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Axial Cone Beam Reconstruction by Weighted BPF/DBPF and Orthogonal Butterfly Filtering

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Abstract

Goal—The backprojection-filtration (BPF) and the derivative backprojection filtered (DBPF) algorithms, in which Hilbert filtering is the common algorithmic feature, are originally derived for exact helical reconstruction from cone beam (CB) scan data and axial reconstruction from fan beam data, respectively. These two algorithms can be heuristically extended for image reconstruction from axial CB scan data, but induce severe artifacts in images located away from the central plane determined by the circular source trajectory. We propose an algorithmic solution herein to eliminate the artifacts.

Methods—The solution is an integration of three-dimensional (3D) weighted axial CB-BPF/DBPF algorithm with orthogonal butterfly filtering, namely axial CB-BPF/DBPF cascaded with orthogonal butterfly filtering. Using the computer simulated Forbild head and thoracic phantoms that are rigorous in inspecting reconstruction accuracy and an anthropomorphic thoracic phantom with projection data acquired by a CT scanner, we evaluate performance of the proposed algorithm.

Results—Preliminary results show that the orthogonal butterfly filtering can eliminate the severe streak artifacts existing in the images reconstructed by the 3D weighted axial CB-BPF/DBPF algorithm located at off-central planes.

Conclusion—Integrated with orthogonal butterfly filtering, the 3D weighted CB-BPF/DBPF algorithm can perform at least as well as the 3D weighted CB-FBP algorithm in image reconstruction from axial CB scan data.

Significance—The proposed 3D weighted axial CB-BPF/DBPF cascaded with orthogonal butterfly filtering can be an algorithmic solution for CT imaging in extensive clinical and preclinical applications.

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Keywords

Backprojected-filtration; CT; Derivative backprojection; Filtered backprojection; Reconstruction

I. Introduction

THE algorithms for approximate image reconstruction from cone beam (CB) projection data acquired along a circular trajectory (namely axial CB scan) have been extensively employed in x-ray CT for medical and industrial applications [1, 2]. However, since the circular trajectory does not satisfy the data sufficiency condition (DSC) [3], CB artifact appears and deteriorates with increasing cone angle. Though the reconstruction algorithms associated with helical/spiral [4-9] or ‘circle plus’ [10-11] trajectories have been proposed in the literature to meet the DSC, axial CB scan is still the most desirable for physiological and interventional imaging in the clinic, which drives the community of CT image reconstruction to explore solutions for reducing, if not eliminating, the CB artifacts existing in the images reconstructed from the projection data acquired in axial CB scan, especially when the cone angle becomes relatively large [12-19].

The algorithm derived by Katsevich in [6] (namely, the Katsevich algorithm) was a breakthrough in pursuing theoretically exact algorithms for image reconstruction from the CB projection data acquired along a helical/spiral source trajectory (namely helical/spiral CB scan). Another milestone was made by Pan and Zou in their so-called backprojection-filtration (BPF) algorithm [7], in which, based on the concepts of PI-line and PI-line segment, an exact reconstruction algorithm on the support of PI-line segments was derived. Compared to the Katsevich algorithm, the BPF algorithm made at least three conceptual impacts on exact image reconstruction in helical/spiral CB scan: (*i*) filtering in an analytic image reconstruction can be reordered to even post, i.e., after, the backprojection; (*ii*) filtering can be carried out along PI-lines, rather than the curves defined in the detector that records the projection data; (*iii*) the helical/spiral projection data needed by the exact reconstruction on a PI-line segment is minimal. By adopting the concepts of virtual axial source trajectory and virtual PI-line segment, the BPF algorithm was heuristically extended for reconstruction in a region of interest (ROI) [20] from axial CB scan data. Such an extension enables the BPF algorithm to perform comparably with the existing FDK algorithm, though they behave differently in certain aspects, e.g., the intensity dropping along the longitudinal direction [20].

