Published in final edited form as:

Int J Numer Method Biomed Eng. 2017 April ; 33(4): . doi:10.1002/cnm.2815.

Fluid-Structure Interaction and Structural Analyses using a Comprehensive Mitral Valve Model with 3D Chordal Structure

Milan Toma^a, Daniel R. Einstein^b, Charles H. Bloodworth IV^a, Richard P. Cochran^c, Ajit P. Yoganathan^a, and Karyn S. Kunzelman^c

^aWallace H. Coulter Department of Biomedical Engineering, Georgia Institute of Technology and Emory University, Technology Enterprise Park, Suite 200, 387 Technology Circle, Atlanta GA, 30313-2412, U.S.A

^bDepartment of Mechanical Engineering, St. Martin's University, 5000 Abbey Way SE, Lacey WA, 98503, U.S.A

^cDepartment of Mechanical Engineering, University of Maine, 219 Boardman Hall, Orono ME, 04469-5711, U.S.A

Abstract

Over the years, three-dimensional models of the mitral valve have generally been organized around a simplified anatomy. Leaflets have been typically modeled as membranes, tethered to discrete chordae typically modeled as one-dimensional, non-linear cables. Yet, recent, high-resolution medical images have revealed that there is no clear boundary between the chordae and the leaflets. In fact, the mitral valve has been revealed to be more of a webbed structure whose architecture is continuous with the chordae and their extensions into the leaflets. Such detailed images can serve as the basis of anatomically accurate, subject-specific models, wherein the entire valve is modeled with solid elements that more faithfully represent the chordae, the leaflets, and the transition between the two. These models have the potential to enhance our understanding of mitral valve mechanics, and to re-examine the role of the mitral valve chordae, which heretofore have been considered to be “invisible” to the fluid and to be of secondary importance to the leaflets. However, these new models also require a rethinking of modeling assumptions. In this study, we examine the conventional practice of loading the leaflets only and not the chordae in order to study the structural response of the mitral valve apparatus. Specifically, we demonstrate that fully resolved 3D models of the mitral valve require a fluid-structure interaction analysis to correctly load the valve *even* in the case of quasi-static mechanics. While a fluid-structure interaction mode is still more computationally expensive than a structural-only model, we also show that advances in GPU computing have made such models tractable.

Keywords

Fluid-structure interaction; Mitral valve; Forces; Comprehensive computational model; Papillary muscle; Chordal structure; Chordae tendineae

Conflict of Interest

No benefits in any form have been or will be received from a commercial party related directly or indirectly to the subject of this manuscript.

1. Introduction

The anatomy of the mitral valve (MV) is complex; it includes two asymmetric leaflets attached to the left ventricle at the annulus, and numerous chordae tendineae that serve to attach the leaflets to the left ventricle via the papillary muscles. The chordae attach in a transitional way to both the free edge of the leaflet and to the ventricular surface. Some smaller chords do not connect to the leaflet at all but rather provide connections between adjacent chords. The function of the valve throughout the cardiac cycle is complex, and involves interaction among all of these structures. To this day, a complete understanding of the complex mechanical function of the normal MV remains elusive.

Three-dimensional computational mechanics models have been developed over the years to better understand both normal mitral valve function,^[1, 2, 3, 4, 5] dysfunction in the presence of disease,^[6, 7, 8, 9] or function/dysfunction in surgically repaired states^[10, 11, 12, 13] Modeling outcomes are typically quantitative predictions of tissue stress or hemodynamic alterations^[9, 3, 14, 15] resulting from disease or surgery. Only in the latter case have researchers attempted to model fluid-structure interactions (FSI), since, clearly, the mitral valve exists as a submerged structure in a highly dynamic fluid environment. Still, researchers have long foregone the complexity of an FSI analysis and focused on static and quasi-static loading of the valve during closure by simply applying a pressure boundary condition on either the atrial or ventricular sides of the leaflets.^[16]

This simplification has been possible because the leaflets have been modeled as two-dimensional elements (shells or membranes) and because the chordae have been modeled as separate, discretely connected one-dimensional elements (cables or beams).^[17, 18, 19, 20, 21, 22, 23, 24, 25, 26] This conceptualization facilitates applying a pressure boundary condition to the ‘leaflets’ and ignoring the chordae. So ingrained has this thinking become that it is now commonplace to conceptualize the chordae as strings that are ‘invisible’ to the fluid. However, this is not an accurate assumption.

