Using compressive sensing to recover images from PET scanners with partial detector rings

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(Received 11 March 2014; revised 13 November 2014; accepted for publication 15 November 2014; published 18 December 2014)

Purpose: Most positron emission tomography/computed tomography (PET/CT) scanners consist of tightly packed discrete detector rings to improve scanner efficiency. The authors' aim was to use compressive sensing (CS) techniques in PET imaging to investigate the possibility of decreasing the number of detector elements per ring (introducing gaps) while maintaining image quality.

Methods: A CS model based on a combination of gradient magnitude and wavelet domains (wavelet-TV) was developed to recover missing observations in PET data acquisition. The model was designed to minimize the total variation (TV) and L1-norm of wavelet coefficients while constrained by the partially observed data. The CS model also incorporated a Poisson noise term that modeled the observed noise while suppressing its contribution by penalizing the Poisson log likelihood function. Three experiments were performed to evaluate the proposed CS recovery algorithm: a simulation study, a phantom study, and six patient studies. The simulation dataset comprised six disks of various sizes in a uniform background with an activity concentration of 5:1. The simulated image was multiplied by the system matrix to obtain the corresponding sinogram and then Poisson noise was added. The resultant sinogram was masked to create the effect of partial detector removal and then the proposed CS algorithm was applied to recover the missing PET data. In addition, different levels of noise were simulated to assess the performance of the proposed algorithm. For the phantom study, an IEC phantom with six internal spheres each filled with F-18 at an activity-to-background ratio of 10:1 was used. The phantom was imaged twice on a RX PET/CT scanner: once with all detectors operational (baseline) and once with four detector blocks (11%) turned off at each of 0°, 90°, 180°, and 270° (partially sampled). The partially acquired sinograms were then recovered using the proposed algorithm. For the third test, PET images from six patient studies were investigated using the same strategy of the phantom study. The recovered images using WTV and TV as well as the partially sampled images from all three experiments were then compared with the fully sampled images (the baseline). Comparisons were done by calculating the mean error (%bias), root mean square error (RMSE), contrast recovery (CR), and SNR of activity concentration in regions of interest drawn in the background as well as the disks, spheres, and lesions.

Results: For the simulation study, the mean error, RMSE, and CR for the WTV (TV) recovered images were 0.26% (0.48%), 2.6% (2.9%), 97% (96%), respectively, when compared to baseline. For the partially sampled images, these results were 22.5%, 45.9%, and 64%, respectively. For the simulation study, the average SNR for the baseline was 41.7 while for WTV (TV), recovered image was 44.2 (44.0). The phantom study showed similar trends with 5.4% (18.2%), 15.6% (18.8%), and 78% (60%), respectively, for the WTV (TV) images and 33%, 34.3%, and 69% for the partially sampled images. For the phantom study, the average SNR for the baseline was 14.7 while for WTV (TV) recovered image was 13.7 (11.9). Finally, the average of these values for the six patient studies for the WTV-recovered, TV, and partially sampled images was 1%, 7.2%, 92% and 1.3%, 15.1%, 87%, and 27%, 25.8%, 45%, respectively.

Conclusions: CS with WTV is capable of recovering PET images with good quantitative accuracy from partially sampled data. Such an approach can be used to potentially reduce the cost of scanners while maintaining good image quality. © 2015 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4903291>]

Key words: compressive sensing, positron emission tomography (PET), image recovery, image reconstruction, gap-filling

1. INTRODUCTION

The majority of current positron emission tomography/computed tomography (PET/CT) scanners consists of tightly packed detector blocks arranged in rings to detect the annihilation photons from the decay of a positron-emitting radionuclide. These systems currently cost $\$1\text{--}3 \times 10^6$,¹ depending on the scanner model and thus are prohibitively expensive for some imaging centers in the U.S. and for the majority of centers in developing countries. Decreasing the number of detectors per ring by introducing detector gaps (while maintaining the same scanner diameter) could potentially lower the cost of the scanner, thereby making this technology more accessible to physicians and patients. However, decreasing the detectors could also reduce image quality because fewer observations (samples) of the corresponding object are used to generate the final image.

Several approaches have been proposed to estimate missing samples from tomographic data.^{2–4} These techniques can be divided into two main groups. The first group relies on various forms of interpolation,^{2–4} such as truncated and windowed sinc, bilinear, bicubic, Gaussian, model-based function fitting, or interpolation filtering. The main drawback of these interpolation algorithms is their high sensitivity to local variation, which results in substantial error in the data-fitting process. Interpolating functions also oscillate severely at the end points of the data range.⁵

The second group of recovery techniques utilizes a statistical framework such as maximum likelihood expectation maximization (MLEM) or maximum *a posteriori* (MAP).^{6,7} These approaches are based on fitting a statistical model to the observed data while striving to maximize the most probable data to fill the missing observations in an iterative manner.⁸ Conventional expectation maximization (EM) algorithms, unfortunately, have slow convergence rates and noisy reconstruction owing to error augmentation as the number of iterations increases.⁷ There are several varieties of EM algorithms, such as multiresolution and wavelet spline-based approaches that try to avoid these problems.⁸ A comparison between the two main recovery groups shows that EM techniques outperform the interpolation methods but the computational time required for EM techniques is much longer.⁸

