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A Tikhonov Regularization Scheme for Focus Rotations with Focused Ultrasound Phased Arrays

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Abstract

Phased arrays have a wide range of applications in focused ultrasound therapy. By using an array of individually-driven transducer elements, it is possible to steer a focus through space electronically and compensate for acoustically heterogeneous media with phase delays. In this paper, the concept of focusing an ultrasound phased array is expanded to include a method to control the orientation of the focus using a Tikhonov regularization scheme. It is then shown that the Tikhonov regularization parameter used to solve the ill-posed focus rotation problem plays an important role in the balance between quality focusing and array efficiency. Finally, the technique is applied to the synthesis of multiple foci, showing that this method allows for multiple independent spatial rotations.

I. Introduction

Since the first use of phased arrays over a century ago, they are now notably used with antennas [1], radar [2], optics [3], and ultrasound [4]. Phased array ultrasonics, specifically, has advantages over a single ultrasound transducer because the increased degrees of freedom allow for the electronic steering of the beam and the compensation for propagation through heterogeneous media by using phase delays to reconstruct a focus. This has led to the use of phased arrays in both imaging and therapeutic ultrasound [4]. Here, we will concern ourselves with the therapeutic ultrasound.

The success of focused ultrasound phased arrays in clinical treatments in recent years has led to a growth in research in the field. Clinical studies using focused ultrasound phased arrays to treat a range of diseases, including liver and kidney cancer [5], uterine fibroids [6], neuropathic pain [7], and essential tremor [8], [9], among others, have demonstrated the promise of focused ultrasound as a non-invasive treatment modality. As the use of focused ultrasound phased arrays continues to grow, increasing efforts have been expended in optimizing treatments and postulating methods for improved patient outcomes, including the use of quantitative temperature images to predict the optimal power for focused ultrasound surgery [10], the application of Zernike polynomials to adaptive focusing [11], optimizing thermal dose distribution using numerical models [12], investigating standing wave formation in transcranial focused ultrasound [13], and a study on the effect of near- and far-field heating on patient safety in uterine fibroid treatments [14]. In addition, new techniques have been introduced to control the acoustic field emitted from a phased array [15], [16].

This work aims to expand on the ideas presented in [15] to control the spatial manifestation of the focus generated from a phased array.

The idea of steering and focusing an ultrasound phased array can be formulated as an inverse problem: determining the phase delays required on a set of elements to produce a tight focus at a desired location. This problem has been well-studied [4], [17]. The focus is therefore treated as a single point in space. The inverse of the backwards propagation from the focus then determines the required phase delays to produce the forward problem. Using this technique, there is always a unique solution. Whether this focus is satisfactory, however, is a much more complex consideration, and is based on many factors, including transducer element spacing and steering range [4].

The spatial manifestation of the focus is highly variable and can change based on the array f-number, propagation through an aberrating medium, as well as steering off the main axis of the array. Therefore, the focus can be elongated or rotated. Since focused ultrasound relies heavily on the precision of the beam and the ability to induce biological changes in a limited space, the ability to control the spatial manifestation of the focus allows for additional control over the beam. This work adds to a growing volume of literature on phased array controls [18]–[20].

In reality, the focus does not manifest as a point in space, but rather as a considerable volume of heating, typically on the order of a wavelength. The manifestation of the focus can be considered a Gaussian distribution in three dimensions. Using this idea, the focus can then be described as having an "orientation" in space, by considering how the focus is distributed relative to a fixed coordinate system. Therefore, by sampling points around the focus and solving the inverse problem with multiple control points [15], it should be possible to control the spatial distribution of the focus.

Taking a sample of points around the focus poses an additional problem, however, since the focusing problem can become ill-posed. This is always the case, for instance, when a larger number of points are sampled at the focus than there are transducers in the array. Tikhonov regularization has been used successfully to solve numerous inverse problems in a range of fields [21], and has previously been applied to focused ultrasound to minimize heating at the base of the skull by using the concept of the anti-focus [22]. In this paper, the focusing problem is expanded to include the desired orientation of the focus in space.

