.06-.15 gamete per generation or, with 10^4 loci per gamete, $6-15 \times 10^{-6}$ per locus.

* Department of Genetics, University of Wisconsin, Paper No. 634.

¹ G. B. L. Arner, *Columbia Univ. Studies in History, Economics, and Public Law*, **31**, No. 3, 1-99, 1908.

² H. J. Muller, *Bull. N.Y. Acad. Med.*, **24**, 447-469, 1948.

³ H. M. Slatis, *Am. J. Human Genet.*, **6**, 412-418, 1954.

⁴ J. Sutter and L. Tabah, *Population*, **7**, 249-266, 1952; **8**, 511-526, 1953.

⁵ S. M. Bemiss, *Trans. Am. Med. Assoc.*, **11**, 319-425, 1858.

⁶ Here and later we are making the simplifying assumption that a pre-existing mutant allele has the same effect on viability as the new mutant. This is justified as a first approximation by the observation that compounds of mutants are often near the average of the mutant homozygotes. This formula is given for only two alleles, but the extension to a larger number is obvious and the linearity in F remains. In applying this to a real population, we assume $F = 0$ for all individuals not known to come from a consanguineous marriage.

⁷ S. Wright, *Am. Naturalist*, **56**, 330-338, 1922; *Ann. Eugen.*, **15**, 323-354, 1951.

⁸ H. J. Muller, *Am. J. Human Genet.*, **2**, 111-176, 1950.

⁹ H. J. Muller, *J. Cellular Comp. Physiol.*, **35**, suppl. 1, 205-210, 1950, and unpublished data of Muller and Campbell.

¹⁰ C. Stern, G. Carson, M. Kinst, E. Novitski, and D. Uphoff, *Genetics*, **37**, 413-449, 1952.

¹¹ Referred to on p. 41 of Th. Dobzhansky and S. Wright, *Genetics*, **26**, 23-51, 1941.

¹² H. J. Muller, *Harvey Lectures, Ser.* **43**, pp. 165-229, 1947-1948.

¹³ S. G. Levit, *J. Genetics*, **33**, 411-434, 1936.

¹⁴ F. Seto, *Am. Naturalist*, **88**, 373-378, 1954.

¹⁵ The "single locus" rate in *Drosophila* may, of course, be the total rate among a series of pseudoalleles. In man there is not only this difficulty but the fact that mutants at different loci with phenotypically similar effects may be counted as mutants at a single locus.

¹⁶ J. Neel and W. Schull, *Human Heredity* (Chicago: Univ. Chicago Press, 1954), p. 144.

TWO LATE-GLACIAL DEPOSITS IN SOUTHERN CONNECTICUT

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One of the recent trends in Pleistocene research has been the study of pollen in deposits that might record late-glacial vegetation changes. To my knowledge there are now only two North American localities that clearly provide such evidence. Not including the type locality in Wisconsin for the Two Creeks interstadial flora,¹ these are George Reserve, Michigan,² and Aroostook County, Maine.³

Evidence from these sections suggests that during the Two Creeks interstadial the central parts of Wisconsin and Michigan were forested with spruce and fir, while at the same time northern Maine had only local stands of spruce. During the ensuing ice advance, eastern Michigan and northern Maine suffered great alterations of climate and may have been characterized by steppe-like vegetation.

The New Evidence.—The late-glacial deposits described briefly here (to be more fully detailed in a later publication) lie less than 20 miles south of the late-Wisconsin drift border described by Flint,⁴ near Middletown, Connecticut. One of these is a kettle, Totoket Bog, earlier analyzed by Deevey,⁵ and the other is Durham Meadows, a formerly lacustrine basin which lies in a small valley tributary to the Connecticut River. The pollen sequences obtained from these are different from

the established Connecticut chronology in that they include previously undescribed oscillations within the so-called "postglacial spruce zone" (zone A of Deevey) as well as two entirely new pollen zones at the base of the sequence. The profiles described here provide evidence of two major reversals of vegetation type and suggest, therefore, that the early parts of the New England postglacial chronology are far more complex than was previously realized.

Methods.—To prepare the core sediments for pollen analysis, the bromoform flotation method⁶ was used. Surface silts from Durham were treated with HF; acetolysis followed. Identifications were based on acetolysed preparations of herbarium material, including all eastern species of the genera *Alnus*, *Lycopodium*, and *Betula*.

Pollen Zones.—The diagrams of Figure 1 outline the broad chronology. The two new sequences (*below*) resemble the typical Connecticut chronology (*above*) in

FIG. 1.—New England pollen sequences.

that they show an early herb phase,⁷ spruce (A), pine (B), and three deciduous zones (C: oak, hickory, and chestnut). The Durham profiles indicate that marsh conditions prevailed locally during the oak maximum.

