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Evaluation of the transverse oscillation technique for cardiac phased-array imaging: A theoretical study

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Abstract

The transverse oscillation (TO) technique can improve the estimation of tissue motion perpendicular to the ultrasound beam direction. TOs can be introduced using plane wave (PW) insonification and bi-lobed Gaussian apodisation (BA) on receive (abbreviated as PWTO). Furthermore, the TO frequency can be doubled after a heterodyning demodulation process is performed (abbreviated as PWTO*). This study is concerned with identifying the limitations of the PWTO technique in the specific context of myocardial deformation imaging with phased arrays and investigating the conditions in which it remains advantageous over traditional focused (FOC) beamforming. For this purpose, several tissue phantoms were simulated using Field II, undergoing a wide range of displacement magnitudes and modes (lateral, axial and rotational motion). The Cramer-Rao lower bound (CRLB) was used to optimize TO beamforming parameters and theoretically predict the fundamental tracking performance limits associated with the FOC, PWTO and PWTO* beamforming scenarios. This framework was extended to also predict performance for BA functions which are windowed by the physical aperture of the transducer, leading to higher lateral oscillations. It was found that windowed BA functions resulted in lower jitter errors compared to traditional BA functions. PWTO* outperformed FOC at all investigated SNR levels but only up to a certain displacement, with the advantage rapidly decreasing when SNR increased. These results suggest that PWTO* improves lateral tracking performance, but only when inter-frame displacements remain relatively low. The study concludes by translating these findings to a clinical environment by suggesting optimal scanner settings.

Index Terms

Transverse oscillations; phased-arrays; Cramer-Rao Lower Bound; simulation

I. Introduction

Myocardial deformation imaging based on ultrasound (US) is widely used to investigate cardiac performance non-invasively. Most commercial US vendors currently offer software packages able to extract both 2D and 3D cardiac strain [1][2]. Over the past decade, the potential for cardiac deformation imaging to support clinical-decision making has been demonstrated in a variety of scenarios [3]. Its gain in popularity on the clinical side went hand in hand with global research efforts on the engineering side to develop more robust motion estimators to assess cardiac mechanics.

From a conceptual point-of-view these technological advances can be classified according to those leading to improvements on the image acquisition side (i.e. US hardware and beamforming developments), and those that operate on the post-processing side (i.e. US signal and image processing strategies). While often seen as separate research fields requiring two different expertises, advances in both areas are intrinsically linked as existing image processing techniques are expected to naturally benefit from an improved US image quality (i.e. a better spatial and higher temporal resolution), and vice versa. For example, obtaining robust deformation estimates perpendicular to the US insonification (i.e. in the transversal direction) is intrinsically more difficult due to the lower image resolution and low transverse frequency content of traditional US images. It has been shown that transverse oscillations (TO) can be introduced in several ways *during* the beamforming process, which naturally lead to better transverse motion estimates, e.g. [4]. This technique has been applied in the context of tissue imaging [5] and blood flow imaging [6], with the majority of work tailored to 2D linear arrays. More recently, the TO technique has been extended to phased-array [7] and convex array geometries [8] for blood flow imaging. The latest reports have demonstrated their use in the context of 3D imaging [9][10]. A comparison of different tracking algorithms for TO imaging was given in e.g. [11]. As an alternative, TOs can also be generated *after* the beamforming process, i.e. synthetically without the use of dedicated beamformers, either by performing signal manipulations in the frequency domain [12][13] [14] or by image filtering in the frequency domain [15][16][17].

This paper investigates the use of the TO technique in the specific context of myocardial deformation imaging. Translating this technique to cardiac phased arrays for tissue imaging is particularly challenging due to the small transducer footprint, the spread of the US beams over depth and the relatively low operating frequency (due to the required penetration depth). These are all factors limiting the maximal attainable TO frequency. Furthermore, choosing the optimal TO beamforming parameters is not straightforward and can impact tracking performance. An optimisation strategy for blood flow imaging on phased arrays was described in e.g. [7]. However, the optimisation process in the context of tissue imaging with phased arrays has remained underexplored. While several (phase-based) algorithms were specifically designed to take advantage of the lateral frequencies present in the TO

images and were shown to perform better than traditional speckle tracking algorithms (e.g. in [10][18][19][20]), these studies did not necessarily investigate the impact of the TO parameter choice on tracking performance as a single TO parameter combination was used. This paper aims to address this underexplored area.

We propose the use of the Cramer-Rao lower bound (CRLB) to optimize TO beamforming parameters and predict the fundamental tracking performance limits associated with each beamforming sequence. This allows us to look at trends without being influenced by the specific implementation or assumptions of a particular motion estimator, e.g. some of them require an exact estimate of the lateral frequency produced by beamforming in order to extract motion (e.g. [21]).

The presented CRLB formulation is an extension of previous work [22] to include windowed aperture functions, allowing us to predict performance for apodisation functions beyond those that traditional phase-based motion estimators typically work with (due to the required analytic signal assumptions). Also, the CRLB formulation now takes the effect of aperture apodisation on electronic signal-to-noise ratio (SNR) into account. We investigate performance over a much larger range of motions and with different beamformers (compared to a single motion and beamformer in [22]). Finally, we apply the CRLB to phased arrays as opposed to linear arrays in the original work [22]. As will be shown, the presence of speckle decorrelation when imposing lateral motion will now play a more pronounced role in predicting tracking performance.

This paper is organized as follows. Sect. II gives an overview of the different strategies described in the literature to generate and analyze TOs. In particular, subsections II-A to II-D describe and motivate the three beamforming sequences that are compared in this study. Subsection II-E highlights the observations that led to this work, in particular the considerations and specific challenges faced when moving from a linear to a phased array. The extensions made to the CRLB formulation are the subject of Sect. III. The different in-silico scenarios that are evaluated are described in Sect. IV. Results for each scenario are then discussed in Sect. V. The paper concludes with the implications of these results in determining optimal scanner settings in a clinical environment.

II. Beamforming Strategies

A. Ultrasound image formation

The formation of a 2D ultrasound (US) radio-frequency (RF) image can be modelled as a linear acoustic system which is fully characterized by its point spread function $p(x, z)$ (PSF) [23]. The two-way PSF can be considered separable in space, as well as in transmit and receive [24]:

$$p(x, z) = p_x(x) \cdot p_z(z) = p_x^t(x) \cdot p_x^r(x) \cdot p_z(z) \quad (1)$$

with the lateral radiation pattern $p_x(x)$ being the product of its lateral components in transmit $p_x^t(x)$ and receive $p_x^r(x)$ and with $p_z(z)$ being the pulse-echo axial response.

According to the Fraunhofer approximation, the lateral pressure pattern in either transmit or receive can be estimated by taking the Fourier transform of the respective aperture function $w(\xi)$ evaluated at spatial frequencies $f_x = \frac{x}{\lambda_z z}$, where λ_z is the transmitted pulse wavelength, and ξ is a variable describing the spatial position on the aperture [25]. Equivalently, for two-way beamforming this profile can be obtained by the Fourier transform of the convolution of the transmit $w^t(\xi)$ and receive $w^r(\xi)$ aperture. This approximation is valid at the focal point of a focused transmit sequence or in the far field of an unfocused transmit event, and assumes a narrowband transmit. This leads to a general expression for the PSF:

$$p(x, z) = \mathcal{F}_{\xi} \{w^t(\xi) \otimes w^r(\xi)\}(f_x) \cdot e^{-\pi \left(\frac{2z}{\sigma_z}\right)^2} \cos(4\pi f_z z) \quad (2)$$

where the axial profile is determined by the transmitted pulse, here represented by a Gaussian-weighted sinusoid with width σ_z and axial frequency $f_z = \frac{1}{\lambda_z}$. Note the factor of 2 in the axial direction since the round-trip distance is considered.

B. Conventional focused beamforming

In conventional imaging a series of pulse-echo measurements are made in different directions to cover the region of interest. Conventionally, one image line is reconstructed per transmit event (i.e. single line acquisition, SLA). In order to obtain an optimal lateral resolution the ultrasound beam is typically focused by using the full aperture, followed by dynamic focusing on receive and applying receive apodisation to reduce side lobes. This sequence will hereafter be abbreviated as **FOC**. For example, Fig. 1 (column 1) shows a typical PSF for a FOC-SLA sequence acquired with a phased array, without transmit apodisation and with receive Hanning windowing. The PSF is spatially invariant under lateral motion. The parameter settings for these Field II simulations [26] are described more in detail in section IV.

C. Transverse oscillation beamforming

As can be seen in Eq. (2) using the Fraunhofer approximation, a laterally oscillating field can be generated by choosing the appropriate apodisation functions during one-way (in either transmit or receive) or two-way beamforming [11][5]. The latter approach leads to higher lateral frequencies but can also be limiting as the transmit focus is fixed, unless a full synthetic aperture dataset is used for beamforming. However, beamforming these datasets in real-time and under conditions of motion remains challenging.