In fact, the physical distinction between the chordae and the leaflets is not always clear, a point that has become more apparent with the increasing availability of high-resolution imaging data of the mitral valve. With no clear boundary between chordae and leaflets, *a priori* application of a pressure boundary condition to the leaflets becomes problematic in the absence of fluid loading.

To investigate the validity of simplified applied pressure boundary conditions with geometrically faithful, imaging-based models, we developed a fully 3D model of the mitral valve wherein the entire mitral apparatus was directly determined from the segmentation of high-resolution CT data.^[4, 5] In this model, there are no anatomical delineations between chordae and leaflet. Moreover, all mechanical behavior of the valve apparatus is governed by a unified constitutive model with spatially varying parameters. Unlike previous models of the mitral valve, the chordae are fully three-dimensional and their transition into the leaflets is anatomically accurate. In this paper, two models are analyzed. In the first model, abbreviated as FSI, the entire valve apparatus is mechanically coupled to the fluid. Thus, the

interfacial pressure develops organically. In the second model, abbreviated as STR, a pressure boundary condition, equivalent to the one in the FSI model, is applied to the ventricular side of the ‘leaflets’ and no fluid-structure interaction is considered. We then compare model predictions to measured forces that develop in the chordae.

2. Methods

Model development, including μ CT in-vitro scanning, image processing and mesh generation have been previously described and validated.^[4, 5] In short, explanted ovine mitral valves were mounted in an in-vitro setup, and structural data for the mitral valve was acquired with μ CT. Experimental data from the in-vitro ovine mitral valve system were used to validate the computational model.

2.1. Tissue

The constitutive model for the material properties utilized a three-dimensional splay invariant, based on an approximation of a three-dimensional Gaussian distribution of fibers.^[27] The model is defined in terms of the 2nd Piola-Kirchhoff stress, $\mathbf{S}$, as follows

$$\mathbf{S} = \kappa J(J-1)\mathbf{C}^{-1} + \mu J^{-2/3} \mathbf{Dev}[(I - \bar{\mathbf{C}}^{-2})/4] + \sum_{i=1}^n \sigma_i(\lambda_i) J^{-2/3} \mathbf{Dev}[\mathbf{K}_i] + \sum_{i=1}^n \varepsilon(\lambda_i) J^{-2/3} \mathbf{Dev}[\mathbf{K}_i],$$

where J is Jacobian, κ is bulk modulus, σ_i is passive fiber stress-strain rule for the i^{th} (1 or 2) fiber population, ε is active fiber stress-strain rule, $\mathbf{Dev}$ is deviatoric projection operator, and $\mathbf{C}$ is right-Cauchy deformation, and λ_i is dispersed fourth invariant defined as

$$\lambda_i^2 = \text{tr}(\mathbf{K}\bar{\mathbf{C}}),$$

where $\bar{\mathbf{C}}$ is the isochoric part of the right-Cauchy deformation, $\mathbf{K}$ is called the dispersion tensor or anisotropy tensor and is given in global coordinates; note, as $\mathbf{K}$ embodies the concept of dispersion, λ is not a stretch in a classical sense. The matrix $\mathbf{K}$ is governed by splay parameters described in literature.^[27] The biaxial stress-strain behavior of the mitral leaflets (Fig. 1(a)) is characterized by the structural constitutive equation.

The passive fiber model is defined in the fiber coordinate system, and is given by a micro-structural model^[27] based on the physiology of crimped collagen fibers (see Alg. 1, Fig. 1(b)); specifically, given a fiber stretch λ , this model returns the true stress σ/λ and tangent modulus $d\sigma/d\lambda$ of the fiber. There are three physiologic parameters (material constants defined at the top of the algorithm) that the user must supply. The discontinuity in the $d\sigma/d\lambda$ curve indicates that the model presented in Alg. 1 predicts a stress σ response that is continuous and once-differentiable in stretch λ up to fiber failure.

