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Resolving Phase Ambiguity in Dual-Echo Dixon Imaging Using a Projected Power Method

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Abstract

Purpose—To develop a fast and robust method to resolve phase ambiguity in dual-echo Dixon imaging.

Methods—A major challenge in dual-echo Dixon imaging is to estimate the phase error resulting from field inhomogeneity. In this work, a binary quadratic optimization program was formulated to resolve the phase ambiguity. A projected power method was developed to efficiently solve the optimization problem. Both the 1-peak fat model and 6-peak fat model were applied to 3D datasets. Additionally, the proposed method was extended to dynamic MRI applications using the 6-peak fat model. With IRB approval and patient consent/assent, the proposed method was evaluated and compared with region growing on 29 consecutive 3D high-resolution patient datasets.

Results—Fast and robust water/fat separation was achieved by the proposed method in different representative 3D datasets and dynamic 3D datasets. Superior water/fat separation was achieved using the 6-peak fat model compared with the 1-peak fat model. Compared to region growing, the proposed method reduced water/fat swaps from 76% to 7% of the patient cohort.

Conclusion—The proposed method can achieve fast and robust phase error estimation in dual-echo Dixon imaging.

Keywords

water-fat separation; fat suppression; two-point Dixon; binary quadratic optimization; dynamic Dixon

Introduction

Dixon imaging offers robust water/fat separation in the presence of static field inhomogeneity (1–4). Due to chemical shift, water and fat signals can be separated from multiple datasets acquired at different echo times. Among various Dixon techniques, dual-

echo Dixon imaging is often desirable because of its high scan efficiency and flexibility (5–12).

A major challenge in dual-echo Dixon imaging is to estimate the phase error resulting from field inhomogeneity. Due to field inhomogeneity, two ambiguous phase error (or phasor) candidates can be analytically calculated at each image pixel (5,6,8,9). Since field inhomogeneity is usually smoothly varying, different types of methods can be applied to resolve this phase ambiguity. The first type of these methods uses a region growing approach, in which initial seeds are identified first, followed by a sequential estimation of the phase error (5,12). The second type, such as regional iterative phasor extraction (RIPE), applies a spatial smoothing to promote the smoothness of field inhomogeneity (6). The third type formulates the problem into a global optimization of smooth phase errors in local regions (9,13,14). These global optimization problems can be solved by graph cut (14) and tree-reweighted message-passing (TRW-S) (9) algorithms, and have shown to be robust in areas with large field inhomogeneity. However, these algorithms can be computationally intensive, especially for high dimensional datasets, such as 3D datasets or dynamic MRI datasets.

In this work, we formulate a binary quadratic optimization program to resolve the phase ambiguity in dual-echo Dixon imaging. A projected power method is proposed to efficiently solve the binary quadratic optimization. The proposed method is investigated in six different patient applications, in which high-resolution 3D dual-echo Dixon imaging is often an essential part of the corresponding imaging protocols. Both the 1-peak fat model and 6-peak fat model are compared. Additionally, the proposed method is applied in three different dynamic 3D MRI applications, in which the feasibility of fast joint space-time water/fat separation with the 6-peak fat model is investigated. To validate the robustness of the proposed method, a clinical study with 29 consecutive patients focusing on one application is performed. Water/fat separation is carried out using both a commercially available region growing method and the proposed method. The reconstructed images are compared and evaluated.

Theory

Dual-echo Dixon imaging can be modeled as (8):

$$\begin{aligned} S_1 &= (W + c_1 F) e^{i\phi_1}, \\ S_2 &= (W + c_2 F) e^{i\phi_2}, \end{aligned} \quad [1]$$

where S_1 and S_2 are complex composite signals acquired at two echo times, W and F are real-valued water and fat signals in image space, c_1 and c_2 are dephasing factors for fat with respect to water using a 1-peak fat model or multiple-peak fat model (usually known *a priori*) corresponding to the two echo times, and ϕ_1 and ϕ_2 are additional phases attributed to field inhomogeneity. In this work, the dephasing factors c_1 and c_2 are calculated by (8,15,16):

