CSF Pressure and Velocity in Obstructions of the Subarachnoid Spaces

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SUMMARY – According to some theories, obstruction of CSF flow produces a pressure drop in the subarachnoid space in accordance with the Bernoulli theorem that explains the development of syringomyelia below the obstruction. However, Bernoulli's principle applies to inviscid stationary flow unlike CSF flow. Therefore, we performed a series of computational experiments to investigate the relationship between pressure drop, flow velocities, and obstructions under physiologic conditions. We created geometric models with dimensions approximating the spinal subarachnoid space with varying degrees of obstruction. Pressures and velocities for constant and oscillatory flow of a viscous fluid were calculated with the Navier-Stokes equations. Pressure and velocity along the length of the models were also calculated by the Bernoulli equation and compared with the results from the Navier-Stokes equations. In the models, fluid velocities and pressure gradients were approximately inversely proportional to the percentage of the channel that remained open. Pressure gradients increased minimally with 35% obstruction and with factors 1.4, 2.2 and 5.0 respectively with 60, 75 and 85% obstruction. Bernoulli's law underestimated pressure changes by at least a factor 2 and predicted a pressure increase downstream of the obstruction, which does not occur. For oscillatory flow the phase difference between pressure maxima and velocity maxima changed with the degree of obstruction. Inertia and viscosity which are not factored into the Bernoulli equation affect CSF flow. Obstruction of CSF flow in the cervical spinal canal increases pressure gradients and velocities and decreases the phase lag between pressure and velocity.

Introduction

An obstruction of the subarachnoid space, such as ectopic position of the cerebellar tonsils in Chiari I malformation, increases pressure gradients and velocities in the CSF during the cardiac cycle¹⁻⁴. These changes have not yet been adequately quantified because of deficiencies in measurement techniques and because of the confounding effect of inter-individual variations. The role of flow obstruction in Chiari and syringomyelia has not been adequately evaluated⁵.

Previous studies do not clarify whether viscous and inertial forces must be considered in the analysis of CSF flow. In one theory of syringomyelia pathogenesis, CSF pressure and velocity have an inverse relationship as predicted by the Bernoulli equation, that lacks terms for

inertia and viscosity^{6,7}. Other literature suggests that viscous and inertial forces have a significant effect on CSF dynamics⁸. In a physical model of obstruction in the spinal subarachnoid space, pressure diminishes gradually with distance along unobstructed portions of the model and more steeply in regions of obstruction without an increase in pressure downstream from the obstruction, predicted in the absence of viscous forces⁹. Studies of CSF pressure and velocity performed with computational fluid dynamics (CFD) show phase differences between peak CSF pressure and peak velocity, an indication that viscous and inertial effects have a role in CSF flow¹⁰⁻¹². Recent theories suggest that the timing of the velocity and pressure pulsations affects the CSF flow in the perivascular spaces¹³, and obstructions disturb the phase-shift be-

Figure 1 Color display of velocities in the midline plane → at maximal flow (top) and flow reversal (bottom) in the 0% (left) and 75% obstructed (right) models. At the time of maximal flow, the velocities in the fluid space have a maximum in the center of the channel and near zero flow at the edge of the channel. In the unobstructed model, velocities increased from zero at the edge to its maximum in the center of the channel. In the unobstructed model, velocities remain constant along the length of the model. In the obstructed model, fluid flows with greater velocity at the level of the obstruction. Above and below the obstruction, flow has the same velocities and velocity profile as in the unobstructed model. Mean flux, the integral over the cross sectional area is constant along the length of the model. At the time of flow reversal, velocities are positive in direction near the wall of the fluid chamber and negative in the center of the fluid space, indicating bidirectional flow. When flow reverses direction again, the negative and positive velocities occur again, with their locations with respect to the wall reversed. The mean velocity is zero at this time. In the obstructed model at the time of flow reversal, velocities at the level of obstruction are negative only. Above and below the obstruction, both positive and negative velocities are present (bidirectional flow).

Figure 2 Color display of pressures in the midline at the time of peak flow (left) and flow reversal (right) in the 0% and 75% obstructed models. In the unobstructed model, pressure decreases progressively along the length of the model at the time of peak flow. In the obstructed model at peak flow, the pressure gradient is steeper at the level of obstruction than above and below the obstruction. At the time of flow reversal, pressure gradients in the unobstructed model are larger than at the time of peak flow. In the obstructed model the pressure decreases less steeply over the obstruction.

tween these pulsations¹². A systematic study of the nonlinear relation between velocity, pressure and obstruction under reasonable physiological conditions has not been performed.