A variant of the statistical group are those algorithms that utilize penalty terms in the statistical optimization process (MAP algorithms) in an effort to improve the resultant reconstructed image.^{9–11} These algorithms, however, are not designed to recover missing data *per se*, but are primarily utilized to suppress image noise and improve image quality. For example, Kisilev *et al.*⁹ evaluated the effect of adding either the wavelet or the total variation (TV) domains as a (prior) penalty term in their EM algorithm. Their results showed that including these prior terms improves image quality as compared to the regular EM. Similar results were also observed by Jonsson *et al.*¹⁰ when using TV regularization, as well as Zhou *et al.*¹¹ and Raheja *et al.*,⁸ when employing wavelets with MAP-EM.

More recently, compressive sensing (CS) techniques have been used as a third approach to recover a PET image

from an incomplete dataset.¹² The general idea of CS is to reproduce the exact signal from fewer samples than is required (twice the maximum frequency exhibited by signal). This can be achieved by transforming the signal to a new domain (called sparsifying domain) such that the corresponding coefficients that represent the signal in the new domain are very sparse (few nonzero elements). The specific domain can be wavelet, Fourier, bandlet, curvelet, or several predefined domains, depending on the signal at hand. One promising domain, often used for representing medical images, is the gradient magnitude domain (GMD), which has been shown to generate very accurate image reconstructions even when few samples are available.¹³ However, regardless of the selected domain, the success of CS recovery depends on finding the sparsest set of coefficients that expresses the signal in the desired domain or, in other words, minimizing a cost function corresponding to that domain.

Ahn *et al.*¹² recently used the CS technique to fill the missing data in images from the high-resolution research tomograph (HRRT) PET scanner. In these scanners, the octagonal shape of the PET system introduces gaps among the detectors, creating undesired missing data in the sinogram that accounts to about 10% of the sinogram space. In their implementation, Ahn *et al.*¹² used the GMD as a sparsifying domain and applied total variation minimization to find the sparsest coefficients. Although Ahn *et al.*¹² showed that CS can recover the gaps inherent in the HRRT scanner design, their model relied on only minimizing the TV and they did not model the noise in their formulation.

In this paper, we improve on the model of Ahn *et al.*¹² by adding the wavelet domain to the GMD in an effort to further sparsify the observed image. We also include the inherent Poisson noise present in the measured observations as a penalty term in the minimization of the cost function. The resultant model is a novel cost function which is a linear combination of TV, wavelet transform, and Poisson ML estimation. We believe this is the first time such a model has been proposed to recover missing data in PET imaging. The modified approach, which henceforth we will refer to as the wavelet-TV (WTV) approach, was tested using simulations, phantoms, and six patients' studies and compared with a generalized TV approach similar to that of Ahn *et al.*¹²

2. MATERIAL AND METHODS

2.A. Compressive sensing

CS recovery using GMD and TV minimization of PET images can be written as

$$\operatorname{argmin} \|m\|_{\text{TV}} = \sum_i \sum_j |\nabla m(i,j)|, \quad (1)$$

$$\text{subject to } G \cdot m = y, \quad (2)$$

where m is the reconstructed image, G is the system matrix of the PET scanner, y is the partial observation (which in this case is in sinogram space), and ∇ is the discrete gradient operator.

There are different approaches to solve this minimization problem. One approach is to include the data fidelity term in the minimization problem and solve the following equation in one step:

$$\operatorname{argmin}_m \sum_i \sum_j |\nabla m(i,j)| + \lambda (\|Gm - y\|_2^2), \quad (3)$$

where λ is the Lagrangian constant. The other approach is to divide the optimization into two parts and perform it iteratively:^{12,13} first, using an initial image reconstructed from partially sampled data, the TV term is minimized without any constraint (unconditional optimization); second, an image m is found that satisfies Eq. (2) under the condition that the TV minimization of this m is represented by the solution of Eq. (1). This process is then iterated until convergence. Differences between these models are related to convergence speed and dependence on the value assigned to the hyperparameter λ . In the work of Ahn *et al.*, the second approach (two steps) was used and will be the method that our proposed approach will be compared to.

In our approach, we modified the CS recovery process in the following manner to represent a sparser signal and to gain a better recovery of the image:

$$\operatorname{argmin}_m \sum_i \sum_j |\Phi(m(i,j))| + \left(\sum_{k=1}^N ((e_k^T Gm) - y_k \log((e_k^T Gm))) \right), \quad (4)$$

where $\Phi(m(i,j)) = \alpha |\psi(m(i,j))| + \beta |\nabla m(i,j)|$ is a linear combination of wavelet and TV functions, y_k is the k th element of the vectorized observation (y) and e_k is the k th unit basis vector. $\psi(m)$ and ∇m are the wavelet and the gradient of image m , while α and β are regularization parameters (hyperparameters) that affect the weighting of the wavelet and gradient operators during the optimization. These parameters are chosen empirically and change depending on the object being reconstructed. The wavelet that we used in this work was Daubechies4.¹⁴ Further description of these terms and their values can be found in Sec. 4.