$$c_n = \sum_m w_m e^{i2\pi \Delta f_m t_n}, \quad n=1, 2, \quad [2]$$

where t_n represents the echo times, f_m is the resonance frequency offset for fat at different peaks of the fat model, and w_m is the corresponding weighting that sums up to 1. For Dixon imaging with in-phase and opposed-phase echo times and a 1-peak fat model, c_1 and c_2 are simply +1 and -1 respectively, while for multiple-peak fat models, c_1 and c_2 may differ in both magnitude and phase. For the rest of this paper, a 6-peak fat model is applied to approximate the multifrequency fat spectrum (15). The frequency offsets (in ppm) and the corresponding weightings are: -3.80 (8.7%), -3.40 (69.3%), -2.60 (12.8%), -1.94 (0.4%), -0.39 (3.9%), and 0.60 (4.8%). Please note that the 1-peak fat model in this work is referred to the dominant peak in the fat spectrum (-3.5 ppm), not to be confused with the “single peak” situation where an image pixel only contains a single component (water or fat) (17,18).

The main problem in dual-echo Dixon imaging is to estimate the phase error $\phi = \phi_2 - \phi_1$ (or phasor $P = e^{i\phi}$) between two echoes. Two pairs of solutions (W_1, F_1) and (W_2, F_2) can be obtained analytically (8), yielding two potential solutions for P .

$$\begin{aligned} P_1 &= \frac{S_1^* S_2}{(W_1 + c_1^* F_1)(W_1 + c_2 F_1)}; \\ P_2 &= \frac{S_1^* S_2}{(W_2 + c_1^* F_2)(W_2 + c_2 F_2)}. \end{aligned} \quad [3]$$

The problem then converts to choosing the correct P from P_1 and P_2 at each image pixel. Methods like spatial smoothing (6), region growing (5,12), and global optimization (9,13,14) can be applied to solve this problem.

A global optimization can be formulated as (9, 14):

$$\begin{aligned} &\text{minimize} \quad \sum_r \sum_{s \in \mathcal{N}_r} v(P(r), P(s)) \\ &\text{subject to} \quad P(r) \in \{P_1(r), P_2(r)\}, \quad \forall r, \end{aligned} \quad [4]$$

where r is an index of image pixels, $P(r)$ indicates the phasor associated with the pixel r , s represents pixels in the neighboring pixel set $\mathcal{N}_r$ of r , and v is a discontinuity cost function of phasor pair $(P(r), P(s))$. For example, a quadratic penalty (9, 14) with spatial weighting can be applied:

$$v(P(r), P(s)) = \frac{\min(|S_1(r)|, |S_1(s)|)}{d(r, s)} |P(r) - P(s)|^2, \quad [5]$$

where $d(r, s)$ represents the Euclidean distance between r and s . Note that Eq. [4] is non-convex and in general NP-hard.

Resolving Phase Ambiguity Using a Projected Power Method

In this work, we reformulate Eq. [4] into a binary quadratic optimization program.

Specifically, define a vector variable $X(r) = \begin{pmatrix} X_1(r) \\ X_2(r) \end{pmatrix} \in \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ for each pixel r , which indicates the binary selection of the phasor candidates. As illustrated in Fig. 1, we concatenate all $X(r)$ into a long vector X . Assume for 3D datasets, the image matrix size is $N_x \times N_y \times N_z$, then the dimensionality of X is $2N_x N_y N_z$. Eq. [4] can then be recast as:

$$\begin{aligned} & \text{minimize}_X \quad f(X) := X^{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{n}}} V X = \sum_{r,s} X(r)^{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{n}}} V(r, s) X(s) \\ & \text{subject to} \quad X(r) \in \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \forall r, \end{aligned} \quad [6]$$

where we use each sub-block $V(r, s) \in \mathbb{R}^{2 \times 2}$ of $V \in \mathbb{R}^{2N_x N_y N_z \times 2N_x N_y N_z}$ to encode all 4 possible values of $v(\cdot, \cdot)$ for any pixel pair (r, s) . To be more precise, for each pair (r, s) we set

$$V(r, s) = \begin{cases} \begin{bmatrix} v(P_1(r), P_1(s)), & v(P_1(r), P_2(s)) \\ v(P_2(r), P_1(s)), & v(P_2(r), P_2(s)) \end{bmatrix}, & \text{if } s \in \mathcal{N}_r, \\ 0, & \text{otherwise,} \end{cases} \quad [7]$$

where $P_1(\cdot)$ and $P_2(\cdot)$ are the two phasor candidates at image pixel r and s . This ensures

$$v(P(r), P(s)) = X(r)^{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{n}}} V(r, s) X(s), \quad \forall s \in \mathcal{N}(r), \quad [8]$$

justifying the equivalence between Eq. [4] and Eq. [6].