In addition, the Totoket sequence includes two early phases, represented by a spruce maximum and an older herb maximum denoted in Figure 1 by the new zone symbol *T*. The new sequences, furthermore, provide added detail within spruce zone A of Deevey. In my pollen profiles, that zone contains two spruce maxima, not one, and in each case the first peak is greater than the second. The two major herb and two major spruce zones in my general chronology I have chosen to call "Durham" and "Pre-Durham" spruce and "Younger" and "Older" herb zones. These are described according to their primary features.

Herb Zones.—The chief components of these zones are nonarbooreal. The families Cyperaceae, Gramineae, Compositae, Ericaceae, Saxifragaceae, and Umbelliferae and the forb genera *Artemisia*, *Rumex*, and *Plantago* constitute up to 80 per cent of the total pollen and have an average frequency of 66 per cent. The shrubs *Salix*, *Ilex*, and *Alnus* represent up to 25 per cent of the total pollen. Associated with the Younger herb zone are spores of two subarctic or alpine forms of *Lycopodium*: *L. selago* and *L. annotinum* vars. *alpestre* or *pungens*.⁸

Within the herb zones tree pollen is of less numerical importance (i.e., pine, oak, and hardwoods, each constituting up to 10 per cent of the total pollen) than anywhere else in the cores and, because of the inorganic nature of the sediments, may in part represent secondary pollen or contamination.⁹ However, at the borders of the herb zones pine frequencies appear to be more significant, being as high as 25 per cent of the total pollen.

The Younger herb zone is delineated at its lower and upper borders by major peaks in the *Betula* curve. The lower of these is composed of especially small *Betula* pollen (modal size, 20 μ), which suggests that this is the pollen of the dwarf birch *B. glandulosa* of subarctic and alpine distribution.¹⁰ At the upper border birch pollen is of a larger type, probably representing an arboreal form.

Spruce Zones.—The spruce zones have in common pollen of the following woody associates: pine, fir, birch, and alder (listed in decreasing order of importance). These genera have frequencies that are approximately reciprocal to those of spruce. Herb pollens are less frequent than those of trees.

The Pre-Durham spruce maximum contains spores of the same *Lycopodium* species that characterize the Younger herb zone (see above), while the *Lycopodium* spores of the Durham spruce zone are mainly of the *L. complanatum* and *L. sabinaefolium* groups. Sediments of the Durham spruce zone are organic, while below this they are silty clay and sand.

Pine Zones.—Though pine pollen is present throughout the sequence in moderate numbers, there are two intervals during which it is notable in the pollen count. One of these occurs after the last spruce but before the deciduous phases at the top of the cores and has a stratigraphic position equivalent to that of the well known "Boreal pine period" (zone B of Deevey). The other falls between the two spruce maxima of the Durham spruce zone and represents an earlier pine interval. As in the Boreal pine period, pine is associated with small amounts of spruce, oak, *Ostrya-Carpinus*, hazel, and alder and therefore appears to represent a climate warmer than that of the spruce intervals.

Deciduous Zones.—In both deposits the terminal pollen phases correspond well with those of the established Connecticut chronology (zone C of Deevey): (1) oak, (2) hickory, and (3) oak and chestnut.

Carbon-14 Dating.—Five C^{14} dates obtained from these deposits are listed below with the corresponding zone letter of Deevey's chronology.⁵

Oak maximum (zone C-1); Durham Meadows; $4,800 \pm 400$ years¹¹

Pine maximum (zone B); Durham Meadows; $7,570 \pm 250$ years¹¹

Pine maximum (zone B); Durham Meadows; $8,155 \pm 410$ years¹²

Lower Durham spruce zone (lower zone A); Durham Meadows: $12,700 \pm 280$ years¹¹

Lower Durham spruce zone (lower zone A); Totoket: $13,550 \pm 460$ years¹²

Climatic Interpretation.—In the herb zones there is a predominance of plants that characterize open vegetation, suggesting that the region was not densely forested during these intervals; they suggest an association of grasses, sedges, and forbs, with scattered shrubs like alder, willow, holly, and open stands of birch and pine. All these plants have high light requirements. Cold-loving forms of *Lycopodium* and probably *Betula glandulosa* were present at least during the Younger herb interval. This combination of high-light- and low-temperature-loving plants strongly implies open and perhaps steppe-like conditions during a relatively cold interval.