The so-called two-step Hilbert transform method, which was later named as the derivative backprojection filtered (DBPF) algorithm, was derived by Noo *et al* in [21]. Compared to the BPF algorithm, the DBPF algorithm brings about at least one more conceptual impact: the freedom in choosing the direction along which the post-backprojection (post-BP) Hilbert filtering is carried out, which leads to the success of dealing with latitudinal data truncation in practical situations wherein data truncation is unavoidable. Later on, via a factorization to build a family of plane of interest (POI), the DBPF algorithm was extended to reconstruct volume of interest (VOI) from the projection data acquired in an axial CB scan [22]. The extended DBPF algorithm is based on the fundamental relationship, which is an integral

equation, between an intermediate function and the density function of an object to be reconstructed. However, there exists no analytic solution for such an integral equation. By formulating it as optimization of a constrained least-square with regularization, a VOI supported by a family of POI can be reconstructed numerically [22], using an iterative algorithm in the fashion of steepest descending.

It has been shown in [22, 23] that certain artifacts appear in images reconstructed by the extended BPF algorithm [20], especially at the locations where the object to be reconstructed varies abruptly. By averaging multiple reconstructed images corresponding to various Hilbert filtering directions, a heuristic approach is proposed in [23] to reduce the orientation-specific artifacts. The extended DBPF algorithm proposed in [22] is a solution that is more theoretical sounding and thus of better performance in suppressing the artifacts, since the derived integral equation is a more rational mathematical treatment. However, implementation of the extended DBPF algorithm is significantly more complicated than the original one, because, as indicated in [22], the extended DBPF algorithm is actually in the fashion of iterative reconstruction. In comparison to the original one, the extended DBPF algorithm is substantially less efficient in computation, though the needed number of iterations is relatively small, e.g., 10 [22]. Moreover, extra effort in parameter tweaking and optimization is needed in iterative image reconstruction to assure that the algorithm's behavior is robust over applications. Though tremendous investment has been made by our CT image reconstruction community in the development of iterative image reconstruction algorithms, we believe that CT image reconstruction algorithms in the fashion of FBP and/or BPF/DBPF are still of relevance, mainly because they are appealing in the robustness of their performance over applications and relative ease in their implementation. Hence, in this paper, we present a novel algorithm for image reconstruction from projection data acquired in axial CB scan, with motivation to avoid the algorithmic architecture of iteration and accordingly save the effort in computation.

Recently, the so-called butterfly filtering has been proposed by Pack *et al* to reduce the artifacts caused by data null and redundancy in a partial axial CB scan of 270° scan range [24]. With resort to a pair of orthogonal butterfly filters in spatial-frequency domain (namely orthogonal butterfly filter henceforth), the artifacts caused by data null and redundancy can be mitigated to an unnoticeable level. Recognizing the fact that the artifacts existing in the extended BPF/DBPF algorithm are orientation-specific [20, 23, 25-26] and the orthogonal butterfly filtering is good at suppressing orientation-specific artifacts [24], we propose to cascade the extended BPF/DBPF algorithm with the orthogonal butterfly filtering for image reconstruction from axial CB scan data (namely axial CB-BPP/DBPF cascaded with orthogonal butterfly filtering). Using the Forbild head and thorax phantoms [27-29] that are rigorous in inspecting reconstruction accuracy and an anthropomorphic thoracic phantom with projection data acquired by a CT scanner, we evaluate and verify performance of the proposed algorithm. Below is a concise presentation of the proposed algorithm, followed by the preliminary results of performance evaluation and verification.

II. Theoretical Background

The derivation of BPF and DBPF algorithms starts at different points, but ends up with a common algorithmic architecture. The reader who wants to have an in-depth understanding of the BPF/DBPF algorithms is referred to [7, 21]. In this section, for simplicity in expression, we present the proposed algorithm of axial BPF/DBPF cascaded with orthogonal butterfly filtering in the framework under which the DBPF is derived.

A. CB Geometries for Axial Scan and Reconstruction

The native CB geometry is shown in Fig. 1 (a), where O -xyz denotes the coordinate system. S is the focal spot of x-ray source, and $P(x, y, z)$ a point within the object to be imaged. The ray emanating from S and passing $P(x, y, z)$ is determined by its view angle β , fan angle γ , and cone angle α . In the native CB geometry, the circular trajectory is mathematically expressed as

$$ST(\beta) = (R \sin \beta, R \cos \beta, 0), \beta \in (\beta_{\min}, \beta_{\max}), \quad (1)$$

where $\beta_{\min}$ and $\beta_{\max}$ denote the start and end of source trajectory.