CFD in patient-specific geometries has been used and validated in the evaluation of CSF flow¹⁴⁻¹⁶. Utilizing idealized geometries in place of patient-specific models in CFD permits selected variables to be studied in isolation^{8,10,11,17}. We created a simplified model of obstruction and used realistic parameters for CSF viscosity and oscillation. We calculated pressure and velocity with the Navier-Stokes equations which have terms for viscosity and inertia. We compared these results with calculations from the Bernoulli equation, in which viscous and inertial forces are neglected.

Methods

Geometric models

Idealized models of the cervical spinal subarachnoid space with a 1.5 cm diameter outer cylinder and a 1.0 cm diameter inner cylinder both 10 cm long were made in NETGEN¹⁸. The walls of the models were assumed to be impermeable and rigid. In one model the fluid space had the same cross-sectional area from top to bottom. In the other models, we placed an elliptical enlargement of the inner cylinder of 1.2, 1.33, 1.4 or 1.44 cm diameter at the midpoint of the model. The enlargement was 2 cm in length in each case. We converted the models into tetrahedral computational meshes consisting of interconnected nodes in the same program. The meshes consisted of approximately four million cells, with some variation related to the volume of the fluid space. The minimum distance between nodes was 0.2 mm and the maximum 2 mm. The finest resolutions were applied in the obstructed region. A layered mesh structure with higher resolution was used next to walls. The adequacy of the spatial resolution was tested by running the simulations with a coarser and a finer mesh. We calculated the average pressure difference from the time of maximum to the time of minimum pressure at the different mesh resolutions.

Simulations

We simulated constant and oscillatory flow through the models by solving the Navier-Stokes equations. The Navier-Stokes solver

was implemented with the finite element library FEniCS¹⁹ using a semi-implicit pressure correction scheme²⁰ and linear elements for both velocity and pressure. At the inflow and outflow boundaries, i.e., top and bottom of the geometric models, we defined a plug shaped velocity profile, constant in time or varying sinusoidally. The inflow velocity for constant flow was set to 3.5 cm/s. The period of the sinusoidal inflow profile was set equal to 0.85 s to simulate a typical heart rate and the stroke volume was set to 0.85 ml/per heartbeat to achieve a peak velocity at the inlet and outlet of 3.5 cm/s. At the walls of the fluid space we assumed a no-slip boundary condition. The fluid was prescribed as water at body temperature with density $\rho = 1.0 \text{ g/cm}^3$ and viscosity $\mu = 7\text{e-}3 \text{ g s/cm}$. Gravity was neglected under the assumption that it is balanced by hydrostatic pressures. Flow was calculated at 0.25 millisecond intervals for the sinusoidal flow profile and at 10 millisecond intervals for the constant profile, starting from resting conditions. For each time step, the Navier-Stokes equations were solved at every node in the computational mesh. For the constant inflow and outflow profile, the simulations were run until the pressure and velocity reached steady state. For the sinusoidal inflow and outflow rate, simulations were performed for four cycles. Snapshots of velocity and pressure were taken in Paraview²¹ and were displayed as contour plots by means of Matplotlib in Python²². Flow patterns for 0.5 cm at each end of the models, where flow profiles had incompletely developed, were disregarded.

Quantitative comparison of velocities and pressure

Peak velocities at the midpoint of the model and maximal pressure differences between top and bottom were plotted for each model against the percentage obstruction. A curve was fit to the points using `scipy.optimize.leastsq()`²³.

The pressures along the longitudinal axis were tabulated in Paraview for the constant flow rate and at $t = 0.28 \text{ s}$, $t = 0.43$, and $t = 0.64 \text{ s}$ for the sinusoidal flow, with the module Matplotlib in Python²².

Pressure drops at the level of the obstruction were also calculated from Bernoulli's Law

$$\left(\frac{1}{2} \rho (u_1^2 - u_2^2) \right)$$

based on peak velocities in the straight part (u_s) and obstructed part (u_o) of the channel. For constant inflow rate we plotted

$$\frac{1}{2}\rho u^2 + p$$

along the model (ρ = density; u = velocity; p = pressure) using the computed velocities and pressures at the center of the fluid channel.

We calculated local maximal Womersley numbers (W)

$$W = L \sqrt{\frac{2\pi\rho}{T_s\mu}}$$

which describe the ratio of the local fluid acceleration to the viscous forces acting on it. The characteristic length (L) was defined as

$$L = \sqrt{\frac{A}{\pi}},$$

where A represents the cross-sectional area at the midpoint of the model, and T_s the duration of systole, which was set to 0.43 s.

We calculated local maximal Reynolds number, the ratio of inertial to viscous forces, as

$$Re = \frac{\rho u_{max} L}{\mu},$$

where u_{max} is the peak velocity.

