Published in final edited form as:

IEEE Int Conf Robot Autom. 2016 May ; 2016: 4422–4428. doi:10.1109/ICRA.2016.7487641.

Catadioptric Stereo Tracking for Three Dimensional Shape Measurement of MRI Guided Catheters

Russell C. Jackson, Taoming Liu, and M. Cenk Çavuşoğlu

Department of Electrical Engineering and Computer Science (EECS) at Case Western Reserve University in Cleveland, OH, USA

Abstract

The recent introduction of Magnetic Resonance Imaging (MRI)-actuated steerable catheters lays the ground work for increasing the efficacy of cardiac catheter procedures. The MRI, while capable of imaging the catheter for tracking and control, does not fulfill all of the needs required to identify and develop a complete catheter model. Specifically, the frequency response of the catheter must be identified to ensure stable control of the catheter system. This requires a higher frequency imaging than the MRI can achieve. This work uses a catadioptric stereo camera system consisting of a mirror and a single camera in order to track a MRI actuated catheter inside a MRI machine. The catadioptric system works in parallel to the MRI and is capable of recording the catheter at 60 fps for post processing. The accuracy of the catadioptric system is verified in imaging conditions that would be found inside the MRI. The stereo camera is then used to track a catheter as it is actuated inside the MRI.

I. Introduction

Cardiac catheters are an important surgical tool which are used for a wide variety of procedures such as angioplasty and balloon septostomy. Recently, Magnetic Resonance Imaging (MRI) actuated catheters have been developed [1]. The purpose of this paper is to introduce an external camera system that is used to track a catheter as a continuum robot during actuation inside the MRI. The tracking system can be used to validate both MRI tracking and catheter modeling. Validating the MRI tracking and catheter modeling will help lead to the development of an MRI steerable catheter which can be used for MRI catheter interventions. This work distinguishes itself from previous research by using a single camera and a single mirror to create an orthogonal view catadioptric stereo system. In this application, the camera system images the catheter while the MRI provides the primary force of actuation.

The MRI machine is capable of completing real time imaging and is capable of distinguishing different tissue types inside the patient [2], [3]. Since physical characterization of the catheter requires higher frequency sensing than the MRI can provide and internal catheter sensors may affect the catheter characteristics, an external camera system was used to track the catheter at high speeds. This system is intended for catheter

development to ensure that the catheter is accurately modeled and controlled. This imaging system would not be used during an intervention as the catheter would be inside of the patient. Tracking the catheter with a camera provides a set of validation data which can be compared to the results of tracking the catheter with MRI images as well as generate high speed catheter images for use in frequency response analysis. This frequency response model is required to ensure stable control of the catheter system. The external camera is an important component of the development cycle for a MRI actuated catheter.

There are safety precautions associated with MRI machines that make it difficult to use standard camera systems. In particular, the ambient magnetic field of the MRI (around 3 Tesla) is powerful enough to dislodge flecks of metal and convert them into high speed projectiles. Consequently, surgeons, operators, and researchers must exercise caution when working inside MRI suites as introduced metallic objects can be sucked inside the MRI machine if they are too close. The safety concerns of the MRI machine coupled with ferromagnetic material inside most camera systems, necessitate that the camera must stay more than 6 m away from the bore (center) of the MRI machine. At this distance, the forces due to the MRI ambient magnetic field are considered to be safe (< 300 Gauss). This far but necessary distance between the camera and the catheter requires using a long focal length lens. When using a long focal length at a far distance, it can be difficult to detect the movement of the catheter as it moves away and towards the camera (i.e. the catheter can move ± 12.5 cm which is approximately 2% of the 6 m between the camera and the catheter. While one solution would have been to use non-ferrous MRI-compatible cameras inside the MRI bore (such as those by MRC Systems GmbH Heidelberg, Germany or Qualisys AB Gothenburg, Sweden), these cameras do not offer the full HD (1920×1080) resolution at 60 fps that the implemented system contains. Additionally placing the cameras inside the MRI machine may lead to electronic interference which may reduce MRI image quality. To combat this imprecise depth detection while preserving a large resolution image, a mirror was introduced to provide an additional catheter view that is oriented at 90° to the first one. This creates what is known as a catadioptric stereo system.