It has been recognized that a CB reconstruction implemented in the so-called cone-parallel geometry generates more uniform noise in CT images [12]. Hence, the proposed algorithm is presented in the cone-parallel geometry. Via a row-wise fan-to-parallel rebinning, the native CB geometry is converted into the cone-parallel geometry, as illustrated in Fig. 1 (b). Then, the ray emanating from focal spot S and passing through $P(x, y, z)$ is uniquely determined by its view angle β , distance t from the axis of rotation (AOR), and cone angle α .

B. Axial BPF/DBPF Algorithm in Cone-parallel Geometry

Let's define central plane (the plane in solid line in Fig. 2) as the plane determined by the circular source trajectory in an axial CB scan. In a way similar to that in [20], we define a number of virtual planes above and underneath the central plane (Fig. 2). If we denote an image plane by its z-location Z_0 , then $Z_0 = 0$ and $Z_0 \neq 0$ correspond to the central plane and a virtual plane, respectively. Let $s(\alpha, \beta, t; Z_0)$ be the projection of the object to be imaged at the plane denoted by Z_0 . If we further define

$$\tilde{s}(\alpha, \beta, t; Z_0) = \frac{\partial}{\partial t} s(\alpha, \beta, t; Z_0), \quad (2)$$

then, according to [7, 9], we have

$$-2\pi \mathbf{H}_\theta f(x, y, Z_0) \approx b_\theta(x, y, Z_0) = \int_{-\beta_m}^{\beta_m} \cos \alpha \operatorname{sgn}(\sin(\beta - \theta)) \tilde{s}(\alpha, \beta, t; Z_0) d\beta, \quad (3)$$

where $\mathbf{H}_\theta f(x, y, Z_0)$ denotes the Hilbert transform of object function $f(x, y, Z_0)$ along direction $(-\sin\theta, \cos\theta, 0)$ [7, 21]. $b_\theta(x, y, Z_0)$ is obtained using the algorithmic steps that are identical to those in [7, 21]. An exact reconstruction of the object function $f(x, y, Z_0)$ is obtained at the central plane ($Z_0 = 0$), while approximate reconstruction is obtained at virtual plane ($Z_0 \neq 0$).

C. 3D Weighted Axial BFP/DBPF Algorithm

With (3) in hand, we are ready to introduce the 3D weighted axial BFP/DBPF algorithm as

$$-2\pi\mathbf{H}_\theta f(x, y, Z_0) \approx b_\theta(x, y, Z_0) = \int_{-\beta_m}^{\beta_m} \cos(\alpha w_{3d}(\alpha, \beta, t; Z_0) \operatorname{sgn}(\sin(\beta - \theta))) \tilde{s}(\alpha, \beta, t; Z_0) d\beta. \quad (4)$$

$w_{3d}(\alpha, \beta, t; Z_0)$ is a voxel-wise 3D weighting function and satisfies the normalization condition [9, 12]

$$w_{3d}(\alpha, \beta, t; Z_0) + w_{3d}(\alpha_c, \beta_c, t_c; Z_0) = 1, \quad (5)$$

where $(\alpha, \beta, t; Z_0)$ and $(\alpha_c, \beta_c, t_c; Z_0)$ denote a pair of conjugate rays in the cone-parallel geometry.

D. Orthogonal Butterfly Filtering

Pack *et al* proposed the approach of orthogonal butterfly filtering to deal with the null and redundancy of projection data in spatial-frequency domain in the case of a partial $[0^\circ, 270^\circ)$ axial CB scan [24]. Presented in Fig. 3 is an example of the orthogonal butterfly filter pair. It should not be hard for us to understand that all vertical features in an image can be removed if the horizontal butterfly filter W_h is applied, while all horizontal features can be removed if the vertical butterfly filter W_v is applied. The pair of orthogonal butterfly filters is analytically defined as,