At deep locations, the focus can exceed 1–2cm in length [23]. In addition, the orientation of the focus is dictated by the distance from the center of the array, and can overlap with nerves and other critical structures due to the length of the focus. By controlling the orientation of the focus, it is then possible to more accurately target a volume for therapy while sparing surrounding tissue. More specifically, it is shown here that the problem can be posed as a rotation on the focal volume in 3-dimensional space. As an immediate application, this paper focuses on thermal surgery using a 2D focused ultrasound phased array, as proposed in [23].

II. Materials and Methods

A. Transducer Geometry

Following [23], a transducer array consisting of 4572 individually-driven elements in a ring configuration was simulated. The outer diameter was 120 mm, and the inner diameter was 20 mm. The centre was left empty with the idea that it may be filled with an imaging transducer. This geometry is illustrated in Fig. 1. The transducer elements had a center-to-center spacing of $\lambda/2$, where λ is the wavelength of ultrasound in water, in order to ensure full steering capabilities [4]. The domain grid, including the transducer elements, was subdivided into voxels of size $\lambda/5$, in order to ensure low errors in the pressure field estimation. The transducer was sonicating at 500 kHz through tissue in all cases presented, so that the voxel dimensions were $0.6 \text{ mm} \times 0.6 \text{ mm} \times 0.6 \text{ mm}$.

B. Numerical Model

Since the ultrasound is assumed to propagate in a homogeneous medium, a modified Rayleigh-Sommerfeld equation is used to describe its propagation [24], [25], where $p(r)$, the pressure at point r from a source vibrating with a velocity u , is given by

$$p(r) = \int_S \frac{jk_c \rho c}{2\pi \|r - r'\|} e^{-jk_c \|r - r'\|} u(r') dr', \quad (1)$$

where S is the surface of the source, $k_c = k - j\alpha$ is the complex wave number, where α is the attenuation coefficient, c is the speed of sound, and $j = \sqrt{-1}$. (1) was solved numerically. The surface of the transducer elements was broken into surfaces of infinitesimal area, which could be considered point sources, given the relative scale of the propagation distance to the transducer size. (1) could then be expressed as a summation over the infinitesimal point source transducers. This technique is elucidated in [26].

In order to study the resultant heating, the Pennes bioheat equation [27] was solved. The bioheat equation is written as

$$\rho C_t \partial_t T = \kappa \nabla^2 T - \rho_b C_b W_b (T - T_b) + Q, \quad (2)$$

where ρ is the tissue density, C is the specific heat capacity of the tissue, κ is the tissue thermal conductivity, ρ_b is the blood density, C_b is the specific heat capacity of blood, W_b is the perfusion rate in the tissue, T_b is the blood temperature, and Q is a constant heat source, determined from the pressure field. Since the tissue was homogeneous, (2) was solved using a Fourier transform method introduced in [28]. All temperature simulations were done for a sonication time of 20s, in order to allow for sufficient diffusion of heat and perfusion effects of blood. The relationship between pressure in soft tissue, p , and absorbed power density, Q , is stated as [29]

$$Q = \frac{\alpha}{\rho c} |p|^2, \quad (3)$$

where α is the attenuation and c is the speed of sound in the medium.

Table I lists the parameter values used in the acoustic and thermal simulations.

C. Theory

The conventional focusing of ultrasound arrays consists of calculating the phase delays required to steer the beam to one [17] or more [15] points. It is argued here that by expanding the region of interest at the focus to encompass a volume with dimensions on the order of the wavelength, it is possible to manipulate the "orientation" of the focus in space, and as a result, rotate the focus freely as desired.