In this paper, we study the receive-only approach where an unfocused plane wave (PW) is transmitted using the full aperture to influence the lateral PSF as little as possible such that $p_x(x) = \mathcal{F}\{w^r(\xi)\}(f_x)$. Intuitively, $p_x(x)$ will oscillate by choosing $w^r(\xi)$ as the convolution of an arbitrary function $f(\xi)$ with two delta functions symmetrically positioned at $\pm\xi_0$ on the aperture to produce the windowing function $w_{apod}(\xi)$ [4]. Strictly speaking $w_{apod}(\xi)$ should be windowed to the physical aperture by $\text{rect}(\xi/D)$, defined as being 1 between $\pm D/2$ and with D the width of the aperture [4]. In practice however the rect function is usually omitted

as one assumes w_{apod} to be sufficiently small outside the aperture to ensure that signal manipulations can be simplified:

$$\begin{aligned} w^r(\xi) &= \left(f(\xi) \otimes_{\xi} [\delta(\xi - \xi_0) + \delta(\xi + \xi_0)] \right) \cdot \text{rect} \left(\frac{\xi}{D} \right) \\ &= w_{apod}(\xi) \cdot \text{rect} \left(\frac{\xi}{D} \right) \\ &\approx w_{apod}(\xi) \end{aligned} \quad (3)$$

Many arbitrary functions $f(\xi)$ have been proposed, e.g. sinc [4][27], Gaussian [21][28], Hanning [29], Hamming [30] and Blackman [30]. In the context of blood flow imaging, it has been shown that the shape of $f(\xi)$ is not crucial for the performance of a motion estimator [29]. The Gaussian is further examined in this paper, mainly for its prevalent use in the context of tissue imaging which is our intended application area [21]. More specifically, following Eq. (3), the bilobed Gaussian receive apodisation (abbreviated as BA) is given by

$$w^r(\xi) = \frac{1}{2\sigma_0} e^{-\pi \left(\frac{\xi}{\sigma_0} \right)^2} \otimes_{\xi} [\delta(\xi - \xi_0) + \delta(\xi + \xi_0)] \quad (4)$$

with σ_0 and $\pm \xi_0$ being the width and two centers of the Gaussians respectively. The resulting PSF $p(x, z)$ according to Eq. (2) is therefore

$$p(x, z) = e^{-\pi(\sigma_0 f_x)^2} \cos(2\pi \xi_0 f_x) \cdot e^{-\pi \left(\frac{2z}{\sigma_z} \right)^2} \cos(4\pi f_z z) \quad (5)$$

In the case of a linear array, one can obtain a depth-independent 2D PSF function, oscillating at a constant axial and lateral wavelength (resp. λ_z and λ_x), and with an axial and lateral width (determined by resp. σ_z and σ_x),

$$p(x, z) = e^{-2\pi \left(\frac{x}{\sigma_x} \right)^2} \cos \left(2\pi \frac{x}{\lambda_x} \right) \cdot e^{-\pi \left(\frac{2z}{\sigma_z} \right)^2} \cos \left(4\pi \frac{z}{\lambda_z} \right) \quad (6)$$

when aperture growth is considered by making the BA parameters depth-dependent¹:

$$\xi_0 = \frac{\lambda_z z}{\lambda_x}, \sigma_0 = \frac{\lambda_z z \sqrt{2}}{\sigma_x} \quad (7)$$

¹The factor $\sqrt{2}$ is included here to remain in accordance with [21].

The above expressions can be easily translated to a phased array by performing a polar coordinate transformation (e.g. [10]) by setting $z = r$, $\lambda_x = r\lambda_\theta$ and $\sigma_x = r\sigma_\theta$. This implies that the 2D PSF $p(r, \theta)$ now oscillates with constant wavelengths when viewed from polar space (resp. λ_r and λ_θ),

$$p(r, \theta) = e^{-\pi \left(\frac{2r}{\sigma_r}\right)^2} \cos\left(4\pi \frac{r}{\lambda_r}\right) \cdot e^{-2\pi \left(\frac{\theta}{\sigma_\theta}\right)^2} \cos\left(2\pi \frac{\theta}{\lambda_\theta}\right) \quad (8)$$

As such, when one envisions tracking tissue in polar space, BA parameters become constant with depth:

$$\xi_0 = \frac{\lambda_r}{\lambda_\theta}, \sigma_0 = \frac{\lambda_r \sqrt{2}}{\sigma_\theta} \quad (9)$$

With the sound field now oscillating in 2D, several methods have been proposed to more accurately assess lateral motion. Phase-based methods have been popular and estimate the phase-shift between sampled spatial quadrature (SQ) signal pairs by applying complex auto-correlators. These SQ pairs are typically created by parallel receive beamforming, either by steering two beams symmetrically around the transmit beam [4][11] or by beamforming in the transmit direction with two different receive apodisation functions [31]. Rather than directly operating on sampled SQ signals, it is also possible to first beamform the complete US image (showing 2D TOs). Image processing techniques, such as block matching, can then be applied to assess lateral motion between subsequent frames [32]. In the context of myocardial deformation imaging, image processing techniques directly operating on beamformed images have seen most popularity. As such, we further investigate the use (and properties) of beamformed TO images.

Several methods have been described to beamform US images from plane wave (PW) transmits. Fig. 1 compares three common strategies and shows the corresponding PSF functions when transverse oscillations (TO) are generated using receive apodisation only (columns 2–4). The most straightforward beamforming scheme is to send a single PW and reconstruct the whole image (column 2) leading to ultrafast image sequences [33]. The downside is that the resulting images typically have a low resolution and, particularly in the context of cardiac imaging, illuminating the full region of interest with a single transmit is difficult. Furthermore, as shown in Fig. 1 (column 2), the PSF is spatially variant under lateral motion, which makes tracking particularly challenging. As an alternative, and in an effort to increase image resolution while maintaining a relatively high frame rate, it has been proposed to coherently sum different steered PWs [34]. However, during this process the TOs, which were produced on receive for every PW transmit, destructively interfere (column 3). In the third alternative, the image is formed in an SLA fashion [18], i.e. only the line in the PW transmit direction is reconstructed on receive (column 4). As can be seen, this SLA scheme maintains the TOs. Furthermore, in contrast with the single PW transmit (column 2), this scheme leads to a less spatially variant PSF under lateral motion when seen in polar

space. This is a desirable feature for tissue tracking because performance is less dependent on the image position. The latter sequence will be used in the remainder of the paper and will be referred to as **PWTO**.

D. Heterodyning demodulation

The characteristics of the received 2D oscillating US field as described above in Sect. II-C, will be influenced by both the axial and lateral velocity component. Traditionally, a twice 1D scheme is therefore typically adopted, where the two components of the SQ signal are first motion-compensated for in the axial direction, before continuing the tracking process laterally [4][31]. However, any error originating from the axial motion-compensation will deteriorate lateral tracking quality.

In [32], the authors proposed to solve this disadvantage by tracking 2 pairs of phase images, computed from single quadrant analytic images associated with the original image. As an alternative, tracking can also be performed after a heterodyning demodulation process as proposed in [35][36]. This process removes frequency content in one direction while doubling the frequency in the other. It consists of two conceptual stages: (i) the creation of two complex signals r_1 and r_2 , and (ii) mixing their content to produce axial-only and lateral-only oscillations. Both stages can be implemented in numerous ways. Stage (i) can be realized starting from the left/right beam SQ pair described in [36][29], starting from the odd/even apodisation SQ pair formulated in [35] or by using a series of spatial Hilbert transforms detailed in [22]. Stage (ii) can be achieved by multiplying the complex signals of stage (i) as in [35][22] or by combining the phases directly as in [36][29].

