar overall agreement between measured and predicted fine-structure shifts. Like the shifts in the simulated DPOAEs, the data reveal non-monotonic variations of both $\delta f_{\uparrow}(f)$ and $\Delta f_{\uparrow}(f)$. Comparison with the bottom panel demonstrates that these undulations are often correlated with notches or other irregularities in the amplitude of the reflection-source component of the DPOAE. (Although not illustrated here, similar correlations are seen in the simulated data.) When $P_r(f)$ varies rapidly in level, and especially when it falls close to the noise floor, fine-structure interference patterns and their shifts become difficult to quantify.

Our theoretical expressions for the dependence of fine-structure shifts on sweep rate and emission delay rely on assumptions whose validity may vary idiosyncratically across ears or across frequency within an ear. For example, Eq. (8) for $\Delta f_{\uparrow}$ follows from attributing the shifts entirely to changes in component phase while ignoring possible variations in component magnitude. Although this assumption is satisfied exactly by the simple constant-magnitude-and-delay representation of $\hat{P}_{dp}(f)$ [Eq. (9)], the assumption is only approximate for more realistic models and for measured data. Violation of these assumptions—presumably at some frequencies more than others—may account for some of the differences between the measured and predicted shifts evident in Figs. 3 and 4. Other factors that may compromise the evaluation of $\delta f_{\uparrow}(f)$ and $\Delta f_{\uparrow}(f)$ for real data include the inherent irregularity of reflection-source OAE delays (Kalluri and Shera, 2013) and contamination by

measurement noise, whose effects can be exacerbated by the necessity of taking numerical derivatives to evaluate the phase gradients. Finally, some features of the data may reflect nonlinear influences of dispersive wave propagation on emission generation, as originally hypothesized (Henin *et al.*, 2011; AlMakadma *et al.*, 2015); however, their total contribution to the measured fine-structure shifts appears likely to be small.

V. DISCUSSION

The agreement we find between measured and predicted fine-structure shifts implies that most of the observed shift arises as a methodological side effect of applying the LSF procedure to swept-tone measurements involving delays. This conclusion differs from the results of previous studies (Henin *et al.*, 2011; AlMakadma *et al.*, 2015), which argued that the shifts are physiological in origin and likely reflect the properties of cochlear dispersion and nonlinearity. To buttress their arguments, these studies performed an important—but subtly flawed—in-silico control experiment: they analyzed a simulated multi-component (composite) DPOAE waveform and found no difference between sweeps in the two directions. This null result was taken as proof that the shifts observed in real ears cannot be mere artifacts of the LSF procedure, but must arise through physiological mechanisms. Our analysis reveals, however, that the control was unrealistic in one crucial aspect—the simulated DPOAE waveform had no component delays. Indeed, the composite signal was created by amplitude-modulating a swept-tone carrier rather than by summing components with different phase slopes, as done in the models used here. Consequently, the equivalents of τ_d and

τ_r in the simulation were both zero and, as predicted by Eq. (8), no frequency shifts were observed.

A principal benefit of the LSF procedure is its ability to estimate emission magnitude and phase directly from the time-domain waveform. As we have seen, however, this same feature renders the analysis sensitive to interactions between emission delay and sweep rate or direction. The problem is straightforward to address when the emission consists of only a single component. Indeed, Kalluri and Shera (2013) incorporated a variable delay into their LSF model for estimating swept-tone stimulus-frequency emissions (SFOAEs), rendering their procedure largely immune to the sorts of frequency shifts discussed here. For DPOAEs, however, the existence of multiple emission components complicates the situation.³ LSF procedures are not alone in their susceptibility, and other methods that operate on the time waveform, such as heterodyning, are subject to similar artifacts. Simple Fourier analysis, however, is not. For example, the Fourier transform of any waveform, $y(t)$, and of its time-reversal, $y(-t)$, are identical in magnitude. Thus, in contrast to LSF analysis, Fourier analysis of the swept-frequency signal $p_{dp}(t)$ given by Eq. (10) yields the same magnitude fine structure, independent of sweep direction. Despite this drawback of the LSF method, its use for analyzing swept-tone measurements has many compensating advantages (e.g., Long *et al.*, 2008; Kalluri and Shera, 2013).

Although perhaps best understood as a byproduct of LSF analysis, the observed frequency shifts do, of course, depend on cochlear delays and their variation with frequency; thus, strictly speaking, the shifts are indeed physiological in origin. Our analysis demonstrates, however, that although fine-structure shifts require cochlear (and/or middle-ear) delays, they do not require cochlear dispersion (i.e., changes in phase slope or delay with frequency). The simulations reported in Fig. 2 show that DPOAE fine-structure shifts occur even if the component delays are constant, independent of frequency.