Specific parameter values used in this model are given below. For all structures, the bulk modulus, κ , was 2.80×10^4 kPa; the isotropic parameter μ was 10.0 kPa and the fiber stiffness, E_f was 4.8×10^4 kPa. Dimensionless parameters H_0/η_0 and R_0/η_0 were 14.5 and

1.75 for the anterior leaflet, 7.0 and 1.0 for the posterior leaflet, and 8900.0 and 25.0 for the papillary muscles. For all chordae, R_0/t_0 was 5.0. H_0/t_0 varied for most chordae, ranging from 1.35 for the strut chordae to 90 for the marginal chords.

2.2. Fluid

Fluid-structure interaction of cardiac valves is challenging because valve closure represents a change in topology. We chose to adopt an approach based on smooth particle hydrodynamics (SPH) because of its flexibility and because of its compatibility with GPU architectures.^[29] With SPH, the fluid is modeled as a collection of discrete particles. In our approach, the fluid is assumed to be barotropic, weakly-compressible and inviscid, and is governed by a set of ordinary differential equations as follows.

The continuity equation, i.e. conservation of mass, is

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \mathbf{v} \Rightarrow \frac{d\rho_i}{dt} = -\rho_i \sum_{j=1}^{N_p} (\mathbf{v}_i - \mathbf{v}_j) \frac{m_j}{\rho_j} \cdot \nabla_i W_{ij} + 4^{th} \text{ order diffusive term},$$

where W is the Kernel function and the 4th order diffusive term improves the pressure evolution.

The momentum equation, i.e. Euler equation, is

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} + \mathbf{f} \Rightarrow \frac{d\mathbf{v}_i}{dt} = -\sum_{j=1}^{N_p} \left(\frac{P_i + P_j}{\rho_i \rho_j} + \Pi_{ij} \right) m_j \cdot \nabla_i W_{ij} + \mathbf{f}_i,$$

where Π_{ij} is the artificial viscosity. The artificial viscosity is introduced to capture shocks. As a result, the SPH can handle strong shocks and vorticity generation.

The energy equation, i.e. first law of thermodynamics, is

$$\frac{d\varepsilon}{dt} = -\frac{P}{\rho} \nabla \cdot \mathbf{v} \Rightarrow \frac{d\varepsilon_i}{dt} = -\sum_{j=1}^{N_p} \left(\frac{P_i}{\rho_i^2} + \frac{\Pi_{ij}}{2} \right) (\mathbf{v}_i - \mathbf{v}_j) m_j \cdot \nabla_i W_{ij}.$$

This equation describes changes in ε due to entropy changes and due to volume changes (compression, expansion). Hence, each particle carries either the energy or the entropy per unit mass as independent variable.

To be able to integrate the above system of common differential equations, additional information about the motion of the fluid particles themselves is needed in form of equations of motion and state. The motion equation is

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \Rightarrow \frac{d\mathbf{r}_i}{dt} = \sum_{j=1}^{N_p} (v_i - v_j) \frac{m_j}{\rho_j} W_{ij}$$

and the state equation

$$P = P(\rho) \Rightarrow P_i = c_0^2 (\rho_i - \rho_0).$$

Equation of motion for fluids with stiff equation of state, i.e. almost incompressible fluids, help the fluid particles retain a relative controlled regular composition, which means that the particles move more orderly.

The artificial viscous term above, Π_{ij} , is defined as

$$\Pi_{ij} = \begin{cases} \frac{-\alpha c_{ij} \phi_{ij} + \beta \phi_{ij}^2 (\mathbf{r}_i - \mathbf{r}_j)}{\rho_{ij}} & \text{for } (\mathbf{r}_i - \mathbf{r}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) < 0 \\ 0 & \text{for } (\mathbf{r}_i - \mathbf{r}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) \geq 0 \end{cases},$$

where

$$\begin{aligned} \phi_{ij} &= \frac{h_{ij} (\mathbf{r}_i - \mathbf{r}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j)}{\|\mathbf{r}_i - \mathbf{r}_j\|^2 + \eta}, \\ c_{ij} &= \frac{1}{2} (c_i + c_j), \\ \rho_{ij} &= \frac{1}{2} (\rho_i + \rho_j), \\ h_{ij} &= \frac{1}{2} (h_i + h_j). \end{aligned}$$

In the viscous term above, the usual constants α and β for FSI water free surface are 0.03 and 0.001 respectively, c is speed of sound, $\eta = 0.01 h_{ij}^2$ is to prevent numerical divergence when two particles approach each other and $\mathbf{v}$ represents the particle velocity vector. The viscosity is further treated using laminar viscous term in Navier-Stokes equation as follows