The catadioptric stereo system developed in this work places the mirror about 30 cm from the target while the camera is by necessity approximately 6 m from both the tracking target and mirror. In this paper, the geometry and physical characteristics of the system are explored. The geometric accuracy and tracking capabilities of the system are validated using test patterns and successful catheter tracking is demonstrated.

The remaining paper is organized as follows. Section II discusses prior work with catheter control and tracking followed up with prior work on catadioptric stereo systems. Section III shows and discusses the implemented catadioptric stereo system while explaining its geometry and calibration methods. Section IV details the experimental validation of the catadioptric system. Section V introduces the catheter model and demonstrates successful tracking. Finally, section VI outlines the main results and how it will be used in support of further catheter development and testing.

II. Background

This project spans two distinct areas of research. Specifically, catheter modeling and tracking as well as the geometry of catadioptric stereo imaging. The next two subsections explore both aspects in detail.

A. Catheter Control & Tracking

Catheter robotics are being researched as a viable supplement to the current manual control that surgeons are currently required to utilize during catheter procedures [1], [4], [5]. One particular procedure that would greatly benefit from catheter automation is Cardiac Ablation which is used to treat Atrial Fibrillation [6]. During typical treatment for Atrial Fibrillation, a surgeon will manually direct a catheter up the femoral vein, inside the right atrium, and finally through the interatrial septum into the left atrium. Once there, the surgeon will use the catheter to ablate the cardiac tissue. The purpose of the ablation is to stop extraneous pacing signals from transmitting through the left atrium and causing irregular heart beats [6]. The path of ablation is carefully picked and automating the catheter during the ablation would allow for that path to be followed precisely as well as ensuring that the ablated paths are continuous and well defined. High quality path execution reduces the risk of fibrillation recurrence as well as complications [6].

Many catheters are underactuated which means that in order to accurately control the catheter, it is important that the catheter and its local range of motion are correctly identified [7]. Since, unlike fully actuated robots, the workspace is limited by the current catheter pose, catheter geometry identification is a prerequisite for effective path planning and execution.

Previous authors have looked at the problem of detecting catheters by treating them as continuum robots. These robots are often actuated using cannulas or guide wires [8], [9]. Catheters are long and thin, this can lead to complications when using stereoscopic vision. In particular, the long and thin structure of the catheter may fall on the epipolar lines of the stereo vision system [10]. This may lead to ambiguities when trying to deproject the catheter images into 3-dimensional Euclidean space. One solution is to use a set of orthogonal cameras. When the view points of the stereo camera system are at 90° , the epipolar lines correspond to nearly orthogonal directions in camera space; this reduces the ambiguity that often occurs during stereo imaging and tracking. While one common problem with disparate views during stereo vision is the radically different appearance of the tracked object in each image, the symmetric shape of the catheter means that it is easily detectable in both images even when using, orthogonal viewpoints [11].

The restrictive nature of the MRI required utilizing a specialized stereo catadioptric system. A planar mirror allowed two orthogonal camera views to be utilized while no camera was in fact placed near the MRI bore.

B. Catadioptric Stereo

Catadioptric stereo imaging systems have some distinct advantages when compared to a dual camera stereo system. One of the main advantages is that since the left and right cameras both share the same physical camera, the geometry is shared (i.e. the camera intrinsic and

distortion parameters are the same in both the main image and the stereo image). One other common problem in stereo camera systems is image synchronization. In a two camera system, the cameras must be synchronized using an external clock. Since there is only one imaging device in a catadioptric stereo system, synchronization is guaranteed. Previous work on catadioptric camera systems validate the geometry and test the results of the calibration [12], [13], [14], [15]. Many of these examples used multiple mirrors to define the system geometry. Some catadioptric stereo systems even use nonplanar mirrors[12]. A diagram of the planar catadioptric system is shown in Fig. 1. Notice that the virtual camera is in a left handed coordinate frame. This is caused by the mirror flipping the direction of both the x and z vectors leaving y in the same direction. The transformation between the real camera frame and the virtual camera frame is defined as follows [13]:

$$G = \begin{bmatrix} I - 2nn^T & 2dn \\ 0 & 1 \end{bmatrix}. \quad (1)$$

Here I is the 3×3 identity matrix, n is the mirror normal vector, and finally, d is the distance between the camera and the mirror (minimum distance between the plane and the origin point of the camera). The determinant, $|G| = -1$, captures fact that the transform changes camera frame from a right handed one to a left handed one. It is also of note that G is its own inverse (i.e. $G^{-1} = G$). There are several aspects of this system design that distinguish it from previous work. While catadioptric stereo systems have been designed and analyzed, this paper looks at using such a system in order to track and model a 3d object using 2 nearly orthogonal views.

III. Catadioptric System Geometry

The camera system being utilized is also unique in that the mirror is much closer to the object being tracked than the camera system. Notice that the mirror is angled at 45° to the camera (i.e. $n = [\sqrt{2}/2, 0, \sqrt{2}/2]^T$). This means that $d = 3\sqrt{2}\text{m}$. The result is that the transform is nominally:

$$G = \begin{bmatrix} 0 & 0 & -1 & 6 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

While this is the nominal transformation is created by the mirror, there is variability in both the camera distance and mirror mounting angle. The actual stereo calibration must be robust against these variations.

The camera lens had a variable focal length (12.5 – 75 mm) and was set to be approximately 50 mm. The focal length may vary between experiments but it is always fixed during the experiment. The scene being imaged was in fact two scenes (original + mirror image) which are at different distances away from the camera. The difference is approximately the distance

from the scene to the mirror as shown by the point p' in Fig. 1. Since telephoto lenses, such as, the one used for imaging the catheter tend to have a narrower depth of field, (i.e. a narrower range of distances where the scene appears to be in focus), the f-stop (defined as ratio of the focal length to the aperture diameter) had to be increased in order to increase the depth of field of the lens. Increasing the f-stop shrinks the lens iris and restricts the amount of light hitting the camera sensor. Doing this increases the depth of field so that both the catheter and its mirror image are in focus. The eventual goal of the camera tracker is to track the frequency response of the catheter. To that end, a 60 fps high definition USB3 camera with a resolution of 1080×1920 pixels was used. The camera was manufactured by Point Grey (Richmond, BC, Canada).

The combination of the high frame rate, telephoto lens, and a large f-stop required significantly more light than typically needed for imaging applications. Due to the constraints imposed by the MRI, lighting sources had to be kept 6 m away from the MRI bore as well. While some light sources are not considered to be ferromagnetic, the DC power supply current may induce RF interference as well as introduce its own magnetic fields inside the MRI due to the DC current. Minimizing electrical interference inside the MRI is nearly as important as minimizing the dangers of ferrous materials inside the MRI suite. The long lighting distance coupled with the minimal light inside the MRI room and the high lighting requirements needed high powered spot lights to provide the necessary light at the correct distance. A pair of 100 Watt LED's outputting approximately 9000 lumens each were equipped with plastic Fresnel lenses in order to adequately light the scene. Customized lighting was used to avoid introducing a significant amount of ferromagnetic material into the MRI suite because while it is true that the lights and camera are a safe distance away from the MRI unit, it is still a safety risk that must be respected. Pictures of the system components are shown in Fig. 2. A diagram of the vision components inside the MRI suite is shown in Fig. 3.