$$W_h(r_\rho, \theta_\rho) = \frac{1}{2} \begin{cases} 1 - 0.1^{\tan(|\theta_\rho - \pi/4|)}, 0 \leq \theta_\rho < \pi/4, \\ 0.1^{\tan(|\theta_\rho - \pi/4|)}, \pi/4 \leq \theta_\rho < \pi/2, \\ 0.1^{\tan(|\theta_\rho - 3\pi/4|)}, \pi/2 \leq \theta_\rho < 3\pi/4, \\ 1 - 0.1^{\tan(|\theta_\rho - 3\pi/4|)}, 3\pi/4 \leq \theta_\rho < \pi, \\ 1 - 0.1^{\tan(|\theta_\rho - 5\pi/4|)}, \pi \leq \theta_\rho < 5\pi/4, \\ 0.1^{\tan(|\theta_\rho - 5\pi/4|)}, 5\pi/4 \leq \theta_\rho < 3\pi/2, \\ 0.1^{\tan(|\theta_\rho - 7\pi/4|)}, 3\pi/2 \leq \theta_\rho < 7\pi/4, \\ 1 - 0.1^{\tan(|\theta_\rho - 7\pi/4|)}, 7\pi/4 \leq \theta_\rho < 2\pi, \end{cases} \quad (6)$$

$$W_v(r_\rho, \theta_\rho) = 1 - W_h(r_\rho, \theta_\rho),$$

where (r_ρ, θ_ρ) is the polar coordinate in the 2D spatial-frequency domain.

E. Axial CB-BPF/DBPF Cascaded with Orthogonal Butterfly Filtering

With each algorithmic component presented above, we are ready to introduce the algorithm of axial CB-BPF/DBPF cascaded with orthogonal butterfly filtering for image reconstruction in axial CB scans.

Given an image to be reconstructed in either the central or virtual plane, shown in Fig. 4 are the algorithmic steps after the data acquired in the native CB geometry is converted into the cone-parallel geometry via row-wise fan-to-parallel rebinning [12, 25]. When the projection data are available in a full axial CB scan (0° – 360°), both horizontal and vertical Hilbert transforms (namely HHT and VHT, respectively) are applied on the intermediate images, i.e., the left side of (3), followed by vertical and horizontal butterfly filtering correspondingly (Fig. 4 (a)). In the case of partial axial CB scan (0° – 270°), the projection data are grouped into two sets corresponding to 0° – 180° and 90° – 270° angular ranges, respectively (Fig. 4 (b)). The image reconstructed from the 1st group of data with HHT goes through vertical butterfly filtering, while that from the 2nd group of data with VHT goes through horizontal butterfly filtering. Subsequently, the reconstructed two sub-images are pixel-wise added to form the final image, as specified in the last step of the algorithmic flowchart (Fig. 4 (a) and (b)). The formation of final image can be analytically defined as

$$f(r_\rho, \theta_\rho, Z_0) = (f_h(x, y, Z_0) + f_v(x, y, Z_0)) / 2 \quad (7)$$

$$f_h(r_\rho, \theta_\rho, Z_0) = \frac{1}{2\pi} BF_{0^\circ} b_{90^\circ}(x, y, Z_0) \quad (8)$$

$$f_v(r_\rho, \theta_\rho, Z_0) = \frac{1}{2\pi} BF_{90^\circ} b_{0^\circ}(x, y, Z_0), \quad (9)$$

where $b_{0^\circ}(x, y, Z_0)$ and $b_{90^\circ}(x, y, Z_0)$ are the BPF/DBPF reconstructions corresponding to the Hilbert transform carried out along 0° and 90° directions, respectively, and BF_{0° and BF_{90° represent the Butterfly filtering carried out along 0° and 90° . Two points should be made here: (i) the 3D backprojection is carried out in the cone-parallel geometry, and (ii) the rebinned projection data in the angular range 90° – 180° are utilized twice.

