Published in final edited form as:

Med Phys. 2007 November ; 34(11): 4281–4284. doi:10.1118/1.2794171.

Uniform attenuation correction using the frequency-distance principle

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Abstract

The frequency-distance principle (FDP) is a well-known relationship that relates the distance between the object and the detector to the slope in the two-dimensional Fourier transform of the projection sinogram. This relationship has been previously applied to compensation of the distance dependent collimator blurring in SPECT (single photon emission computed tomography) in the literature. This paper makes an attempt to use the FDP to correct for uniform attenuation in SPECT. Computer simulations reveal that this technique works well for objects consisting of point sources but does not work well for distributed objects.

Keywords

attenuation correction; SPECT; medical imaging; Fourier transform

I. INTRODUCTION

The frequency-distance principle (FDP) was first discovered by Edholm, Lewitt, and Lindholm¹ when they studied the two-dimensional (2D) Fourier transform of the sinogram of the Radon transform of a 2D object. The group then applied their FDP to correction of depth-dependent collimator blurring in SPECT.^{2,3} Many researchers utilized a frequency space approach to improve reconstruction followed by conventional filtered backprojection. Hawkins *et al.*⁴ applied the FDP to their circular harmonic transform (CHT) algorithm for quantitative SPECT reconstruction. In the CHT algorithm, the far-field frequency domain data are replaced by the near-field data, so that the data are less blurred and less attenuated. The primary effect of the FDP was to improve the noise and aliasing characteristics of the attenuated backprojector. Hawkins's method utilized the near-field signal only that resulted in backprojection that attenuated, rather than amplified, the backprojection. This had a beneficial effect in improving the SNR of the reconstruction as well as reducing collimator blur. The similar idea was also used in Metz and Pan's work.⁵ The Metz–Pan algorithms also utilized frequency space interpolation to obtain a sinogram corrected for attenuation. The Metz–Pan algorithms determined the optimal stochastic averaging of near- and far-field signals for a particular data set to obtain the greatest improvement in SNR in the reconstructed image. It made no attempt to correct for collimator blur. Glick *et al.*⁶ used the

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FDP to compensate for the collimator blurring effect then used Bellini's filtered backprojection algorithm⁷ to correct for constant attenuation. The Bellini method utilized opposing views, a frequency space weighting, and finished with ordinary ramp filtering and backprojection. Glick used Bellini's method in a frequency space interpolation method to obtain a projection without attenuation. The reconstruction could be finished with the backprojector of choice. Iterative algorithms were used in the last step. This resulted in better stochastic behavior than the near-field only approach, but somewhat less than optimal behavior for eliminating collimator blur because opposing views were used. In fact, it may have introduced the arc artifacts that Soares *et al.*⁸ observed and analyzed. Kohli *et al.*⁹ used the FDP to preprocess the data to correct for the collimator blurring effect and used the iterative OS-EM (ordered-subset expectation maximization) algorithm¹⁰ to reconstruct the image with attenuation correction. The FDP has also been extended to a slat collimator imaging geometry.¹¹

This article investigates a different application of the FDP, that is, the use of FDP in constant attenuation correction. The FDP is briefly reviewed in Sec. II where a constant attenuation compensation method is also introduced. Some numerical examples are presented in Sec. III. The FDP attenuation correction results are compared with the results obtained from the Tretiak–Metz filtered backprojection algorithm.¹² Finally, Sec. IV concludes the paper.

II. METHODS

II.A. Review of the frequency-distance principle

The FDP is briefly introduced as follows. Let $p(s, \theta)$, be the parallel-beam projections of a 2D object, where θ is the view angle and s is the coordinate on the detector. We take the Fourier transform with respect to s and the Fourier series expansion with respect to θ of $p(s, \theta)$ (for the sake of convenience, we refer to this combined transform as a 2D Fourier transform), and we get

$$P(\omega, n) = \frac{1}{2\pi} \int_0^{2\pi} \int_{-\infty}^{\infty} p(s, \theta) e^{-i(\omega s + n\theta)} d\theta ds. \quad (1)$$

For a point source δ at (r, φ) the corresponding Radon transform is

$$p(s, \theta) = \delta(s - r \cos(\varphi - \theta)), \quad (2)$$

and the distance from the point source to the detector is given as (see Fig. 1)

$$\text{dist}(\theta) = R + r \sin(\varphi - \theta), \quad (3)$$

where R is the distance from the center of rotation to the detector. For this particular object, we have

$$P(\omega, n) = \frac{1}{2\pi} \int_0^{2\pi} e^{-i(\omega r \cos(\varphi - \theta) + n\theta)} d\theta. \quad (4)$$