According to Eq. (7), fine-structure shifts Δf_{fs} in the composite DPOAE depend on the summed frequency shifts, $\Delta f_d + \Delta f_r$, of the distortion and reflection components individually. In practice, the phase gradient of the distortion component is generally considerably smaller than that of the reflection component (e.g., Abdala and Dhar, 2012), so that $|\tau_d| \ll |\tau_r|$ and, as a result, $|\Delta f_d| \ll |\Delta f_r|$. Thus, our analysis predicts that the frequency shifts observed in composite DPOAE fine-structure are dominated by underlying shifts in the reflection component, and specifically its phase. These relationships are consistent with the trends observed experimentally (Henin *et al.*, 2011; AlMakadma *et al.*, 2015). As a corollary, we expect differences in component delays between subject groups—such as between newborns and adults (Abdala and Dhar, 2012)—to manifest themselves as differences in fine-structure shifts, and vice versa.

Our analysis makes the unexpected prediction that the magnitudes and phases of DPOAE components are shifted by different amounts. In particular, Eqs. (1) and (5) imply that the shifts differ by a factor of two ($\Delta f \cong \frac{1}{2} \Delta f_{dp}$). Figure 5 tests this prediction by comparing DPOAE fine-structure and reflection-component magnitude shifts for sweeps of

FIG. 5. Differing frequency shifts for DPOAE fine structure and reflection-component magnitude. Up- and down-sweeps for both the total DPOAE (P_{dp}) and its reflection-source component (P_r) are shown using thick black and thin gray lines, respectively. The short black scale bars give the approximate size of the average fine-structure shift on this frequency interval. The short gray bars are twice as long—as predicted by Eqs. (1) and (5), they provide a good estimate of the shift in reflection-component magnitude. The data are from the same subject as Fig. 1.

± 2 oct/s. Because $|\tau_d/\tau_r| \ll 1$ in the frequency range examined, distortion-component phase shifts make a negligible contribution to the total fine-structure shift, which therefore provides an excellent proxy for the shift in reflection-component phase.⁴ In the figure, the gray scale bars shown adjacent to the reflection component are drawn twice as long as the black scale bars that measure the mean shift in DPOAE fine structure. Although the frequency shifts are variable, and in some regions somewhat ill-defined (e.g., for $|P_r|$ near 1.6 kHz in the figure), the overall shift in reflection-component magnitude appears roughly twice that characterizing the shift in fine structure, as predicted.

A. Compensating for frequency shifts

For many applications, the frequency shifts produced by the use of swept tones are small enough to be neglected. For example, the data in Fig. 4 indicate that at a sweep rate of 1 octave/s, the measured DPOAE fine-structure shifts δf_{fs} amount only to about 10 cents (i.e., $\frac{1}{12}$ of a semitone, where the semitone itself is $\frac{1}{12}$ of an octave). When greater precision is desired, our analysis suggests a simple *post hoc* procedure for compensating swept-tone measurements (e.g., $P_{dp}^{\uparrow\downarrow}$) to obtain estimates of the DPOAEs measured at other sweep rates or using discrete tones ($\hat{P}_{dp}$).

Because the two principal components of the total DPOAE have different phase-gradient delays, and therefore require different amounts of compensation, the procedure begins by unmixing the total DPOAE into components ($P_d^{\uparrow\downarrow}$ and $P_r^{\uparrow\downarrow}$). Given these individual components, the compensation amounts to associating each component with its own, actual frequency—as estimated using the frequency shift derived from its phase-gradient delay—and then remixing the compensated components to synthesize an estimate of the steady-state DPOAE. Because magnitudes and phases are shifted by different amounts [cf. Eqs. (1) and (5)], these too must be isolated, shifted separately, and then recombined.