A. Catadioptric Calibration Process

In order to guarantee that the mirror calibration is robust even in the MRI, a calibration pattern was placed in the scene with the catheter (as shown in Fig. 2 and Fig. 6). The pattern, which is a grid of black circles, is located such that it is out of the way of the catheter for nearly its full range of motion. When deployed inside the MRI, the camera intrinsic parameters are found using a chessboard calibration pattern. The intrinsics are found at the beginning of each experiment to account for variation in actual camera distance, lens focus, and lens zoom. The actual intrinsics are computed using functions found in the OpenCV computer vision library [16]. Once the intrinsic camera parameters are found, the camera image is undistorted. The mirror calibration pattern is then manually selected from the undistorted image to seed the stereo calibration. Once the seed points in the front view are selected, the transform between the calibration pattern and the camera is computed in free space. This transform is G_{lo} . The next step is to find the transform between the calibration pattern and the mirror image. This transform is:

$$G_{ro} = G G_{lo} \quad (3)$$

Here G is the transformation between the camera and its mirror, as defined in (1) can be constructed in terms of spherical coordinates: $u = [\theta, \phi, r]^T$ where

$$\begin{aligned} \mathbf{n}(u) &= [\cos(\theta) \cos(\phi), \sin(\theta) \cos(\phi), \sin(\phi)]^T \\ d(u) &= r. \end{aligned} \quad (4)$$

The advantage of defining d and n using (4) is that the normal vector n is guaranteed to be a unit vector. This geometry naturally embeds itself into describing the catadioptric calibration. Solving for the spherical point u will also characterize the catadioptric stereo geometry. The derivative of the normal vector n and the distance d can be computed in terms of the spherical coordinates. Further computation (which will be omitted for brevity) are performed to compute the image derivatives of p' in terms of the spherical coordinates.

$$\frac{\partial p'}{\partial u} = \frac{\partial p'}{\partial G} \begin{bmatrix} \frac{\partial G}{\partial n} & \frac{\partial G}{\partial d} \end{bmatrix} \begin{bmatrix} \frac{\partial n}{\partial u} \\ \frac{\partial d}{\partial u} \end{bmatrix} \quad (5)$$

The matrix $\frac{\partial p'}{\partial G}$ is the derivative of the image point p' with respect to the mirror transform G . The derivative of G is coupled with the derivative of d and n to create the derivative of the image point as a function of u . If multiple points are known, then the derivative of each point can be computed and combined.

$$\begin{bmatrix} \frac{\partial p'_0}{\partial u} \\ \frac{\partial p'_1}{\partial u} \\ \vdots \\ \frac{\partial p'_n}{\partial u} \end{bmatrix} = \begin{bmatrix} \frac{\partial p'_0}{\partial G} \\ \frac{\partial p'_1}{\partial G} \\ \vdots \\ \frac{\partial p'_n}{\partial G} \end{bmatrix} \begin{bmatrix} \frac{\partial G}{\partial n} & \frac{\partial G}{\partial d} \end{bmatrix} \begin{bmatrix} \frac{\partial n}{\partial u} \\ \frac{\partial d}{\partial u} \end{bmatrix} \quad (6)$$

The point set Jacobian can be defined as

$$J = \begin{bmatrix} \frac{\partial p'_0}{\partial G} \\ \frac{\partial p'_1}{\partial G} \\ \vdots \\ \frac{\partial p'_n}{\partial G} \end{bmatrix} \begin{bmatrix} \frac{\partial G}{\partial n} & \frac{\partial G}{\partial d} \end{bmatrix} \begin{bmatrix} \frac{\partial n}{\partial u} \\ \frac{\partial d}{\partial u} \end{bmatrix}. \quad (7)$$

If enough points are selected, the Jacobian becomes over determined and a pseudoinverse can be used to update the spherical point u .

$$\mathbf{u}' = \lambda J^+ \left(P'_{measured} - P'_{estimated} \right) + \mathbf{u} \quad (8)$$

Here $\mathbf{u}'$ is the new mirror information while $P'_{measured}$ is the set of points detected in the image and $P'_{estimated}$ is the set of points estimated from G . The variable λ is a scaling parameter used to ensure stability. This allows the calibration to constantly iterate against new images by minimizing the error between the calculated points of $p'_{estimate}$ (estimated from the previous calibration) and the detected points, $p'_{measured}$. After $\mathbf{u}$ converges, the normal vector $\mathbf{n}$ and the mirror distance d can be computed.