Throughout the rest of this paper, we set the default neighboring pixel set as $\mathcal{N}_r = \{s : d(r, s)$

$\leq 5\}$ (56 pixels in 3D), and the spatial weighting in Eq. [5] was applied (Fig. 2). Note that V is large but highly sparse, and VX can be implicitly represented as simple pixel-wise operations (details shown in Appendix). In this work, an iterative projected power method is developed to solve Eq. [6]. The major steps are summarized below:

1. Set $P(r)$ to unity (zero phase error) (8), and initialize X based on:

$$X(r) = \begin{cases} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, & |P(r) - P_1(r)| < |P(r) - P_2(r)|, \forall r. \\ \text{otherwise} & \end{cases} \quad [9]$$

2. Repeat until convergence or the iteration number reaches the maximum (by default 100):

a. Update $X \leftarrow VX$;

b.

$$\text{Set } X(r) \leftarrow \begin{pmatrix} \frac{X_2(r)}{X_1(r) + X_2(r)} \\ \frac{X_1(r)}{X_1(r) + X_2(r)} \end{pmatrix}, \forall r.$$

3. Repeat until convergence or the iteration number reaches the maximum (by default 100):

a. Update $X \leftarrow VX$;

b.

$$\text{Set } X(r) = \begin{cases} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, & \text{if } X_1(r) < X_2(r), \\ \text{otherwise}, & \end{cases} \quad \forall r.$$

In Step 2 and Step 3, convergence is declared when the relative ℓ_2 error of X at two consecutive iterations (denoted by X^{k-1} and X^k) is smaller than a pre-defined threshold ε ,

i.e. $\frac{\|X^k - X^{k-1}\|}{\|X^{k-1}\|} < \varepsilon$. The final phasor is $P(r) = X(r)^{\hat{a}\hat{T}\hat{n}} \begin{bmatrix} P_1(r) \\ P_2(r) \end{bmatrix}, \forall r$. Water and fat can then be separated by solving a pair of linear equations (8):

$$\begin{aligned} W' + c_1 F' &= S_1, \\ W' + c_2 F' &= S_2 P^*, \end{aligned} \quad [10]$$

where $W' = We^{i\phi_1}$ and $F' = Fe^{i\phi_1}$ are complex-valued water and fat signals.

Relationship to Power Method for Eigenvalue Decomposition

The proposed method shares some similarity with the power method commonly used for eigenvalue decomposition or principal component analysis. If Eq. [6] is relaxed and modified as:

$$\begin{aligned} &\text{minimize} && X^{\hat{a}\hat{T}\hat{n}}(-V)X \\ &\text{subject to} && \|X\|=1, \end{aligned} \quad [11]$$

then the problem reduces to finding the eigenvector corresponding to the largest eigenvalue of $-V$, which can be solved efficiently by a power method (19). Specifically, the leading eigenvector X of $-V$ can often be calculated iteratively by repeating the following steps:

1. $X \leftarrow (-V)X$;
2. Normalize X .

The problem described in Eq. [6] adds an additional binary constraint for the choice of X to Eq. [11]. In the proposed algorithm, Step 2 serves as an intermediate step, which is equivalent to normalizing X at each image pixel (instead of the entire vector), and Step 3 enforces the actual binary constraint. Therefore, we term the proposed method a *projected power method*.

Note that when $P_1 = -P_2$ (e.g., when $c_1 = 1$ and $c_2 = -1$), the problem becomes completely ambiguous. Specifically, if X is a solution to Eq. [6], then $\mathbf{1} - X$ (where $\mathbf{1} \in \mathbb{R}^{2N_x N_y N_z}$) is also an optimal solution. The convergence of the proposed method in this case relies on the initialization, and the complete ambiguity between X and $\mathbf{1} - X$ can result in global water/fat swaps (20).