More formally, the compensation proceeds as follows. For simplicity, we illustrate the procedure for estimating emissions $\hat{P}_{dp}$ measured in the steady state ($b = 0$); the details are easily generalized to obtain estimates at other

rates. Given a set of up- and/or down-swept measurements $P_{\text{dp}}^{\uparrow\downarrow}(f_i)$ associated with nominal DP frequencies, f_i , we unmix into components and write $P_{\text{dp}}^{\uparrow\downarrow}(f_i) = P_{\text{d}}^{\uparrow\downarrow}(f_i) + P_{\text{r}}^{\uparrow\downarrow}(f_i)$. The corresponding values of $\hat{P}_{\text{dp}}(f_i)$ are then estimated as

$$\hat{P}_{\text{dp}}^{\uparrow\downarrow}(f_i) \cong \hat{P}_{\text{d}}^{\uparrow\downarrow}(f_i) + \hat{P}_{\text{r}}^{\uparrow\downarrow}(f_i), \quad (16)$$

where

$$\hat{P}_{\text{d}}^{\uparrow\downarrow}(f_i) \cong e^{S(f_i, 2\Delta f_{\text{d}}^{\uparrow\downarrow})\{\ln|P_{\text{d}}^{\uparrow\downarrow}|\} + iS(f_i, \Delta f_{\text{d}}^{\uparrow\downarrow})\{\angle P_{\text{d}}^{\uparrow\downarrow}\}}, \quad (17)$$

and an analogous equation applies for $\hat{P}_{\text{r}}^{\uparrow\downarrow}(f_i)$. Although the $\hat{P}_{\text{dp}}^{\uparrow\downarrow}(f_i)$ are both estimates of $\hat{P}_{\text{dp}}(f_i)$, a quantity independent of sweep direction, we maintain the superscripted arrows as a reminder of their respective origins. The shifting operator $S(f_i, \Delta f)\{H\}$ takes a set of measurements $H(f_i)$, nominally measured at frequencies f_i (with $i = 1, 2, \dots, N$), regards them as actually measured at frequencies $f_i - \Delta f(f_i)$, and then interpolates them back to the f_i .⁵ The frequency shifts $\Delta f_{\text{d}}^{\uparrow\downarrow}$ and $\Delta f_{\text{r}}^{\uparrow\downarrow}$ can be estimated from the phase-gradient delays τ_{d} and τ_{r} using Eq. (5)⁶:

$$\Delta f_{\text{d,r}}^{\uparrow\downarrow}(f_i) = \pm \frac{1}{2} |b| \tau_{\text{d,r}}. \quad (18)$$

In practice, it pays to reduce the detrimental effects of noise, as well as of the frequency scatter inherent in reflection-source OAE delays arising from their mechanisms of generation (Shera and Bergevin, 2012). Here, we do this by using locally averaged values of $\Delta f_{\text{d}}^{\uparrow\downarrow}$ and $\Delta f_{\text{r}}^{\uparrow\downarrow}$, computed as in Eq. (15).

Figure 6 illustrates application of the compensation procedure to the swept-tone data of Fig. 1. The solid lines show the results of compensating these DPOAE measurements, which were measured using sweep rates of ± 2 oct/s, to obtain estimates measured at a rate 16 times slower ($\frac{1}{8}$ oct/s). The factor of $|b|$ appearing in Eq. (18) for $\Delta f_{\text{d,r}}^{\uparrow\downarrow}(f_i)$ was therefore replaced by $2 - \frac{1}{8} = 1.875$ oct/s, the difference between

FIG. 6. Compensating for frequency shifts to estimate emissions measured at another sweep rate. The thin dotted lines show DPOAE spectra from Fig. 1 measured using fast sweeps (2 oct/s) in both directions. The solid lines show the same data compensated to estimate the DPOAEs measured at a rate approaching the sinusoidal steady state ($\frac{1}{8}$ oct/s). The procedure involves unmixing the total DPOAE into distortion and reflection components, compensating each component for frequency shifts derived from its respective phase-gradient delay, and then recombining them. The thick dashed line shows the average of the up- and down-sweeps measured at the slower rate.

the measured and the target rates. DPOAEs measured at rates as slow as $\frac{1}{8}$ oct/s show negligible frequency shifts and are nearly indistinguishable from measurements obtained in the steady state; to avoid further cluttering the plot, the dashed line therefore shows the average of the sweeps measured in the two directions. The figure demonstrates that the compensation procedure largely eliminates the fine-structure shifts induced by the use of swept tones. The magnitude compensation is generally more accurate near fine-structure peaks than near fine-structure dips, especially the deeper ones. Because the dips represent near cancellation between the components, DPOAE levels are acutely sensitive to small errors in component estimation at these frequencies. Although exhibited here for composite DPOAE level shifts, the same procedure compensates both magnitude and phase.

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¹To avoid the confusing anachronism, we labeled the abscissa in Fig. 1 as “emission frequency”; here in Sec. II, where we enjoy the benefit of hindsight, we see that the axis represents the *nominal* emission frequency.

²The numerical value of the expected fine-structure shift given in the text applies to human ears. Equation (8) predicts that fine-structure shifts will typically be smaller in laboratory animals such as guinea pigs and chinchillas, where the delay τ_{r} is shorter by a factor of 2–3 (e.g., Shera and Guinan, 2003).