The above expression holds for an ideal collimator without blurring. Let

$$\Phi(\theta) = \omega r \cos(\varphi - \theta) + n\theta, \quad (5)$$

then

$$\Phi'(\theta) = \omega r \sin(\varphi - \theta) + n. \quad (6)$$

The principle of stationary phase implies that the largest contribution to the integral at the right-hand side of Eq. (4) occurs when the phase $\Phi(\theta)$ changes most slowly. Letting $\Phi'(\theta) = 0$ yields

$$r \sin(\varphi - \theta) = -\frac{n}{\omega}. \quad (7)$$

Notice that the distance from the detector to the point object is $\text{dist}(\theta) = R + r \sin(\varphi - \theta)$, and we have

$$\text{dist}(\theta) = R - \frac{n}{\omega}. \quad (8)$$

The above relationship is often referred to as the FDP. This principle is based on the observation that the function $\exp(-i\Phi(\theta))$ is oscillating with high frequencies except for the points where $\Phi'(\theta) = 0$. Therefore the contribution to the component $P(\omega, n)$ in Eq. (4) is mainly from the object activities at the distance determined by Eq. (8) at the detector's view angle θ .

To illustrate this point, two examples of the real part of the function $\exp(-i\Phi(\theta))$ are shown in Fig. 2, where $r=1$, $\varphi = \pi/2$, and $n=3$. In Fig. 2(a) $\omega=50$, and in Fig. 2(b) $\omega=5$. The regions with fast oscillation of $\exp(-i\Phi(\theta))$ have little contribution to $P(\omega, n)$, and the regions with slowest oscillation (i.e., when $\Phi'(\theta) = 0$) have the most contribution.

The FDP Eq. (8) is an approximation that assumes that the integral in Eq. (4) is determined by only the values of θ when $\Phi'(\theta) = 0$. Solutions exist for $\Phi'(\theta) = 0$ only if

$$|\omega| \geq |n|/r. \quad (9)$$

Here r is the distance from the point of interest to the origin. Figure 2 shows that the FDP is more accurate for higher frequencies ω and less accurate for lower frequencies ω for a fixed r .

II.B. Frequency-distance principle for attenuation correction

Now we assume that the projections are uniformly attenuated with a linear attenuation coefficient μ . We assume that the boundary of the attenuator is also known; thus after multiplying by a scaling factor, the projection data are the exponential Radon transform of the object. Equivalently, this prescaling procedure sets the detector at the axis of rotation, that is, $R=0$. For a point source δ at (r, φ) the corresponding exponential Radon transform is

$$p_a(s, \theta) = e^{\mu r \sin(\varphi - \theta)} \delta(s - r \cos(\varphi - \theta)), \quad (10)$$

where $r \sin(\varphi - \theta)$ is the negative of the distance [see Eq. (3)], and $e^{\mu r \sin(\varphi - \theta)}$ is the attenuation factor for the point source.

All discussion in Sec. II A can be applied here. Following the same steps as in Sec. II A, we have

$$P_a(\omega, n) = \frac{1}{2\pi} \int_0^{2\pi} e^{\mu r \sin(\varphi - \theta)} e^{-i(\omega r \cos(\varphi - \theta) + n\theta)} d\theta. \quad (11)$$

After using the FDP, we get

$$P_a(\omega, n) \approx e^{-\mu n/\omega} P(\omega, n), \quad (12)$$

where $P(\omega, n)$ is for attenuation-free data. Relationship (12) may be useful for attenuation compensation. Due to large approximation errors in FDP for low frequency components, the attenuation compensation method suggested by Eq. (12) may not work well for a distributed object but may work well for point source type objects. We will use computer simulations to verify this hypothesis next. When Eq. (12) is used for attenuation correction it is rewritten as $P(\omega, n) \approx e^{\mu n/\omega} P_a(\omega, n)$, which is singular when $\omega=0$. In implementation, a regularization method is adopted by forcing $P(\omega, n)=0$ as $\omega=0$.

III. COMPUTER SIMULATIONS

In this section, the FDP attenuation correction technique (12) is applied to three computer generated phantoms. One phantom is a large uniform disk with a diameter of 21 cm, and the attenuator is the same size of the source. The source and the attenuator are concentric. The center of the disk is the center of the detector rotation. The attenuation coefficient of water i.e., $\mu=0.15$ cm at 140 keV is assumed.