The second term in Eq. (4) is the fidelity term and represents the Poisson noise in PET imaging. The MLEM method assumes that the system model (G) accurately relates the probability density function (PDF) of the estimated image data to the Poisson-distributed sinogram data according to^{15,16}

$$P(y|Gm) = \prod_{k=1}^N \frac{(e_k^T Gm)^{y_k}}{y_k!} \exp(-e_k^T Gm), \quad (5)$$

where N is the number of elements in the sinogram. Thus, our method seeks to maximize the likelihood term or equivalently minimize the negative log likelihood function given by

$$\begin{aligned} \xi(m) &= -\log(P(y|Gm)) \\ &= -\left(\sum_{k=1}^N \log((e_k^T Gm)^{y_k}) - \sum_{k=1}^N \log(y_k!) \right. \\ &\quad \left. + \sum_{k=1}^N \log(\exp(-e_k^T Gm)) \right). \end{aligned} \quad (6)$$

Since the second term in Eq. (6) is not a function of m , it serves as a constant and does not have any role in the minimization, so we ignore it in the cost function. Further simplification of Eq. (6) gives

$$\begin{aligned} \xi(m) &= -\left(\sum_{k=1}^N y_k \log((e_k^T Gm)) + \sum_{k=1}^N (-e_k^T Gm) \right) \\ &= \sum_{k=1}^N ((e_k^T Gm) - y_k \log((e_k^T Gm))). \end{aligned} \quad (7)$$

For the WTV approach, we used the partially sampled image reconstructed using MLEM (20 iterations) as the initialization point and iterated 150 times due to the complexity of the proposed cost function while using the steepest descent (SD) algorithm¹⁷ to reach the global minimum. For each iteration, the update of image is given by

$$\begin{aligned} m^{k+1} &= m^k - \alpha \nabla f(m), \\ &= m^k - \alpha \sum_i \sum_j \nabla |\Phi(m(i,j))| \\ &\quad - \alpha \left(\sum_{k=1}^N \left((e_k^T G) - \frac{y_k}{(e_k^T Gm + \varepsilon)} (e_k^T G) \right) \right), \end{aligned} \quad (8)$$

where m is the image, ∇ is the discrete gradient operator, ε is set to a constant so the denominator is always nonzero, α is the step size, determined with the line-search algorithm.¹⁸ The pseudocode of the SD algorithm is shown below. More details about this algorithm and its convergence can be found in Freund.¹⁷

ALGORITHM 1. The pseudo code of the steepest descent algorithm to find minimum value of a function.

Steepest Descent Algorithm

Input: $f(m)$, $\nabla f(m)$, ε

Objective: $m^* = \operatorname{argmin}_m f(m)$

1: $k \leftarrow 0$, $m^0 \leftarrow$ initial image

2: $d^k = -\nabla f(m^k)$. if $\|d^k\| \leq \varepsilon$, stop, $m^* = m^k$

3: $\alpha^k = \operatorname{argmin}_\alpha f(m^k + \alpha d^k)$ to determine the stepsize (α^k) using line-search algorithm

4: $m^{k+1} \leftarrow m^k + \alpha^k d^k$, $k \leftarrow k+1$

5: Goto step 1

In general, the number of iterations required for the algorithm to converge varies depending on the complexity of the image and noise. To achieve convergence, we usually evaluate whether a change between the image and its update is below a preset threshold (which in this case is calculated based on the ratio of the cost function and its update and was set to 10^{-4}). However, for practical purposes, we also set a maximum number of iterations as a stopping criterion

FIG. 1. (a) Simulated PET scanner configuration, showing all detector modules (two blocks/module). First block in module numbers 0, 4, 8, 13, 17, 22, 26, and 31 was turned off (black) from total of 35 modules. (b) Block diagram of our proposed compressed sensing iterative recovery process.

as well. This is useful for cases where the optimization does not reach the desired convergence criteria (preset threshold). Our choice of the partially sampled image as the initial image helps achieve convergence since the partially sampled image is believed to be in the local neighborhood of the true solution.

The CS framework that we developed in Eq. (4) was applied in two dimensions (2D) for all experiments—simulation, phantom, and patient studies. The optimization was solved in one step.

2.B. Experimental setup

We evaluated our CS recovery algorithm using three tests: a simulation, a phantom, and six patients' studies.

2.B.1. Simulation study

We simulated a PET image consisting of six disks of 55, 44, 33, 27, 22, and 15 mm in a warm background. The disk-to-background ratio was set to 5:1. The simulated images were multiplied by the system matrix to generate the corresponding sinograms and then Poisson noise was added. The added Poisson noise was based on the mean of the sinogram counts. The image matrix size was set to 128×128 ,

and the corresponding sinogram was set to 367×315 to represent the General Electric Discovery RX (DRX) PET scanner (GE Healthcare; Waukesha, WI) which consists of 35 detector modules (each module has two detector blocks) arranged in a circular fashion of 88.6 cm diameter [Fig. 1(a)]. The resulting sinogram was then masked such that 11% of the data was set to zero to simulate the effect of detector block gaps. The detector blocks that we turned off were located at 0° , 90° , 180° , and 270° and corresponded to the first block in module numbers 0, 4, 8, 13, 17, 22, 26, and 31 as shown in Fig. 1(a). The resultant sinograms were then reconstructed using our image recovery technique (WTV) as well as a simple TV reconstruction similar to the approach of Ahn *et al.*¹² A block diagram of the recovery process is shown in Fig. 1(b).