III. Numerical Evaluation

A. Parameters of CT System, Scan and 3D Reconstruction

The proposed algorithm of axial CB-BPF/DBPF cascaded with orthogonal butterfly filtering is initially evaluated using the Forbild head and thorax phantoms [27-29] simulated in an axial scan under the native CB geometry and rebinned into the cone-parallel geometry [12, 25]. In simulation, the distance from x-ray source to the axis of rotation (AOR) of CT gantry is assumed as 541.0 mm. The detector z-dimension is assumed 64×0.625 mm and 128×0.625 mm at AOR, respectively, and correspondingly, the center of Forbild head phantom is

located at (0, 0, 20) mm and (0, 0, 40) mm. Noise is simulated corresponding to 4×10^8 photons/cm² in Poisson distribution for each x-ray emanating from the source. 1,160 projection views are acquired along the angular range [0°, 360°). The image matrix is 512×512 , field of view (FOV) 256 mm $\times$ 256 mm, and voxel size 0.5 mm $\times$ 0.5 mm $\times$ 0.625 mm. The performance of the proposed algorithm is also evaluated using projection data of an anthropomorphic thoracic phantom acquired by a CT scanner. The geometry of the CT scanner is the same as that used in the simulation, but only a total of 984 projection views are acquired along the angular range [0°, 360°). The image matrix is also 512×512 , but FOV 512 mm $\times$ 512 mm, and voxel size 1.0 mm $\times$ 1.0 mm $\times$ 0.625 mm.

B. Scan Range and post-BP Hilbert Filtering Orientation

Both full [0°, 360°) and partial [0°, 270°) axial CB scan data are employed to evaluate the proposed algorithm of axial CB-BPF/DBPF cascaded with orthogonal butterfly filtering. Since a full scan corresponds to angular range [0°, 360°), there is no limitation on the orientation along which the post-BP Hilbert filtering is carried out [7, 21]. We carry out the post-BP Hilbert transform and orthogonal butterfly filtering along 0° (horizontal in reconstructed image) and 90° (vertical) directions, respectively. In the case of partial axial CB scan, the post-BP Hilbert transform can only be implemented along the 0° (horizontal) and 90° (vertical) directions, respectively, followed by the orthogonal butterfly filtering correspondingly.

IV. Evaluation Results

In the performance evaluation and verification of the proposed algorithm, the outermost slice is selected as the reconstruction plane. The performance evaluation by simulated data is initially carried out using 64-detector row axial scan data, followed by 128-detector row data. Then, the algorithm's performance is verified using 64-detector row data acquired by a CT scanner. Presented in Fig. 5 (a-a') and (b-b') are images reconstructed by the proposed algorithm from the projection data acquired in a full axial CB scan (64-detector row) with the Hilbert transform carried out along 0° and 90°, respectively. Orthogonal butterfly filtering is applied correspondingly on the images shown in Fig. 5 (a-a') and (b-b'), and the output images are presented in Fig. 5 (c-c'). It is clearly observed that severe horizontal and vertical streak artifacts existing in the images reconstructed by the axial CB-BPF/DBPF algorithm can be eliminated by the cascaded orthogonal butterfly filtering. The layout of Fig. 6 is identical to that in Fig. 5, and the only difference between the corresponding images is that the images in Fig. 6 are reconstructed by the proposed algorithm with 3D weighting. To highlight the efficacy of the 3D weighting scheme in the proposed algorithm, the difference between the images reconstructed without 3D weighting (Fig. 5 (c-c')) and those with 3D weighting (Fig. 6 (c-c')) is presented in Fig. 7 (a) and (b), respectively, which demonstrates that the 3D weighting indeed reduces CB artifacts significantly.

The images corresponding to the cases of 128-detector row are presented in Figs. 8 – 10, respectively. Note that, though the cone angle is virtually doubled, the proposed axial CB-BPF/DBPF algorithm cascaded with orthogonal butterfly filtering still perform very well in

artifact suppression, in comparison to the results corresponding to 64-detector row data acquisition that are shown in Figs. 5 – 7.

The images reconstructed by the proposed axial CB-BPF/DBPF algorithm corresponding to angular scan ranges 0° – 180° and 90° – 270° are shown in Fig. 11 (a-a') and (b-b'), respectively. It is obvious that severe horizontal and vertical streak artifacts exist but can be eliminated by the proposed orthogonal butterfly filtering, as demonstrated in Fig. 11 (c-c'). It should be noted that no 3D weighting is applied in the reconstruction of getting these two images, as virtually no redundant data is available within each angular scan range. As a result, CB artifacts exist in the images shown in Fig. 11 (c-c') (artifacts surrounding the high attenuating components). The results corresponding to the cases of 128-detector row are presented in Fig. 12. It is noted that, at almost doubled cone angle, the proposed orthogonal butterfly filtering still perform very well in eliminating the artifacts caused by Hilbert filtering. Again, as virtually no redundant projection data is available and thus 3D weighting cannot be applied, CB artifacts remain in the images displayed in Fig. 12 (c-c').