1) Quantifying a focal volume—In order to quantify *focus orientations* and *rotations*, it is necessary to introduce some formalism. It was assumed that the focused ultrasound acoustic pressure field could be well-fitted to a 3-dimensional Gaussian distribution, defined as

$$G(r) = (2\pi)^{-3/2} |\Sigma|^{-1/2} e^{-\frac{1}{2}(r-\mu)^T \Sigma^{-1} (r-\mu)}, \quad (4)$$

where μ is the location of the peak ultrasound intensity, and Σ is the weighted sample covariance matrix, defined as

$$\Sigma = \frac{\sum_i w_i}{(\sum_i w_i)^2 - \sum_i w_i^2} \sum_i w_i (r_i - \mu)^T (r_i - \mu). \quad (5)$$

The weighted sample covariance matrix is defined such that $w_i = |p(r_i)|$, where $p(r_i)$ is the pressure at sample point r_i , so that (5) may be rewritten as

$$\Sigma = \frac{\sum_i |p(r_i)|}{(\sum_i |p(r_i)|)^2 - \sum_i |p(r_i)|^2} \times \sum_i |p(r_i)| (r_i - \mu)^T (r_i - \mu). \quad (6)$$

It can then be thought that μ describes the focus, and Σ describes the focal volume. In this way, by arbitrarily thresholding the value of G , it is possible to extract a focal volume with the shape of an ellipse. From the natural definition of an ellipse in 3-dimensional space, it is then possible to assign major and minor axes, which can be taken as the eigenvectors of the covariance matrix defined in (5). The eigenvector corresponding to the largest eigenvalue is considered the major axis. Representing the major axis eigenvector in spherical coordinates, the orientation is naturally defined as (θ, ϕ) . This is illustrated in Fig. 2. In future reference,

the value of θ will be referred to as the *obliquity*. The meaning of the major axis eigenvalue is more ambiguous, and its precise definition will not be of concern here.

Since the focus is traditionally considered a point source, the phases assigned to the transducer elements are such that there is constructive interference at this point. In this paper, it is argued that there are benefits to considering the problem where the focal volume is considered. In this paper, the set of sampled points, called $\{r_i\}$, are taken in a regular Cartesian grid surrounding the target focus, with a sampling rate of 5 points per wavelength. Since the set of points is taken on the order of a wavelength, a total of $(2 \times 5 + 1)^3 = 11^3 = 1331$ points are taken. That is, $N = 1331$. It is assumed that there are $K = 4752$ independent elements, but the following formalism is valid for all K . Each transducer element corresponds to a specific $u = u(r)$ describing the phasing vector, which is subdivided into points of size $\lambda/5$.

2) Formalisms—The phasing problem when considering a set of points in the focal region can be described as solving the equation [15]

$$Hu = b, \quad (7)$$

for the phasing vector $u = u(r)$, where

$$H = \begin{pmatrix} H_p(r_1) \\ \vdots \\ H_p(r_N) \end{pmatrix}_{N \times K}, \quad (8)$$

and

$$H_p(r_n) = (p^1(r_n) \dots p^K(r_n))_{1 \times K} \quad (9)$$

and

$$b = \begin{pmatrix} p(r_1) \\ \vdots \\ p(r_N) \end{pmatrix}, \quad (10)$$

where $p^m(r_n)$ describes the pressure contribution at r_n from element m , so that

$$p^m(r) = \int_{S_m} \frac{jk_c \rho c}{2\pi \|r - r'\|} e^{-jk_c \|r - r'\|} u_m(r') dr', \quad (11)$$

and S_m is the surface of the transducer of element m .

3) Rotation of a Focus—Let R be a rotation matrix such that $\tilde{p}(x) = p(R^{-1}x)$, where $\tilde{p}$ is the pressure field after application of the rotation. That is, the pressure field has been rotated by R . The problem may then be stated as follows: Given a focal volume V , what distribution of power and phase, $\tilde{u}$, would return V rotated by some rotation matrix R ? Using the above formalism, it is possible to define a matrix problem

$$H[R]\tilde{u}=b, \quad (12)$$

where

$$H[R]=\begin{pmatrix} H[R]_p(r_1) \\ \vdots \\ H[R]_p(r_N) \end{pmatrix}_{N \times K}, \quad (13)$$

and

$$H[R]_p(r_n)=(p^1(R^{-1}(r_n - \mu)+\mu), \dots, p^K(R^{-1}(r_n - \mu)+\mu))_{1 \times K} \quad (14)$$

and b is defined as in (10). $H[R]$ corresponds to the forward propagator mapping the transducer velocity phasors, $\tilde{u}$, to the focal volume V .