For the WTV approach, we used the partially sampled image reconstructed using MLEM (20 iterations) as the initialization point and iterated 150 times due to the complexity of the proposed cost function while using the steepest descent algorithm¹² to reach the global minimum. In this implementation, the α and β values were set to 0.24 and 0.63, respectively. For the TV minimization, we also used the partially sampled image for initialization but iterated 20 times while also employing the steepest descent algorithm, as described in the work of Ahn *et al.*¹² No attenuation or scatter effects were simulated in both cases.

The recovered images for both approaches as well as the partially sampled images were then compared with the fully sampled image [no gaps, also generated using MLEM (20 iterations)] by evaluating the mean error (bias) and root mean square error (RMSE) in regions of interest (ROIs) between corresponding images for each of the six disks and background. Background in this case was defined as the whole area of the phantom excluding the six disks. RMSE was calculated as follows:

$$\%RMSE(m^{rcvd}) = \sqrt{\frac{\sum_x \sum_y (m_{x,y}^{org} - m_{x,y}^{rcvd})^2}{\sum_x \sum_y (m_{x,y}^{org})^2}} \times 100. \quad (9)$$

The mean error (bias) was calculated as

$$\%mean\ error(m^{rcvd}) = \frac{\sum_x \sum_y (m_{x,y}^{rcvd} - m_{x,y}^{org})}{\sum_x \sum_y (m_{x,y}^{org})} \times 100. \quad (10)$$

In addition, contrast recovery (CR) and SNR of the disks were also calculated where

$$Contrast\ Recovery = \frac{\frac{H}{B} - 1}{\frac{H_{true}}{B_{true}} - 1}, \quad (11)$$

$$SNR = \frac{Mean(H)}{Standard\ deviation(B)}, \quad (12)$$

where H stands for mean of the measured disk activity (hot), B is the mean for the measured background, and “true” refers to the known values in the disks and the background. The standard deviation in the SNR calculation was based on five ROIs drawn in the background. We also generated difference images between the recovered image and the fully sampled image (gold standard) and plotted the histograms of these differences.

Finally, we studied the value of the Poisson noise term using two scenarios. First, by replacing the Poisson noise term in our WTV model [Eq. (4)] by Gaussian noise [Eq. (13)]. Second, by comparing our WTV model including the Poisson noise term to a TV approach using Gaussian term [Eq. (4)] similar to the work of Ahn *et al.* In both scenarios, the recovery performance was assessed by calculating the mean error (bias) under increasing Poisson noise levels (0.05 – 5σ , where σ is the standard deviation of the mean counts in the sinogram)

$$\operatorname{argmin}_m \sum_i \sum_j |\Phi(m(i,j))| + (\|Gm - y\|_2^2). \quad (13)$$

2.B.2. Phantom study

For the phantom study, we used an International Electrotechnical Commission (IEC) image-quality phantom with six internal spheres that were filled with Fluorine-18 water with a sphere-to-background ratio of 10:1. The activity concentration of the spheres and background was 37 kBq/cm^3 ($1\text{ }\mu\text{Ci/cm}^3$) and 3.7 kBq/cm^3 ($0.1\text{ }\mu\text{Ci/cm}^3$), respectively. The phantom was imaged twice on a DRX PET/CT scanner (GE Healthcare; Waukesha, WI): once with all detectors operational (baseline), and once with four detector blocks (11%) turned off at each of 0° , 90° , 180° , and 270° (partially

sampled). To turn off detector blocks, the corresponding photomultiplier tube (PMT) gains were set to zero prior to data acquisition. The two PET data acquisitions were performed immediately one after the other to minimize the effect of radioactivity decay using one field of view (FOV) of 3 min which corresponds to our clinical imaging protocol.

The partially sampled sinograms were used to generate a mask that captures the exact geometrical effect of detector removal in the sinogram space. The fully sampled sinograms were then reconstructed using MLEM (20 iterations), with all data corrections (random, scatter, attenuation, normalization, decay, etc.), performed within the reconstruction loop to maintain the integrity of the Poisson statistics. These images were then forward-projected and masked using the template derived from the partially sampled sinograms. Although this approach does not truly simulate the imaging conditions of a scanner with detector gaps (see Sec. 4), it eliminates the need to apply data correction (for normalization, attenuation, scatter, randoms, decay, etc.) because the resulting sinograms were generated from images that included these corrections. The corresponding partially sampled sinograms were then reconstructed using WTV, simple TV, and without any correction (partially sampled) using similar parameters described in Sec. 2.B.1. The resultant images were compared with the fully sampled image (no gaps) by evaluating the mean error (bias), RMSE, CR, and SNR in ROIs drawn on the corresponding images for each of the six spheres and the background. In addition, difference images between the recovered and the fully sampled images were generated and plotted. For this evaluation, α and β were empirically set to 0.3 and 0.56, respectively.