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} + \mathbf{f} + \frac{1}{\rho} \nabla \cdot (\mu \nabla \mathbf{v}),$$

where the laminar viscous term becomes

$$\frac{1}{\rho} \nabla \cdot (\mu \nabla \mathbf{v}) \Rightarrow \sum_{j=1}^{N_p} \frac{m_j (\mu_i + \mu_j) (\mathbf{r}_i - \mathbf{r}_j)}{\rho_i \rho_j (\|\mathbf{r}_i - \mathbf{r}_j\|^2 + \eta)} \cdot \nabla_i W_{ij} (\mathbf{v}_i - \mathbf{v}_j).$$

In the weakly compressible SPH formulation, the speed of sound for each particle must be chosen carefully to ensure that the fluid is very closely incompressible. To determine the

values for c , we consider compressibility effects scale with $O(M^2)$, where M denotes the Mach number of the flow. So, we obtain

$$\frac{|\Delta\rho|}{\rho_0} \approx \frac{\|\mathbf{v}\|^2}{c^2}.$$

Thus, we can ensure that $|\rho/\rho_0 - \eta|$ if the speed of sound is assumed to be large enough such that

$$c > \frac{\|\mathbf{v}\|}{\sqrt{\eta}}.$$

Typically, η is set to 0.01, i.e. following density variations of the order of 1%.

2.3. Fluid-Structure Interaction

The SPH particles interact with the structural finite elements using a penalty based contact algorithm. In penalty-based contact, when a penetration is found, a force proportional to the penetration depth is applied to resist and eliminate penetration. Linear contact spring stiffness is based on the nodal masses that come into contact and the time step size as follows

$$k = 0.1 \frac{m}{\max(\Delta t_{global}^2, \Delta t_{SPHcontact}^2)}.$$

The resulting contact stiffness is independent of the material constants so it is well suited for treating contact between fluid and structure.

2.4. Boundary Conditions

For the FSI analysis, fluid particles were confined in a pipe-like rigid structure surrounding the MV model and prescribed velocity boundary conditions were applied to the open ends to develop clinically relevant pressures, see Fig. 2. The prescribed velocity profiles used in the simulations were based on the data used in the in-vitro measurements.^[30] The nodes at the bottom of the papillary muscles were fixed in all three directions, as well as the nodes on the annular attachment. Pressure develops from flow of the fluid particles. During the systole, the fluid particles are driven towards the atrium. As a result, the pressure builds up only on the ventricular surface of the valve and subsequently drives the closure of the leaflets. In the STR analysis, physiologically equivalent pressure fields were applied directly to the ventricular surface of the leaflets to simulate the closure.

2.5. Solution

At run time, 346,676 discrete particles were created in the fluid domain. Fluid motion and boundary interaction was solved with the the IMPETUS Afea SPH Solver[®], while large

deformation in the solid mitral valve was simultaneously solved with the IMPETUS Afea Solver[®]. Both the solvers use a commodity GPU for parallel processing. All solid elements were fully integrated removing the possibility of hourglass modes and element inversion that plagues the classic under-integrated elements.

Both fluid and solid domains and their interaction were solved with an explicit integration scheme. All simulations were solved on a standard workstation. Parallel acceleration was achieved with a Tesla K40 GPU with 12 GB of Graphic DDR memory and 2880 CUDA Cores. To confirm that convergence was reached, h-refinement of the finite element mesh was performed and the solution was found to yield same results.

3. Results

The leaflet and chordal stresses computed by the STR and FSI simulations showed a distinct increase in stress magnitude at full closure (Fig. 3b), as compared to the open state (Fig. 3a). The stress was not symmetrically distributed, a result of the valve specific geometry, which is itself not symmetric. The local maxima of the principal stress values (1 MPa) were found primarily on the chordae tendineae and their connections to the leaflets.