³If the component delays $\tau_{\text{d}}(f)$ and $\tau_{\text{r}}(f)$ were somehow known in advance, LSF algorithms could account for them by incorporating these delays into the model fit to the measured waveform. Unfortunately, component delays appear too variable across subjects to make this strategy generally viable for DPOAEs.

⁴We make use of this proxy simply for graphical convenience.

⁵In MATLAB, the shifting operator $S(f_i, \Delta f)\{H\}$ might be implemented as follows:

```
Hi_shifted = interp1(fi - dfi, Hi, fi);
```

⁶If the frequency shifts employed in the compensation procedure were twice as large as those given by Eq. (18), the shift operator would effectively convert up-swept components into down-swept components, and vice versa (e.g., $P_{\text{r}}^{\uparrow\downarrow}$ into $P_{\text{r}}^{\downarrow\uparrow}$). The shift operator would then implement a generalization of the “phase alignment” procedure described by AlMakadma *et al.* (2015), which acts by constructing “chimeric” DPOAE components (e.g. by combining the phase of $P_{\text{r}}^{\downarrow\uparrow}$ with the magnitude of $P_{\text{r}}^{\uparrow\downarrow}$).

Abdala, C., and Dhar, S. (2012). “Maturation and aging of the human cochlea: A view through the DPOAE looking glass,” *J. Assoc. Res. Otolaryngol.* **13**, 403–421.

Abdala, C., Dhar, S., and Mishra, S. (2011). “The breaking of cochlear scaling symmetry in human newborns and adults,” *J. Acoust. Soc. Am.* **129**, 3104–3114.

Abdala, C., Luo, P., and Shera, C. A. (2015). “Optimizing swept-tone protocols for recording distortion-product otoacoustic emissions in adults and newborns,” *J. Acoust. Soc. Am.* **138**, 3785–3799.

Abdala, C., Luo, P., and Shera, C. A. (2016). “Comparison of methods for DPOAE component separation,” *Assoc. Res. Otolaryngol. Abs.* **39**, PS664.

AlMakadma, H. A., Henin, S., Prieve, B. A., Dyab, W. M., and Long, G. R. (2015). “Frequency-change in DPOAE evoked by 1 s/octave sweeping primaries in newborns and adults,” *Hear. Res.* **328**, 157–165.

Bennett, C. L., and Özdamar, Ö. (2010). “Swept-tone transient-evoked otoacoustic emissions,” *J. Acoust. Soc. Am.* **128**, 1833–1844.

