Angular coherence in ultrasound imaging: Theory and applications

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The popularity of plane-wave transmits at multiple transmit angles for synthetic transmit aperture (or coherent compounding) has spawned a number of adaptations and new developments of ultrasonic imaging. However, the coherence properties of backscattered signals with plane-wave transmits at different angles are unknown and may impact a subset of these techniques. To provide a framework for the analysis of the coherence properties of such signals, this article introduces the angular coherence theory in medical ultrasound imaging. The theory indicates that the correlation function of such signals forms a Fourier transform pair with autocorrelation function of the receive aperture function. This conclusion can be considered as an extended form of the van Cittert Zernike theorem. The theory is validated with simulation and experimental results obtained on speckle targets. On the basis of the angular coherence of the backscattered wave, a new short-lag angular coherence beamformer is proposed and compared with an existing spatial-coherence-based beamformer. An application of the theory in phase shift estimation and speed of sound estimation is also presented. © 2017 Acoustical Society of America. [<http://dx.doi.org/10.1121/1.4976960>]

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I. INTRODUCTION

Plane-wave synthetic transmit aperture (PWT-SA) is an ultrasound imaging technique that utilizes plane wave transmits at multiple steering angles to form high resolution images by synthetically focusing the transmit beam at all image points.¹ In the literature, it is also referred as “coherent plane wave compounding.”¹ The use of plane waves for transmit is equivalent to imaging with virtual source synthetic transmit aperture^{2,3} with the virtual sources placed in the far field, although in this case the positions of the virtual elements are unknown. Recently, this technique has been proposed in a number of applications. For example, it has been used in “ultrafast” imaging⁴ applications and transient elastography,¹ in color-encoded speckle imaging,⁵ to characterize blood flow,⁴ to analyze speckle coherence in anisotropic media using backscatter tensor imaging,⁶ to obtain functional brain imaging on rodents,^{7,8} and more recently in speed-of-sound estimation.⁹ Through these applications, PWT-SA has been popularized as a method to achieve high frame rate synthetic transmit aperture imaging^{1,10} to improve image resolution everywhere, and to improve sensitivity in Doppler imaging.⁴ Despite of the large number of applications, the angular coherence property of signals backscattered from spatially incoherent targets insomnified by the plane waves at different transmit angles has not been elucidated. This property has significant implications in several applications.

Previous theory on the coherence of medical ultrasound signals was adapted from the van Cittert Zernike (VCZ) theorem to describe the spatial coherence of backscattered waves in ultrasound imaging.¹¹ This theory was then utilized by Mallart and Fink¹² to develop a focusing quality metric for phase aberration estimation to correct phase aberration. Liu and Waag analyzed the assumptions of the adapted VCZ theorem¹³ and used this theory develop a phase aberration correction method.¹⁴ Pinton further described the impact of phase aberration¹⁵ and reverberation clutter on the spatial coherence of ultrasound signals.¹⁶ A variety of techniques utilizing spatial coherence have been developed based on this adapted theorem. This theory also provides support for other phase aberration correction algorithms, including nearest-neighborhood cross-correlation,^{17,18} where it shows that signals received on neighboring elements are highly correlated so that accurate time-delay estimation can be performed. In addition, the short-lag spatial coherence (SLSC) beamformer uses spatial coherence information to produce images with better lesion detection than the delay-and-sum beamformer.^{19,20} Flow detection with the spatial coherence of blood signals has also been utilized to obtain higher sensitivity of slow flow and small vessels.^{21,22}

However, the current theory of spatial coherence of medical ultrasound signals does not apply to the coherence of PWT-SA signals obtained with different transmit angles. The VCZ theorem indicates that the spatial coherence of ultrasound signals obtained with focused transmit sequences is related to the transmit aperture function. For plane wave transmits, the effective “virtual transmit sources” are at infinity and therefore the theory of spatial coherence cannot

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give a direct result for PWT-SA signals. The theory proposed by Bottenus *et al.*²³ on acoustic reciprocity and spatial coherence explores spatial coherence imaging (SLSC) in the transmit aperture domain in ultrasound imaging. However, this theory does not hold for PWT-SA and needs to be generalized to account for synthetic transmit sources at infinity.

This paper presents an analysis on the coherence of the PWT-SA signals in the framework of the angular coherence theory using the principle of acoustic reciprocity. The analysis is validated with simulation and experimental data. Furthermore, based on the angular coherence of the signals, a short-lag angular coherence (SLAC) beamformer is proposed and compared with the SLSC beamformer. The potential application of the theory in phase shift estimation and speed of sound estimation is also discussed. The paper is arranged as follows: Sec. II presents the angular coherence theory in the context of medical ultrasound, a summary of coherent plane-wave compounding sequences, and an analysis of the correlation of coherent plane-wave compounding signals; Sec. III provides simulation and experimental validations of the theory; Sec. IV introduces two applications of the theory: the SLAC beamformer and speed-of-sound estimation.

II. THEORY

The angular correlation function was first introduced by Walther *et al.*²⁴ in the context of optics. The function was later more rigorously defined and studied by Wolf *et al.*²⁵ to describe the angular coherence properties of the optical field produced by a spatially incoherent radiation source. In Sec. II, we adapt the angular correlation function and the angular coherence theory presented by Wolf²⁵ to the angular coherence properties of ultrasonic signals.

A. Angular correlation functions of backscattered ultrasound waves from speckle targets

The field in consideration of the original theory proposed in Ref. 25 is generated by a spatially incoherent source and propagates into a half-plane. In medical ultrasound imaging, an isochronous volume of a homogeneous medium of diffuse scatterers insonified by an ultrasound transducer can be considered a spatially incoherent source.¹¹

The pressure field at an arbitrary point $\mathbf{P} = (x, y, z)$ in the half-space ($z \geq 0$) where the backscattered ultrasonic wave propagates can be represented as a complex function $v(\mathbf{P}; \omega)$ in the temporal frequency domain. The angular spectrum representation of $v(\mathbf{P}; \omega)$ is²⁶

$$\begin{aligned} v(\mathbf{P}; \omega) &= v(x, y, z; \omega) \\ &= \int \int_{-\infty}^{+\infty} a(p, q; \omega) \exp[ik(px + qy + mz)] dp dq, \end{aligned} \quad (1)$$

where, k is the wave number and p, q , and m are normalized spatial frequencies in the x, y , and z dimension, respectively. p, q , and m satisfy the following equation:

$$p^2 + q^2 + m^2 = 1, \quad (2)$$

$a(p, q; \omega)$ is the angular spectrum of the field $v(\mathbf{P}; \omega)$. Physically, $a(p, q; \omega) \exp[ik(px + qy + mz)]$ is the complex representation of the plane wave component of the field propagating at the direction determined by the vector (p, q, m) , and $a(p, q; \omega)$ represents the complex envelope of the plane wave. In the following, because m can be uniquely determined from (p, q) according to Eq. (2), the vector $\vec{r} = (p, q)$ is used to represent the direction of the plane wave component $a(p, q; \omega) \exp[ik(px + qy + mz)]$ in Eq. (1).