2.B.3. Patient study

Six patients (one man and five women; mean age 54 ± 21 yr) referred for PET/CT imaging were also evaluated to test the performance of the proposed WTV approach. All patients fasted for 4 h before being injected intravenously with 296–444 MBq (8–12 mCi) of FDG. Imaging started 60–90 min after injection. The imaging process consisted of a scout followed by a regular whole-body CT scan and a whole-body PET scan covering the area from the patients’ orbits to thighs. The duration of the PET scan was 3 min per bed position. During the imaging process, the patients were requested to breathe regularly. The acquired sinograms were then reconstructed using MLEM (20 iterations) to generate the corresponding images. These images were then forward-projected and masked to create the effect of partial detector removal (11% off) in a similar manner to the phantom study. The corresponding partially sampled sinograms were then reconstructed using the WTV approach, the simple TV approach, and without any recovery (partially sampled) with the same parameters described before and compared with the fully sampled (no gaps) image. Comparison was done by evaluating the mean error, RMSE, CR, and SNR in ROIs drawn on the lesions of the corresponding images. α and β for these patients’ studies ranged between 0.1–0.5 and 0.5–0.85, respectively.

FIG. 2. The simulation study comparing two methods of recovering partially sampled PET images: (a) the baseline image, (b) the partially sampled image, (c) the WTV-recovered image, and (d) the TV-recovered image. All images are displayed in the same scale, and the units are arbitrary intensity level ($\alpha = 0.24$, $\beta = 0.63$).

3. RESULTS

3.A. Simulation

Results from the fully sampled, partially sampled, the WTV, and TV approaches are shown in Fig. 2. The convergence of the simulation study for WTV was achieved by reaching the preset difference threshold between iterations. The error in activity concentration of each internal disk and background between the partially sampled image, the WTV-recovered image, and the TV-recovered image relative to the fully sampled image is shown in Table I. C1–C6 represent the six disks, where C1 and C6 are the largest and smallest disks, respectively. Total refers to the entire image.

The error in SNR and contrast of each internal disk in the partially sampled image, the WTV-recovered image, and the TV-recovered image is shown in Fig. 3. These results show

that both WTV and TV have improved performance over the partially sampled image.

Figure 4(a) shows a difference image between the WTV-recovered and fully sampled images. Figure 4(b) shows a similar difference image between the TV-recovered and the fully sampled images. These figures clearly show that the WTV approach has less error than the TV approach. Furthermore, the error in the WTV approach is mainly constrained to the disks, whereas it is widely dispersed through the entire image in the TV approach.

The histogram of these errors is shown in Figs. 5(a) and 5(b). As can be seen, the error is more concentrated around zero in the WTV approach, suggesting more accurate recovery for this approach compared to the TV approach.

The plot comparing the two noise models of Gaussian and Poisson [as described by Eqs. (4) and (13)] is shown in Fig. 6.

TABLE I. RMSE and mean error of the simulation study for the six disks: C1 (largest) to C6 (smallest); results are with respect to the fully sampled (baseline) image.

		C1	C2	C3	C4	C5	C6	Bkg
Partially sampled	Mean error (%)	−33.2	−31.9	−33.9	−36.6	−38.5	−42.3	−20.1
	RMSE (%)	34.9	33.3	34.9	37.3	39.1	42.4	49.4
TV	Mean error (%)	1.1	0.2	−0.1	−0.2	−0.2	−1.3	−0.3
	RMSE (%)	2.6	1.7	1.8	1.9	1.9	1.6	8.4
WTV	Mean error (%)	0.5	0.0	0.2	0.0	−0.2	−0.7	−0.2
	RMSE (%)	1.3	0.8	0.8	0.8	0.8	0.9	2.7

FIG. 3. (a) SNR of the simulation study for the six disks: C1 (largest) to C6 (smallest); PS: partially sampled, BS: baseline. (b) Contrast recovery of the simulation study for the six disks: C1 (largest) to C6 (smallest); PS: partially sampled, BS: baseline.

Also, the plot of mean error versus noise for the six disks using WTV and TV is shown in Fig. 7. Both plots clearly show that for any noise level and disk size, WTV (solid lines) has less bias than TV (dashed).

3.B. Phantom study

Figure 8 shows the result of the phantom study. The partially sampled image exhibits streak artifacts owing to the missing observations (samples), whereas the WTV and TV images show less such artifacts. Convergence of the phantom study was achieved by stopping at the maximum number of iterations (150 for WTV and 20 for TV). The RMSE and mean error of activity concentration values of each sphere between the fully sampled, partially sampled, and recovered images for the WTV and TV approaches are shown in Table II. Figure 9(a) shows a difference image between the WTV-recovered image and the fully sampled image. Figure 9(b) shows a similar difference image between the TV-recovered image and the fully sampled image. All images are displayed using the same scale. As can be seen in Table II and Fig. 8, the WTV approach results in a lower RMSE than does the TV approach. Furthermore, the resultant difference image of the WTV approach is smaller than the difference image of the TV approach [Figs. 9(a) and 9(b)].