In this paper we adopt the formulation of [22]. Expressed in terms of the PSF $p(x, z)$ we obtain for stage (i):

$$r_{even}(x, z) = p(x, z) + i\mathcal{H}_z\{p(x, z)\} \quad (10)$$

$$r_{odd}(x, z) = \mathcal{H}_x\{r_{even}(x, z)\} \quad (11)$$

$$r_1(x, z) = r_{even}(x, z) + ir_{odd}(x, z) \quad (12)$$

$$r_2(x, z) = r_{even}(x, z) - ir_{odd}(x, z) \quad (13)$$

where $\mathcal{H}_z$ and $\mathcal{H}_x$ stand for the partial Hilbert transform in the z and x direction respectively. Stage (ii) produces a complex PSF $p_{ax}(x, z)$ and complex PSF $p_{la}(x, z)$ oscillating only in the axial and lateral direction respectively, when these operations are performed:

$$p_{ax}(x, z) = r_1(x, z) \cdot r_2(x, z) \quad (14)$$

$$p_{lat}(x, z) = r_1(x, z) \cdot \overline{r_2(x, z)} \quad (15)$$

Substituting Eq. (15) with the specific PSF considered in this paper (defined in Eq. (6)), performing a polar coordinate transform, and taking the real component of p_{lat} leads to

$$\Re\{p_{lat}(r, \theta)\} = e^{-8\pi \left(\frac{r}{\sigma_r}\right)^2} \cdot e^{-4\pi \left(\frac{\theta}{\sigma_\theta}\right)^2} \cdot \cos\left(4\pi \frac{\theta}{\lambda_\theta}\right) \quad (16)$$

Compared to Eq. (8), it can be seen that the lateral frequency is doubled. The resulting image is used in the remainder of this paper and is abbreviated as **PWTO*** to indicate this is the result after heterodyning. A list of abbreviations is given in Table I. The last column in Fig. 2 shows an example of the resulting PSF function.

E. Linear vs phased array TO beamforming

Translating a TO sequence from a linear to a phased array has two implications.

First, as addressed in Sect. II-C, the apodisation function should become fixed with depth. This in itself is not a novel finding and has been described previously in [7][18].

The second implication is related to the effect of speckle decorrelation on tracking performance due to the underlying motion. In order to avoid confusion, the following definitions are first adopted in the remainder of the paper. Motion is always defined in Cartesian space, i.e. motion orthogonal or parallel to the transducer surface is termed axial versus lateral motion respectively. Tracking is always performed in beamspace and defined as either along or orthogonal to the beamdirection, i.e. in the radial or transverse direction respectively. For a linear array these definitions coincide: tracking a uni-axial/uni-lateral tissue motion remains a 1D problem in beamspace. For a phased array on the other hand, uni-axial or uni-lateral motion is translated to a 2D motion field in beamspace. While this is purely the result of geometry, it has greater consequences on tracking performance than has been reported in the context of TO beamforming.

These consequences can be observed in Fig. 2 by looking at the interference pattern change of 2 PSFs undergoing a uni-lateral or rotational motion. This was quantified by computing the normalized cross-correlation between two subsequent speckle patterns similar to what is done in traditional block matching [37]. Important to note is that the blocks were deformed according to the underlying imposed motion, and that observed decorrelation differences are therefore only due to differences in beamforming and array configuration. The same Field II settings were used as in Fig. 1. For FOC, regardless of the array configuration and the underlying motion, speckle patterns remained correlated. Moving to PWTO changed these

trends. More specifically, underlying motion causing a shearing component in beamspace lead to decorrelation. For the linear and phased array this happened for a rotational and lateral motion respectively. Again, this result is not surprising and these effects have been described before, e.g. [38]. However, its effect is more pronounced for the PWTO sequence compared to a FOC sequence since the attempt to add more transverse oscillations caused the PSF to become wider.

In the context of blood flow imaging this effect is not as disruptive. While blood velocities can be fairly high, relative lateral motion remains fairly small as it is estimated between SQ signals sampled at a pulse repetition frequency in the order of kHz [6]. On the other hand, if tracking is based on beamformed PWTO images, the frame rate is substantially lower. Since the early adoptions of this technique for phased-array tissue imaging [18], the consequences of this observation on tracking performance have, to the best of our knowledge, not been extensively reported.

III. Cramer-Rao lower bound formulation

A. Traditional formulation

The CRLB predicts the minimum attainable standard deviation of the jitter of the displacement estimates of an unbiased motion estimator. Jitter errors can be seen as slight displacements of the true peak of the cross-correlation function caused by inherent uncertainties of the tracking process. These uncertainties can not be removed, and can be attributed to the combined effects of signal decorrelation, noise and the use of finite window lengths and bandwidths during motion estimation. Jitter therefore places a fundamental lower limit on the performance of displacement estimators [39], and can be used as a guide to select optimal TO parameters. In this paper we study the jitter on the lateral motion estimate, and therefore assume that axial motion has been corrected for already (similar to the strategy of e.g. [4]).

The CRLB formulation in this paper is based on the signal model and assumptions chronologically described in [39] and [22], leading to an expression of the standard deviation of the jitter σ for a transverse motion estimate $\hat{x}$ with underlying true motion x :

$$\sigma(\Delta x - \Delta \hat{x}) = \left(\int_{-\infty}^{+\infty} \frac{4\pi^2 T u^2}{\frac{1}{\rho^2} \left(1 + \frac{1}{\text{SNR}^2} \cdot \frac{G_{NN}(u)}{G_{SS}(u)} \right)^2 - 1} du \right)^{-\frac{1}{2}} \quad (17)$$

This expression provides insights into the fundamental tradeoffs between three parameters: speckle decorrelation expressed as the normalized signal decorrelation ρ computed over a kernel with length T , electronic signal-to-noise ratio (SNR) of the beamformed signals expressed as a fraction (not in decibels), and image frequency content $g(u)$ here represented by the power spectrum $G(u)$. In particular, the term $G_{SS}(u)$ is the normalized signal power spectrum defined as:

$$G_{SS}(u) = \frac{g_{SS}^2(u)}{\int_{-\infty}^{+\infty} g_{SS}^2(u) du} \quad (18)$$

with a similar expression for the noise power spectrum $G_{NN}(u)$. The latter is assumed constant over the full frequency range. From Eq. (17) it can be seen that the standard deviation of the lateral jitter σ becomes worse when decorrelation is higher (i.e. when ρ decreases), SNR is lower, or when less transverse frequency content is available (i.e. the $G_{SS}(u)$ terms contributes more at lower frequencies).

In order to properly evaluate the CRLB, an expression for the frequency spectrum $g_{SS}(u)$ is required. This can be obtained using numerical simulations, experimental analysis or, more elegantly, analytical expressions. Given a known 2D analytical expression for the PSF $p(x, z)$, the 2D frequency spectrum $P(u, v)$ is simply found by its Fourier transform

$$P(u, v) = \mathcal{F} \{p(x, z)\}(u, v) \quad (19)$$

The lateral frequency spectrum was calculated for the dominant central transmit frequency f_0 as

$$g_{SS}(u) = P(u, f_0) \quad (20)$$

After substituting the analytic PSF functions derived in the previous sections in Eq. (20), the following relationships can be computed for each imaging scenario.

The frequency space covered by the FOC imaging scenario with Hanning on receive has the shape of a Hanning window extending up to approximately $\pm D(2\lambda_z z)$,

$$g_{SS}(u) = \frac{\lambda_z z}{2} \left[1 - \cos \left(2\pi \left(\frac{\lambda_z z}{D} u + \frac{M+1}{2(M-1)} \right) \right) \right] \quad (21)$$

where M is the number of array elements.

The shape of the frequency spectrum for the PWTO beamformed images is a Gaussian with its shape depending on the TO parameters:

$$g_{SS}(u) = \frac{\sigma_x}{\sqrt{2}} \left[e^{-\pi \cdot \left(\frac{\sigma_x}{\sqrt{2}} \right)^2 \cdot \left(u - \frac{1}{\lambda_x} \right)^2} + e^{-\pi \cdot \left(\frac{\sigma_x}{\sqrt{2}} \right)^2 \cdot \left(u + \frac{1}{\lambda_x} \right)^2} \right] \quad (22)$$

Finally, after heterodyning, the PWTO* imaging case sees its frequency shape widened and shifted towards twice the frequency compared to Eq. (22):

$$g_{SS}(u) = \begin{cases} \frac{\sigma_x}{\sqrt{2}} \left[e^{-\pi \cdot (\frac{\sigma_x}{\sqrt{2}})^2 \cdot (u - \frac{2}{\lambda_x})^2} + e^{-\pi \cdot (\frac{\sigma_x}{2})^2 \cdot (u + \frac{2}{\lambda_x})^2} \right] & u > 0 \\ 0 & u < 0 \end{cases} \quad (23)$$

We now propose two further refinements to the CRLB formulation to more appropriately evaluate the different TO scenarios. The motivation for both refinements are given in the respective sections.