Dynamic Dual-Echo Dixon Imaging

In certain clinical applications, such as dynamic contrast-enhanced (DCE) MRI (21) or cardiac cine MRI (22), dual-echo Dixon imaging needs to be repeatedly performed at different time points (contrast phases or cardiac phases). In these cases, water/fat separation is usually carried out separately at each time point, which makes it susceptible to global water/fat swaps at certain time points. Since the field map varies smoothly both in space and time, the discontinuity cost function in Eq. [5] can also include the time dimension. The proposed projected power method can be easily extended to dynamic MRI applications by including neighboring pixels in both time and space. In fact, a smaller neighborhood in time is usually needed. For the dynamic applications in this work, 58 neighboring pixels in total (56 in space and 2 in time) were included (as shown in Fig. 2).

Overall Water/Fat Separation Workflow

Here we summarize the overall workflow for water/fat separation in dual-echo Dixon imaging.

1. Phase-preserved coil combination

Dual-echo Dixon imaging is commonly accelerated using parallel imaging (23, 24) and/or compressed sensing (25). For accelerated datasets, parallel imaging and/or compressed sensing reconstruction is performed prior to water/fat separation. SENSE type of reconstruction already includes phase-preserved coil combination in the reconstruction (23, 26). For other coil-by-coil reconstructions, such as GRAPPA (24) and ARC (27), a final SENSE-type coil combination (with an acceleration factor of 1) is required to preserve the phase information for water/fat separation. In this case, coil sensitivity maps need to be estimated and applied for both echoes.

2. Downsampling of the coil-combined images

Since field inhomogeneity is smooth, we can downsample the original images to low-resolution images and speed up the reconstruction (28–31). In this work, all images are downsampled to an empirically determined voxel size of $6 \times 6 \times 6 \text{ mm}^3$.

3. Phasor calculation and selection

Solve Eq. [3] to obtain two phasor candidates, then perform the proposed projected power method to select the correct phasor candidate.

4. Upsampling of the selected phasor

The resolved phase error is spatially smoothed (8) then upsampled back to the original voxel size, followed by water/fat separation in Eq. [10].

Methods

With institutional review board approval and informed patient consent/assent, all the following dual-echo Dixon datasets were acquired on 3T GE MR750 scanners (GE Healthcare, Waukesha, USA) using dual-echo 3D spoiled-gradient echo sequences with DISCO (21), variable density Poisson disk (32), or variable density radial view ordering (22) sampling patterns.

Parameter Selection for Projected Power Method

Phasor Initialization and Phase Offset—To test the sensitivity of the phasor initialization (Step 1 of the proposed method) as well as additional signal phase offset, the projected power method was performed on a 3D pelvis dataset (acquisition parameters listed in Tab. 1) under four scenarios: (i) phasor P was initialized with unity (zero phase error) at each pixel; (ii) P was initialized with -1 (180° phase error) at each pixel; (iii) P was initialized with a random phase error at each pixel; (iv) P was initialized with unity and S_2 in Eq. [1] was added with an additional 180° phase offset. Both the 1-peak fat model and 6-peak fat model were applied in all scenarios. For the 1-peak fat model, two sets of dephasing factors c_1 and c_2 were applied: (a) c_1 and c_2 were manually set to $+1$ and -1 ; (b) c_1 and c_2 were calculated from Eq. [2] to be $0.9999 - 0.0132i$ and $-0.9998 + 0.0197i$ respectively based on the actual echo times. For the 6-peak fat model, c_1 and c_2 were also calculated from Eq. [2] to be $0.7975 - 0.2352i$ and $-0.6088 + 0.2007i$ respectively. The other default parameters (downsampling/upsampling, 200 iterations in total, 56 neighboring pixels in 3D) were applied in all scenarios.

Neighboring Pixel Set—To determine the necessary neighboring pixel set, the proposed method was performed on a 3D breast dataset and a 3D abdomen dataset (acquisition parameters listed in Tab. 1) with three different neighboring pixel sets: (i) $|\mathcal{N}_r| = 6$ ($d = 1$), reported in (9), (ii) $|\mathcal{N}_r| = 26$ ($d = 3$), 3D extension of the 2D second-order neighborhood reported in (14), and (iii) $|\mathcal{N}_r| = 56$ ($d = 5$). The other default parameters (unity initialization, downsampling/upsampling and 200 iterations in total) were applied in all experiments.