SNR and contrast recovery of each internal disk in the partially sampled image, the WTV-recovered image, and the TV-recovered image for the phantom study are shown in Fig. 10. These tables show that on average, WTV had better performance than TV for all reported metrics.

3.C. Patient study

Results from one patient study (patient #6) is shown in Fig. 11. The fully sampled, partially sampled, and recovered image using the WTV and TV methods for this patient are shown in Figs. 11(a)–11(d), respectively. The RMSE and mean error of the lesions in the six patient studies for the WTV, simple TV, and partially sampled images are shown in Table III. Corresponding SNR and CR of the lesions in the six patient studies is shown in Fig. 12. Two out of six patient studies (#2 and #5) converged based on a preset threshold, while the remaining four converged based on maximum number of iterations. The results clearly show that WTV has a much lower RMSE compared to the partially sampled as well as simple TV images. Figure 13(a) shows a difference image between the WTV-recovered and fully sampled images. Figure 13(b) shows a similar difference image between the simple TV-recovered and fully sampled images. The difference images clearly show that the TV recovery approach results in noisier images, whereas the WTV approach shows less noise but more

FIG. 4. Difference images from the simulation study (a) difference between the baseline image and the WTV-recovered image; (b) difference between the baseline image and the TV-recovered image. Both images are displayed in the same scale, and the units are arbitrary intensity level.

FIG. 5. Histogram of difference images from the simulation study. (a) Histogram of the difference between the baseline image and the WTV-recovered image; (b) histogram of difference between the baseline image and the TV-recovered image. The x -axis is pixel intensity (with arbitrary intensity unit), and the y -axis is intensity population.

structured streak artifacts. However, quantitatively, the WTV approach has a lower error than the simple TV approach over all six patients as shown in Table III.

4. DISCUSSION AND CONCLUSIONS

We developed a signal recovery model based on CS that utilizes the sparsity in the GMD and the wavelet domains to recover missing PET data owing to detector gaps. Furthermore, our model included a Poisson noise term in its formulation to suppress the inherent noise in the measured data. The model was tested using simulation, phantom, and patient studies, and our results showed very comparable performance to fully sampled (no detector gaps) data.

Previous efforts of signal recovery in PET imaging using CS techniques have focused on using GMD and its TV optimization.¹² However, despite the effectiveness of such an approach in data recovery, TV-based algorithms often have undesirable streak artifacts in the recovered image and tend to over-smooth image details and textures.^{19,20} In an effort to overcome these drawbacks while generating a qualitatively more desirable and quantitatively more accurate image, we made use of both GMD and wavelet (in our implementation Daubechies4) domains in our CS design. Advantages of using both domains rather than one include a sparser representation and modeling the smooth as well as the rough components of the image.²¹

Our choice of wavelets as a second domain in the design of our CS approach was primarily due to the wavelet's ability to capture small image details with very few coefficients. The wavelet transform is an effective tool to analyze images in a multiscale framework to capture localized image details in both space and frequency domains.^{14,22} Our results in Tables I–III show that the inclusion of the two domains as presented by the WTV approach results in only 1.2%, 10.1%, and 7.2% average differences (in RMSE) when compared to the fully sampled images for simulations, phantom, and patient studies, respectively. In addition, these results show that the WTV approach provides images that are 1.7%, 10.4%, and 8% better in RMSE than a simple TV (GMD only) approach. Furthermore, the mean error between the WTV-recovered and the original image was 0.26%, 5.4%, and 0.7% for simulation, phantom, and the patients' studies while the contrast recovery was 97%, 78%, and 92%, respectively. In addition, the mean error between the TV-recovered and the original image was 0.48%, 18.2%, and 1.3% for simulation, phantom, and the patients' studies while the contrast recovery was 96%, 60%, and 87%, respectively.

In addition to using two domains, a noise Poisson log likelihood term was also included in the model to suppress the inherent noise in the recovered images. When solving inverse problem such as image reconstruction, an accurate modeling of the noise is essential to limit its propagation through the entire image. In this regard, a Poisson log likelihood term

FIG. 6. Performance of the two different noise-limiting cost functions using the WTV approach. The plot shows the mean error for each of the six disks for increasing noise levels (0.05–5 σ where σ is the standard deviation of the mean counts in the sinogram) using two cost functions Poisson (solid) and Gaussian (dashed).

FIG. 7. Performance of two different data recovery approaches TV and WTV. The plot shows the mean error for each of the six internal disks for increasing noise levels (0.05–5 σ where σ is the standard deviation of the mean counts in the sinogram). TV is shown with dashed lines and WTV with solid lines.

FIG. 8. The phantom study comparing two methods of recovering partially sampled PET images: (a) the baseline image, (b) the partially sampled image, (c) the WTV-recovered image, and (d) the simple TV-recovered image. All images are displayed with the same scale ($\alpha = 0.3$, $\beta = 0.56$), units are $100 \times \text{Bq/mL}$.