The angular correlation function $\mathcal{A}(p_1, q_1; p_2, q_2; \omega)$ is introduced as the cross-correlation between the complex envelopes of two angular components (i.e., plane wave components) of the field

$$\mathcal{A}(p_1, q_1; p_2, q_2; \omega) = \langle a(p_1, q_1; \omega) a^*(p_2, q_2; \omega) \rangle. \quad (3)$$

The angular correlation function is closely related to another second-order coherence function, namely, the cross-spectral density function. The cross-spectral density at two spatial locations $\mathbf{P}_1$ and $\mathbf{P}_2$ is defined as²⁷

$$W(\mathbf{P}_1, \mathbf{P}_2; \omega) = \langle v(\mathbf{P}_1; \omega) v^*(\mathbf{P}_2; \omega) \rangle. \quad (4)$$

Using the angular spectrum representation of $v(\mathbf{P}; \omega)$ in Eq. (1), the cross-spectral density function can be written as

$$\begin{aligned} W(\mathbf{P}_1, \mathbf{P}_2; \omega) &= \int \int \int \int_{-\infty}^{+\infty} \langle a(p_1, q_1; \omega) a^*(p_2, q_2; \omega) \rangle \\ &\quad \times \exp[ik(p_1 x_1 + q_1 y_1 + m_1 z_1 \\ &\quad - p_2 x_2 - q_2 y_2 - m_2 z_2)] dp_1 dq_1 dp_2 dq_2 \\ &= \int \int \int \int_{-\infty}^{+\infty} \mathcal{A}(p_1, q_1; p_2, q_2; \omega) \\ &\quad \times \exp[ik(p_1 x_1 + q_1 y_1 + m_1 z_1 \\ &\quad - p_2 x_2 - q_2 y_2 - m_2 z_2)] dp_1 dq_1 dp_2 dq_2. \end{aligned} \quad (5)$$

The cross-spectral density function $W(\mathbf{P}_1, \mathbf{P}_2; \omega) \equiv W(x_1, y_1, z_1; x_2, y_2, z_2; \omega)$ with respect to x_1, x_2, y_1, y_2 , can be expressed as a four-dimensional Fourier integral:

$$\begin{aligned} W(x_1, y_1, z_1; x_2, y_2, z_2; \omega) &= \int \int \int \int_{-\infty}^{+\infty} \hat{W}(f_1, g_1, z_1; f_2, g_2, z_2; \omega) \\ &\quad \times \exp[i(f_1 x_1 + g_1 y_1 + f_2 x_2 + g_2 y_2)] df_1 dg_1 df_2 dg_2, \end{aligned} \quad (6)$$

where f and g are spatial frequencies and $f = kp, g = kq$.

The Fourier conjugate of the cross-spectral density function $W(\mathbf{P}_1, \mathbf{P}_2; \omega)$ is

$$\begin{aligned} \hat{W}(f_1, g_1, z_1; f_2, g_2, z_2; \omega) &= \frac{1}{(2\pi)^4} \int \int \int \int_{-\infty}^{+\infty} W(x_1, y_1, z_1; x_2, y_2, z_2; \omega) \\ &\quad \times \exp[-i(f_1 x_1 + g_1 y_1 + f_2 x_2 + g_2 y_2)] dx_1 dy_1 dx_2 dy_2. \end{aligned} \quad (7)$$

Although the Fourier transforms utilized here appear unconventional, their forms are convenient to solving the angular coherence function and are widely utilized in statistical optics.^{25,27,28} A detailed description of these conventions can be found in Appendix A of Goodman.²⁸

Comparing Eq. (5) and Eq. (6), it can be seen that the angular correlation function $\mathcal{A}(p_1, q_1; p_2, q_2; \omega)$ can be expressed as a scaled version of $\hat{W}(f_1, g_1, z_1; f_2, g_2, z_2; \omega)$ multiplied by a complex phase factor associated with the positions of the points $\mathbf{P}_1$ and $\mathbf{P}_2$ in the z dimension.

$$\mathcal{A}(p_1, q_1; p_2, q_2; \omega) = k^4 \hat{W}(f_1, g_1, z_1; f_2, g_2, z_2; \omega) \times \exp[-ik(m_1 z_1 - m_2 z_2)]. \quad (8)$$

Equation (8) is valid for any choice of $\mathbf{P}_1$ and $\mathbf{P}_2$ in the half-plane ($z \geq 0$) in which the backscattered wave propagates. Consider two points $\mathbf{P}_1$ and $\mathbf{P}_2$ on the transmit focal plane ($z=0$), as shown in Fig. 1. The transmit focal plane can be considered as the source plane for backscattered waves, and these two points $\mathbf{P}_1$ and $\mathbf{P}_2$ on the transmit focal plane can be considered as spatially incoherent sources of backscattered waves.¹¹ Therefore, for $\mathbf{P}_1$ and $\mathbf{P}_2$ on the transmit focal plane, where $z_1 = z_2 = 0$, Eq. (8) becomes

$$\mathcal{A}(p_1, q_1; p_2, q_2; \omega) = k^4 \hat{W}(f_1, g_1, 0; f_2, g_2, 0; \omega). \quad (9)$$

FIG. 1. (Color online) Illustration for the focused-transmit case. The transducer T_0 focuses a transmit beam (shown in blue) at the linear incoherent homogeneous scattering media (M). The backscattered wave (shown in red) propagates into a half space. In the far field, the backscattered wave at any direction are plane waves and can be considered as an angular component of the backscattered wave. Two such components corresponding to transverse spatial frequencies $k \cdot p_1$ and $k \cdot p_2$ are shown in the figure. Two transducers, T_1 and T_2 , can be used to detect the two plane waves. The transmit focal plane is denoted as $z=0$, and the half-plane where the backscattered waves propagates is denoted as $z>0$. $\mathbf{P}_1$ and $\mathbf{P}_2$ are two spatial locations within the incoherent media on the transmit focal plane.