The solution to this problem may be obtained using a Tikhonov regularization. Assuming that u is the solution to (7), then $\tilde{u}$ in (12) can be found by minimizing the functional

$$f(\tilde{u})=\|H[R]\tilde{u} - b\|^2 + \alpha\|\tilde{u} - u\|^2, \quad (15)$$

for some regularization parameter α . The regularization parameter, in this case, takes units of Pa^2 , since u is the velocity phasor and H is measured in Pascals, and b is measured in $\text{Pa} \cdot \text{m} / \text{s}^2$.

In this case, α plays the role of a Tikhonov regularization parameter, and can be considered a cost function for deviations of $\tilde{u}$ from u . The minimum norm solution to (15) is given by [22]

$$\tilde{u}=(H[R]^*H[R]+\alpha I)^{-1}(H[R]^*b+\alpha u), \quad (16)$$

where I is the identity matrix of size K and $*$ denotes the Hermitian transpose. It is important to note here that the result for general isometries would be derived identically, since isometries in a 3-dimensional Cartesian space are linear operators.

4) Generalizations—Given a sufficient number of degrees of freedom, it is possible to also define independently-rotated multiple foci. In this case, suppose that there are M focal volumes, labeled as V_i for $i = 1 \dots M$. Then (14) can be written

$$H[R] = \begin{pmatrix} H_{V_1}[R_1] \\ \vdots \\ H_{V_i}[R_i] \\ \vdots \\ H_{V_M}[R_M] \end{pmatrix}_{(\sum_{i=1}^M N_i) \times K} \quad (17)$$

and

$$H_{V_i}[R]_p(r_{i,n}) = (p^1(R_i^{-1}(r_{i,n} - \mu_i) + \mu_i), \dots, p^K(R_i^{-1}(r_{i,n} - \mu_i) + \mu_i))_{1 \times K}, \quad (18)$$

where R_i and μ_i correspond to the rotation matrix and mean position of the i^{th} focal volume, V_i , respectively. It is assumed here that $r_{n,i} = r_{m,j}$ for $m = n$, for all i, j .

D. Error Analysis

The effect of error in the phasing controls on the resultant acoustic field was analyzed. Various levels of random error were added to the transducer power assigned to each element in the array, as well as the phasing used. Low ($\langle \text{SNR} \rangle = 5$), moderate ($\langle \text{SNR} \rangle = 2$), and high ($\langle \text{SNR} \rangle = 4/3$) levels of error were analyzed, and the resultant pressure field normalized to the intensity of the error-free case, were described.

III. RESULTS

A. Focus Rotations

Fig. 3 demonstrates the distribution of temperature resulting from the rotation of a focus to predetermined angles, where in each case the normalized temperature distribution is shown. As a reference, Fig. 3a shows the original focus without rotation, using the conventional point-source phasing technique. Figs. 3b, 3c, and 3d show the rotation of the focus by 15° , 30° , and 45° about the y-axis, respectively. In all three cases, the inverse problem was solved with $\alpha = 10^3$. To give an idea of the spatial distribution of heating using this method, Fig. 4 provides a 3-dimensional representation of a 45° rotation in the yz-plane, contrasted to the focus distribution using conventional phasing techniques. In this case, the temperature is thresholded at half the maximum temperature in the focal region. From this figure, it is evident that the focusing is maintained while the focal volume is rotated. An issue of note is that the volume of heating increases as the angle of rotation increases. In the three cases presented in Fig. 3, a 15° rotation results in an 18% increase in the focal volume, a 30° rotation results in an 60% increase in the focal volume, and a 45° rotation results in an 141% increase in the focal volume, as compared to the focus obtained from standard focusing. The

focal volume in all three cases was taken as the volume inside the 50% contour of the peak temperature.

Fig. 5 demonstrates the role of the regularization parameter α on the quality of the focusing and the rotation of the focus, as well as the distributions of transducer velocities and phases to obtain these pressure maps. In this case, the problem is set up such that the focus should be rotated 30° about the y-axis. The first row illustrates the focus obtained for various α -values. The second and third rows show the transducer velocity magnitude and phase, respectively, of the transducer elements in the array, determined by solving (12).