Comparable closure is reached using the STR analysis only, but the behavior of the chordae tendineae is obviously different and not in agreement with in-vivo (or in-vitro) observations.^[30] By comparing the STR and FSI results in Fig. 3 at two different time points, a clear difference can be seen not only in the stress distributions, but also in the shape of the chordae tendineae, especially at $T = 0.75 \cdot T_{sys}$ where full closure has not been reached yet and is expected to be reached at T_{sys} ; as demonstrated by the dashed lines tracing two of the chordae (Fig. 3(a)). The chordae in Fig. 3(a) for STR analysis are visibly arched while the chordae in Fig. 3(a) for FSI analysis are straight. Thus, if we examined the forces at each chordal attachment and plot the resulting vectors, their directions would naturally be different, as shown in Fig. 4.

Furthermore, the time course of chordal attachment forces along all three axes were in very good agreement between computational FSI and experimental measurements (Fig. 5(b)).^[4] The same is not shown in the case of the structural analysis (Fig. 5(a)).

On top of that, the FSI analysis can provide additional fluid-related results that STR analysis cannot. The Fig. 6 shows the regurgitant volume measured as the mass of the fluid particles crossing the area at the level of the MV annulus. No regurgitation is observed from the point when both the leaflets come in contact together, whereas the highest level of regurgitation is observed at time point $T = 0.3 \cdot T_{sys}$.

4. Discussion

This work addresses the question whether FSI analysis is actually necessary when simulating the MV closure. It is shown that the time course of the papillary muscle forces is not in agreement with the experimental measurements when the fluid domain is omitted from the simulations. It is, however, assumed that the difference would be smaller if simplified representation of the chordae tendineae was used. However, in that case it would

not be possible to validate this model with the experimental results^[30] that are directly related to the subject-specific chordal structure unless closely tuned intentionally. Moreover, in simplified models, the distribution of stress, particularly at the “junction” between chords and leaflets, is an artifact of modeling choices rather than patient-specific properties, including geometry. Thus, when chordae related pathologies are to be investigated and valve-specific geometry is used for that purpose, the FSI is necessary for accurate analysis.

It is important to note that, while prior to closure the magnitudes and directions of chordal force are different, e.g. at $T = 0.75 \cdot T_{sys}$, at closure, i.e. at $T = T_{sys}$, the magnitudes and directions of chordal forces are comparable. However, select chordal forces and directions are but one set of validation measures. Fig. 3 illustrates that stress distribution in both the chordae and the leaflets are quite dissimilar.

Finally, an FSI analysis permits the investigation of the influence of chordae tendinae on fluid flow in both health and disease. To our knowledge, this question has never been looked at. Even casual consideration of MV geometries now resolvable with high-resolution imaging indicate that the assumption that chords are ‘invisible’ to ventricular blood is inaccurate. It is a distinct advantage of FSI methods that they can realistically model regurgitant blood flow and volume (Fig. 6). Furthermore, the FSI analysis has a potential to investigate vortices that form upstream and downstream of the valve.

There are good reasons why most finite element modeling of the MV includes simple chordal geometry. The current cardiac imaging techniques with 3D echo and MRI are not nearly as detailed as the images that can be obtained in a μ CT. Much work remains to be done to improve and streamline image-collection and image-processing strategies to make patient-specific MV modeling clinically relevant. However, increases in computational power have rendered such simulations possible. It is worth mentioning here that patient specific and clinically relevant models are two distinctive goals. Our model is not clinically relevant as of now, but it is patient/subject specific; and, therefore, suitable for analyzing different pathologies, especially when related to chordal structure.

The combination of SPH, to govern the fluid motion, and finite element method, to govern the motion and deformation of the solid parts, provides a unique advantage that allows the user to run these complex simulations on single GPU station and the closure of the MV is reached in a reasonable run time. And, most importantly, the simulations are stable and producing converged solutions without the need for remeshing. Currently, each simulation presented herein took about twelve hours of computing time to reach T_{sys} . Moreover, GPU technology and its performance is increasing rapidly. Although, the SPH formulation used here assumed the fluid to be inviscid, we have made the comparison by introducing viscosity in one of the simulations and it did not have an influence on the MV closure and the results presented in this paper. Introducing viscosity will be more important when the fluid flow and formation of vortices is to be analyzed, which is the focus of our future work.