IV. Experimental Validation

The geometric theory laid above must be validated before the stereo system can be used in catheter tracking. In order to ensure that the results of tracking the catheter are meaningful, the accuracy of the stereo system is verified using a test pattern that demonstrates the accuracy of the catadioptric stereo system. The MRI uses water molecules to complete its imaging. Since the catheter contains no water intrinsically, the catheter had to be submerged in a water bath in order to be visible to the MRI. The water while necessary for MRI imaging creates a water/plastic/air interface which can refract light; the catadioptric validation was completed with and without water to show that the presence of water does not significantly affect the calibration results.

The accuracy of the calibration was validated by utilizing black and white lego bricks mounted on an XYZ translational stage. The size of a single 1×1 brick is $8 \times 8 \times 9.6$ mm. There were a total of 3 black lego bricks spaced throughout a face of white bricks. Three bricks were mounted together to create a test pattern on a plane that spans a rectangle of size 48.0×67.2 mm. The calibration is verified as the test pattern is moved throughout a $17\text{mm} \times 10\text{mm} \times 10\text{mm}$ volume using a high precision linear stage. While it is true that the lego motion volume is on the order of the size of one lego brick, the lego bricks themselves were oriented such that they created a large area for validation. The high precision stage was thus able to test small motion detection accuracy. This allowed for the validation of both the local precision and global accuracy of the catadioptric stereo system. Three different test runs were recorded; in the first run there was no water in the tank, in the second run, the tank was halfway filled with water, in the final run, the tank was completely filled with water. Filling the tank halfway showed that the alignment between the dry blocks and the wet blocks was not affected significantly by the refraction of the water/plastic/air interface. The different water levels with the bricks are shown in Fig. 4. While the images were not captured inside an MRI machine, the lighting and system geometry mimicked the conditions inside the MRI suite. Table I summarizes the results. Here n_i is the total number of images processed during the motion, $\bar{e}_{rms}$ is the average rms error between the test pattern and the fitted points (The fit was found using a SE(3) transform), σ_{rms}^2 is the variance of the rms error, and $\max e_{rms}$ is the maximum error over the course of the calibration.

From the results, it can be seen that the rms error is less than 1 mm when the blocks are either dry or fully submerged. The only time that the error peaks above 1 mm is when the black block is at the waterline. Due to internal reflection, the block distorts. This causes the tracker to eventually track the waterline instead of the black block. Since the focus of this experiment was to measure the potential error caused by the water's refraction and not track objects near the water line, the results of this tracking were considered a success; even when the object is fully submerged, it still has a linear transform between it and the camera. It is however important to note the risks that are associated with tracking objects near the water line. Whenever possible, tracked objects should be either dry or completely submerged. The stereo system is now validated both with and without water immersion.

V. Catheter Tracking

Now that the catadioptric stereo system has been validated, it can be used to track the catheter. The catheter, shown in Fig. 5, has a total of 3 coils: one axial and 2 side. All three coils are located at nearly the same location on the catheter. The coils are embedded in the models used by collaborators [17], [7], [18], [19]. What follows is a brief description of the catheter model.

A. Catheter Model

The catheter is a long narrow tube. As the deflection angle is sometimes large, this long tubing could not be analyzed as a whole beam by the beam theory. Therefore, finite differences approach is applied to analyze the deflection of the catheter by dividing it into short segments. For each finite segment, a quasi-static torque-deflection equilibrium equation is calculated using beam theory and Bernoulli-Euler law. By using the deflection displacements and torsion angles, the kinematic model of the catheter system is derived by considering each segment as a robot link with the two ends of the segment as the link's joints. The homogeneous transformation relationship between the two ends of the segments is calculated. A more detailed analysis of the catheter is available in [17], [7], [18], [19].