- Brown, A. M., Harris, F. P., and Beveridge, H. A. (1996). "Two sources of acoustic distortion products from the human cochlea," *J. Acoust. Soc. Am.* **100**, 3260–3267.
- Choi, Y.-S., Lee, S.-Y., Parham, K., Neely, S. T., and Kim, D. O. (2008). "Stimulus-frequency otoacoustic emission: Measurements in humans and simulations with an active cochlear model," *J. Acoust. Soc. Am.* **123**, 2651–2669.
- Dhar, S., and Abdala, C. (2010). "Differences in distortion product otoacoustic emission phase recorded from human neonates using two popular probes," *J. Acoust. Soc. Am.* **128**, EL49–EL55.
- Dhar, S., Rogers, A., and Abdala, C. (2011). "Breaking away: Violation of distortion emission phase-frequency invariance at low frequencies," *J. Acoust. Soc. Am.* **129**, 3115–3122.
- Heitmann, J., Waldman, B., Schnitzler, H. U., Plinkert, P. K., and Zenner, H.-P. (1998). "Suppression of distortion product otoacoustic emissions (DPOAE) near $2f_1 - f_2$ removes DP-gram fine structure—Evidence for a secondary generator," *J. Acoust. Soc. Am.* **103**, 1527–1531.
- Henin, S., Long, G. R., and Talmadge, C. L. (2011). "The effect of sweep rate on estimates of distortion-product otoacoustic emission (DPOAE) fine structure and properties of separated DPOAE components obtained with continuously sweeping primaries," *Assoc. Res. Otolaryngol. Abs.* **34**, 100.
- Kalluri, R., and Shera, C. A. (2001). "Distortion-product source unmixing: A test of the two-mechanism model for DPOAE generation," *J. Acoust. Soc. Am.* **109**, 622–637.
- Kalluri, R., and Shera, C. A. (2013). "Measuring stimulus-frequency otoacoustic emissions using swept tones," *J. Acoust. Soc. Am.* **134**, 356–368.
- Kemp, D. T., and Brown, A. M. (1983). "An integrated view of cochlear mechanical nonlinearities observable from the ear canal," in *Mechanics of Hearing*, edited by E. de Boer and M. A. Viergever (Martinus Nijhoff, The Hague), pp. 75–82.
- Kim, D. O. (1980). "Cochlear mechanics: Implications of electrophysiological and acoustical observations," *Hear. Res.* **2**, 297–317.
- Kohlrausch, A., and Sanders, A. (1995). "Phase effects in masking related to dispersion in the inner ear. II. Masking period patterns of short targets," *J. Acoust. Soc. Am.* **97**, 1817–1829.
- Lee, J., Dhar, S., Abel, R., Banakis, R., Grolley, E., Lee, J., Zecker, S., and Siegel, J. H. (2012). "Behavioral hearing thresholds between 0.125 and 20 kHz using depth-compensated ear simulator calibration," *Ear Hear.* **22**, 315–329.
- Long, G. R., Talmadge, C. L., and Lee, J. (2008). "Measuring distortion-product otoacoustic emissions using continuously sweeping primaries," *J. Acoust. Soc. Am.* **124**, 1613–1626.
- Mauermann, M., Uppenkamp, S., van Hengel, P. W. J., and Kollmeier, B. (1999). "Evidence for the distortion product frequency place as a source of distortion product otoacoustic emission (DPOAE) fine structure in humans. I. Fine structure and higher-order DPOAE as a function of the frequency ratio f_2/f_1 ," *J. Acoust. Soc. Am.* **106**, 3473–3483.
- Oxenham, A. J., and Dau, T. (2001). "Towards a measure of auditory-filter phase response," *J. Acoust. Soc. Am.* **110**, 3169–3178.
- Recio, A., and Rhode, W. S. (2000). "Basilar membrane responses to broadband stimuli," *J. Acoust. Soc. Am.* **108**, 2281–2298.
- Schroeder, M. R. (1970). "Synthesis of low peak-factor signals and binary sequences with low autocorrelation," *IEEE Trans. Inf. Theory* **16**, 85–89.
- Shera, C. A., and Bergevin, C. (2012). "Obtaining reliable phase-gradient delays from otoacoustic emission data," *J. Acoust. Soc. Am.* **132**, 927–943.
- Shera, C. A., and Guinan, J. J. (1999). "Evoked otoacoustic emissions arise by two fundamentally different mechanisms: A taxonomy for mammalian OAEs," *J. Acoust. Soc. Am.* **105**, 782–798.
- Shera, C. A., and Guinan, J. J. (2003). "Stimulus-frequency-emission group delay: A test of coherent reflection filtering and a window on cochlear tuning," *J. Acoust. Soc. Am.* **113**, 2762–2772.
- Shera, C. A., Guinan, J. J., and Oxenham, A. J. (2002). "Revised estimates of human cochlear tuning from otoacoustic and behavioral measurements," *Proc. Natl. Acad. Sci. USA* **99**, 3318–3323.
- Siegel, J. H., Cerka, A. J., Recio-Spinoso, A., Temchin, A. N., van Dijk, P., and Ruggero, M. A. (2005). "Delays of stimulus-frequency otoacoustic emissions and cochlear vibrations contradict the theory of coherent reflection filtering," *J. Acoust. Soc. Am.* **118**, 2434–2443.
- Sisto, R., Moleti, A., and Shera, C. A. (2007). "Cochlear reflectivity in transmission-line models and otoacoustic emission characteristic time delays," *J. Acoust. Soc. Am.* **122**, 3554–3561.
- Smith, B. K., Sieben, U. K., Kohlrausch, A., and Schroeder, M. R. (1986). "Phase effects in masking related to dispersion in the inner ear," *J. Acoust. Soc. Am.* **80**, 1631–1637.
- Summers, V., de Boer, E., and Nuttall, A. L. (2003). "Basilar-membrane responses to multicomponent (Schroeder-phase) signals: Understanding intensity effects," *J. Acoust. Soc. Am.* **114**, 294–306.
- Talmadge, C. L., Tubis, A., Long, G. R., and Piskorski, P. (1998). "Modeling otoacoustic emission and hearing threshold fine structures," *J. Acoust. Soc. Am.* **104**, 1517–1543.
- Wojtczak, M., and Oxenham, A. J. (2009). "On- and off-frequency listening by Schroeder-phase complexes," *J. Assoc. Res. Otolaryngol.* **10**, 595–607.