Today, it is possible to investigate the mechanics of the mitral valve throughout the cardiac cycle, considering all of the complexities and all of the interactions among its components. Proper understanding of these complex interactions is axiomatic. In the near term, on this

foundation, we will turn our attention to the investigation of chordae tendineae-related pathologies, e.g. chordal rupture and its surgical repair techniques, and their influence on transvalvular hemodynamics. It is the balance between stress and flow, tissue and blood, closure and regurgitation that determines functional equilibrium for cardiac valves. An FSI analysis permits the study of that equilibrium or its loss in the context of surgical repair. Our work has shown that that FSI analysis is now tractable without omitting the role and complexity of the mitral valve chordae.

Acknowledgments

This study was supported by a grant from the National Heart Lung and Blood Institute (R01-HL092926).

References

1. Dahl SK, Vierendeels J, Degroote J, Annerel S, Hellevik LR, Skallerud B. Fsi simulation of asymmetric mitral valve dynamics during diastolic filling. *Computer methods in biomechanics and biomedical engineering*. 2012; 15:121–130. [PubMed: 21086206]
2. Kunzelman KS, Cochran RP, Chuong CJ, Ring WS, Verier ED, Eberhart RC. Finite element analysis of the mitral valve. *J Heart Valve Dis*. 1993; 2:326–340. [PubMed: 8269128]
3. Einstein DR, Reinhall PG, Kunzelman KS, Cochran RP. Nonlinear finite element analysis of the mitral valve. *J Heart Valve Dis*. 2005; 3:376–385.
4. Toma M, Jensen MO, Einstein DR, Yoganathan AP, Cochran RP, Kunzelman KS. Fluid-structure interaction analysis of papillary muscle forces using a comprehensive mitral valve model with 3D chordal structure. *Ann Biomed Eng*. 2016; 44(4):942–953. [PubMed: 26183963]
5. Toma M, Bloodworth CH, Einstein DR, Pierce EL, Cochran RP, Yoganathan AP. High resolution subject-specific mitral valve imaging and modeling: Experimental and computational methods. *Biomechanics and Modelling in Mechanobiology*. 2016 [Online ahead of print].
6. Kunzelman KS, Cochran RP, Chuong CJ, Ring WS, Verier ED, Eberhart RC. Finite element analysis of mitral valve pathology. *J Long Term E3 Med Implants*. 1993; 3:161–179.
7. Kunzelman KS, Reimink MS, Cochran RP. Annular dilatation increases stress in the mitral valve and delays coaptation: A finite element computer model. *Cardiovasc Surg*. 1997; 5:427–434. [PubMed: 9350801]
8. Kunzelman KS, Reimink MS, Cochran RP. Flexible versus rigid ring annuloplasty for mitral valve annular dilation: A finite element model. *J Heart Valve Dis*. 1998; 7:108–116. [PubMed: 9502148]
9. Einstein DR, Kunzelman KS, Reinhall PG, Nicosia MA, Cochran RP. The relationship of normal and abnormal microstructural proliferation to the mitral valve closure sound. *Transactions of the ASME*. 2005; 127:134–147.
10. Cochran RP, Kunzelman KS. Effect of papillary muscle position on mitral valve function: relationship to mitral homografts. *Annals of Thoracic Surgery*. 1998; 66(Suppl):S155–161. [PubMed: 9930439]
11. Reimink MS, Kunzelman KS, Verier ED, Cochran RP. The effect of anterior chordal replacement on mitral valve function and stresses. *ASAIO Transactions*. 1995; 41:M754–M762.
12. Kunzelman KS, Reimink MS, Verier ED, Cochran RP. Replacement of mitral valve posterior chordae tendineae with expanded polytetrafluoroethylene suture: A finite element study. *J Card Surg*. 1996; 11:136–145. [PubMed: 8811408]
13. Reimink MS, Kunzelman KS, Cochran RP. The effect of chordal replacement suture length on function and stresses in repaired mitral valves: A finite element study. *J Heart Valve Dis*. 1996; 5:365–375. [PubMed: 8858500]
14. Chandran KB, Kim H. Computational mitral valve evaluation and potential clinical applications. *Ann Biomed Eng*. 2015; 43(6):1348–1362. [PubMed: 25134487]
15. Domenichini F, Pedrizzetti G. Asymptotic model of fluid–tissue interaction for mitral valve dynamics. *Cardiovascular engineering and technology*. 2015; 6:95–104. [PubMed: 26577230]