TABLE II. RMSE and mean error of the phantom study for six spheres: C1 (largest) to C6 (smallest). Results are with respect to the fully sampled image (baseline).

		C1	C2	C3	C4	C5	C6	Bkg
Partially sampled	Mean error (%)	−35.7	−30.1	−28.4	−11.8	−42.2	−34.6	−34
	RMSE (%)	36.2	35.3	39	29.1	27.7	17.3	31.6
TV	Mean error (%)	−7.0	−2.6	−28.4	−12.	−36.1	23	12.8
	RMSE (%)	18.9	21.8	24.1	16.2	19.8	12.2	26.9
WTV	Mean error (%)	−3.5	−7.2	−2.4	−4.5	−5.9	9.1	17.5
	RMSE (%)	3.9	5.8	1.7	13.9	1.13	11.9	26.1

FIG. 9. Difference images of the phantom study (a) difference between the baseline image and the WTV-recovered image; (b) difference between the baseline image and the TV-recovered image. Both images are displayed in the same scale; units are $100 \times \text{Bq/mL}$.

FIG. 10. (a) SNR of the phantom study for the six disks: C1 (largest) to C6 (smallest); PS: partially sampled, BS: baseline. (b) Contrast recovery of the phantom study for the six disks: C1 (largest) to C6 (smallest); PS: partially sampled, CR: contrast recovery, BS: baseline.

FIG. 11. One axial slice of a patient study (patient 6) comparing two methods of recovering partially sampled PET images. (a) The baseline image, (b) the partially sampled image, (c) the WTV-recovered image, and (d) the simple TV-recovered image. All images are displayed with the same scale. Units are in natural log Bq/mL ($\alpha = 0.5$, $\beta = 0.85$).

was included in the cost function to model the radioactivity decay process, which is inherently described by a Poisson distribution.¹⁵ Not considering noise or modeling the noise using a Gaussian distribution results in worse outcomes, as can be seen in Figs. 2(d), 8(d), and 11(d). In these figures, the TV approach did not include a noise term and hence resulted in more reconstructed image noise than in the WTV-recovered image. These findings are further supported by the results in Figs. 6 and 7 showing that WTV has a better recovery compared to TV at all noise levels.

Combining wavelet with GMD requires carefully weighting each domain (α and β) in our proposed formulation [Eq. (4)]. Furthermore, this weighting should also balance the Poisson log likelihood term (data fidelity) such that each term is well represented in the final results. The data fidelity errors are mainly determined by the noise levels of the scan, whereas the optimal TV and wavelet values depend

on the spatial variation of the true object. As a result, α and β need to be carefully adjusted on scans with different settings (scan time, injected activity, postinjection, time activity distribution, etc.) and/or on different objects/patients (size, anatomy, tissue type). Although many investigators have spent considerable effort on an automatic selection of α and β , a generalized solution is still not available.²³ In this regard, the clinical performance of WTV regularization depends on iterative and subjective tweaking of the α and β parameters. In the simulation study, we chose values of 0.24 and 0.63 for α and β , respectively, while for the phantom study these values were 0.3 and 0.56 and for the patient study these values ranged between 0.1 and 0.5 for α and 0.5 and 0.85 for β . These selections were based on trial and error such that the gradient of the cost function remained negative (so the value it takes in each step is equal or smaller than the current step) while balancing the contribution of each term in the cost function. Probably the best choice of α and β should be adaptively based on varying these values in each iteration to help reach convergence with techniques similar to those presented by Barzilai–Borwein.^{24–26} If α is set to zero (or very small), the solution tends to converge to TV-CS recovery (with Poisson noise suppressed), and if β is set to zero (or very small) the solution diminishes the impact of TV and converges to wavelet recovery.

For the majority of our results, the mean error (bias) of the WTV approach underestimated the activity concentration compared with the fully sampled image. This was particularly the case in small ROIs (or lesions) that had few pixels. In this case, the comparison is harder and more erroneous owing to fewer pixels being measured. We believe that this might

TABLE III. The RMSE, ME for PS, WTV, TV images for six ROI in six patient studies. ME = mean error.

Patient #	Partially sampled		TV		WTV	
	RMSE%	ME%	RMSE%	ME%	RMSE%	ME%
1	25.2	−21.1	15.3	−0.2	4.7	−0.1
2	25.6	−25.7	18.9	−2.5	4.7	−1.4
3	21.3	−32.9	13.8	−2.9	10.4	−2.0
4	26.2	−31.1	9.7	−1.6	1.8	−0.8
5	23.9	−20.9	9.6	−0.2	1.4	−0.0
6	32.4	−32.5	23.4	−0.2	20.3	−0.3

FIG. 12. (a) SNR of the patient study. P1:P6 patient number; PS: partially sampled, BS: baseline. (b) Contrast recovery for the six patients; PS: partially sampled, CR: contrast recovery, BS: baseline.

be due to the effect of suppressing the noise provided by the Poisson log likelihood term. Further investigation of this issue is warranted. This issue, however, was not seen in the patient studies, perhaps because the lesion sizes were rather large.