The focus obtained using conventional point-source focusing is shown in Fig. 5a. It is shown in Fig. 5b that in the underregularized case, since the problem is ill-posed, the focusing disappears, and there is a large deposition of energy in the peripheral regions. On the other hand, with a sufficiently high α , the cost of deviation from the initial phasing is too high, and hence there is minimal change from the initial pressure pattern, as shown in Fig. 5d. Fig. 5c shows that for certain intermediate α , it is possible to obtain a good balance between focusing and the desired focal rotation. This is also illustrated by contrasting Figs. 5e and 5i to Figs. 5h and 5l. The high α -value results in a phase and amplitude plot nearly identical to the case of conventional phasing. The intermediate cases of $\alpha = 10^{-4}$ and $\alpha = 10^4$ in Figs. 5f and 5g illustrate significant amplitude deviations from the original cases. It is clear from the phase plots in Figs. 5j and 5k that in the underregularized case, the phase deviates significantly from the conventional phasing case, resulting in the non-existent focusing illustrated in Fig. 5b. Figs. 5b–d are normalized about the mean transducer velocity, and thresholded below twice this mean, in order to have sufficient contrast.

Fig. 6 plots the normalized peak pressure, $|p|$, the focal obliquity, θ , and the standard deviation of the element powers for a range of values of α . The values are normalized such that the total power outputs of the transducer array are identical for different α . In addition, the standard deviation is normalized to the maximum deviation of powers. As suggested in Fig. 5, there is a complex relationship between α , θ , P , and $|p|$. Ideally, the solution to the problem should render a high peak pressure, minimal change in the distribution of power amongst the elements, and the focus should be oriented in the exact direction spatially. As shown in Fig. 6, however, the peak pressure is considerably lower as the angle of obliquity increases, and the powers vary more significantly. This is to be expected, of course, since the problem posed in (15) is a balance between the conventional phase controls and the minimum-norm solution of the obliquity problem, which can be ill-posed.

Fig. 7 quantifies the effect of α on the peak acoustic pressure for $x = 0, 20$, and 40 mm laterally, and at depths of $z = 50$ and 100 mm. As can be seen from the figure, a higher α value, corresponding to higher levels of regularization, leads to a higher peak pressure. Although it has been previously been shown that the pressure magnitude of lateral targets is lower than central targets for this transducer array configuration [23], for a given rotation, the relative amplitude for each rotation remains relatively constant.

Fig. 8 demonstrates the achieved rotation angle for each target angle at various positions. It can be seen that for lower α , the obtained obliquity is closer to the target angle of obliquity,

illustrated by the diagonal line in the figure, which indicates an optimal attainment of the target angles between 0 and 45°. The peak achievable angle of rotation, however, is greatly diminished for lateral positions, illustrated at $x = 20$ and 40 mm.

Fig. 9 demonstrates the ability of this focusing method to re-orient the focal volume along the z -direction of the array. In Fig. 9a, foci at depths of 50 and 100 mm, and 0, 20, and 40 mm laterally were simulated using conventional focusing. Consistent with [23], the foci off-axis are oblique, and the angle of obliquity increases as the steering angle increases, as expected. By fitting each focus to a Gaussian, it is possible to determine the rotation matrix R that will re-orient the focus along the z -axis. Fig. 9b illustrates the re-orientation of the foci along the z -axis of the array using this method. In all six cases, a value of $\alpha = 10^3$ was used when solving (16).

B. Multiple Independent Rotations

The synthesis of multiple independently-rotated foci using (17) is illustrated in Fig. 10. Fig. 10a demonstrates the phasing pattern using the conventional multiple focus synthesis technique introduced in [15]. The rotation of the left and right foci by 45° and -45°, respectively, is demonstrated in Fig. 10b. In this case, a value of $\alpha = 10^3$ was used.

C. Near-Field Heating Analysis

The potential for near-field heating is a substantial concern for focused ultrasound treatments, and has been studied previously for conventional phasing techniques and uterine fibroid ablation [14]. Fig. 11 demonstrates the role of α on near field heating, for α values of 10^3 , 10^4 , and 10^5 . It can be seen from the figure that a decreased level of regularization, manifesting as smaller α , has the effect of increased near-field heating. This is of concern to any clinical application of this technique for ablation, where the use of high temperature elevation at the focus must be accompanied by low levels of heating elsewhere, in the interest of patient safety.