16. Kheradvar A, Groves EM, Falahatpisheh A, Mofrad MK, Alvi SH, Tranquillo R, Dasi LP, Simmons CA, Grande-Allen KJ, Goergen CJ, Baaijens F, Little SH, Canin S, Griffith B. Emerging trends in heart valve engineering: Part iv. computational modeling and experimental studies. *Ann Biomed Eng.* 2015; 43(10):2314–2333. [PubMed: 26224522]
17. Gao H, Ma XS, Qi N, Berry C, Griffith BE, Luo XY. A finite strain nonlinear human mitral valve model with fluid structure interaction. *International Journal for Numerical Methods in Biomedical Engineering.* 2014; 30:1597–1613. [PubMed: 25319496]
18. Rim Y, Laing ST, McPherson DD, Kim H. Mitral valve repair using eptfe sutures for ruptured mitral chordae tendineae: A computational simulation study. *Annals of Biomedical Engineering.* 2013
19. Mansi T, Voigt I, Georgescu B, Zheng X, Mengue EA, Hackl M, Ionasec R, Noack T, Seeburger J, Comaniciu D. An integrated framework for finite-element modeling of mitral valve biomechanics from medical images: Application to mitralclip intervention planning. *Medical Image Analysis.* 2012; 16:1330–1346. [PubMed: 22766456]
20. Schievano S, Kunzelman KS, Nicosia MA, Cochran RP, Einstein DR, Khambadkone S, Bonhoeffer P. Percutaneous mitral valve dilatation: Single balloon versus double balloon. a finite element study. *Journal of Heart Valve Disease.* 2009; 18:28–34. [PubMed: 19301550]
21. Wenk JF, Zhang Z, Cheng G, Malhotra D, Bolton GA, Burger M, Suzuki T, Saloner DA, Wallace AW, Guccione JM, Ratcliffe MB. First finite element model of the left ventricle with mitral valve: Insights into ischemic mitral regurgitation. *Annals of Thoracic Surgery.* 2010; 89:1546–1554. [PubMed: 20417775]
22. Maisano F, Redaelli A, Soncini M, Votta E, Arcobasso L, Alfieri O. An annular prosthesis for the treatment of functional mitral regurgitation: Finite element model analysis of a dog bone-shaped ring prosthesis. *Annals of Thoracic Surgery.* 2005; 79:1268–1275. [PubMed: 15797061]
23. Stevanella M, Maffessanti F, Conti CA, Votta E, Arnoldi A, Lombardi M, Parodi O, Caiani EG, Redaelli A. Mitral valve patient-specific finite element modeling from cardiac mri: Application to an annuloplasty procedure. *Cardiovascular Engineering and Technology.* 2011; 2(2):66–76.
24. Mansi, T., Voigt, I., Mengue, EA., Ionasec, R., Georgescu, B., Noack, T., Seeburger, J., Comaniciu, D. *Medical Image Computing and Computer-Assisted Intervention*, chapter Towards Patient-Specific Finite-Element Simulation of MitralClip Procedure. Springer; Berlin Heidelberg: 2011.
25. Lau KD, Diaz V, Scambler P, Burriesci G. Mitral valve dynamics in structural and fluid–structure interaction models. *Medical Engineering & Physics.* 2010; 32:1057–1064. [PubMed: 20702128]
26. Prot V, Haaverstad R, Skallerud B. Finite element analysis of the mitral apparatus: annulus shape effect and chordal force distribution. *Biomechanics and modeling in mechanobiology.* 2009; 8(1): 43–55. [PubMed: 18193309]
27. Freed AD, Einstein DR, Vesely I. Invariant formulation for dispersed transverse isotropy in aortic heart valves: an efficient means for modeling fiber splay. *Biomech Model Mechanobiol.* 2005; 4:100–117. [PubMed: 16133588]
28. Kunzelman KS, Einstein DR, Cochran RP. Fluid–structure interaction models of the mitral valve: function in normal and pathological states. *Phil Trans R Soc B.* 2007; 362:1393–1406. [PubMed: 17581809]
29. Liu, GR., Liu, MB. *Smoothed Particle Hydrodynamics: A Meshfree Particle Method.* World Scientific Publishing Company; 2003.
30. Jensen MO, Fontaine AA, Yoganathan AP. Improved in vitro quantification of the force exerted by the papillary muscle on the left ventricular wall: Three-dimensional force vector measurement system. *Annals of Biomedical Engineering.* 2001; 29:406–412. [PubMed: 11400721]

Appendix

Algorithm 1

Given H_0/r_0 , R_0/r_0 , E_f and λ_u , where H_0 is the initial wavelength of crimp, R_0 is the initial amplitude of crimp, r_0 is the initial fibril radius, E_f is the elastic modulus of the fiber in the linear region, and λ_u is its ultimate stretch, then:

Set $r_0 = 1$ so that $H_0 \equiv H_0/r_0$ & $R_0 \equiv R_0/r_0$.