The second phantom is also a uniform disk but with a diameter of 8.4 cm. The attenuator is the large disk as described above. The center of the source disk is 5.04 cm off center.

The third phantom also uses the same attenuator. The phantom consists of two dots, that is, small disks of diameter 0.63 cm. One dot is 5.04 cm off center, and the other dot is 1.05 cm off center. These two dots have the same emission concentration.

In all computer simulations in this article, the projection data are generated analytically using closed-form expressions. The detector has 128 bins, and the bin size is 0.175 cm. The detector rotates around the phantom 360° with 128 view angles. The images are reconstructed in a 128×128 array.

When the third phantom is used, the projection sinograms are shown in Fig. 3, where the FDP correction method (12) is shown to be effective to correct for photon attenuation. The corresponding reconstructed images are shown in Fig. 4. No noise is added to the projections. The reconstruction algorithm is the regular filtered backprojection (FBP) algorithm with a ramp filter.

In Fig. 5, the proposed method is compared with the Tretiak–Metz FBP algorithm, which is able to correct for uniform attenuation, using noisy attenuated projections. The noise is Poisson distributed. The background noise in the reconstructed images has been studied by evaluating the variances in a rectangular background region defined by the pixel coordinates [18:63, 18:107]. The background variance for the image reconstructed with proposed method is 93.4, and it is 140.0 for the image reconstructed with the Tretiak–Metz method. The image obtained with the Tretiak–Metz method has more severe noisy rays than the image that uses the FDP method to compensate for the attenuation.

As expected, the FDP method does not perform well as the object gets larger. Figure 6 shows the reconstructions using a mid-size phantom and a large-size phantom. As the objects gets larger, the FDP method tends to over correct for the attenuation effect. This is because relation (12) is more accurate for high frequency components than for low frequency components.

IV. CONCLUSIONS

The well-known FDP provides an approximate relation between the imaging distance and the frequency components of the projection sinogram. It is interesting to see whether this approximate relation can be used to compensate for attenuation in SPECT. Since this relation is more accurate for high frequency components, it is expected that the FDP method can be used to correct for uniform attenuation for small objects. Our computer simulations verified this hypothesis. It has also been observed that for large objects the FDP attenuation correction method overcorrects. For an extended source, an accurate attenuation correction may not be obtained by using relation (12) alone. If relation (12) is not used, but the FDP is used as a guideline, a pretty good attenuation correction can be obtained as previously done, for example, by Hawkins *et al.* (1991) and Glick *et al.* (1994).

The proposed attenuation correction method has been compared with the Tretiak–Metz method using noisy projections. It seems that the FDP-based method provides less noisy images. One explanation is that the Tretiak–Metz method uses an exponential backprojector, and the exponential factor may amplify noise. On the other hand, the FDP-based attenuation correction method uses a regular FBP algorithm to reconstruct the image, and its backprojector does not contain an exponential factor.

Acknowledgments

This work was partially supported by NIH grant 1R21 EB003298. The author thanks Dr. Roy Rowley for English editing of this manuscript.

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Fig. 1.
A detector at view angle θ measures a point source.

Fig. 2. Plots of function Φ as defined in Eq. 5, with $r=1$, $\varphi=\pi/2$, $n=3$. (a) $\omega=50$ (b) $\omega=5$.

Fig. 3.

Sinograms of the two-point phantom. (a) Sinogram of the attenuated projections. (b) Sinogram after the FDP attenuation correction. (c) Sinogram from attenuation-free projections. (d) Two-point phantom. All images are displayed after the maximum image values are scaled to 255, that is the maximum brightness.

Fig. 4.

Reconstructions (with profiles) of the two-point phantom. (a) FBP reconstruction without attenuation compensation. (b) FBP reconstruction with FDP attenuation compensation. (c) FBP reconstruction with attenuation-free data. All images are displayed after the maximum image values are scaled to 255, that is the maximum brightness. All profiles are drawn at the same vertical location.

Fig. 5. Reconstructions with noisy data. (a) FDP-based attenuation correction followed by the FBP algorithm (image background noise variance 93.4). (b) Tretiak–Metz FBP algorithm with attenuation correction (image background noise variance 140.0).

Fig. 6.

Reconstructions (with profiles) of mid-size and large-size phantoms. The large phantom (c) and (d) has the same size as the attenuator. The mid-size phantom (a) and (b) has a radius that is 40% of that of the attenuator, and is off-centered. (a) and (c) are reconstructed without attenuation compensation. (b) and (d) use the FDP-based attenuation correction. The dotted profiles are from the true phantom.