The spatial incoherent property of the diffuse scatterers can be expressed using the scattering function $\chi(x, y, z; \omega)$ as¹¹

$$\langle \chi(x_1, y_1, 0; \omega) \chi^*(x_2, y_2, 0; \omega) \rangle = \chi_0(\omega) \delta(x_1 - x_2) \delta(y_1 - y_2), \quad (10)$$

where, $\chi(x, y, 0; \omega)$ is the scattering function at location $\mathbf{P} = (x, y, 0)$ on the focal plane at frequency ω , and $\chi_0(\omega)$ is the frequency-dependent scattering amplitude for the homogeneous diffuse scattering medium. Physically, Eq. (10) indicates that the cross-correlation of the scattering function of the medium is non-zero only when the two sample points have the same spatial location. The backscattered field at location $\mathbf{P} = (x, y, 0)$ on the focal plane is $v(x, y, 0; \omega) = H(x, y, 0; \omega) \chi(x, y, 0; \omega)$, where $H(x, y, z; \omega)$ is the transmit field at location (x, y, z) .

The cross-spectral density function $W(x_1, y_1, 0; x_2, y_2, 0; \omega)$ on the focal plane is

$$\begin{aligned} W(x_1, y_1, 0; x_2, y_2, 0; \omega) &= \langle v(x_1, y_1, 0; \omega) v^*(x_2, y_2, 0; \omega) \rangle \\ &= \langle H(x_1, y_1, 0; \omega) \chi(x_1, y_1, 0; \omega) H^*(x_2, y_2, 0; \omega) \\ &\quad \times \chi^*(x_2, y_2, 0; \omega) \rangle \\ &= \langle H(x_1, y_1, 0; \omega) H^*(x_2, y_2, 0; \omega) \rangle \chi_0(\omega) \\ &\quad \times \delta(x_1 - x_2) \delta(y_1 - y_2). \end{aligned} \quad (11)$$

Therefore, the cross-spectral density function $W(x_1, y_1, 0; x_2, y_2, 0; \omega)$ is non-zero if and only if $(x_1, y_1) = (x_2, y_2)$. This can be expressed as¹¹

$$\begin{aligned} W(x_1, y_1, 0; x_2, y_2, 0; \omega) &= \langle H(x_1, y_1, 0; \omega) H^*(x_2, y_2, 0; \omega) \rangle \delta(x_1 - x_2) \delta(y_1 - y_2) \\ &= S(x_1, y_1, 0; \omega) \delta(x_1 - x_2) \delta(y_1 - y_2), \end{aligned} \quad (12)$$

where, $S(x, y, z; \omega)$ is the spectral density function at point $\mathbf{P}$, defined as

$$S(x, y, z; \omega) = \langle \chi_0(\omega) H(x, y, z; \omega) H^*(x, y, z; \omega) \rangle. \quad (13)$$

From Eqs. (7) and (12), it can be seen that

$$\begin{aligned} \hat{W}(f_1, g_1, 0; f_2, g_2, 0; \omega) &= \frac{1}{(2\pi)^4} \iiint \int_{-\infty}^{+\infty} S(x_1, y_1, 0; \omega) \delta(x_1 - x_2) \delta(y_1 - y_2) \\ &\quad \times \exp[-i(f_1 x_1 + g_1 y_1 + f_2 x_2 + g_2 y_2)] dx_1 dy_1 dx_2 dy_2 \\ &= \frac{1}{(2\pi)^4} \iint_{-\infty}^{+\infty} S(x_1, y_1, 0; \omega) \\ &\quad \times \exp\{-i[(f_1 - f_2)x_1 + (g_1 - g_2)y_1]\} dx_1 dy_1. \end{aligned} \quad (14)$$

From Eqs. (9) and (14), the angular correlation function can be represented as the Fourier transform of the spectral density function at the source plane:

$$\begin{aligned} \mathcal{A}(p_1, q_1; p_2, q_2; \omega) &= \frac{k^4}{(2\pi)^4} \int \int_{-\infty}^{+\infty} S(x_1, y_1, 0; \omega) \\ &\times \exp\{-ik[(p_1 - p_2)x_1 \\ &+ (q_1 - q_2)y_1]\} dx_1 dy_1. \end{aligned} \quad (15)$$

Equation (15) can be rewritten in a different form. The spatial Fourier transform of the spectral density function is defined as

$$\begin{aligned} \hat{S}(f, g, z; \omega) &= \frac{1}{(2\pi)^2} \int \int_{-\infty}^{+\infty} S(x, y, z; \omega) \\ &\times \exp[-i(fx + gy)] dx dy. \end{aligned} \quad (16)$$

$$\begin{aligned} \int_{-\infty}^{\infty} \hat{S}(\Delta p, \Delta q, z; \omega) d\omega &= \int_{-\infty}^{\infty} \left\{ \frac{1}{(2\pi)^2} \int \int_{-\infty}^{+\infty} S(x, y, z; \omega) \exp[-i(k\Delta p x + k\Delta q y)] dx dy \right\} d\omega \\ &= \frac{1}{(2\pi)^2} \int \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{\infty} S(x, y, z; \omega) d\omega \right] \exp[-i(k\Delta p x + k\Delta q y)] dx dy \\ &= \frac{1}{(2\pi)^2} \int \int_{-\infty}^{+\infty} I(x, y, z) \exp[-i(k\Delta p x + k\Delta q y)] dx dy \\ &= \hat{I}(k\Delta p, k\Delta q, z), \end{aligned} \quad (19)$$

in which, $\Delta p = p_1 - p_2$ and $\Delta q = q_1 - q_2$, $I(x, y, z)$ is the spatial distribution of source intensity defined as $\int_{-\infty}^{\infty} S(x, y, z; \omega) d\omega$, and $\hat{I}(f, g, z)$ is the spatial Fourier transform of the intensity function with respect to x and y dimensions. Integrating both sides of Eq. (17) over temporal frequency and using the relations in Eqs. (18) and (19), we have

$$\tilde{\mathcal{A}}(p_1, q_1; p_2, q_2) = \frac{k^4}{(2\pi)^2} \hat{I}[k(p_1 - p_2), k(q_1 - q_2), 0]. \quad (20)$$

Equation (20) indicates that the angular correlation function $\tilde{\mathcal{A}}(p_1, q_1; p_2, q_2)$ and the spatial distribution of intensity at the source plane form a spatial Fourier transform pair, and the Fourier transform variables are proportional to the angular differences $p_1 - p_2$ and $q_1 - q_2$.

Note that up to Eq. (20), the theory applies to an isochronous volume of a homogeneous medium insonified by an ultrasound transducer, and does not make any assumption regarding the transmitted wave. If the transmit beam is focused onto the medium, as shown in Fig. 1, the spatial distribution of backscattered field intensity at the focus is related to the transmit pressure field, $\mathcal{H}(x, y) = \int_{-\infty}^{\infty} H(x, y, 0; \omega) d\omega$, and scattering function, $\chi(x, y)$, at the focus.¹¹

$$I_{FT}(x, y, 0) = |\mathcal{H}(x, y)\chi(x, y)|^2, \quad (21)$$

in which, the subscript *FT* is used to denote focused transmit.