B. Refinements to the signal-to-noise ratio term

The representation of the CRLB in Eq. (17) did not take into account the effects of individual channel apodisation on electronic SNR and assumed it would remain constant regardless the imposed apodisation. In practice, however, this is not entirely true. For example, lowering the BA width σ_0 while keeping the BA offset ξ_0 constant, will decrease the received signal from the outer apodisation elements, and will therefore also affect (i.e. decrease) the SNR of the overall sum-and-delayed signal. In order to incorporate this effect we perform the following substitution in Eq. (17):

$$\text{SNR} = \alpha^t \cdot \alpha^r \cdot \text{SNR}' \quad (24)$$

where SNR' is the expected SNR when no transmit or receive apodisation is performed, and with α^t and α^r factors representing the percentage drop in SNR after applying individual channel apodisation on either transmit or receive respectively. In order to simplify the subsequent derivation, we further assume that we can increase the scanner power output of the PWTO sequences to realize the same signal amplitude on transmit as the FOC sequence, i.e. $\alpha^t = 1$. Finally, we define α^r as follows:

$$\alpha^r = \frac{\sqrt{\sum_i^M \sum_j^M w_i w_j}}{\sqrt{M \sum_i^M w_i^2}} \quad (25)$$

for a 1D-array with M elements where w_i is the receive apodisation of element i . The reader is referred to Appendix (A) for the derivation of this expression.

C. Refinements to the frequency spectrum term

The current workflow to obtain the expressions for the frequency spectrum terms in Eqs. (21)–(23) was based on having an analytical expression for the PSF $p(x, z)$ available, which in turn was easily realized by the assumption of Eq. (3) stating that contributions of the apodisation functions outside the physical aperture were negligible, i.e. $w^r(\xi) \approx w_{apod}(\xi)$.

In this paper, we now generalize this by allowing apodisation functions to be windowed by the aperture, i.e. $w^r(\xi) = w_{apod}(\xi) \cdot \text{rect}\left(\frac{\xi}{D}\right)$. Since obtaining an analytical expression for the PSF $p(x, z)$ is not straightforward anymore, we modify the workflow, and work directly in the aperture domain to compute the frequency spectrum $g_{SS}(u)$ term as:

$$\begin{aligned}
 g_{SS}(u) &= \mathcal{F}\{p(x)\}(u) \\
 &= \mathcal{F}\left\{\mathcal{F}\{w^r(\xi)\}\left(\frac{x}{\lambda_z z}\right)\right\}(u) \\
 &= \mathcal{F}\left\{\mathcal{F}\{w_{apod}(\xi)\}\left(\frac{x}{\lambda_z z}\right) \otimes_x D \text{sinc}\left(D \frac{x}{\lambda_z z}\right)\right\}(u) \\
 &= \mathcal{F}\left\{\mathcal{F}\{w_{apod}(\xi)\}\left(\frac{x}{\lambda_z z}\right)\right\}(u) \cdot \mathcal{F}\left\{D \text{sinc}\left(D \frac{x}{\lambda_z z}\right)\right\}(u) \\
 &= (\lambda_z z)^2 \cdot \text{rect}\left(\frac{\lambda_z z}{D} u\right) \cdot w_{apod}(\lambda_z z u)
 \end{aligned} \tag{26}$$

This expression holds true for both the FOC and PWTO sequence. It leads to the previously derived expressions for non-windowed aperture functions of Eqs. (21) and (22) when dropping the terms associated with the rect windowing and substituting w_{apod} appropriately.

Following this workflow and as shown in Appendix (B), the frequency spectrum of the heterodyned PWTO* sequence can also be expressed in terms of $w^r(\xi)$ as follows:

$$g_{SS}(u) = 4(\lambda_z z)^4 \left(\left[\text{rect}\left(\frac{\lambda_z z}{D} u\right) \cdot w_{apod}(\lambda_z z u) \cdot S(u) \right] \otimes_u \left[\text{rect}\left(\frac{\lambda_z z}{D} u\right) \cdot w_{apod}(\lambda_z z u) \cdot S(u) \right] \right) \tag{27}$$

where $S(u)$ is the Heaviside (step) function. Given that the normalization operation in Eq. (18) will remove the scalar weighting term, this expression thus states that the shape of $g_{SS}(u)$ after heterodyning can simply be found by the self-convolution of the right-hand side of the windowed apodisation function. This is under the assumption that the apodisation function is symmetric around the center of the array.

IV. Simulation environment

To gain insight into the performance of the 3 investigated beamforming scenarios (FOC, PWTO and PWTO*), simulations were performed using Field II [26]. A 64-element phased array (320 μm pitch, 20 μm kerf, 5 mm element height) was simulated emitting a Gaussian pulse at 3.0 MHz and 70% fractional bandwidth (1.08 μs pulse length). A uniform rectangular speckle-generating in-silico phantom was simulated having 80-by-40 mm axial-by-lateral dimensions, centrally positioned at 60 mm depth, using 25 scatterers per resolution cell. Acoustic responses were computed using a sampling rate of 120 MHz, and channel data for each element was stored individually at a sampling rate of 60 MHz.

All three beamforming scenarios used the same SLA scan sequence, imaging the phantom with an opening angle of 200° (from -10° to 10°), using steps of 0.25° to ensure spatial lateral sampling was above the Nyquist criterium. Apodisation was performed according to

the state-of-the-art. More specifically, the FOC sequence used a fixed axial focus of 60 mm, and beamformed images without transmit apodisation, followed by Hanning apodisation on receive. The PWTO and PWTO* cases transmitted unfocused plane waves with Hanning on transmit, and a bilobed Gaussian on receive.

For every simulated image pair undergoing a specific displacement, the CRLB was computed according to the procedure outlined in Sect. III. The normalized cross-correlation ρ was computed over rectangular windows having an axial-by-lateral size of 1.5-by-6 mm, approximately four times the resolution in the axial and lateral dimension of the FOC beamformer at a depth of 60 mm. The window size was chosen arbitrarily as a compromise, on the one hand to be large enough to contain enough speckle patterns to properly evaluate speckle decorrelation with induced motion, and on the other hand to remain small enough to ensure a sufficient spatial resolution of these decorrelation estimates [40]. Decorrelation was computed in polar space, with the window for the displaced image, deformed according to the ground truth for reasons explained in Sect. II-E. The window length T in the CRLB computations was set equal to the lateral window length.

V. Results and Discussion

The series of experiments that were carried out are summarized in Table II, together with the relevant beamforming parameters. Results for each imaging case are discussed separately. Subsection V-A starts with demonstrating the general workflow to obtain CRLB curves. Subsection V-B is concerned with determining the optimal TO parameter settings in two cases: when the bilobed Gaussian falls within the aperture (abbreviated as **PWTO90**, according to the description in Sect. III-A), or when its windowed by the aperture (termed **PWTO50** with frequency spectrum determined as in Sect. III-C). The numbers ‘50’ and ‘90’ refer to the group of apodisation functions with at least 50% or 90% of their nonzero contributions falling within the aperture. The influence of SNR, and the displacement type is the topic of subsections V-C and V-D. The results section continues with extrapolating how the results of this paper could impact choosing appropriate beamformers for in-vivo scanning (section V-E). Finally, limitations are discussed in section V-F.

A. Sample CRLB curves

Fig. 3 illustrates the general workflow to obtain CRLB curves. Panels (a), (b) and (c) show the different factors going into the CRLB equation leading to the jitter error curves depicted in panel (d). For demonstration purposes, a single BA window on receive was chosen for the PWTO and PWTO* sequence ($\lambda_\theta = 5.75^\circ$ $\sigma_\theta = 10.5^\circ$) with 20 dB SNR.

For reasons mentioned in Sect. II-E, note that speckle patterns decorrelated at a much faster rate with PWTO compared to FOC (0.76 and 0.98 respectively for a lateral displacement of 1000 μm and 20 dB SNR). In this example, the negative effect on tracking performance was not compensated for by the higher TO frequency content, i.e. the FOC sequence produced a lower jitter error for the full displacement range.

After heterodyning, speckle patterns in the PWTO* sequence decorrelated at an even faster rate than PWTO (only 0.54 at 1000 μm). However, this was counteracted by the increased

signal spectral content (e.g. as shown in panel c), resulting in a better performance of PWTO* over PWTO over the whole range of considered displacements. Compared to the FOC sequence, PWTO* performed better up to 127 μm displacement.

B. Optimisation of TO parameters

In the example of Fig. 3, the position of the jitter curves for the PWTO and PWTO* scenario depend on the BA parameters λ_θ and σ_θ . In order to find their optima, the range was varied between $\lambda_\theta = 3 - 12^\circ$, and $\sigma_\theta = 0.5 - 20^\circ$. The minimum λ_θ was chosen to result in Gaussians which were offset such that 50% remained within the aperture. The range for σ_θ was selected to realize a wide variety of apodisation shapes as shown in Fig. 4. The optimal BA parameters were determined by computing jitter errors from image pairs undergoing a fixed 200 μm lateral displacement to be within a zone of acceptable jitter errors. Beamforming was performed according to the PWTO* sequence.