Results

Geometric model

The model had cross sectional areas of the fluid space reduced by 35%, 60%, 75%, and 85% respectively for diameters of 1.2, 1.33, 1.4 and 1.44 cm. The models had a smooth transition from unobstructed to obstructed regions. For simulations at three mesh resolutions the average pressures differed by 0.002, 0.02, 0.02, 0.03, and 0.09 cmH₂O for 0%, 35%, 60%, 75%, and 85% obstruction respectively. Peak velocities varied by up to 10%. Varying the resolution did not affect the timing of peak velocities and pressures or the qualitative behavior of the solution.

Simulations

With flow started from rest, pressure and velocity stabilized by 3 s in the constant flow simulations. For the sinusoidal flow simula-

tions, the velocity and pressure plots over time from the first to the fourth cycles were indistinguishable in all models. The mean volume flux, that is the integral of the normal velocity over the cross sectional area, was constant along the length of the models, showing that volume (mass) conservation was fulfilled in the simulations. Axial images showed that velocity varied with distance from the walls, while pressure did not.

Unobstructed model

For the constant inflow and outflow rate, the velocity had an unchanging profile along the length of the model. In the axial plane the velocity peaked in the center of the channel and progressively decreased towards the walls. For a sinusoidal inflow and outflow rate, velocity varied both with location in the channel and phase of the cycle. During times of high velocity, velocities varied from zero at the wall to maximal in the center of the channel (Figure 1). When flow reversed, flow in the center of the channel maintained the direction of flow during the last half cycle as flow near the wall began to move in the opposite direction resulting in synchronous bidirectional flow.

For constant inflow, pressure changed linearly from top to bottom along the length of the model. With sinusoidal inflow and outflow, the pressure gradient oscillated continuously with peaks at $t = 0.03$ s and $t = 0.46$ s and zero gradient at $t = 0.24$ s and $t = 0.66$ s.

Obstructed models

In the obstructed models, the velocity profiles in the obstructed regions differed from those in the unobstructed model while in the regions away from the obstruction they did not differ visibly from those in the unobstructed model (Figure 1). With constant inflow and outflow rate, larger peak velocities were present in the narrowed region of the fluid space than in the regions upstream and downstream from the obstruction. For sinusoidal inflow and outflow conditions, velocities were greater in the region of obstruction than upstream or downstream from the obstruction, except during the phases of the cycle with least flow volumes. Peak velocities increased at the level of obstruction as the degree of obstruction increased. As in the unobstructed model, the velocity peaked in the center of the channel and decreased towards the wall, where it was

Figure 3 A) Plot of pressure in cmH₂O during constant flow along the unobstructed and obstructed models. Pressure decreases steeply at the level of obstruction (4-6 cm from the top of the model) and diminishes less steeply along the unobstructed portions of the models. Pressure is assumed to be 0 at the bottom end of the model. B) In the Bernoulli equation, $\frac{1}{2}\rho u^2 + p$, is assumed constant along a streamline. The deviation of this quantity from its value at the outlet shows the error in Bernoulli's law as a function of the distance from the outlet. In unobstructed regions the deviations are relatively small, e.g., for the unobstructed model the deviation reaches a factor 3.5 at the top of the model. At the obstruction (4-6 cm from the top) the deviations are large, e.g. the deviation reaches a factor 60 at the level of an 85% obstruction.

Figure 4 Plot of peak velocities as a function of obstruction of the fluid space. The dots represent the computed peak velocities and the solid line the fitted curve. The velocity increases at 35% obstruction, and continues to increase inversely proportional to the percentage of the channel remaining open.

Figure 5 Plot of pressure difference between top and bottom of the models as a function of percentage obstruction of the fluid space. Pressure differences increase slowly for obstructions of 60% or less and steeply and more steeply for obstructions of 75 and 85%. The curve shows that the pressure is nearly inversely proportional to the percentage of the channel remaining open.

zero. At the time when flow direction reversed, bidirectional flow was evident upstream and downstream from the obstruction and at the obstruction in models with less than 75% obstruction (Figure 1). Away from the obstruction, velocity profiles did not change along the long axis of the model.

For the constant flow condition, pressure varied along the length of the obstructed models nonlinearly, with a greater decrease per unit length in regions of obstruction. The pressure decrease in the obstructed region varied with the degree of obstruction. For oscillatory inflow and outflow rate, the pressure varied

Figure 6 Plot of pressure difference between top and bottom of the models as a function of time in the cycle. As the level of obstruction increases, the magnitude of pressure difference increases. As the degree of obstruction increases, the time of the maximal pressure processes to a later time in the cycle. In the unobstructed model, the pressure gradient peaks at 0.46 s; in the most obstructed model, it peaks at 0.61 s, a change of 65 degrees in the cardiac cycle.