The theory of *k*-space in ultrasound imaging indicates that the transmit pressure field intensity at the focus and the

The angular correlation function is

$$\mathcal{A}(p_1, q_1; p_2, q_2; \omega) = \frac{k^4}{(2\pi)^2} \hat{S}[k(p_1 - p_2), k(q_1 - q_2), 0; \omega]. \quad (17)$$

In medical ultrasound imaging, the signal under investigation is usually the amplitude or intensity of the field, rather than the spectral density function. The quantities on both sides of Eq. (17) can be integrated over the temporal frequency dimension to give

$$\tilde{\mathcal{A}}(p_1, q_1; p_2, q_2) = \int_{-\infty}^{\infty} \mathcal{A}(p_1, q_1; p_2, q_2; \omega) d\omega. \quad (18)$$

autocovariance function of the transmit aperture function form a Fourier transform pair²⁹

$$C_{tx}(u, v) = \int \int_{-\infty}^{\infty} |\mathcal{H}(x, y)|^2 \exp\left(-2\pi j \frac{xu + vy}{z\lambda}\right) dx dy, \quad (22)$$

where (x, y) is the spatial dimension in the transmit focal plane, (u, v) represents both the Fourier conjugate variable of (x, y) and the spatial dimension in the transmit aperture plane, $C_{tx}(u, v)$ is the autocovariance function of the transmit aperture function.

Therefore, by taking the Fourier transform and ensemble average of Eq. (21), we have

$$\hat{I}_{FT}(k\Delta p, k\Delta q, 0) = \chi_0 C_{tx}(k\Delta p, k\Delta q), \quad (23)$$

where $C_{tx}(k_p, k_q)$ is the autocorrelation function of the transmit aperture function, and $\chi_0 = \int_{-\infty}^{\infty} \chi_0(\omega) d\omega$ is the scattering amplitude of the homogeneous media.

Using this relationship, we find that for focused transmit ultrasound imaging, the angular coherence function of the backscattered field from the homogeneous diffuse scattering medium at the focus is proportional to the spatial autocovariance function of the transmit aperture function.

$$\tilde{\mathcal{A}}_{FT}(p_1, q_1; p_2, q_2) = \frac{k^4 \chi_0}{(2\pi)^2} C_{tx}[k(p_1 - p_2), k(q_1 - q_2)]. \quad (24)$$

Normalizing the angular coherence function with its maximum, the normalized angular coherence function can be found as

$$\begin{aligned}\bar{\mathcal{A}}_{FT}(p_1, q_1; p_2, q_2) &= \frac{\tilde{\mathcal{A}}(p_1, q_1; p_2, q_2)}{\tilde{\mathcal{A}}(p_1, q_1; p_2, q_2)|_{p_1=p_2, q_1=q_2}} \\ &= \frac{C_{tx}[k(p_1 - p_2), k(q_1 - q_2)]}{C_{tx}(0, 0)}. \quad (25)\end{aligned}$$

This conclusion has a similar form as the normalized spatial autocovariance function described in Ref. 11; however, there are different physical meanings between Eq. (25) and that of Ref. 11. The spatial autocovariance function in Ref. 11 is a function of the spatial locations in the receive aperture, and the angular coherence function above is a function of the normalized spatial frequencies of the plane wave components of backscattered waves.

B. Acoustic reciprocity and the correlation function of backscattered signals in plane-wave transmit synthetic aperture sequences

The acoustic reciprocity theorem is a result of the time-reversal and space-reversal symmetry nature of the wave equation,³⁰ which only contains second-order derivatives of the time variable and the space variable. The theorem was described by Rayleigh^{31,32} and Helmholtz³³ in acoustics and then extended to electromagnetic waves³⁴ and optics.³⁵ One form of the theorem indicates that “interchanging the source and the receiver does not alter the resulting field.”³⁶ A further detailed introduction to the theorem can be found in Fokkema *et al.*³⁷ In acoustics, the theorem has been applied in time-reversal acoustics,^{36,38} seismic measurement³⁹ and marine acquisitions.⁴⁰ In modern scattering theory, the reciprocity theorem is expressed by the symmetry of the scattering matrix.³⁴ The theorem is valid for forward and inverse scattering,^{41,42} as well as in the presence of absorption³⁸ and multiple scattering.^{43,44} Recently, Bottenus *et al.*²³ proved its validity in acoustic spatial coherence. Here we provide a brief proof for its validity with respect to the angular coherence of ultrasound, and then apply the theorem to explore the coherence of the signals obtained with plane wave transmission at different angles.

For simplicity, consider a linear, homogeneous, and weakly scattering media M in two-dimensional (2D) space and three transducers as shown in Fig. 1. The center transducer (T0) can transmit and focus waves into the medium. The two other transducers (T1 and T2) are placed in the far-field and can receive plane waves. The apertures of T1 and T2 are tilted at α_1 and α_2 degrees with respect to T0. The transmit angle α is related with the normalized spatial frequency p by $p = -\tan(\alpha)$.

For the focused-transmit case, denote the initial complex amplitude of transmit pulse from T0 to be $v_{10} = |a_{10}|e^{i[k_0 r + \phi_{10}]}$, with initial phase ϕ_{10} , and focal length r . Because T1 and T2 are in the far field, they receive two distinct components of the angular spectrum of backscattered waves from M with propagation angle α_1 and α_2 .⁴⁵ These

two components of the angular spectrum are in the form of plane waves that can be described as $v_{11} = |a_{11}|e^{i[k(p_1 x + m_1 z) + \phi_{11}]}$, and $v_{12} = |a_{12}|e^{i[k(p_2 x + m_2 z) + \phi_{12}]}$. The correlation between the complex amplitudes of the signals received by T1 and T2 is

$$\mathcal{A}_{FT}(\alpha_1, \alpha_2) = |a_{11}a_{12}|\exp[i(\phi_{11} - \phi_{12})]. \quad (26)$$

For the plane wave transmit case, we consider the reciprocal of the focused transmit case, shown in Fig. 2. With all transducer locations unchanged, two plane wave are transmitted separately by T1 and T2, with transmission angle α_1 and α_2 into sample space. The pulses can be represented as $v_{20} = |a_{20}|e^{i[k(-p_i x - m_i z) + \phi_{20}]}$, $i = 1, 2$. These two plane waves have the same amplitude $|a_{20}|$ and initial phase ϕ_{20} and different transmission angles. After each transmit, the backscattered wave is received by T0 and is DAS beamformed at the sample location. For the two transmit events, the received signals are $v_{21} = |a_{21}|e^{i[k(-p_1 x - m_1 z) + \phi_{21}]}$, and $v_{22} = |a_{22}|e^{i[k(-p_2 x - m_2 z) + \phi_{22}]}$, respectively. The correlation between the complex amplitudes of the received signals is

$$\mathcal{A}_{PWT}(\alpha_1, \alpha_2) = |a_{21}a_{22}|\exp[i(\phi_{21} - \phi_{22})]. \quad (27)$$

Because of acoustic reciprocity,

$$\begin{aligned}\phi_{10} - \phi_{11} &= \phi_{20} - \phi_{21}, \\ \phi_{10} - \phi_{12} &= \phi_{20} - \phi_{22}.\end{aligned} \quad (28)$$

FIG. 2. (Color online) Illustration for the plane-wave transmit case. The transducers T1 and T2 transmit plane waves (shown in red) corresponding to transverse spatial frequencies $-k \cdot p_1$ and $-k \cdot p_2$, respectively, into the half space. For each transmit, the linear homogeneous diffuse scattering media (M) backscatters the wave. The backscattered wave (shown in red) propagates into a half space. The transducer T0 focuses a receive beam (shown in blue) in the media M. This is the reciprocal of the focused transmit case shown in Fig. 1.