B. Catheter Imaging & Tracking

The catheter being tracked is labeled with orange paint on the coils as well as the base and the tip. The orange paint created a unique color that was easy to identify in the catheter images captured during MRI experiments. As such the orange points were used as a baseline for tracking the catheter. The detection of curvilinear objects like the catheter has been looked at previously by numerous authors [20], [21], [22]. The orange paint successfully tracks both endpoints along with the coil. The tracked points are fitted to a quadratic spline in order to measure the length of the catheter. The fitted catheter image is shown in Fig. 6. The reason that the catheter is only painted in three locations is that painting creates a coating which changes the material properties of the catheter. Painting the catheter at the ends as well as on the coil allows the catheter to be labeled with a minimal effect on the catheter mechanics. The video accompanying this paper shows the catheter being tracked as it moves due to an energized axial coil. Table II summarizes the measured length of the catheter during tracking. Here n_c is the number of images that the catheter is measured in, $\bar{\ell}$

is the average length of the catheter, σ_ℓ^2 is the variance of the catheter length, while finally $\min \ell$ and $\max \ell$ are the minimal and maximal estimated lengths of the catheter respectively.

The average length of the imaged catheter was 107 mm while the actual measured length of the catheter was 110 mm. This is within 2.7 % of the catheter length. The average measurement is likely short because the quadratic curve is only a 2nd order polynomial fit and does not adequately approximate the curvature of the catheter geometry. Utilizing a higher order model or even a NURBS model as well as incorporating geometric models might estimate the catheter length more accurately. The still image shown in Fig. 6 shows the catheter as it moves and is tracked during energization.

VI. Conclusions & Future Work

Catadioptric stereo tracking proved to be a viable solution to the unique problems that are associated with conducting research inside an MRI machine. The lego marker tracking shows that the catadioptric stereo system is accurate to within 1 mm. The catheter motion video shows that the tracking system is capable of following the catheter as it moves inside the MRI suite.

This research is part of an ongoing project. The next step of the vision tracking is to align the camera coordinate frame to a base frame defined by the catheter holder. This would allow the relative position of the catheter to be identified using an intuitive reference frame. Another future plan is to improve the performance of catheter tracking by implementing a catheter model which is then fitted to the observed image and incorporated with the catheter kinematic model in order to complete predictive catheter tracking. This includes using a higher order curve to track the catheter. Additional segmentation of the catheter body, acting in conjunction with the marker segmentation and a stochastic filter would allow the entire catheter to be accurately tracked. One final future goal is to use the computer vision tracking to identify the frequency response characteristics of the catheter during coil excitation. This will be accomplished by synchronizing the catheter excitation with the camera. Completing the above steps will allow for characterization and control of an MRI actuated catheter system.

Acknowledgments

This work was supported in part by the National Science Foundation under grant IIS-1524363, and National Institutes of Health under grant R01EB018108.

REFERENCES

- [1]. Hetts SW, Saeed M, et al. Magnetically-Assisted Remote Controlled Microcatheter Tip Deflection under Magnetic Resonance Imaging. J Vis Exp. 2013; (74)
- [2]. Campbell-Washburn, AE., Faranesh, AZ., Lederman, RJ., Hansen, MS. MR Sequences and Rapid Acquisition for MR-Guided Interventions. Magnetic Resonance Imaging Clinics of North America. Aug. 2015 [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S106496891500077X>
- [3]. Ratnayaka K, Faranesh AZ, et al. Interventional cardiovascular magnetic resonance: still tantalizing. Journal of cardiovascular magnetic resonance: official journal of the Society for Cardiovascular Magnetic Resonance. 2008; 10:62. [PubMed: 19114017]