Compute the constant parameters:

$$L_0 = ((2\pi R_0)^2 + H_0^2)^{\frac{1}{2}}, \quad \Lambda = L_0/H_0 < \lambda_u,$$

$$E_s = E_f / \{ (H_0/L_0)^2 + [H_0(L_0 - H_0)/L_0^2] [1 + 37/6\pi^2 + 2(L_0/\pi r_0)^2] \},$$

where L_0 is the chord length of helix over one wavelength, while Λ is the stretch and E_s is the secant modulus at the transition between the toe and linear regions. If $\lambda < \Lambda$ Then

$$\xi = 6(\pi r_0)^2 [\Lambda^2 + (4\pi^2 - 1)\lambda^2] \lambda$$

$$/ \{ \Lambda \{ 3H_0^2(\Lambda^2 - \lambda^2) [3\Lambda^2 + (8\pi^2 - 3)\lambda^2] + 8(\pi r_0)^2 [10\Lambda^2 + (3\pi^2 - 10)\lambda^2] \} \},$$

$$d\xi/d\lambda = \{ 18H_0^2(\pi r_0)^2 [3\Lambda^6 + (28\pi^2 - 3)\Lambda^4\lambda^2 + (32\pi^4 - 8\pi^2 - 3)\Lambda^2\lambda^4$$

$$+ (32\pi^4 - 20\pi^2 + 3)\lambda^6] + 48(\pi r_0)^4 [10\Lambda^4$$

$$+ (117\pi^2 - 20)\Lambda^2\lambda^2 + (12\pi^4 - 43\pi^2 + 10)\lambda^4] \}$$

$$/ \{ \Lambda \{ 3H_0^2(\Lambda^2 - \lambda^2) [3\Lambda^2 + (8\pi^2 - 3)\lambda^2]$$

$$+ 8(\pi r_0)^2 [10\Lambda^2 + (3\pi^2 - 10)\lambda^2] \}^2 \},$$

$$\sigma/\lambda = E_s \xi (\lambda - 1) / \lambda^2 \quad \& \quad d\sigma/d\lambda = E_s [\xi / \lambda^2 + (d\xi/d\lambda)(\lambda - 1) / \lambda]$$

Else If $\Lambda < \lambda < \lambda_u$ Then

$$\sigma/\lambda = E_s (\Lambda - 1) / \Lambda \lambda + E_f (\lambda - \Lambda) / \lambda \quad \& \quad d\sigma/d\lambda = E_f$$

Else Fibril Failure

$$\sigma/\lambda = 0 \quad \& \quad d\sigma/d\lambda = 0.$$

Return σ/λ and $d\sigma/d\lambda$.

Figure 1.

Anterior mitral valve stress-strain behaviour, fiber and cross-fiber directions. Arrows indicate the effect of increasing strain levels in the orthogonal direction (a)^[28, 9] and the fiber direction at each point of the mitral valve mesh (b).

Figure 2.

The fluid particles are confined in a pipe-like rigid structure surrounding the MV model and prescribed velocity boundary conditions are applied to the open ends via the use of moving pistons in z-direction.

Figure 3.

First deviatoric principal stress; STR and FSI simulations results at two different time points.

Figure 4.

Numbering of the individual anterior papillary muscle (APM) and posterior papillary muscle (PPM) chordal attachment points (a). The individual force vectors of the APM and PPM chordal attachment points are shown in red and the average force vectors are shown in blue, at $T = 0.75 \cdot T_{sys}$. The difference between the STR (b) and FSI (c) analyses can be observed.

Figure 5.

Individual components of posterior-medial papillary muscle force from FSI and STR analyses compared with experimental measurements.^[30]

Figure 6.
Regurgitant volume measured as the mass of the fluid particles crossing the area at the level of the MV annulus.