Combining these two relationships, we have $\phi_{11} - \phi_{12} = \phi_{21} - \phi_{22}$. The correlation function of the delay-compensated received signals from plane wave transmits at two angles α_1 and α_2 is proportional to the correlation function of the received plane wave signals measured at α_1 and α_2 in the focused transmit case.

$$\mathcal{A}_{\text{PWT}}(\alpha_1, \alpha_2) \propto \mathcal{A}_{\text{FT}}(\alpha_1, \alpha_2). \quad (29)$$

In the plane-wave transmit case, because the transmit and receive are interchanged compared to the focused-transmit case, the angular correlation function of signals in the plane-wave transmit case is proportional to the autocorrelation function of the receive aperture function, instead of the transmit aperture function. Applying Eq. (29) to 2D aperture, we define the normalized angular coherence function as

$$\bar{\mathcal{A}}_{\text{PWT}}(p_1, q_1; p_2, q_2) = \frac{C_{rx}[k(p_1 - p_2), k(q_1 - q_2)]}{C_{rx}(0, 0)}. \quad (30)$$

Note that the right-hand side of Eq. (30) depends only on the difference between the transmit angles, and it can be rewritten as

$$\bar{\mathcal{A}}_{\text{PWT}}(\Delta p, \Delta q) = \frac{C_{rx}(k\Delta p, k\Delta q)}{C_{rx}(0, 0)}, \quad (31)$$

in which, $\Delta p = p_1 - p_2$, and $\Delta q = q_1 - q_2$.

The physical implication of Eq. (31) is that the cross-correlation function of the backscattered signals from plane-wave transmits at different angles and a spatially incoherent homogeneous medium is proportional to the normalized autocorrelation of the receive aperture function. This can be considered as an extension to the van Cittert Zernike theorem.

Conventionally, the receive aperture function is expressed as a function of the spatial location of the receive elements. The conversion of the receive aperture function to be a function of receive angle is straightforward. The angle (θ_x, θ_y) corresponding to the receive element at $(x_e, y_e, 0)$ with sample at (x_s, y_s, z) is

$$\begin{aligned} (\theta_x, \theta_y) &= (\arctan(p), \arctan(q)) \approx (p, q) \\ &= \left(\frac{x_e - x_s}{z}, \frac{y_e - y_s}{z} \right). \end{aligned} \quad (32)$$

Because the receive elements in linear transducers in medical ultrasound are uniformly located in space, the angular sampling by linear transducers is non-uniform.

The transmit angular spacing $(k\Delta p, k\Delta q)$ in Eq. (31) can be expressed as fractions of the maximum angle sampled by the receive aperture $(\eta_p, \eta_q) = (k\Delta p/kp_{\max}, k\Delta q/kq_{\max})$ as

$$\bar{\mathcal{A}}_{\text{PWT}}(\eta_p, \eta_q) = \frac{C_{rx}(\eta_p, \eta_q)}{C_{rx}(0, 0)}, \quad (33)$$

in which, $0 \leq \eta_p, \eta_q \leq 1$. If the transmit angular spacing is greater than the maximum angle sampled by the receive aperture (i.e., η_p or $\eta_q > 1$), $\bar{\mathcal{A}}_{\text{PWT}}(\eta_p, \eta_q) = 0$.

C. F-numbers and the sampling range of the angular coherence function

One interesting conclusion that can be deduced from Eq. (33) is the impact of transmit F-number (F_{tx}) and receive F-number (F_{rx}) on the sampling of the angular coherence function. The transmit F-number determines the maximum transmit angle on both sides of the transducer axis as

$$\Theta_{tx} = \pm \arccot(2F_{tx}). \quad (34)$$

The maximum angular spacing between any two transmit plane waves is $2\Theta_{tx}$. Similarly, the receive F-number determines the maximum angular range of receive as $2\Theta_{rx}$. The maximum angular spacing in transmit can then be expressed as a fraction of the maximum receive angular spacing as

$$\begin{aligned} \eta_{\max} &= \frac{2\Theta_{tx}}{2\Theta_{rx}} \\ &= \frac{\arccot(2F_{tx})}{\arccot(2F_{rx})}. \end{aligned} \quad (35)$$

Equation (35) represents the maximum sampling range of the angular coherence function, determined by the pair of transmit and receive F-numbers.

Moreover, the minimum value of the angular coherence function using a PWT sequence with a pair of transmit and receive F-numbers can also be found. According to Eq. (33), the normalized correlation function has a minimum value at $\eta_{\max}$, and the minimum value can be predicted by sampling the normalized autocorrelation function of the receive aperture function at $\eta_{\max}$. Therefore, for a given receive apodization function, the transmit F-number and the receive F-number determine the minimum value of the angular coherence function.

For example, if a constant apodization of 1 is used across a one-dimensional (1D) receive aperture, the normalized correlation function of the receive aperture function is a triangular function with a maximum value of 1. According to Eqs. (33) and (35), the minimum angular coherence value can be predicted as

$$\min \bar{\mathcal{A}}_{\text{PWT}} = \begin{cases} 1 - \frac{\arccot(2F_{tx})}{\arccot(2F_{rx})}, & F_{rx} < F_{tx} \\ 0, & F_{rx} \geq F_{tx}. \end{cases} \quad (36)$$

III. SIMULATION AND EXPERIMENTAL VALIDATION STUDIES

A. Simulation studies

1. Simulation methods

Field II^{46,47} is used to demonstrate the angular coherence theory proposed in Sec. II. In these simulations, a 0.2 mm pitch, 128 element linear array was used to transmit 8 MHz plane wave pulses into a homogeneous phantom. The backscattered radio-frequency (RF) signals were sampled at 160 MHz and dynamically focused in receive to reconstruct

TABLE I. Simulation parameters.