A

B

Figure 7 Plots of pressure along the length of the unobstructed and obstructed models to illustrate different pressure fields during oscillatory flow. One pattern (A) has pressure diminishing with distance along the model more steeply near the obstruction than at a distance from the obstruction. This pattern occurs at peak inflow ($t = 0.21$). At another time in the cycle (B), pressure has a bimodal fluctuation along the models, with pressure decreasing with distance near the obstruction. This occurs when flow is in the process of reversing ($t = 0.28$ s). It reflects the fact that the pressure wave has a later phase near the obstruction than away from it. Another pattern (C) has pressure increasing along the models, more steeply near the obstruction than at a distance from it. This plot illustrates pressures at $t = 0.43$ s.

C

with time and with position along the long axis of the model. As with the constant inflow rate, the pressure decreased more rapidly at the level of the obstruction than downstream or

upstream from the obstruction during most of the cycle (Figure 2). The pressure profile along the length of the model varied with the phase of the cycle, having less steep gradients at the

level of obstruction when the flow diminished and reversed (Figure 1). At times in the cycle, the pressure gradient was biphasic, that is it varied in one direction between the top and bottom of the model and varied in a different direction over the obstruction (Figure 7).

Quantitative comparison of velocities and pressure

Constant flow

Velocity for the constant inflow condition in the unobstructed model ranged from 0 at the wall to 5.3 cm/s in the center of the channel anywhere along the length of the model. In obstructed models, peak velocities in the obstructed regions were 7.9, 13.7, 23.1, and 34.0 cm/s with 35, 60, 75 and 85% obstruction respectively.

Pressure differences for constant flow were 0.04 cmH₂O in the unobstructed model and 0.06, 0.14, 0.43, 1.17 cmH₂O with a 35%, 60%, 75% and 85% obstruction respectively (Figure 3a).

By the Bernoulli equation

$$\left(\frac{1}{2} \rho u^2 + p = \text{constant} \right)$$

the pressure difference was 0.04, 0.08, 0.2 and 0.3 cmH₂O with a 35%, 60%, 75% and 85% obstruction respectively, considerably lower than the pressure differences calculated for constant flow with the Navier-Stokes equations. Using the computed velocities and pressures, the term

$$\frac{1}{2} \rho u^2 + p,$$

which is assumed invariable in the Bernoulli equation, deviated from the value at the inflow by a factor of 3.5 to 80 in the obstruction models (Figure 3B).

Oscillatory flow

For oscillating inflow, peak velocity in the unobstructed model was 5.2 cm/s, near the center of the channel anywhere along the length of the model. When flow reversed, velocities of 1 cm/s were present in one direction with a velocity of 0.7 cm/s in the opposite direction. In obstructed models, peak velocities were 7.7, 13.8, 23.4, and 35 cm/s respectively (Figure 4). The velocity was inversely proportional to the percentage of cross sectional area remaining. The obstructed

models had bidirectional flow upstream and downstream from the obstruction of similar magnitude as in the unobstructed model. In the obstructed region, bidirectional flow was apparent in models with up to 60% obstruction.

Pressure differences reached 0.3, 0.4, 0.6, and 1.3 cmH₂O for the 35, 60, 75 and 85% obstruction respectively compared to 0.26 cmH₂O in the unobstructed model (Figure 5). The timing of the maximal pressure differential varied with degree of obstruction. For the 0% obstruction the pressure gradient peaked at $t = 0.46$ s, a 75 degree phase shift compared to velocity (Figure 6). With increasing obstruction, the phase difference changed to 72, 59, 35 and 10 degrees for 35%, 60%, 75% and 85% obstruction respectively. Concordantly, pressure differences at peak velocity changed as the degree of obstruction changed. At peak flow, the pressure difference over the obstacle increased from 0.03, 0.1, 0.4 to 1.2 cmH₂O respectively for the 35%, 60%, 75% and 85% obstruction models (Figure 6). The change in pressure gradient at the level of obstruction had a different temporal pattern than it had upstream and downstream from the obstruction (Figure 7B).

Deviations of the term

$$\left(\frac{1}{2} \rho u^2 + p \right)$$

in the Bernoulli equation at locations along the model from inflow were greater for oscillatory flow than for constant flow.