3D Dual-Echo Dixon Imaging

To test the robustness of the default reconstruction parameters for the proposed method, six different representative datasets (pelvis, breast, abdomen, chest, foot, leg) were selected. Patient demographics and acquisition parameters are listed in Tab. 1. As summarized in the overall water/fat separation workflow, coil combination, downsampling, phasor selection, and upsampling were sequentially performed for the proposed method. Both the 1-peak fat model and 6-peak fat model were applied, with the corresponding dephasing factors calculated from Eq. [2].

Dynamic Dual-Echo Dixon Imaging

To investigate the feasibility of joint space-time water/fat separation on dynamic MRI applications, three representative datasets were selected, including 3D breast DCE MRI, 3D cardiac cine MRI, and 3D abdominal DCE MRI. Patient demographics and acquisition parameters are listed in Tab. 2. The cardiac cine dataset was acquired with prospective ECG-triggering and retrospectively interpolated into 20 cardiac phases. The proposed method with the default parameters (unity phasor initialization, downsampling/upsampling, 200 iterations in total, and 58 neighboring pixels in space and time) and the 6-peak fat model was performed.

Robustness Evaluation

To evaluate the robustness of the proposed method, we focused on one clinical application and recruited 29 consecutive patients (21 female and 8 male, mean age 10.1 years) at our institution, who were referred for extremity MRI with concerns of soft tissue tumor, bone tumor or inflammation. Due to the large field of view required to address the imaging concern (coverage ranges from lower pelvis to toes), 3D high-resolution dual-echo Dixon imaging was performed as a crucial part of our institutional imaging protocol. The Dixon imaging parameters were: TE1/TE2 = 2.2/3.3 ms, TR = 6.3 - 7.3 ms, and bandwidth = ± 100 kHz. Patient demographics and the rest of the acquisition parameters are listed in Supporting Table S2. The proposed method was performed for water/fat separation, and the corresponding results were compared with the commercial reconstruction using the region growing method (GE MR750 software version DV25) (5). Only the 1-peak fat model was applied in both methods. Two board-certified radiologists, who were blinded to the type of reconstructions, reviewed both reconstructions in consensus. For each patient, two readers identified whether water/fat swaps (global and/or local) appeared in each reconstruction. Additionally, the preference between the region growing method and the proposed method was recorded. The preferred reconstruction was retrospectively identified as region growing or the proposed method. The preference criteria were: (1) region growing was preferred over the proposed method; (2) no preference was made between the two methods; (3) projected power was preferred over the region growing method.

To reduce the reconstruction time, coil compression (with 6 virtual coils) was applied to all datasets at the beginning of the reconstruction prior to parallel imaging and compressed sensing reconstruction and water/fat separation (33). To preserve the phase between two echoes, the same coil compression matrices were applied to data from both echoes. Depending on the sampling pattern, ARC (27), ESPIRiT (26), or compressed sensing

parallel imaging reconstruction (22) was performed to reconstruct the undersampled datasets. When applicable, SENSE-type coil combination (with acceleration factor of 1) was performed on the virtual coils prior to the water/fat separation. For the 3D dual-echo Dixon datasets, the proposed method was implemented in C/C++. For the dynamic dual-echo Dixon datasets, the proposed method was carried out in Matlab (The Mathworks, Natick, MA). All reconstructions were performed on a PC with two Intel Xeon E5-2640 processors and 64 GB DRAM. Reconstruction times for phasor selection and the entire water/fat separation reconstruction times were recorded. A demonstration of the proposed method can be found at <http://web.stanford.edu/~tzhang08/software.html>. The region growing method in the clinical evaluation was performed directly on the scanner using the commercial reconstruction. Water/fat separation results in all 3D datasets and dynamic 3D datasets were also subjectively evaluated by the two radiologists.

Results

Parameter Selection for Projected Power Method

Phasor Initialization and Phase Offset—The water/fat separation results are shown in Fig. 3. The experiments with the 1-peak fat model (a) $c_1 = 1$ and $c_2 = -1$ and (b) $c_1 = 0.9999 - 0.0132i$ and $c_2 = -0.9998 + 0.0197i$ were labeled as “1-Peak I” and “1-Peak II” respectively. No global or local water/fat swaps were observed in case with 1-Peak II and the 6-peak fat model. However, for 1-Peak I ($P_1 = -P_2$), global or local swaps appeared under different initialization conditions and additional phase offset.