Fig. 5 shows jitter errors when varying the BA parameters. Based on the specific parameter combinations, the graph can be divided in two areas satisfying either the PWTO90* or PWTO50* conditions. For PWTO90*, the error distribution is consistent with other studies [21][22], i.e. errors tend to be smaller when the TO wavelength λ_θ becomes lower or when the PSF width σ_θ becomes smaller. Similarly, the trends for PWTO50* are in correspondence with the phase-based tracking results reported in [22]. Optimal values for the PWTO90* and PWTO50* were respectively $\lambda_\theta = 5.25^\circ$, $\sigma_\theta = 12.0^\circ$ and $\lambda_\theta = 3.00^\circ$, $\sigma_\theta = 3.00^\circ$.

The final CRLB curves, using these optimal parameters, are shown in Fig. 6. Similar trends as in Fig. 3 can be observed, i.e. heterodyning always lowered jitter errors. Lateral tracking performance in PWTO images is lower because axial and lateral motion are intrinsically linked, i.e. displacement that occurs in the axial direction deteriorates the ability to accurately track laterally. PWTO* images do not suffer from this disadvantage as axial frequencies are removed during the process, while only the lateral frequency is retained (and doubled). The CRLB formulation formally shows that this higher lateral frequency content more than makes up for the higher decorrelation rates of PWTO* compared to PWTO because the jitter levels are lower in the former case. Fig. 3 also shows that allowing the apodisation functions to be windowed reduced jitter errors further. Given these trends, we decided to focus our analysis in the remainder of the paper on the heterodyned scenarios only, i.e. PWTO90* and PWTO50*. PWTO90* was the preferred beamformer compared to FOC up to a displacement of 125 μm , whereas PWTO50* extended this further up to 472 μm .

C. Influence of SNR

Three parameters determine the jitter in the CRLB equation. The influence of decorrelation ρ and frequency content g_{SS} was discussed previously. This section is concerned with identifying the influence of SNR on jitter by varying its contribution from 10 dB to 30 dB. CRLB curves are computed using the optimal BA parameters from the previous section and are shown in Fig. 7. As expected, higher SNR led to lower jitter errors for the FOC, PWTO90* and PWTO50* scenarios (bottom row).

Interestingly, when imaging conditions were more difficult, i.e. at combinations of higher displacement and lower SNR, the use of TO beamforming over FOC imaging was justified longer (Fig. 7; top row). Indeed, the SNR-term in Eq. (17) modulates the relative contributions of g_{SS} and ρ towards jitter error. When SNR becomes higher, the contribution of g_{SS} is smaller, and the advantage of having a higher frequency content is therefore less pronounced. Translating this to absolute jitter errors (Fig. 7; bottom row), it can be seen that the PWTO90* sequence is preferred over FOC imaging up to 382 μm at 10 dB SNR, but only up to 31 μm for 30 dB SNR. The PWTO50* sequence warranted use above 1000 μm at 10 dB, and up to 151 μm at 30 dB.

D. Influence of displacement type

Fig. 8 shows that tissue being displaced in the axial direction shows negligible speckle decorrelation. Note that the displacement magnitude was the same as in the previous section where lateral tissue motion was investigated. A similar insignificant decorrelation rate was seen for tissue undergoing a rotation up to 4° (Fig. 9). By means of comparison, at a depth of 60 mm for example, this rotation corresponds to a maximal axial displacement of 146 μm towards the transducer and a maximal lateral displacement of 4185 μm . In both displacement scenarios, the ‘shearing’ effect between neighboring PSF functions causing decorrelation as explained in Fig. 2 is not present. As such, as the result of Eq. (17), the trends in the lateral jitter error between the beamforming scenarios are dominated by the amount of lateral frequency content. For both displacement scenarios, the PWTO* beamformer is therefore preferred over FOC imaging for all investigated SNR. Furthermore, PWTO50* performed again better than PWTO90*, especially when SNR was lower (see bottom rows of Figs. 8 and 9).

E. Potential clinical implications

The results in this paper can be used as a guide to determine optimal scanner settings in a clinical setting when images are to be acquired with the goal of cardiac deformation imaging in mind. More specifically, the insights gained from the lateral motion scenario (in Fig. 7) are used for this purpose as this motion mode produced the largest jitter errors and the largest differences between the FOC and PWTO beamformers, and therefore poses the most severe system design constraints.

As an example, we consider performing strain imaging in a 4 chamber view of a normal volunteer. For illustration purposes, we assume an imaging depth of 10 cm, and a line density of 3 beams per degree to definitely satisfy the spatial Nyquist criteria. According to [41], the maximum lateral velocity during the cardiac cycle is 8.6 cm/s and is reached during early diastole. We want to determine whether images should be acquired using FOC or PWTO50*, i.e. the best performing PWTO sequence. Tracking quality, here quantified by jitter, will be influenced by image SNR and the amount of inter-frame displacements. The former is difficult to control in clinical practice as it is intrinsically determined by the patient, the acoustic window, and by the probe output power. The inter-frame displacement is determined by the frame rate and, having fixed the line density, can be modified by changing the opening angle.

Fig. 10(a) translates this information and shows the preferred beamformer in terms of SNR and imaging opening angle. According to the theory developed in this paper, the use of PWTO beamforming remains only warranted for small opening angles. Increasing the opening angle (e.g. to image both walls of the ventricle), quickly selects the traditional FOC beamformer as the preferred imaging scheme, especially for high SNR values. The main cause is the higher interframe displacement and associated decorrelation that puts PWTO at a disadvantage. When one envisions determining only systolic function, conditions relax as shown in Fig. 10(b). Since the maximum lateral velocity during systole is lower (4.32 cm/s; [41]), the use of PWTO50* is justified longer.

F. Limitations

This paper analysed tracking performance from a theoretical point-of-view. This allowed to gain further insights without being biased towards a single motion estimator, or being dependent on a particular implementation. It is important to note that the described trends are valid for unbiased motion estimators only. Biased motion estimators, on the other hand, have the potential to further improve performance [42]. Extending analysis to this family of algorithms, however, falls outside the scope of this paper.

The CRLB assumed the only factor affecting image quality was SNR. In the context of cardiac deformation imaging however it is known that image quality in transthoracic echocardiography tends to be relatively poor in part due to the presence of reverberations and (stationary) clutter from multipath and off-axis scattering. Further research is required to investigate the behavior of TO beamforming and its effect on tracking performance in the presence of these image artefacts. This could be studied by introducing artefacts into these simulations, e.g. by following the strategy described in [43].

In order to more intuitively understand trends in tracking performance related to beamforming, motion modes were purposely restricted to bulk motion only. Nevertheless, in the context of strain imaging, further tests have to be performed on simulated elastography phantoms with inclusions (e.g. similar to the ones in [44]) to finetune observations. These phantoms will result in larger 2D displacement (and strain) gradients, potentially making tracking more challenging. Nevertheless, it should be noted that, in this study, 2D displacement gradients were already present in the lateral/axial displacement scenarios as tracking was performed in beamspace.

Finally, the PWTO beamforming sequence was implemented in an SLA fashion for reasons explained earlier in Sect. II-C. As such, a fair comparison between PWTO and FOC could be performed since the frame rate was the same. This led to the clinical trends shown in Fig. 10. However, given that PWs cover a larger field of view with every transmit, this also (artificially) limited frame rate and may therefore still underestimate potential of PWTO. Further research will focus on investigating faster PWTO imaging schemes in combination with parallel beamforming to increase frame rate. For example, fewer PW emissions could be used to insonify the same region of interest and coherently summed afterwards. Unfortunately, as shown in Fig. 1b, the receive-only PWTO approach used in this paper is incompatible with this scheme. Instead, the TOs could be introduced after a coherent summing scheme by filtering in the frequency domain [15][16][17]. However, this technique

has only been described in the context of linear arrays, and further research is required to extent it to a sector scan as there may not be enough overlap between neighboring PWs to warrant the TO filtering approach. As an alternative, it has been suggested to coherently sum diverging waves instead as they would intrinsically cover a larger field-of-view [45]. The theoretical CRLB performance of these filtering approaches remains the topic of future research.

VI. Conclusions

We have contrasted several transverse oscillation techniques against traditional focused beamforming to predict the fundamental tracking performance limits associated with both beamforming scenarios. The CRLB formulation in this paper provides a theoretical framework to select optimal TO parameters. It was found that bi-lobed Gaussian apodisation functions which are windowed by the physical aperture of the transducer, resulted in a better tracking performance compared to those who fall completely inside the aperture. Furthermore, the results presented in this paper highlight the importance of the fundamental trade-offs between having a higher transverse image frequency content when using PWTO but at the cost of a higher decorrelation rate with increased lateral tissue displacement. These findings suggest that PWTO (and PWTO*) improve lateral tracking performance over FOC imaging but only when inter-frame displacements remain relatively low.