- [4]. Lillaney P, Caton C, et al. Comparing deflection measurements of a magnetically steerable catheter using optical imaging and MRI. *Med Phys.* Feb.2014 41(2):22305.
- [5]. Penning, RS., Jung, J., Borgstadt, JA., Ferrier, NJ., Zinn, MR. Towards closed loop control of a continuum robotic manipulator for medical applications; *Intelligent Robots and ...* 2011. p. 5139-5146.[Online]. Available: ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=5980233
- [6]. Calkins H, Kuck KH, et al. 2012 HRS/EHRA/ECAS expert consensus statement on catheter and surgical ablation of atrial fibrillation: Recommendations for patient selection, procedural techniques, patient management and follow-up, definitions, endpoints, and research trial design. *Journal of Interventional Cardiac Electrophysiology.* 2012; 33(2):171–257. [PubMed: 22382715]
- [7]. Greigarn, T., Cavusoglu, MC. Pseudo-rigid-body model and kinematic analysis of MRI-actuated catheters; 2015 IEEE International Conference on Robotics and Automation (ICRA); IEEE. may 2015; p. 2236-2243.[Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=7139495>
- [8]. Ayvali, E., Desai, JP. Accurate in-plane and out-of-plane ultrasound-based tracking of the discretely actuated steerable cannula; 2014 IEEE International Conference on Robotics and Automation (ICRA); IEEE. may 2014; p. 5896-5901.[Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6907727>
- [9]. Camarillo DB, Milne CF, Carlson CR, Zinn MR, Salisbury JK. Mechanics modeling of tendon-driven continuum manipulators. *IEEE Transactions on Robotics.* 2008; 24(6):1262–1273.
- [10]. Cham, T-J., Cipolla, R. Stereo coupled active contours; *Proceedings of IEEE Computer Society Conference on Computer Vision and Pattern Recognition*; jun 1997; p. 1094-1099.
- [11]. Camarillo, DB., Loewke, KE., Carlson, CR., Salisbury, JK. Vision based 3-D shape sensing of flexible manipulators; 2008 IEEE International Conference on Robotics and Automation; 2008; p. 2940-2947.[Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=4543656>
- [12]. Geyer C, Daniilidis K. Catadioptric projective geometry. *International Journal of Computer Vision.* 2001; 45(3):223–243.
- [13]. Gluckman, J., Nayar, S. Planar catadioptric stereo: geometry and calibration; *Proceedings. 1999 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (Cat. No PR00149)*; IEEE Comput. Soc. 1999; p. 22-28.[Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=786912>
- [14]. Gluckman J, Nayar S. Rectified catadioptric stereo sensors. *IEEE Transactions on Pattern Analysis and Machine Intelligence.* 2002; 24(2):224–236. [Online]. Available:<http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=982902>.
- [15]. Wu H-HP, Lee M-T, Weng P-K, Chen S-L. Epipolar geometry of catadioptric stereo systems with planar mirrors. *Image and Vision Computing.* Jul; 2009 27(8):1047–1061. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0262885608001996>.
- [16]. Bradski G. The OpenCV Library. *Dr Dobbs Journal of Software Tools.* 2000; 25:120–125. [Online]. Available: <http://opencv.willowgarage.com>.
- [17]. Liu, T., Cavusoglu, MC. Three Dimensional Modeling of an MRI Actuated Steerable Catheter System; *IEEE International Conference on Robotics and Automation: ICRA: [proceedings] IEEE International Conference on Robotics and Automation*; jan 2014; p. 4393-4398.[Online]. Available: <http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6907499>
- [18]. Greigarn, T., Cavusoglu, MC. Task-Space Motion Planning of MRI-Actuated Catheters for Catheter Ablation of Atrial Fibrillation; *Proceedings of the ... IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE/RSJ International Conference on Intelligent Robots and Systems*; sep 2014; p. 3476-3482.[Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/25485168>
- [19]. Liu T, Poirot N, Lombard, et al. Modeling and Validation of the Three Dimensional Deflection of an MRI-Compatible Magnetically-Actuated Steerable Catheter. *IEEE transactions on bio-medical engineering.* Dec.2015 (99):1. vol. PP. [Online]. Available: <http://www.ncbi.nlm.nih.gov/pubmed/26731519>.
- [20]. Jackson, RC., Yuan, R., Chow, D-L., Newman, W., Cavusoglu, MC. Automatic initialization and dynamic tracking of surgical suture threads; 2015 IEEE International Conference on Robotics

and Automation (ICRA); IEEE. may 2015; p. 4710-4716.[Online]. Available: <http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=7139853>

- [21]. Javdani, S., Tandon, S., Tang, J., O'Brien, JF., Abbeel, P. Modeling and perception of deformable one-dimensional objects; Robotics and Automation (ICRA), 2011 IEEE International Conference on; may 2011; p. 1607-1614.
- [22]. Padoy, N., Hager, GD. 3D thread tracking for robotic assistance in tele-surgery; Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on; sep 2011; p. 2102-2107.