Neighboring Pixel Set—The water/fat separation results with three different neighboring pixel sets of the breast dataset and the abdomen dataset are shown in Fig. 4. For the breast dataset, the proposed method converged in all cases. However, for the abdomen dataset, a large neighboring pixel set was necessary for the proposed method using the 1-peak fat model.

3D Dual-Echo Dixon Imaging

The reconstruction times for phasor selection and the entire water/fat separation is shown in Tab. 1. Fast reconstruction times were achieved for all 3D datasets. Examples of water and fat images using the proposed method are shown in Fig. 3 (pelvis), Fig. 4 (breast), Fig. 4 (abdomen), Fig. 5(a) (chest), Fig. 5(b) (foot), and Fig. 5(c) (leg). No global or local water/fat swaps were observed for the 1-peak fat model (dephasing factors calculated from Eq. [2]) and the 6-peak fat model. Significant better water/fat separation results were achieved using the 6-peak fat model in the pelvis, breast, foot, and leg datasets.

Dynamic Dual-Echo Dixon Imaging

The reconstruction times for phasor selection and the entire water/fat separation using the proposed method are shown in Tab. 2. The proposed method was slower due to the non-optimized implementation in Matlab compared to the C/C++ implementation for the 3D datasets. Examples of water and fat images are shown in Fig. 6 (breast DCE), Fig. 7 (cardiac cine), and Supporting Figure S1 (liver DCE). Excellent water/fat separation was achieved for

all datasets using the proposed method. No global or local water/fat swaps were observed at any contrast/cardiac phases.

Robustness Evaluation

Out of the 29 cases included in this study, local swaps were observed in 22 cases (76%) using the region growing method and 2 cases (7%) cases using the proposed method. For the proposed method, the local swaps appear at the edge of the images for the two cases. No global swaps were observed in any case in either reconstruction. In 11 cases (38%), the proposed method was preferred over the region growing method. In the other 18 cases (62%), two readers did not have any preference between the two methods. Region growing method was never preferred over the proposed method. Representative results are shown in Fig. 8: Fig. 8(a) both methods achieved similar results and no preference was made; Fig. 8(b) region growing suffered from local swaps and no preference was made; Fig. 8(c) region growing suffered from local swaps and the proposed method was preferred.

Discussion

Resolving phase ambiguity is a crucial step in dual-echo Dixon imaging. Inaccurate phase error estimation can lead to global and/or local water/fat swaps. In this work, the phase estimation was formulated as a binary quadratic optimization, and a projected power method was proposed to achieve fast phase error estimation. The proposed method shares similarity with other global optimization algorithms such as graph cut (14) and TRW-S (9), which have shown to be robust in the presence of large field inhomogeneity. The proposed method exploits phase continuity between neighboring pixels and can be solved by simple point-wise operations. As demonstrated in Appendix, the computational complexity of VX is proportional to the number of neighboring pixels in $\mathcal{N}_r$, and therefore is $\mathcal{O}(N_r N)$ (where N is the total number of pixels and N_r is the number of neighboring pixels). Downsampling, assuming the phase inhomogeneity is smooth, reduces N and hence the phasor selection time (28–31). Since the number of neighboring pixels only increased from 56 to 58 when the projected power method was extended from 3D to 4D in dynamic MRI applications, faster reconstruction times are expected using the proposed method when implemented in C/C++. The advantage of this extension is that field inhomogeneity is also enforced to be smoothly varying in time, preventing global water/fat swaps in single time points, which is more likely to happen if water/fat separation is performed independently at each time point.