Number of transducer elements	128
Lateral pitch	0.2 mm
Elevation pitch	7.5 mm
Transmit center frequency	8 MHz
fractional bandwidth	0.6
Transmit pulse window type	Gaussian
Sampling frequency	160 MHz

the delay-and-summed (DAS) signals corresponding to a set of receive beams with a lateral sampling spacing of 0.2 mm. The parameters used in the simulations are summarized in Table I.

The reconstructed RF signals for each transmit angle were then converted to complex IQ signals using the Hilbert Transform. The correlation function between the IQ signals produced with different transmit angles were computed according to Eq. (18) for each point in the imaging grid and then spatially averaged over the center 10 mm in the lateral field-of-view (FOV), where the scatterers were incident upon by all transmit plane waves. Because the signals produced with the homogeneous diffuse scatterers are spatially ergodic, the spatial average is equivalent to the ensemble average in Eq. (18). The angular correlation function was then normalized to its maximum at 0 angular difference to produced the normalized angular coherence function.

Two effects predicted by the angular coherence theory were explored in the simulation studies. First, to demonstrate the impact of the receive aperture function on the angular coherence function, indicated by Eq. (30), three different receive apodization functions were used in the simulations. A total of 62 plane waves were used to uniformly sample an angular range of $\pm 14^\circ$, corresponding to a transmit F-number of 2. The three apodization cases are (a) Gaussian apodization with a standard deviation of 7.81% of the receive aperture size; (b) rectangular apodization with a window width of 7.81% of the receive aperture size; and (c) rectangular apodization with a window width of 23.44% of the receive aperture size.

Second, the impact of transmit F-number (F_{tx}) and receive F-number (F_{rx}) on the minimum coherence value

was examined. A uniform apodization of 1 was used across the 1D receive aperture, and two cases were simulated. In the first case, a fixed receive F-number of 4 was achieved using aperture growth and dynamic receive focusing and the transmit F-numbers were varied from 4 to 7 using different ranges of transmit angles calculated from Eq. (34). For example, for a transmit F-number of 4, the transmit angle range is $\pm 7.13^\circ$, and for a transmit F-number of 7, the transmit angle range is $\pm 4.09^\circ$. In the second case, a fixed transmit F-number of 6 was used and the receive F-number was varied from 4 to 7. For both cases, the angular coherence functions were measured and compared with the theoretical predictions of Eq. (36). Five different speckle realizations were simulated, and the averages and standard deviations of the measurement were computed.

In Figs. 1 and 2, three transducers were used to provide a more intuitive model of sequences using plane-wave transmits and dynamically focused receive. In the simulation and experimental validations, a single transducer is used for both dynamic receive and plane wave transmits at multiple angles by choice of the transmit timing of individual elements. This is common practice in current methods utilizing plane wave synthetic aperture imaging.^{4,7} It is obvious that the ostensibly different transducer settings in the theory section and the validations section are equivalent.

2. Simulation results

Figure 3 shows the normalized correlation curves from both simulation measurement and theoretic predictions according to Eq. (31) as the normalized autocorrelation functions of the apodization functions for comparison. The results produced with three types of receive apodization functions are shown: (a) Gaussian apodization with a standard deviation of 7.81% of the receive aperture size; (b) rectangular apodization with a window width of 7.81% of the receive aperture size; and (c) rectangular apodization with a window width of 23.44% of the receive aperture size. For all three cases, the theoretic predictions of Eq. (31) match the measured coherence curves.

Figure 4 shows the angular coherence as functions of the transmit angular spacings between plane wave transmits for three different transmit F-numbers: (a) 4, (b) 5, and (c) 7.

FIG. 3. (Color online) Angular coherence as functions of angular spacing between plane wave transmits measured from simulations. The angular spacing is expressed as a fraction of the receive angular range (28° in this case). The red curves represent theoretical predictions, and the blue curves represent measurement average and standard deviation from 5 simulations with independent speckle realizations. Three types of receive apodization functions were used: (a) Gaussian apodization with a standard deviation of 7.81% of the receive aperture size; (b) rectangular apodization with a window width of 7.81% of the receive aperture size; and (c) rectangular apodization with a window width of 23.44% of the receive aperture size. For all three cases, the theoretic predictions closely match the measured coherence curves.

FIG. 4. (Color online) The angular coherence as functions of angular spacing between plane wave transmits for three different transmit F-numbers: (a) 4, (b) 5, and (c) 7. A receive F-number of 4 is used for all cases. The angular spacing is expressed as a percentage of the receive angular range. A uniform apodization function of 1 is used for all cases. In (a), the transmit and receive F-numbers are the same, therefore, the transmit and receive beams cover the same angular range and the angular coherence function decreases from 1 to 0 as predicted by the theory. (b)-(c) As transmit F-number increases, the transmit angular range decreases and the angular coherence function samples a smaller angular range.

The same receive F-number of 4 is used for all figures. The transmit angular spacing is expressed as a percentage of the receive angular range. A uniform apodization function of 1 is used for all cases. In Fig. 4(a), the transmit and receive F-numbers are the same, therefore, the transmit and receive beams cover the same angular range. The angular coherence function is a triangle function that decreases from 1 to 0, as predicted by the theory. As transmit F-number increases, the transmit angular range decreases and the angular coherence function samples a smaller angular range. For example, with a receive F-number of 4, a transmit F-number of 5 corresponds to a sampling range of 0–80.5%, and a transmit F-number of 7 corresponds to a sampling range of 0–57.9%. The reduction of sampling range caused by the increase of transmit F-number is shown in Figs. 4(b) and 4(c). The curve and the error bars represent the average and standard deviation of the measurement from five different speckle realizations.

Figure 4 further shows that the minimum value of the angular coherence function is related to the transmit and receive F-numbers because of the change in angular sampling range. The relation between the minimum value of the angular coherence function and the F-numbers is described in Eq. (35) in Sec. II. Figure 5 shows the minimum values of the angular coherence function with different transmit and receive F-numbers. Two cases are presented in this figure: (a) constant receive F-number of 4 and varying transmit F-number of 4 to 7, and (b) constant transmit F-number of 6 and varying receive F-number of 4 to 7. The average measured minimum values are shown in blue. The error bars

represent the standard deviations of measurements from 5 different speckle realizations. The theoretical predictions of Eq. (36) are shown in red. All measurement values closely match the theoretical predictions.

Note that the impact of F-numbers on the sampling range of the angular coherence function is independent of the receive aperture function. However, the minimum value of the angular coherence function is dependent upon the receive aperture function, because the receive aperture function determines the angular coherence function. The results from the rectangular receive aperture function is used here as an example.