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References

1. Jasaityte R, Heyde B, D'hooge J. Current state of three-dimensional myocardial strain estimation using echocardiography. *J Am Soc Echocardiogr*. 2013; 26(1):15–28. [PubMed: 23149303]
2. Voigt J, Pedrizzetti G, Lysyansky P, Marwick T, Houle H, Baumann R, Pedri S, Ito Y, Abe Y, Metz S, Song J, Hamilton J, Sengupta P, Kolias T, D'hooge J, Aurigemma G, Thomas J, Badano L. Definitions for a common standard for 2D speckle tracking echocardiography: consensus document of the EACVI/ASE/Industry Task Force to standardize deformation imaging. *Eur Heart J Cardiovasc Imaging*. 2015; 16(1):1–11. [PubMed: 25525063]
3. Marwick T, Leano R, Brown J, Sun JP, Hoffmann R, Lysyansky P, Becker M, Thomas J. Myocardial strain measurement with 2-dimensional speckle-tracking echocardiography: definition of normal range. *J Am Coll Cardiol: Cardiovasc Imaging*. 2009; 2(1):80–84.
4. Jensen J, Munk P. A new method for estimation of velocity vectors. *IEEE Trans Ultrason Ferroelectr Freq Control*. 1998; 45(3):837–851. [PubMed: 18244236]
5. Liebgott H, Basarab A, Gueth P, Friboulet D, Delachartre P. Transverse oscillations for tissue motion estimation. *Ultrasonics*. 2010; 50(6):548–555. [PubMed: 20005551]
6. Jensen A. New developments in vector velocity imaging using the transverse oscillation approach. *SPIE Med Imaging*. 2013; 8675:86750F.
7. Pihl M, Marcher J, Jensen J. Phased-array vector velocity estimation using transverse oscillations. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2012; 59(12):2662–2675. [PubMed: 23221215]
8. Jensen A, Brandt A, Nielsen M. Convex array vector velocity imaging using transverse oscillation and its optimization. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2015; 62(12):2043–2053.

9. Pihl M, Jensen A. A transverse oscillation approach for estimation of three-dimensional velocity vectors, part I: concept and simulation study. *IEEE Trans Ultras Ferroelectr Freq Control*. 2014; 61(10):1599–1607.
10. Salles S, Liebgott H, Garcia D, Vray D. Full 3-D transverse oscillations: a method for tissue motion estimation. *IEEE Trans Ultras Ferroelectr Freq Control*. 2015; 62(8):1473–1485.
11. Sumi C. Displacement vector measurement using instantaneous ultrasound signal phase - multidimensional autocorrelation and doppler methods. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2008; 55(1):24–43. [PubMed: 18334311]
12. Chen X, Zohdy M, Emelianov S, O'Donnell M. Lateral speckle tracking using synthetic lateral phase. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2004; 51(5):540–550. [PubMed: 15217232]
13. Hasegawa H, Kanai H. Phase-sensitive lateral motion estimator for measurement of artery-wall displacement - phantom study. *IEEE Trans Ultras Ferroelectr Freq Control*. 2009; 56(11):2450–2462.
14. Liebgott H. Fourier domain beamforming for transverse oscillations. *IEEE Int Ultrasonics Symposium*. 2010:1755–1758.
15. Varray F, Liebgott H. An alternative method to classical beamforming for transverse oscillation images: application to elastography. *ISBI - International Symposium on Biomedical Imaging*. 2013:716–719.
16. Salles S, Chee A, Garcia D, Yu A, Vray D, Liebgott H. 2-D arterial wall motion imaging using ultrafast ultrasound and transverse oscillations. *IEEE Trans Ultras Ferroelectr Freq Control*. 2015; 62(6):1047–1058.
17. Jensen J, Stuart M, Jensen J. High frame rate vector velocity estimation using plane waves and transverse oscillation. *IEEE Int Ultrasonics Symposium*. 2015:1–4.
18. Liebgott H, Basarab A, Marincas S, Bernard O, Friboulet D. Tangential oscillations for motion estimation in echocardiography. *IEEE Int Ultrasonics Symposium*. 2008:1761–1764.
19. Liebgott H, Salem A, Basarab A, Gao H, Claus P, D'hooge J, Delachartre P, Friboulet D. Tangential sound field oscillations for 2D motion estimation in echocardiography. *IEEE Int Ultrasonics Symposium*. 2009:498–501.
20. Alessandrini M, Basarab A, Bousset L, Guo X, Serusclat A, Friboulet D, Kouamé D, Bernard O, Liebgott H. A new technique for the estimation of cardiac motion in echocardiography based on transverse oscillations: a preliminary evaluation in silico and a feasibility demonstration in vivo. *IEEE Trans Med Imaging*. 2014; 33(5):1148–1162. [PubMed: 24770919]
21. Liebgott H, Wilhelm J, Jensen J, Vray D, Delachartre P. PSF dedicated to estimation of displacement vectors for tissue elasticity imaging with ultrasound. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2007; 54(4):746–756. [PubMed: 17441584]
22. Bottenus N, Trahey G. Evaluation of the transverse oscillation method using the Cramer-Rao Lower Bound. *IEEE Trans Ultras Ferroelectr Freq Control*. 2015; 62(11):2009–2017.
23. Bamber J, Dickinson R. Ultrasonic B-scanning: a computer simulation. *Phys Med Biol*. 1980; 25(3):463–479. [PubMed: 7403261]
24. Lockwood G, Pai-Chi L, O'Donnell M, Foster S. Optimizing the radiation pattern of sparse periodic linear arrays. *IEEE Trans Ultras Ferroelectr Freq Control*. 1996; 43(1):7–14.
25. Cobbold, R. *Foundations of biomedical ultrasound*. Oxford university press; 2007.
26. Jensen A. FIELD: a program for simulating ultrasound systems. *Med & Biolog Eng & Comput*. 1998; 34(1 part 1):351–353.
27. Liebgott H, Fromageau F, Wilhelm J, Vray D, Philippe D. Beamforming scheme for 2d displacement estimation in ultrasound imaging. *EURASIP Journal on Advances in Signal Processing*. 2005; 8:1–9.
28. Sumi C. Our recent strain-measurement-based shear modulus reconstruction. *IEEE Int Ultrasonics Symposium*. 2005:1771–1776.
29. Udesen J, Jensen A. Investigation of the transverse oscillation method. *IEEE Trans Ultras Ferroelectr Freq Control*. 2006; 53(5):959–971.
30. Kerr R, Anderson M. Aperture design analysis for vector flow imaging. *IEEE Int Ultrasonics Symposium*. 2002:1511–1516.

31. Anderson M. Multi-dimensional velocity estimation with ultrasound using spatial quadrature. *IEEE Trans Ultrason Ferroelectr Freq Control*. 1998; 45(3):852–861. [PubMed: 18244237]
32. Basarab A, Gueth P, Liebgott H, Delachartre P. Phase-based block matching applied to motion estimation with unconventional beamforming strategies. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2009; 56(5):945–957. [PubMed: 19473913]
33. Tanter M, Fink M. Ultrafast imaging in biomedical ultrasound. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2014; 61(1):102–119.
34. Montaldo G, Tanter M, Bercoff J, Benech N, Fink M. Coherent plane-wave compounding for very high frame rate ultrasonography and transient elastography. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2009; 56(3):489–506.
35. Anderson M. A heterodyning demodulation technique for spatial quadrature. *IEEE Int Ultrasonics Symposium*. 2000; 2:1487–1490.
36. Jensen A. An improved estimation and focusing scheme for vector velocity estimation. *IEEE Int Ultrasonics Symposium*. 1999:1465–1470.
37. Trahey G, Allison J, Von Ramm O. Angle independent ultrasonic detection of blood flow. *IEEE Trans Biomed Eng*. 1987; 34(12):965–967. [PubMed: 2961682]
38. Friemel B, Bohs L, Nightingale K, Trahey G. Speckle decorrelation due to two-dimensional flow gradients. *IEEE Trans Ultrason Ferroelectr Freq Control*. 1998; 45(2):317–327.
39. Walker W, Trahey G. A fundamental limit on delay estimation using partially correlated speckle signals. *IEEE Trans Ultrason Ferroelectr Freq Control*. 1995; 42(2):301–308.
40. Lubinski M, Emelianov S, O'Donnell M. Speckle tracking methods for ultrasonic elasticity imaging using short-time correlation. *IEEE Trans Ultrason Ferroelectr Freq Control*. 1999; 46(1): 82–96. [PubMed: 18238401]
41. Kowalski M, Kukulski T, Jamal F, D'hooge J, Weidemann F, Rademakers F, Bijmens B, Hatle L, Sutherland G. Can natural strain and strain rate quantify myocardial deformation? a study in healthy subjects. *Ultrasound Med Biol*. 2001; 27(8):1087–1097. [PubMed: 11527595]
42. Byram B, Trahey G, Palmeri M. Bayesian speckle tracking. Part II: Biased ultrasound displacement estimation. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2013; 60(1):144–157. [PubMed: 23287921]
43. Byram B, Shu J. A pseudo non-linear method for fast simulations of ultrasonic reverberation. *SPIE Med Imaging*. 2016; 9790:97900U.
44. Varray F, Liebgott H. Multi-resolution transverse oscillation in ultrasound imaging for motion estimation. *IEEE Trans Ultrason Ferroelectr Freq Control*. 2013; 60(7):1333–1342.
45. Joos, P., Salles, S., Vray, D., Nicolas, B., Liebgott, H. Cardiac motion estimation based on transverse oscillation and ultrafast diverging wave imaging. *IEEE Int Ultrasonics Symposium*; 2015. p. 1-4.