The determination of the sparsest representation of the observed data in CS techniques requires the minimization of a cost function. In our study, this optimization was performed using the steepest descent technique²⁴ with small step sizes; albeit, such an approach suffers from a longer processing time due to the computational complexity of the cost function. To avoid potential local minima, the partially sampled image was provided as an initial condition. The use of a good initial condition makes convergence to a global minimum closer to the real solution.²⁷

We recovered the missing PET data by first reconstructing the images with no gap, forward-projecting these images with the system matrix, and then applying a mask corresponding to preset detector voids/gaps. In doing so, the resultant sinograms included all data corrections, albeit with detector gaps. Although this approach does not represent an actual measured sinogram with missing data, we believe that our approach does not affect the objective of our investigation since it affects the evaluation in a similar manner when either WTV- or TV-recovered images are compared with the fully sampled data. In practice, to correct the sinogram that is obtained from the scanner with the missing samples, correction factors for attenuation, decay, random, scatter,

and normalization should also be included as inputs to our recovery algorithm.

The results from our WTV approach show that the recovered image is quantitatively more accurate than a standard TV approach. This improvement, however, comes at the cost of computational complexity and longer processing time. For the WTV approach, 150 iterations were required for convergence while the TV technique used 20 iterations. It is noteworthy that the number of iterations between the two approaches is not directly comparable due to differences in determining these values. While the WTV approach recovered the missing data in a single optimization step using 150 iterations, the TV minimization utilized a two step solution with 20 iterations, as described by Ahn *et al.*¹² However, each step in the TV minimization was done using 20 iterations, resulting in an overall number of iterations of $20 \times 40 = 800$. In this evaluation, no investigation was performed to optimize the processing time of the WTV or TV algorithm. The recovery algorithms were implemented in MATLAB ver. 7.9 (R2009b), used a Dual 6c Xeon E5-2667 CPU machine with 2.9 GHz and 64 GB memory running Linux 2.6.32. The WTV algorithm had an average run time of 10–20 min per image (4–6 s/iteration) while the TV run time was 5–30 min (15–90 s/iteration) depending on the complexity of the image.

Owing to computational complexity, the CS framework in this investigation was developed in 2D, and the reconstruc-

FIG. 13. Difference images from one of the patient studies (patient 6). (a) Difference between the baseline image and the WTV-recovered image; (b) difference between the baseline image and the TV-recovered image. Both images are displayed using the same scale, and units are Bq/mL.

tion was performed slice by slice. Each slice was recovered independently with no information obtained from adjacent slices. Since the input to the algorithm was a 2D-corrected sinogram, the signal recovery and image reconstruction performed in this evaluation were conducted in 2D mode as well. Extending the approach to 3D is straightforward but requires more computational time.

Decreasing the number of detectors per ring while employing CS recovery techniques can be beneficial in several ways. This technique could lower the cost of the scanner because the scintillator and the associated photomultiplier tube are among the most expensive components of PET scanners. Less expensive scanners with fewer detectors would be more accessible to patients and physicians. Another potential advantage of the CS techniques is to decrease the scan time by increasing scanner sensitivity through exchanging the removed detectors between the transverse FOV and the axial FOV. Increasing the axial FOV increases the scanner sensitivity as well as the extent of body coverage, both of which lead to shorter scan times while maintaining image quality. Furthermore, decreasing the scan time could increase patient throughput and decrease the likelihood of patient movement during the scan. Finally, increasing scanner sensitivity could also be traded for lower injected activity and hence patient radiation dose, while maintaining similar image quality.

In addition to cost reduction and/or improving scanner performance, our proposed approach might also have potential applications in open PET geometries, such as some designs used in positron emission mammography,²⁸ in-beam PET for *in-vivo* dosimetry in hadron therapy,^{29,30} or when novel PET prototypes or demonstrators are developed and consist of few detector heads.^{31,32} These systems, however, have large detector gaps which might not be adequately recovered using our proposed approach. In this regard, an in depth investigation about the value of our proposed approach to such systems is warranted.

One limitation of this study is the examination of only 11% detector removal. Finding the optimum percentage of detector removal while retaining reliable image recovery depends on two factors: the sparsifying domain and the measurement space (e.g., sinogram). The correlation and dependency between these two domains is a key factor in the quality of image recovery.³³ We will explore this issue in further investigations. Furthermore, the location of the detector removal in this study was limited to the cardinal axes (0, 90, 270, and 360). However, this sampling pattern might not be the best suited. Our future research goals also include finding the best detector configuration to optimize the recovery of the partially sampled image.

In conclusion, CS techniques with Poisson noise modeling using WTV minimization is a promising approach to recover relatively accurate activity concentrations even in the presence of partially sampled data.

ACKNOWLEDGMENTS

The authors thank Jill Delsigne in the Department of Scientific Publications at The University of Texas MD Anderson

Cancer Center. This research was supported in part by the National Institutes of Health through the MD Anderson Cancer Center Support Grant (CA016672).

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