D. Error Analysis

Fig. 12 illustrates the effect of additive noise on the resultant acoustic pressure field, showing (a) the focus obtained through traditional focusing, (b) the rotated focus solved with error-free information from Eq. 16, (c) the solution to Eq. 16 with 20% additive noise, and (d) the solution to Eq. 16 with 50% additive noise. The error was added to the velocity of the transducer elements. As can be seen, the peak acoustic pressure at the focus drops with added error, in a similar fashion to the traditional focusing case, and the sidelobes are more pronounced.

IV. DISCUSSION

The method introduced here to perform an isometry on a focal volume provides an additional degree of control over the manifestation of the focus when performing a thermal focused ultrasound treatment. The possibility of heating extraneous structures necessitates an understanding of the focal orientation and the ability to spare surrounding tissue. As can be seen from the results of the paper, α plays a critical role in the rotation of a focus.

A. The role of α

The phasing technique presented here relies heavily on the value of α used in each solution to (15). In particular, the optimal value of α is problem-specific, and its role as both a Tikhonov regularization parameter and a cost factor between the original phasing pattern and the desired focus isometry further complicates matters. As was demonstrated in Fig. 5, a sufficiently large α -value will result in the focus obtained from conventional phasing, whereas a sufficiently small α -value will result in a complete loss of focusing of the ultrasound. Ideally, it would be useful to develop a technique to implicitly determine the precise value of α for each geometry; however, an optimization over the solution space to each specific problem is more likely to garner useful, albeit less insightful, results. It must be noted that the computational cost of the inverse problem was found to be heavily-dependent on the generation of H and H_R in (8) and (15), rather than the determination of the solution for each α .

A judicious choice of α for any given rotation must balance the desired obliquity, the peak pressure amplitude, and the standard deviation of power. An example of these variables as a function of α is plotted in Fig. 6. In this particular case, a reasonable choice of α would lie between 10^3 and 10^6 , since it is between these values that there is still a significant obliquity while maintaining a relatively equal distribution of power among the elements and a high peak pressure amplitude. The precise value of α , however, must be determined using a metric that quantifies the desired balance between the three factors.

B. Limitations

It must be noted that there are several limitations to the presented technique for rotating foci.

Firstly, only a rudimentary analysis of the effect of error was implemented. It appears, however, that in Fig. 12, random error added to the inverse phasing controls does not significantly affect the focusing quality. There are multiple potential sources of error that could arise in an experimental implementation: transducer surface imperfections, imperfect impedance matching, and errors relating to amplifier gain, to name a few. In addition, as described in [21], the presence of unknown error severely complicates the determination of the Tikhonov regularization parameter, α . If it is assumed that the norm of the error can be well estimated, then the Morozov discrepancy principle can be applied to determine an initial estimate for α [21], which we will call $\alpha_{Morozov}$. This initial estimate will ignore the secondary role that α plays in this application: namely, as a cost parameter. Further deviations from α can then be considered as $\alpha = \alpha' + \alpha_{Morozov}$, where $\alpha' \in (-\alpha_{Morozov}, \infty)$, in order to achieve the focus isometries presented here.

The second limitation is the potential for no good α value to manifest. Naturally, it is possible that there are insufficient degrees of freedom to achieve the desired focal orientation, or to obtain the desired focal orientation without physically unrealistic parameters. It is also possible that the transducer geometry does not allow for the desired angle to be reached. For instance, from the distribution of power illustrated in Fig. 5g, there appears to be a geometric component to the rotation of the focus. As a result, it would seem that a curved or hemispherical transducer array would present a more desirable geometry for

rotation to higher angles. In addition, it is possible that by increasing sufficiently N/K , such that the number of transducer elements is significantly greater than the number of control points, it may be possible to increase the maximum rotation angle achievable, up to a limit.