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He has published and co-authored over 50 scientific papers in international journals and conference proceedings on ultrasound deformation imaging. In 2010, he was nominated for the young investigator award at the IEEE International Ultrasonics Symposium. He is the chair of the ultrasound conference of the SPIE Medical Imaging Symposium since 2015. His current research interests include image registration, beamforming strategies and high frame rate imaging.

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Jan D'hooge received the M.Sc. and Ph.D. degrees in Physics at the University of Leuven (KU Leuven, Belgium) in 1994 and 1999, respectively. His dissertation studied the interaction of ultrasonic waves and biological tissues using computer simulation. As a postdoctoral researcher at the Medical Imaging Computing Laboratory of the University of Leuven, he worked on elastic registration, segmentation, shape analysis, and data acquisition problems related to other modalities (in particular, MRI). In 2006, he was appointed associate professor in the Department of Cardiovascular Diseases of the Medical Faculty.

In 1999, he won the Young Investigator Award of the Belgian Society of Echocardiography, and in 2000, he was nominated for the Young Investigator Award of the European Society of Echocardiography. Since 2009 he has been a part-time visiting professor at the Norwegian Institute of Science and Technology (Trondheim, Norway). He is a member of the Acoustical Society of America, IEEE, and the European Association of Echocardiography. He was the chair of the ultrasound conference of the SPIE Medical Imaging Symposium from 2008 until 2011, technical vice-chair of the IEEE Ultrasonics Symposium from 2008 to

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Gregg Trahey received the B.Sc. and M.Sc. degrees in Electrical Engineering from the University of Michigan (MI, USA) in 1975 and 1979, respectively, and the Ph.D. degree in Biomedical Engineering from Duke University (NC, USA) in 1985. He served in the Peace Corps from 1975 to 1978, and was a Project Engineer with the Emergency Care Research Institute, Plymouth Meeting (PA, USA) from 1980 to 1982. Currently, he is a professor with the Department of Biomedical Engineering, Duke University (NC, USA) and holds a secondary appointment with the Department of Radiology, Duke University Medical Center (NC, USA). His research interests include adaptive phase correction, beamforming and acoustic radiation force imaging methods.

Appendix A. Signal-to-noise ratio after apodisation

In this appendix we quantify the effect of individual channel apodisation on the SNR of the beamformed output signal. Consider a 1D array consisting of M elements, each receiving a signal s_i corrupted by channel noise n_i . We assume a standard beamformer has already properly delayed the individual channel signals s_i . The output beamformed signal S , with associated total noise term N , can be found after appropriately apodising each channel with weight w_i , followed by summing their contributions as:

$$S = \frac{1}{M} \sum_i^M w_i s_i \quad (\text{A.1})$$

$$N = \frac{1}{M} \sum_i^M w_i n_i \quad (\text{A.2})$$

Using the expectance operator E , the variance of the beamformed signal S can be expressed as

$$\sigma_s^2 = E[(S - E[S])^2] \quad (\text{A.4})$$

$$= E[S^2] - (E[S])^2 \quad (\text{A.5})$$

$$= E[S^2] \quad (\text{A.6})$$

where Eq. (A.5) was further simplified to Eq. (A.6) assuming the signals received on each channel have zero mean, i.e. $E[s_i] = 0$. Substituting Eq. (A.1) into Eq. (A.6) leads to

$$\sigma_s^2 = \frac{1}{M^2} \sum_i^M \sum_j^M w_i w_j E[s_i s_j] \quad (\text{A.7})$$

Next, we assume a non-zero and constant variance for each of the received signals on every channel, i.e. $\sigma_{s_i} \equiv \sigma_s$. Eq. (A.7) can then be further simplified by considering that a correlation relation exists between signals across channels as they were already delayed properly:

$$E[s_i s_j] = \sigma_s^2, \quad \forall i \text{ and } \forall j \quad (\text{A.8})$$

The total signal variance σ_s^2 is therefore related to the individual channel variance σ_s^2 as

$$\sigma_s^2 = \frac{\sigma_s^2}{M^2} \sum_i^M \sum_j^M w_i w_j \quad (\text{A.9})$$

In a similar fashion, Eq. (A.7) can be rewritten for the total noise variance σ_N^2 as

$$\sigma_N^2 = \frac{1}{M^2} \sum_i^M \sum_j^M w_i w_j E[n_i n_j] \quad (\text{A.10})$$

where it was assumed that the individual noise channels have zero mean as well, i.e. $\mu_{n_i} = 0$. We also expect a non-zero and constant noise channel variance, i.e. $\sigma_{n_i} \equiv \sigma_n$, but we note that individual channels are uncorrelated with each other:

$$\begin{aligned} E[n_i n_j] &= \sigma_n^2, & \forall i=j \\ E[n_i n_j] &= 0, & \forall i \neq j \end{aligned}$$

Therefore Eq. (A.10) reduces to

$$\sigma_N^2 = \frac{\sigma_n^2}{M^2} \sum_i^M w_i^2 \quad (\text{A.11})$$

Combining Eq. (A.9) and Eq. (A.11) leads to the relation between total signal SNR and individual channel $\text{snr} = \frac{\sigma_s}{\sigma_n}$

$$\text{SNR} = \frac{\sigma_s}{\sigma_N} = \frac{\sqrt{\sum_i^M \sum_j^M w_i w_j}}{\sqrt{\sum_i^M w_i^2}} \cdot \text{snr} \quad (\text{A.12})$$

From this, it follows that the signal SNR without apodisation (abbreviated as SNR') is

$$\text{SNR}' = \sqrt{M} \cdot \text{snr} \quad (\text{A.13})$$

by using $w_i = w_j = 1 (\forall i \text{ and } \forall j)$ in Eq. (A.12).

Finally, without being interested in the individual channel snr, the relative reduction in total signal SNR due to apodisation is therefore given by

$$\alpha^r = \frac{\text{SNR}}{\text{SNR}'} = \frac{\sqrt{\sum_i^M \sum_j^M w_i w_j}}{\sqrt{M \sum_i^M w_i^2}} \quad (\text{A.14})$$

Appendix B. Frequency spectrum for windowed apodisation functions after heterodyning

In this appendix we seek an expression for the signal frequency spectrum $g_{SS}(f)$ in terms of the apodisation $w(\xi)$ after applying heterodyning demodulation. In short, the approach we take is to perform the different heterodyning operations described by Eqs. (10) – (15) directly in the Fourier domain, rather than first computing the resulting PSF $p_{la}(x, z)$ in the spatial domain followed by a Fourier transform to obtain the signal spectrum.

This approach will require performing partial Hilbert transform operations in the Fourier domain. They can be found by considering:

$$\mathcal{F}\{\mathcal{H}_x\{p(x, z)\}\}(u, v) = (-i \operatorname{sgn}(u)) \cdot \mathcal{F}\{p(x, z)\}(u, v) \quad (\text{B.1})$$

$$\mathcal{F}\{\mathcal{H}_z\{p(x, z)\}\}(u, v) = (-i \operatorname{sgn}(v)) \cdot \mathcal{F}\{p(x, z)\}(u, v) \quad (\text{B.2})$$

where the sign function sgn is defined as

$$\operatorname{sgn}(u) = \begin{cases} -1 & u < 0 \\ 0 & u = 0 \\ 1 & u > 0 \end{cases} \quad (\text{B.3})$$

The different heterodyning operations will now be subsequently be performed in Fourier space. Starting with Eq. (10), its Fourier transform is:

$$\begin{aligned} \mathcal{F}\{r_{\text{even}}(x, z)\}(u, v) &= \mathcal{F}\{p(x, z)\}(u, v) + i \mathcal{F}\{\mathcal{H}_z\{p(x, z)\}\} \\ &= P(u, v) \cdot (1 + \operatorname{sgn}(v)) \\ &= P(u, v) \cdot 2S(v) \end{aligned} \quad (\text{B.4})$$

where $S(v)$ is the Heaviside (step) function and is defined as

$$S(v) = \begin{cases} 0 & v < 0 \\ 1/2 & v = 0 \\ 1 & v > 0 \end{cases} \quad (\text{B.5})$$