Fig. 1.

A planar catadioptric system has one real camera (solid) and one ‘virtual’ camera (dashed). A point p in real space projects onto the camera along the red ray. This point reflects off the mirror (black with normal n) along the green ray. The distance between the mirror and the camera origin, d , is aligned to the vector n . The apparent location of p in the mirror is point p' even though the actual point of reflection is p_m . The dashed green rays show how the point is imaged in the virtual camera. Notice that the virtual camera has a left handed coordinate system. This system is not drawn to the same scale as the one being discussed here. In the actual system, the real camera is nearly 6 m away from the point p while the mirror is only 0.3 m from the point p . It is not possible to render the system to scale in the available space.

(a) Catheter Tank and Mirror

(b) Camera and LED

Fig. 2.

The components of the catadioptric stereo imaging system include the water tank and mirror (a) as well as the camera and LED (b). The grid of circles in the tank are for calibrating the mirror. Only one LED is shown in (b).

Fig. 3.

The diagram above gives a sense of the scale of the catadioptric imaging system. The MRI bore, (left cylinder) has a diameter of 70 cm and contains the mirror (dashed box) as well as the catheter (red vertical line in front of the mirror). The camera (far right) is approximately 6 m away from both the catheter and the mirror. The dashed vertical line between the camera and the MRI marks the zone where the ambient magnetic field rises above 300 Gauss. While the dashed line marks a region of reasonable safety, hospital regulations require that any and all potentially hazardous metallic objects are kept as far away from the MRI as possible. The LED lights (not shown) flank the camera.

Fig. 4.

The lego block calibration pattern as imaged by the catadioptric stereo system. (a) has no water in the tank. (b) is halfway filled with water. Finally (c) has the tank filled with water. All three images have been cropped for space. Image (a) and (c) both show good alignment. Image (b) shows good alignment between the blocks that are either fully submerged or dry. The middle block is nearly at the waterline and consequently shows a significant amount of distortion which interferes with the tracking accuracy. The lighting and camera distances matched those of the catheter when it is being imaged inside the MRI machine.

Fig. 5.

The catheter is approximately 110 mm long with orange paint applied to both the base, the coil set and finally the tip. The orange paint acts as a guide for tracking the catheter. The orange and blue wires near the tip, while not used for computer vision tracking allowed the users to determine if the coil reoriented after energizing the side coils. The orange paint creates a hard film upon drying which would interfere with the catheter mechanics if the entire length were to be painted. By only painting the coils and the two ends, the potential effect of the paint on the catheter is minimized.

Fig. 6.

The catheter as it is imaged by the camera in the MRI suite. Notice the position of the stereo calibration markers. They are located as out of the way as reasonably possible. This ensures that the calibration can be iterated even as the catheter moves. The orange paint guides are still visible in both the frontal (left) and lateral (right) view of the catheter. This picture is of the catheter immediately after energizing. Consequently, the catheter is in the process of re-aligning its axial coil to the MRI ambient magnetic field.

TABLE I

Tracking accuracy of the test pattern.

	n_l	$\bar{e}_{rms}$ (mm)	σ_{rms}^2 (mm ²)	$\max e_{rms}$ (mm)
no water	17999	0.20	$3.18\text{E} - 4$	0.27
half water	11343	0.81	0.37	4.44
full water	10665	0.36	$3.78\text{E} - 4$	0.46

TABLE II

Tracked Catheter Length

n_c	$\bar{\ell}(\text{mm})$	$\sigma_{\ell}^2(\text{mm}^2)$	$\min \ell(\text{mm})$	$\max \ell(\text{mm})$
858	107	2	105	111