The spruce intervals, by contrast, apparently represent warmer (but still cool) circumstances and were most certainly characterized by dense forest. The pine intervals probably represent still warmer conditions than the zones which are predominantly spruce.

The pollen profiles described here record two climatic oscillations. The older one, represented by the herb-spruce-herb sequence, probably included cold, cool, and cold phases. Immediately following is the younger oscillation consisting of a shift from spruce to pine to spruce, implying cool, warmer, and then cool phases.

The warmest part of the early oscillation is represented by the Pre-Durham spruce zone, which was preceded and followed by intervals of relatively open vegetation and severe climate. The C^{14} dates of *ca.* 13,500¹² and 12,500¹¹ years from overlying sediments show that the early oscillation terminated significantly before the Two Creeks interval (*ca.* 11,000 years¹¹). The occurrence of this oscillation in southern Connecticut indicates that the region south of Middletown has been free of ice since before the Two Creeks interstadial.

The younger oscillation has as its warmest phase the pine interval in the middle of the Durham spruce zone. The C^{14} dates from the early part of this zone indicate that the younger oscillation began about 13,500 years ago, and thus its inception significantly antedated Two Creeks time. It ended before the Boreal pine period dated by C^{14} here and in other Connecticut deposits at about 8,000 years. Hence the Durham spruce oscillation certainly records the forest phases that introduced the Two Creeks interstadial in Connecticut. The pine phase is probably of Two Creeks age, and the final spruce maximum would then represent the cooling resulting from glacial advance during Valders¹³ time.

The remarkable aspect of this pollen record is that the Two Creeks vegetation of southern Connecticut was so different from that of northern Maine. Late-glacial sections in Maine³ suggest that there spruce was established only locally during that interstadial, while at the same time the southern Connecticut climate was apparently too warm for dense stands of spruce. In New England, therefore, the

densest forests of spruce and fir must have lain between southern Connecticut and northern Maine, i.e., in central New England.

According to the evidence we now have, the glacial advance that brought steppe-like conditions to Aroostook County, Maine, altered the forest composition of southern Connecticut only slightly. The spruce belt which probably occupied central New England during the Two Creeks interval must have migrated southward during Valdres time at least as far as southern Connecticut. The sequences from Durham and Totoket serve, therefore, to date the beginning of the usual "postglacial" chronology and give evidence of a complex series of vegetational and presumably climatic changes that preceded it.

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¹ L. R. Wilson, *Trans. Wisconsin Acad. Sci.*, **27**, 31, 1932.

² Sven T. Andersen, *Danmarks Geol. Undersøgelse*, **II**, **80**, 140, 1954.

³ E. S. Deevey, *Am. J. Sci.*, **249**, 177, 1951.

⁴ R. F. Flint, *Connecticut State Geol. and Nat. Hist. Survey Bull.*, **47**, 1930.

⁵ E. S. Deevey, *Am. J. Sci.*, **237**, 691, 1939.

⁶ David G. Frey, *New Phytologist*, **54**, 257, 1955.

⁷ E. S. Deevey and J. E. Potzger, *Am. J. Sci.*, **249**, 473, 1951.

⁸ Estella B. Leopold, unpublished doctoral dissertation, Yale University, 1955.

⁹ Johs. Iversen, *Danmarks Geol. Undersøgelse*, **IV**, **2**, No. 15, 1936.

¹⁰ Estella B. Leopold, *Grana Palynologica*, **1**, No. 2, p. 140, 1956.

¹¹ Hans E. Suess, *Science*, **120**, 467, 1954.

¹² R. S. Preston, Elaine Person, and E. S. Deevey, *Science*, **122**, 954, 1955.

¹³ R. F. Flint, *Am. J. Sci.*, **254**, 265, 1956.

HYDRODYNAMICAL STABILITY AND POINCARÉ-LYAPUNOV THEORY. I*

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1. *Introduction.*—The purpose of this paper is to initiate a rigorous theory of hydrodynamical stability. Hitherto, stability investigations in this field have been confined to a study, difficult enough, of the linear equations arising from perturbations, without regard to the full nonlinear expressions. Our aim is to establish results corresponding to the classical Poincaré-Lyapunov theory (cf. an earlier book¹) for systems of nonlinear ordinary differential equations, for classes of nonlinear partial differential equations occurring in hydrodynamics. These results parallel those established for differential-difference equations (cf. E. M. Wright,² R. Bellman,³ and, for the nonlinear heat equation, G. Prodi,⁴ and R. Bellman⁵).

To obtain these results, we employ an approach quite distinct from the conventional method based upon separation of variables. Previous investigations of this