The results of performance evaluation by the anthropomorphic thoracic phantom with projection data acquired by a 64-detector row CT scanner are presented in Figs. 13 – 15. By comparing Fig. 13 (b) and Fig. 13 (d), it is observed that, with 3D weighting and orthogonal butterfly filtering, the algorithm proposed in this work performs as well as the 3D weighted axial CB-FBP algorithm [12] in the case of full scan ($[0^{\circ}, 360^{\circ})$). In the case of partial scan ($[0^{\circ}, 270^{\circ})$), as almost no redundant data are available for applying 3D weighting, CB artifacts remain (comparing Fig. 14 (c) with Fig. 14 (d)). The results presented in Fig. 15 are similar to those in Fig. 14, except for the difference in their z-positions (17 mm vs. 8.5 mm), which shows the performance of the proposed algorithm in removing streak artifacts as image plane's z-position varies.

V. Discussions

A 3D weighted axial CB-BPF/DBPF algorithm cascaded with orthogonal butterfly filtering is proposed in this paper for CB image reconstruction from axial CB scan data. The 3D weighting has been previously proposed in [12, 25] for the CB-BPF/DBPF to suppress CB artifacts, while the orthogonal butterfly filtering proposed in [24] is utilized to suppress the severe streak artifacts caused by the post-BP Hilbert filtering.

Since the BPF/DBPF algorithm for image reconstruction in CB axial scans is theoretically approximate, the performance of the proposed algorithm needs to be investigated thoroughly. In this work, we conduct an experimental performance evaluation using the data generated by simulation and acquired by a CT scanner. The preliminary results show that the integration of 3D weighting cascaded with the orthogonal butterfly filtering in the CB-BPF/DBPF algorithm performs well as anticipated, enabling the proposed algorithm for axial CB scan wherein the cone angle is relatively large.

We used to speculate that the occurrence of severe streak artifacts due to the post-BP Hilbert filtering in the 3D weighted CB-BPF/DBPF algorithm [25] might be caused by a corruption by the residual CB artifacts that cannot be removed by the 3D weighting. It is our current

understanding that the BPF/DBPF algorithm was originally derived for exact helical reconstruction on PI-lines or chords and exact axial reconstruction from fan beam scan data, respectively [7, 21, 22]. In axial CB scan, once an image plane is away from the central plane determined by the x-ray source trajectory, the BPF/DBPF algorithm is no longer accurate. Fortunately, the reconstruction inaccuracy mainly manifests itself as streak artifacts aligned with the orientation along which the Hilbert filtering is carried out. Thus, the orthogonal butterfly filtering that is orientation-specific can be utilized to remove the artifacts effectively and efficiently [24].

It is very interesting to note that, as demonstrated in Fig. 16(a), in the image reconstructed by the 3D weighted axial CB-FBP algorithm, the lateral (horizontal) truncation causes severe artifacts at the edges where the truncation occurs, which encroaches on the surrounding areas. However, in the image (Fig. 16(b)) reconstructed by the proposed CB-BPF/DBPF algorithm with the orthogonal butterfly filtering carried out along the 45° and 135° , respectively, the artifacts at the edges of data truncation are significantly milder and do not encroach on surrounding areas. This means that the flexibility of the original BPF/DBPF algorithm in dealing with the issue of data truncation is maintained, or at least partially maintained. It is our speculation that this is mainly because the ramp filtering in the 3D weighted CB-FBP algorithm (or any FBP-based image reconstruction algorithm) is carried out in every projection view, which always corresponds to the worst case in truncation, whereas the post-BP orthogonal butterfly filtering avoids the worst scenario by carrying out along appropriate directions, e.g., along the 45° and 135° directions in the case of horizontal truncation as illustrated in Fig. 16(b). We believe that this is a very important advantage for the proposed 3D weighted axial CB-BPF/DBPF algorithm to reconstruct a 3D region of interest (ROI), and its potential and property definitely deserve a further and thorough experimental investigation in the near future.