B. Experiments

1. Experimental methods

Experimental studies were conducted to validate the angular coherence theory. The experiments were conducted with a Verasonics Vantage 256 system (Verasonics Inc., Redmond, WA) and a Verasonics L12-3v linear transducer with a transmit voltage of 10 Volts. Plane waves with a 7.8 MHz center frequency were transmitted into the homogeneous region of an ATS phantom (model 527, ATS Laboratories, Inc., Bridgeport, CT) at 17 angles covering a $\pm 8^\circ$ range. The speed of sound in the phantom was 1450 mm/s. Signals backscattered from the homogeneous regions were sampled at 40 MHz and dynamically focused. The beamformed signals were processed with the same method as in the simulations. Two apodization functions were used on receive: (a) Gaussian apodization with a

FIG. 5. (Color online) The minimum values of the angular coherence function with different transmit and receive F-numbers. Two cases are presented: (a) constant receive F-number of 4 and varying transmit F-number of 4 to 7, and (b) constant transmit F-number of 6 and varying receive F-number of 4 to 7. The measured minimum values shown in blue closely match the theoretical predictions of Eq. (36) shown in red.

standard deviation of 7.81% of the receive aperture size; (b) rectangular apodization with a window width of 7.81% of the receive aperture size.

2. Experimental results and discussions

Figure 6 shows the normalized correlation curves from both experimental measurement and theoretic predictions according to Eq. (30). The results produced with two types of receive apodization functions were shown: (a) Gaussian apodization and (b) rectangular apodization. For both cases, the theoretic predictions match the measured coherence curves. The results presented in Figs. 3 and 6 show similar agreement between theoretical predictions and measurements.

The analysis of angular correlation functions requires the assumption of the first-order Born approximation. This assumption ensures that the back-scattering is weak, and the forward-scattered waves are not significantly different from the incident waves. Under this assumption, the transmit plane waves are not significantly distorted by the forward-scattering of the particles in the media.

In our Field II simulations and phantom experiments, the back-scattering induced by local heterogeneities is weak and satisfies the first-order Born approximation. For stronger heterogeneities, the impact of aberration and other effects on the angular coherence is a more complicated issue. We expect aberration and reverberation to reduce the coherence in a similar manner to that described by Pinton *et al.*^{16,48}

IV. APPLICATIONS

A. SLAC beamformer

Coherence imaging and coherent flow imaging have been demonstrated to produce images similar to conventional B-mode and power Doppler (PD) images, and provide better suppression of spatially incoherent noise, including thermal noise and reverberation clutter. Coherent flow imaging has also been shown to provide higher sensitivity than PD in small vessel and slow flow detection. The beamformer used in coherence imaging and coherent flow imaging is the SLSC beamformer, which measures the spatial coherence of

backscattered field and forms images of the coherence information. One obstacle in the implementation of SLSC on clinical scanners is that the SLSC beamformer requires radio-frequency (RF) channel signals. Therefore, the built-in delay-and-sum (DAS) computational hardware in clinical scanners needs to be bypassed. This induces a very high computational and material cost in the implementation of SLSC beamformers because a large amount of RF channel data have to be transferred, and a large number of correlations have to be computed.

Based on the angular coherence theory presented here, we propose a SLAC beamformer, which operates on delay-and-summed data rather than channel RF data. This allows the SLAC beamformer to be easily integrated into existing clinical scanners and utilize the built-in DAS hardware to reduce computational costs and achieve higher frame rates. It can be considered as the angular domain equivalent to the SLSC beamformer in the spatial domain. In this section, we describe and demonstrate the SLAC beamformer with a 1D linear array as proof-of-concept.

1. Principles of the SLAC beamformer

In SLAC beamforming, a number (N) of angled plane waves are transmitted into the sample, and backscattered signals from all transmits are delay-and-summed. The transmitted plane waves have a uniform angular sample spacing of δ_θ and cover an angular range of $\pm(N/2)\delta_\theta$ around the transducer axis. This corresponds to an F-number of $(1/2)\text{arccot}[(N/2)\delta_\theta]$.

The delay and summed data are then processed with the SLAC beamformer on a pixel-by-pixel basis for each pixel in the imaging grid, utilizing either RF or IQ data. To be consistent with Sec. II, the delay-and-summed RF data here are demodulated to baseband IQ data. For the pixel at location (x, y) , the N complex data points from N plane wave transmits are denoted as $a_i(x, y)$, $i = 1, 2, \dots, N$. The angular correlation function of the signals at (x, y) at an angular difference (defined as lag) of $\Delta\theta = k \cdot \delta_\theta$ can be calculated as

FIG. 6. (Color online) Angular coherence as functions of angular spacing between plane wave transmits measured from phantom experiments. The angular spacing is expressed as a fraction of the receive angular range (16° in this experiment). The red curves represent theoretical predictions, and the blue curves represent measurement average and standard deviation from 5 images with independent speckle realizations. Two types of receive apodization functions were applied: (a) Gaussian apodization with a standard deviation of 7.81% of the receive aperture size; and (b) rectangular apodization with a window width of 7.81% of the receive aperture size. For both cases, the theoretic predictions closely match the measured coherence curves.

$$\bar{A}_{\text{PWT}}(x, y; \Delta\theta) \approx \frac{1}{N-k} \sum_{i=1}^{N-k} \frac{a_i(x, y) a_{i+k}^*(x, y)}{\sqrt{|a_i(x, y)|^2 |a_{i+k}(x, y)|^2}}. \quad (37)$$

The SLAC signal at point (x, y) is calculated with maximum angular difference (i.e., maximum lag) $\Delta\Theta = K \cdot \delta_\theta$ as

$$R_{\text{SLAC}}(x, y) = \int_0^{\Delta\Theta} \bar{A}_{\text{PWT}}(x, y; \Delta\theta) d\Delta\theta \\ \approx \sum_{k=1}^K \frac{1}{N-k} \sum_{i=1}^{N-k} \frac{a_i(x, y) a_{i+k}^*(x, y)}{\sqrt{|a_i(x, y)|^2 |a_{i+k}(x, y)|^2}}. \quad (38)$$

Typically, the maximum lag $\Delta\Theta$ is 10%–30% of the angular range $N \cdot \delta_\theta$ in accordance with the range used for the SLSC beamformer.

2. Simulation and experiment methods

The SLAC beamformer have been implemented in Field II simulations^{46,47} and on a Verasonics Vantage 256 researcher scanner.

In Field II simulations, a 128-element, 0.2-mm pitch transducer is simulated to transmit 62 plane waves with a center frequency of 8 MHz uniformly sampling an angular range of $\pm 14^\circ$ around the center axis of the transducer. The received signals are sampled at 160 MHz and time-delayed to account for the time of flight. The signals are then demodulated to baseband IQ, and processed with the SLAC beamformer as described in Eq. (38).