In the same light, the value of α has been shown to control multiple aspects of focusing quality, including near-field heating and peak acoustic pressure. From Fig. 8, it is clear that for sufficiently high rotation angle θ , there is no α that allows the focus to reach this obliquity. The maximum attainable angle with the presented geometry appears to be approximately 35° . A curved transducer array could potentially increase the maximum angle achievable because the natural geometry is more conducive to focus rotations.

A third limitation relates to the case presented in Fig. 9, where the foci were re-aligned with the z -axis for steered positions. As can be seen from comparing the foci at $(x = 40, z = 100)$ in Fig. 9a and Fig. 9b, the focal volume thresholded at 60°C in the latter case encompasses a larger area than the former, when using conventional phasing techniques. This shows that as the focus is steered further away from the transducer array, the focus rotations become more costly to the focusing, and the focusing quality can diminish. This is a similar effect as when comparing the focusing at $z = 50$ mm and $z = 100$ mm from the array: as the focus is steered further from the transducer, the focus quality decreases.

As has been shown in transcranial focused ultrasound, the use of amplitude and phase corrections at the focus minimizes the focal volume and concentrates the energy in the most efficient manner. Naturally, one would then assume that as the distribution of power among the array elements increases in range, the deposition of energy at the focus becomes wider. As has been shown in Fig. 8, a lower value of α yields a higher rotation, while the distribution of power among the elements deviates further from the mean. As a result, using this technique, it appears that an increased focal volume is an additional cost. Future techniques may overcome this limitation.

In a similar light, it is evident from Fig. 3 that there are limitations on the achievable angles of obliquity given a transducer array geometry and degrees of freedom. It is possible that other array geometries could be tested in future work for their efficacy in rotating focal volumes.

C. Comparison to Existing Phased Array Control Techniques

Several past works have investigated the used of phased array controls to modify the phasing pattern generated from a phased array of ultrasound transducers [15], [20]. In particular, Hertzberg and Navon demonstrate that it is possible to position multiple foci in order to obtain a desired ultrasound field map using holographic techniques [20], and Ebbini and Cain [15] proposed an iterative multiple focus synthesis technique that allows for an even distribution of acoustic power. The techniques presented here, demonstrated in particular in Fig. 10, can complement these techniques, by adding control on a sub-wavelength scale, allowing for the control of the spatial manifestation of each focus in the acoustic field pattern.

D. Computational Aspects

The implementation of this technique to real-time applications must be considered. Using an NVIDIA GPU for parallel processing and a dual-core computer, the computation of the phasing controls for a single point requires on Matlab (2015a, The MathWorks, Natick, MA, USA) approximately eight minutes. In addition, for a simulation domain spanning $28 \times 28 \times 28 \text{ cm}^3$ and 470^3 grid points, the computation requires approximately 1.6 GB of memory to compute. In future applications, parallelization and faster numerical methods could potentially be explored in order to reduce the computation time for experimental implementation.

V. Conclusions

A method to control the orientation of a focus generated by an ultrasound phased array was introduced. It was shown that, in most cases, by using a Tikhonov regularization with a judicious choice of the hybrid regularization-cost parameter α , it was possible to freely rotate the focus while maintaining a relatively high peak pressure amplitude and having a high array efficiency. It was shown that it is possible to re-orient a steered focus along the main axis of the transducer array using a clinically-relevant focused ultrasound array. It was also shown that it is possible to perform focus isometries on multiple foci independently. Limitations to this technique include the assumption of minimal error, the possibility that no good α -value will manifest, and the diminishing efficacy of the method further from the transducer array, in a manner similar to the conventional point-source focusing technique.

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Biographies

Alec Hughes is a Ph.D. student in the Department of Medical Biophysics at the University of Toronto, Toronto, Canada, working in the Focused Ultrasound Laboratory at the Sunnybrook Research Institute under Dr. Kullervo Hynynen. He obtained his undergraduate degree in mathematics and physics from the same university in 2012. His interests include applications of ultrasound phased arrays and transcranial focused ultrasound.