Similarly, the Fourier transform of r_{odd} defined in Eq. (11) is

$$\begin{aligned} \mathcal{F}\{r_{\text{odd}}(x, z)\}(u, v) &= \mathcal{F}\{\mathcal{H}_x\{r_{\text{even}}(x, z)\}\}(u, v) \\ &= (-i \operatorname{sgn}(u)) \cdot \mathcal{F}\{r_{\text{even}}(x, y)\}(u, v) \\ &= (-i \operatorname{sgn}(u)) \cdot P(u, v) \cdot 2S(v) \end{aligned} \quad (\text{B.6})$$

Next, the Fourier transform of r_1 can now be easily found by using the results of Eqs. (B.5) and (B.6) as

$$\begin{aligned}
\mathcal{F}\{r_1(x, z)\}(u, v) &= \mathcal{F}\{r_{\text{even}}\}(u, v) + i\mathcal{F}\{r_{\text{odd}}\}(u, v) \\
&= P(u, v) \cdot 2S(v) \cdot (1 + \text{sgn}(u)) \\
&= P(u, v) \cdot 4S(v)S(u)
\end{aligned} \tag{B.7}$$

With similar reasoning, the Fourier transform of r_2 becomes

$$\mathcal{F}\{r_2(x, z)\}(u, v) = P(u, v) \cdot 4S(v)S(-u) \tag{B.8}$$

Finally, the frequency content of the lateral heterodyned signal $p_{\text{lat}}(x, z)$ follows

$$\begin{aligned}
\mathcal{F}\{p_{\text{lat}}(x, z)\}(u, v) &= \mathcal{F}\{r_1(x, z) \cdot \overline{r_2(x, z)}\}(u, v) \\
&= \mathcal{F}\{r_1(x, z)\}(u, v) \otimes_{x, z} \mathcal{F}\{r_2(x, z)\}(-u, -v) \\
&= ([P(u, v) \cdot 4S(v)S(u)] \otimes_{x, z} [P(-u, -v) \cdot 4S(-v)S(u)])
\end{aligned} \tag{B.9}$$

Next, we assume $p(x, z)$ to be even in x and z , implying

$$P(u, v) = P(-u, -v) \tag{B.10}$$

The lateral frequency spectrum was calculated for the dominant central transmit frequency by setting $v = f_0$, thereby further simplifying the convolution in Eq. (B.9) to one dimension. This leads to

$$\begin{aligned}
g_{SS}(u) &= \mathcal{F}\{p_{\text{lat}}(x)\}(u) \\
&= ([P(u, f_0) \cdot 4S(u)] \otimes_u [P(u, f_0) \cdot 4S(u)])
\end{aligned} \tag{B.11}$$

Finally, Eq. (27) in the main body of this paper is obtained after substitution of Eq. (26) in Eq. (B.11).

Fig. 1.

Background: PSF for a scatterer at 60 mm depth subjected to a 3 mm lateral motion, imaged using a phased array and beamformed using different scenarios: (a) conventional focused beamforming (FOC) with single line acquisition (SLA); plane wave (PW) insonification with a bi-lobed Gaussian receive apodisation, by either (b) using a single PW, (c) coherently summing PWs emitted under different angles or (d) using an SLA sequence producing transverse oscillations (PWTO); (e) same as in (d) but with heterodyning demodulation (PWTO*). Note that RF images are shown in polar space.

Fig. 2.

Background: Interference pattern of two PSFs subjected to lateral motion (second row) or rotation (third row), showing differences when imaged with a linear array or a phased array. The scanning sequence was either a conventional focused imaging sequence (FOC) or with multiple plane waves appropriately apodised on receive to produce transverse oscillations (PWTO). Note that RF images for the linear and phase array are shown in Cartesian and polar space respectively.

Fig. 3.

Example showing the different variables contributing to the transverse tracking performance predicted by the CRLB for a phantom subjected to lateral motion: (a) correlation ρ at 60 mm depth, (b) relative SNR reduction due to receive apodisation, and (c) the signal spectrum g_{ss} . Performance (d) is compared between the FOC, PWTO and PWTO* imaging sequence, with the resulting B-mode images shown on the top. Colored rectangles indicate the windows where the correlation curves were computed.

Fig. 4.

Range of investigated bi-lobed apodisation functions in terms of λ_θ and σ_θ in Eq. (8). Those apodisation functions that lie within 90% of the aperture have a shaded background. Note that the axes are not linear.

Fig. 5.
Optimal TO parameters when either 90% or 50% of the bi-lobed Gaussians is inside the aperture.

Fig. 6.
Comparison of the tracking performance of the two apodisation scenarios with optimized TO parameters.

Fig. 7.

Influence of SNR on performance in case of lateral tissue motion: PWTO90* vs FOC, and PWTO50* vs FOC. Note that for the jitter-related plots, only the beamformer able to achieve the lowest jitter at any given displacement/SNR combination is shown. Also note that the decorrelation scale is much smaller compared to Fig. 6.

Fig. 8.

Influence of SNR on performance in case of axial tissue motion: PWTO90* vs FOC, and PWTO50* vs FOC. Note that for the jitter-related plots, only the beamformer able to achieve the lowest jitter at any given displacement/SNR combination is shown. Also note that the decorrelation scale is much smaller compared to Fig. 6.

Fig. 9.

Influence of SNR on performance in case of rotational tissue motion: PWTO90* vs FOC, and PWTO50* vs FOC. Note that for the jitter-related plots, only the beamformer able to achieve the lowest jitter at any given displacement/SNR combination is shown.

Fig. 10.

Preferred beamformer according to the CRLB when performing cardiac deformation imaging in a 4 chamber view (imaged up to 10 cm and with 3 beams per degree), when the maximum lateral velocity is (a) 8.6 cm/s or (b) 4.32 cm/s, corresponding to the maximum reached during diastole and systole respectively [41]. Note that the color-coded jitter error for any given opening angle/SNR combination is shown only for the preferred beamformer. The favored operation areas for the PWTO50* and FOC are separated by the curved black line.

TABLE I

List of abbreviations

BA	Bilobed Gaussian Apodisation
CRLB	Cramer-Rao Lower Bound
FOC	Focused (beamforming)
PSF	Point Spread Function
PW	Plane Waves
PWTO	Plane Wave Transverse Oscillation (beamforming)
→PWTO*	→followed by heterodyning
→PWTO90	→with >90% or >50% of the receive apodisation
PWTO50	falling inside the aperture
→PWTO90*	→with >90% or >50% of the receive apodisation
PWTO50*	falling inside the aperture, followed by heterodyning
RF	Radio-Frequency (image)
SLA	Single Line Acquisition
SNR	Signal-to-Noise Ratio
SQ	Spatial Quadrature (signals)
TO	Transverse Oscillations
US	Ultrasound

TABLE II

Overview of the performed simulations and the associated settings

Experiment	Figure	Displacement		Beamforming	SNR [dB]	TO params [deg]	
		Type	Magnitude			λ_θ	σ_θ
V-A. Sample CRLB curves	3	Lateral [μm]	0–1000 [100]	FOC, PWTO, PWTO*	20	5.75	10.5
V-B. Optimizing TO params	4–5	Lateral [μm]	200	FOC, PWTO, PWTO*	20	3–12 [0.25]	0.5–20 [0.5]
Optimum classical TO	6	Lateral [μm]	0–1000 [100]	FOC, PWTO90, PWTO90*	20	5.25	12
Optimum windowed TO	6	Lateral [μm]	0–1000 [100]	FOC, PWTO50, PWTO50*	20	3	3
V-C. Influence SNR	7	Lateral [μm]	0–1000 [100]	FOC, PWTO90*, PWTO50*	10–30 [5]	5.25 ⁽⁹⁰⁾ 3 ⁽⁵⁰⁾	12 ⁽⁹⁰⁾ 3 ⁽⁵⁰⁾
V-D. Influence displ. type	8	Axial [μm]	0–1000 [100]	FOC, PWTO90*, PWTO50*	10–30 [5]	5.25 ⁽⁹⁰⁾ 3 ⁽⁵⁰⁾	12 ⁽⁹⁰⁾ 3 ⁽⁵⁰⁾
	9	Rotation [deg]	0–6 [0.5]	FOC, PWTO90*, PWTO50*	10–30 [5]	5.25 ⁽⁹⁰⁾ 3 ⁽⁵⁰⁾	12 ⁽⁹⁰⁾ 3 ⁽⁵⁰⁾
V-E. Clinical implications	10	Lateral	See ref. [41]	FOC, PWTO50*	10–30	3	3

For those parameter settings that were varied over a certain range, a step size is given between squared brackets. Superscripts (90) and (50) refer to the PWTO90 and PWTO50 beamforming sequences respectively.