Womersley numbers ranged from 7 at the level of 85% obstruction to 18 at unobstructed levels. For Womersley numbers less than 10, pressure and velocity were nearly in phase. For Womersley numbers greater than 10, pressure gradient and velocity differed in phase by as much as 75 degrees. Bidirectional flow occurred in a model only when the Womersley number exceeded 10. The local Reynolds numbers ranged from about 350 to 1050 at peak velocity. Reynolds numbers increased as obstruction increased.

Discussion

Obstruction of the subarachnoid space in the model produced non-linear increases in velocities and pressures. It reduced synchronous bidirectional flow in the vicinity of the obstruction, caused larger and multiphasic pressure

gradients and changed the phase difference between velocity and pressure oscillations. It changed pressures and velocities differently for oscillatory than for constant flow, due to inertial forces. Pressures and velocities did not accord with Bernoulli's law, which neglects inertial and viscous forces in effect in CSF. Specifically pressure did not increase downstream from an obstruction as the Bernoulli law predicts for non-viscous fluids.

To isolate the effect of obstruction of the fluid space, the models were simplified in geometry and in flow patterns, although physiologically correct fluid viscosity and rate of flow oscillation were retained. The dimensions of the models and the length of the cycle accorded with normal human physiologic CSF flow metrics^{8,24}. In all simulations the stroke volume was kept constant. When the finer mesh was employed, peak inflow velocity decreased, because the boundary layer was better resolved. The maximum velocity difference of 10% does not impact the qualitative results or the conclusions. We assumed rigid walls in the model since deformations of the spinal cord have a minor impact on the flow field during the cardiac cycle²⁵. However, with this assumption we eliminated a dampening effect on pressure gradients, neglected possible pressure wave phenomena and ignored volume changes or compliance¹, which are second order effects for velocities and pressure. By eliminating wall permeability, we omit the fluid exchange between the spinal cord and CSF, which occurs normally^{13,26,27}. The amount of fluid exchange is currently impossible to measure, although it factors in several theories on the pathogenesis of syringomyelia²⁷⁻³⁰. Using idealized geometric models and constant or sinusoidal flow as inflow and outflow boundary conditions, we oversimplified CSF flow, probably with the effect of overestimating maximum pressures and velocities in diastole and of underestimating maximum pressures and velocities in systole. The plug shape of the inflow and outflow profile created unphysiologic flow patterns for about 0.5 cm from both ends of the model and therefore these regions were disregarded.

Fluid velocities and pressures in the model are in physiologic ranges. The fluid velocities we observed in the unobstructed model, 5 cm/s, agree with CSF velocities in normal human subjects^{3,31,32}. With the highest degrees of obstruction, velocities in the simulations are higher than some published values for CSF^{3,31-34}, but concordant with recent 4D phase contrast mag-

netic resonance (PCMR) measurements^{4,35}. The calculated pressures and velocities in the models qualitatively agree with results in a physical model of oscillatory flow in the subarachnoid space⁹. In that model, pressure downstream from an obstruction continued to decrease, as in our study but in disagreement with predictions from Bernoulli's law. The maximal pressure differences we observed are in the range pressure calculated from PCMR measurements¹ or for idealized Chiari I subarachnoid space geometries^{10,11}. In our model, obstruction of 30 or 60% produced velocities from 8 to 14 cm/s, in the range of velocities found in vivo in the Chiari I malformation^{3,31}. The pressure differentials in our models were substantially lower than those reported by Martin et al.⁹ although their results are in qualitative agreement with ours. The phase difference between velocity and pressure oscillation diminishing with increasing obstruction accords with PCMR studies in which Chiari I patients have peak velocities at a different time in the cardiac cycle compared to controls^{33,34}. Our results are in agreement with the abnormal phase difference between arterial pulse pressure and CSF pressure in Chiari patients¹³. Synchronous bidirectional flow did not occur at the level of the obstruction in our models but did occur elsewhere. Bidirectional flow increases with increasing Womersley number, i.e., where the flow channel widens. Obstruction of the subarachnoid space has complex effects on CSF pressures, velocities and phase differences between the two. These data indicate that the complexity in CSF dynamics, like the complexity of some blood flow dynamics^{36,37} is not accurately predicted by Bernoulli's law which neglects inertial and viscous forces.

Conclusion

Obstruction of CSF flow in the cervical spinal canal increases pressure gradients and velocities and decreases the phase between pressure and velocity. Inertia and viscous forces affect CSF flow. Bernoulli's law underestimates the magnitude of the pressure gradients at the obstruction and fails to display temporal differences between velocity and pressure. While inertial and viscous forces have been disregarded in some discussions of CSF flow, this study shows their effects must be factored into future discussions of CSF flow. The Bernoulli equation does not adequately explain CSF flow, despite its application in some theories.

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