Kullervo Hynynen received the Ph.D. degree from the University of Aberdeen, Aberdeen, U.K. After completing his postdoctoral training in biomedical ultrasound, also at the University of Aberdeen, he accepted a faculty position at the University of Arizona in 1984. He joined the faculty at the Harvard Medical School, and Brigham and Womens Hospital in Boston, MA, USA, in 1993. There he reached the rank of Full Professor, and founded and directed the Focused Ultrasound Laboratory. In 2006, he moved to the University of Toronto, Toronto, ON, Canada. He is currently the Director of the Physical Sciences research platform and the Centre for Research in Image-Guided Therapeutics at the Sunnybrook Research Institute and a Professor in the Department of Medical Biophysics at the University of Toronto. Dr. Hynynen holds a Tier 1 Canada Research Chair in Imaging Systems and Image-Guided Therapy awarded by the Government of Canada. He was the recipient of the 2014 Rayleigh Award.

Fig. 1. The transducer array with $\lambda/2$ center-to-center element spacing and outer diameter of 12 cm, showing the pressure field at 50 mm in the z -direction.

Fig. 2.
A schematic describing the treatment of the focus as a Gaussian distribution, with orientation described by its major axis.

Fig. 3.

A demonstration of the rotation of the focus at a depth of 50 mm, along the main axis of the array, by (a) 0°, (b) 15°, (c) 30°, and (d) 45°. The normalized temperature profile is shown.

Fig. 4.

A 3-dimensional representation of the new focusing technique introduced, demonstrating (a) conventional focusing techniques, and (b) the focusing technique introduced here which allows for the focus to reorient spatially.

Fig. 5.

The effect of varying α when attempting to rotate the focus 30° about the y-axis. The top row illustrates the resultant normalized acoustic fields. The second and third rows show the transducer velocity magnitude and phase, respectively, of the transducer elements in the array, determined by solving (12).

Fig. 6.

The role of α : The normalized peak pressure, $|p|$, the focal obliquity, θ , and the standard deviation of the element powers, $\sigma(P)$, are plotted. Note the logarithmic scale of α .

Fig. 7.

The effect of α on the peak acoustic pressure at the focus for various rotation angles, at locations 50 and 100 mm in front of, and 0, 20, and 40 mm laterally from the transducer array.

Fig. 8.

The effect of α on the achieved focal obliquity for various target rotation angles, at locations 50 and 100 mm in front of, and 0, 20, and 40 mm laterally from the transducer array.

Fig. 9.

(a) The original foci and (b) the foci realigned with the z axis, at depths of 50 and 100 mm, and 0, 20, and 40 mm laterally. The contour shown encompasses the area with heating above 60°C. $\alpha = 10^3$ was used.

Fig. 10.

(a) Multiple focus synthesis using the technique outlined in [15], and (b) independently rotating the left and right foci by 45° and -45° , respectively. The images and scales depict normalized $|p|^2$. The relative (c) and normalized (d) pressure profiles along the y-axis before (solid line) and after (dashed line) rotation are also plotted, in order to compare the relative peak pressures and the relative sidelobe effects, respectively.

Fig. 11.

The effect of rotation on the near field of the transducer array compared to (a) conventional focusing, for (b) $\alpha = 10^3$, (c) $\alpha = 10^4$, and (d) $\alpha = 10^5$.

Fig. 12.

The effect of additive noise on the resultant acoustic pressure field for a focus rotated by 30° , with regularization parameter $\alpha = 10^4$.

TABLE I

Simulation parameters used in solving (1) and (2), obtained from [4], [30], [31].

Symbol	Meaning	Value
c	Speed of sound in soft tissue	1545 m s^{-1}
ρ	Soft tissue density	1030 kg m^{-3}
C_t	Specific heat capacity of tissue	3600 J kg K^{-1}
α	Tissue attenuation coefficient	$5.0 \text{ Nep m}^{-1} \text{ MHz}^{-1}$
κ	Thermal conductivity of tissue	$0.6 \text{ W m}^{-1} \text{ K}^{-1}$
T_0	Body temperature	37.0°C
ρ_b	Blood density	1030 kg m^{-3}
C_b	Specific heat capacity of blood	3620 J kg K^{-1}
W_b	Perfusion rate in soft tissue	$0.5 \text{ kg m}^{-3} \text{ s}^{-1}$