As demonstrated in the section of Evaluation Results, the proposed 3D weighted axial CB-BPF/DBPF reconstruction algorithm cascaded with orthogonal butterfly filtering works well in the cases of both full scan ($[0^\circ, 360^\circ]$) and partial scan ($[0^\circ, 270^\circ]$). In principle, a partial scan ($[0^\circ, 270^\circ]$) may be able to offer better temporal resolution than a full scan ($[0^\circ, 360^\circ]$). However, as almost no redundant data exists in a partial scan ($[0^\circ, 270^\circ]$), the 3D weighting cannot be applied in image reconstruction. Accordingly, CB artifacts remain in the reconstructed images (see Fig. 11 (c-c'), Fig. 12 (c-c') and Fig. 14 (c)), even though the severe streak artifacts caused by the post-BP Hilbert filtering can be eliminated by the orthogonal butterfly filtering.

As indicated in the Introduction, one of the major motivations to present the 3D weighted axial CB-BPF/DBPF reconstruction algorithm in this paper is to avoid the algorithmic architecture of iteration [22] and accordingly save the effort in computation. We recognize the success of the iterative image reconstruction algorithm presented in [22] in artifact suppression. Meanwhile, we believe that the orthogonal butterfly filtering proposed in our solution should be of similar performance in suppressing artifacts, compared to the algorithm in [22]. However, mainly due to the dramatic difference in the algorithmic framework, i.e., analytic vs. iterative, a direct and thorough comparison between our solution and the algorithm proposed in [22] is beyond the scope of this paper.

It is believed that the proposed 3D weighted axial CB-BPF/DBPF algorithm cascaded with orthogonal butterfly filtering is novel from the standpoint of reconstruction algorithm development, though its extension from the original BPF/DBPF algorithm [7, 21] to the algorithm proposed in this work is quite heuristic. Under the BPF algorithmic framework, an axial CB-BPF reconstruction algorithm has been proposed in the literature [20], in which the derivation of reconstruction formulae is based on the native CB geometry. The 3D weighting and orthogonal butterfly filtering presented in this paper are also applicable to the CB-BPF algorithm published in [20], i.e., in the case without converting the native CB geometry into the cone-parallel geometry.

Further work to investigate the proposed algorithm's robustness over noise and larger cone angle, and especially, its capability of dealing with data truncation for region of interest reconstruction, is underway and the results will be promptly reported in our future publications.

VI. Conclusions

We propose a 3D weighted axial CB-BPF/DBPF algorithm cascaded with orthogonal butterfly filtering for reconstructing images from axial CB scan data. The preliminary results show that the integration of orthogonal butterfly filtering in the 3D weighted axial CB-BPF/DBPF algorithm can generate images at excellent image quality, enabling the proposed algorithm for extensive clinical and preclinical applications wherein axial image reconstruction at relatively large cone angle is needed.

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Fig. 1. Schematic showing the: (a) native CB and (b) cone-parallel geometries.

Fig. 2.
The schematic showing the central plane and virtual planes in the cone-parallel geometry.

Fig. 3. Orthogonal butterfly filters presented in the spatial-frequency domain: (left) horizontal butterfly filter; (right) vertical butterfly filter.

Fig. 4.

Flow chart showing the algorithmic architecture of proposed algorithm implemented in cone-parallel geometry: (a) Full scan ($[0, 360^\circ]$), and (b) Partial scan ($[0, 270^\circ]$) (HHT/VHT: Horizontal/Vertical Hilbert Transform).

Fig. 5. Images of the Forbield head phantom ($z = 20$ mm, outmost slice) reconstructed from full scan ($[0^\circ, 360^\circ)$) 64-row data by the axial CB-BPF/DBPF algorithm (no 3D weighting) with horizontal HT (a, a'), vertical HT (b, b'), and their addition after orthogonal butterfly filtering (c, c') (top: noiseless; bottom: with noise; display: $[-20\ 150]$ HU).