In order to demonstrate the similarity between SLAC and SLSC beamformers, the time-delayed RF signals are also processed with the SLSC beamformer.

The SLAC beamformer with plane wave transmits was implemented on a Verasonics research scanner and an L12-3v linear transducer. In the B-mode sequence, 17 plane waves with 1° angular spacing were used to cover a range of $\pm 8^\circ$ around the transducer axis. The transmit center frequency was 7.8 MHz and the receive data were sampled at 31.2 MHz. To demonstrate the feasibility of using the scanners' built-in DAS hardware with SLAC, the signals are delay-and-summed and demodulated to baseband IQ by the scanner. The baseband IQ data were then used for SLAC calculation as described in Eq. (38).

3. Simulation and experiment results

Figure 7 presents a comparison of simulated SLAC and SLSC images produced from the same data. Both the global

images (the left two) and zoomed-in local images (the right two) show high similarity between SLAC and SLSC images. This is because both SLAC and SLSC images are produced with the same F-number and the same sample numbers (number of angles for SLAC, and number of receive elements for SLSC). The minor differences are due to the different sampling domains by the two beamformers. Because SLAC images are produced with uniform sampling in angle and SLSC images are produced with uniform sampling in the receive aperture, the plane wave signals do not have a direct one-to-one correspondence with the signals in the SLSC beamformer.

Figure 8 shows an example of images produced with the DAS beamformer, the SLSC beamformer, and the SLAC beamformer. The lateral cross-sections of the images at 20 mm depth are also shown. Seventeen transmit angles covering $\pm 8^\circ$ range were used in accordance with typically PWT sequences⁴ to demonstrate SLAC, and thus the sampling in transmit angular domain for SLAC was not directly matched with the sampling in receive aperture domain for SLSC. It can be seen that, compared to the SLSC images, SLAC preserves the image quality, but the background texture is somewhat different.

The SLAC beamformer has been demonstrated with both simulations and on a research scanner as proof-of-concept. The images produced with SLAC beamformer on simulation data show good agreement with those produced with the SLSC beamformer. It is noteworthy that such high agreement can only be achieved when the sampling of both beamformers are close: the angular range for the SLAC beamformer needs to correspond to the F-number for the SLSC beamformer; and the number of angular samples for the SLAC beamformer needs to match the number of active receive elements for the SLSC beamformer. The transmitted plane waves in the PWT case, however, do not exactly correspond to the waves received on the transducer elements in the FT case.

In the experimental studies, the SLAC beamformer is capable of utilizing the built-in DAS beamformer and operated on the delay-and-summed data, which is not possible with the SLSC beamformer. This approach reduces computation time and reduces the amount of data that needs to be transferred from the scanner to the computational components by a factor equal to the number of active receive elements. This is particularly desirable in plane-wave synthetic transmit aperture Doppler imaging where a Doppler image frame is produced from a large amount of channel RF data, i.e., typically more than 100 times the amount of channel RF

FIG. 7. Comparison of SLAC and SLSC images in a plane-wave synthetic transmit aperture sequence in Field II simulations. Both the global images (the left two) and zoomed-in local images (the right two) show high similarity between SLAC and SLSC images. All figures are shown with a normalized linear scale from 0 to 1.

FIG. 8. (Color online) Experimental B-mode images (the first row) produced with the DAS beamformer (left), the SLSC beamformer (middle), and the SLAC beamformer (right). The lateral cross-sections of the images at 20 mm depth are shown in the second row.

data for a B-mode image frame. Moreover, because current clinical scanners do not have the capability of outputting RF channel data from all receive channels, the SLAC beamformer is easier to integrate into current clinical scanners than the SLSC beamformer.

B. Speed of sound estimation

In Sec. II B, the impact of transmit and receive angular ranges, or F-numbers, on the minimum angular coherence values has been analyzed. For a given receive F-number, increasing the transmit angular range leads to lower minimum angular coherence values. This conclusion has profound implications in the applications of PWT sequences where the angular correlation is calculated for phase delay or time delay estimations. We present one example here where the theory can be used to analyze and optimize phase shift estimations in PWT sequences.

According to the Cramer-Rao lower bound, cross-correlation coefficients affect the jitter level in phase-shift estimation. For example, the cross-correlation value for accurate estimation of phase shift is typically above 0.95.⁴⁹ Therefore, the angular coherence theory may provide a method to improve techniques that involves the computation of phase shifts of signals acquired with plane wave transmits. For example, the speed-of-sound estimation technique proposed by Jaeger *et al.*⁹ uses phase-shift estimation on beamformed signals acquired with angled plane waves. One of the transmit angular ranges utilized in Jaeger *et al.*⁹ is -30° to 30° , which corresponds to a transmit F-number of 0.866. In this approach, a maximum angular spacing of 30° was used

in the cross-correlation calculations for phase-shift estimation. Although the receive F-number and the receive aperture function were not specified by Jaeger *et al.*,⁹ we estimated a minimum (i.e., best scenario) receive F-number of 0.5388 from the 20 mm deep object and the 37.12 mm wide transducer for this example. This setting would result in relatively low cross-correlation coefficients between signals acquired with plane wave transmits at any two widely separated angles. For example, the cross-correlation coefficient of signals acquired with transmits at 0° and 30° is no greater than 0.65. In the study by Jaeger *et al.*,⁹ although a coherent summation of signals within 5° around nominal transmit angles was utilized to improve the estimation, the cross-correlation coefficients of the two aforementioned signals would not exceed 0.71, which is considered to be relatively low for phase-shift estimation standard of above 0.95.⁴⁹ Low cross-correlation coefficient values may increase phase shift estimation jitter and negatively impact the accuracy of speed of sound measurement. More information on system noise level, and receive aperture function, as well as frequency and bandwidth for this example, is needed to estimate an optimal angular range. A more thorough study of phase shift estimation with PWT sequences is beyond the scope of this manuscript.

V. CONCLUSION

We have presented an adaptation of the theory of angular coherence for ultrasound. The theory is applicable to plane-wave transmit synthetic aperture imaging and provides an accurate description of the correlation function of

received signals produced with plane-wave transmits at different angles. The theory is validated with both simulations and phantom experiments.

We demonstrate applications of this theory to develop a SLAC beamformer, which utilizes coherence information of backscattered fields in ultrasonic imaging. Compared to the SLSC beamformer, SLAC operates on delay-and-summed RF data instead of RF channel data, which enables the utilization of DAS hardware in scanners and significantly reduces computational costs. The SLAC beamformer is demonstrated in both simulations and experiments of coherence imaging.

Furthermore, we use this theory to analyze the impact of the transmit and receive angular ranges on the minimum values of the angular coherence functions, and presented an example where the analysis can be used to optimize phase shift estimation in the context of sound speed estimation.

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