Fig. 6. Images of the Forbield head phantom ($z = 20$ mm, outmost slice) reconstructed from full scan ($[0^\circ, 360^\circ]$) 64-row data by the axial CB-BPF/DBPF algorithm (3D weighted) with horizontal HT (a, a'), vertical HT (b, b'), and their addition after orthogonal butterfly filtering (c, c') (top: noiseless; bottom: with noise; display: $[-20 \ 150]$ HU).

Fig. 7.
The difference between the images presented in Fig. 5 (c) and Fig. 6 (c) (left) and Fig. 5 (c') and Fig. 6 (c') (right) (display: $[-55\ 55]$ HU).

Fig. 8. Images of the Forbild head phantom ($z = 40$ mm, outmost slice) reconstructed from full scan ($[0^\circ, 360^\circ]$) 128-row data by the proposed algorithm (no 3D weighting) with HHT (a, a'), VHT (b, b'), and their addition (c, c') (top: noiseless; bottom: with noise; display: $[-20\ 150]$ HU).

Fig. 9.

Images of the Forbild head phantom ($z = 40$ mm, outmost slice) reconstructed from full scan ($[0^\circ, 360^\circ]$) 128-row data by the proposed algorithm (3D weighted) with HHT (a, a'), VHT (b, b'), and their addition (c, c') (top: noiseless; bottom: with noise; display: $[-20\ 150]$ HU).

Fig. 10. Difference between the images in Fig. 8 (c) and Fig. 9 (c) (left) and Fig. 8 (c') and Fig. 9 (c') (right) (display: [-55 55] HU).

Fig. 11.

Images of the Forbild head phantom ($z = 20$ mm, outmost slice) reconstructed from data (64-row) in angular scan ranges 0° – 180° and 90° – 270° (b) by the proposed algorithm with HHT (a, a'), VHT (b, b'), and by the proposed algorithm without 3D weighting (c, c') (top row: without noise; bottom row: with noise; display: $[-20\ 150]$ HU).

Fig. 12.

Images of the Forbild head phantom ($z = 40$ mm, outmost slice) reconstructed from data (128-row) in angular scan ranges 0° – 180° and 90° – 270° (b) by the proposed algorithm with HHT (a, a'), VHT (b, b'), and by the proposed algorithm without 3D weighting (c, c') (top row: without noise; bottom row: with noise; display: $[-20\ 150]$ HU).

Fig. 13.

Images of the anthropomorphic thoracic phantom ($z = 17$ mm) reconstructed from full-scan data by the axial CB-FBP algorithm without (a) and with (b) 3D weighting, and the axial CB-BPF/DBPF algorithm cascaded with orthogonal butterfly filtering without (c) and with (d) 3D weighting (display: $[-100\ 100]$ HU).

Fig. 14.

Images of the anthropomorphic thoracic phantom ($z = 17$ mm) obtained by the proposed algorithm from data (64-row) acquired in angular range $[0^\circ, 180^\circ)$ (a) and $[90^\circ, 270^\circ)$ (b), corresponding to Hilbert filtering carried out at 0° and 90° , respectively, and their addition (c). The image in (d) is replication of Fig. 13 (d) for its comparison with (c) (display: $[-100, 100]$ HU).

Fig. 15.

Images of the anthropomorphic thoracic phantom ($z = 8.5$ mm) obtained by the proposed algorithm from data (64-row) acquired in angular range $[0^\circ, 180^\circ)$ (a) and $[90^\circ, 270^\circ)$ (b), corresponding to Hilbert filtering carried out at 0° and 90° , respectively, and their addition (c). The image in (d) is obtained under the same condition as that of Fig. 13 (d) for comparison with (c) (display: $[-100\ 100]$ HU).

Fig. 16.

Images of the Forbild thorax phantom ($z = 0$ mm, central slice) reconstructed from full scan ($[0^\circ, 360^\circ]$) 64-row truncated noisy data by the 3D weighted axial CB-FBP (left) and the proposed 3D weighted axial CB-BPF/DBPF algorithm with the orthogonal Butterfly filtering carried out along 45° and 135° , respectively (right) (display: $[0\ 200]$ HU).