The main reconstruction parameters of the proposed method are the initial phasor, the downsampled voxel size, the neighboring pixel set and the iteration number. The phasor was initialized as unity (zero phase error) in the proposed method, relying on a reasonable shimming suggested by (8). As shown in Fig. 3, when the binary quadratic problem becomes completely ambiguous ($P_1 = -P_2$), the convergence of the proposed method strongly relies on the initial condition. Note that the echo times in most cases included in this work were extremely close to in-phase and opposed-of-phase echo times, however, due to the quantization of actual echo times, the dephasing factors c_1 and c_2 calculated from Eq. [2] may not be exactly +1 and -1, yielding $P_1 \neq -P_2$. Though the problem in Eq. [6] is very ill-conditioned in this case, the proposed method is likely to converge regardless of the initial

condition (Fig. 3). When the multiple-peak fat model is applied, the condition of $P_1 - P_2$ is stronger, which further improves the convergence of the proposed method. As suggested by (6, 8, 9), partially-opposed-phase echo times or flexible echo times could be applied to reduce the ill conditioning of the phase ambiguity. More complicated initialization can also be performed to improve the convergence (6). As demonstrated in Fig. 4, when the 1-peak fat model was applied, a large neighboring pixel set may be necessary because of the ill conditioning of Eq. [6]. The downsampled voxel size and the number of iterations were empirically determined based on representative 3D datasets and kept constant for all datasets included in this work. Automatic reconstruction parameter determination will be the subject of future work.

One of the limitations of this work is that the proposed method has only been compared with the commercial region growing method using the 1-peak fat model in the clinical study. The proposed method can reduce the local water/fat swaps that frequently appeared in the region growing reconstruction. Recent advances of the region growing method can potentially reduce the local swaps and improve the robustness of water/fat separation as well (12). Graph cut (14), TRW-S (9) and the proposed method share similar global optimization formulation, and are expected to achieve similar robustness. A thorough comparison (e.g. computational complexity) between these methods will be the subject of future work.

Another limitation is that the proposed method can only be applied to dual-echo Dixon imaging for water/fat separation. Although not thoroughly evaluated, the proposed method is expected to work with flexible echo times, as evidenced by the 3D chest dataset (TE1/TE2 = 1.3/2.2 ms) and cardiac cine dataset (TE1/TE2 = 1.6/2.2 ms). Because of the similarity, the proposed method can potentially be applied to solve other problems (34), such as velocity unwrapping in phase contrast MRI (35) and phase-sensitive inversion-recovery (PSIR) reconstruction (36).

Conclusion

In this work, a binary quadratic optimization program has been formulated to resolve the phase ambiguity in dual-echo Dixon imaging. A projected power method has been developed to efficiently solve the binary quadratic optimization. Fast reconstruction times and robust water/fat separation have been achieved for high dimensional datasets by the proposed method.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

Acknowledgments

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Appendix A: Fast Calculation of VX

For simplicity, we only demonstrate the point-wise operations of VX in one-dimensional datasets. Consider the scenario where $N_y = N_z = 1$, in which case X reduces to a $2N_x$ -dimensional vector. Here, we assume the neighboring pixel set $\mathcal{N}_r$ of r consists only of its

nearest neighbors, i.e. $\mathcal{N}_r = \{r-1, r+1\}$. Based on the definition of $X(r) = \begin{pmatrix} X_1(r) \\ X_2(r) \end{pmatrix}$, we decompose X into two components

$$X_1 := [X_1(r)]_{1 \leq r \leq N_x} \quad \text{and} \quad X_2 := [X_2(r)]_{1 \leq r \leq N_x}, \quad [\text{A1}]$$

where both X_1 and X_2 are N_x -dimensional vectors, representing the binary selection of P_1 and P_2 respectively. In addition, we set $X' := VX \in \mathbb{R}^{2N_x}$ for simplicity, and decouple X' into two components X'_1 and X'_2 , similar to [A1].

The computation of X' relies on pointwise multiplication with the following phasor difference matrices $D_1, \dots, D_4 \in \mathbb{R}^{N_x \times 1}$:

$$\begin{aligned} D_1 &= [v(P_1(r), P_1(r+1))]_{1 \leq r < N_x}, \\ D_2 &= [v(P_1(r), P_2(r+1))]_{1 \leq r < N_x}, \\ D_3 &= [v(P_2(r), P_1(r+1))]_{1 \leq r < N_x}, \\ D_4 &= [v(P_2(r), P_2(r+1))]_{1 \leq r < N_x}. \end{aligned} \quad [\text{A2}]$$

Specifically, if we define $W(i:j) := [W]_i \dots j$ for any vector W and denote

$$\begin{aligned} X'_{1,1} &= X'_1(1:N_x - 1), \\ X'_{1,2} &= X'_1(2:N_x), \\ X_{1,1} &= X_1(1:N_x - 1), \\ X_{1,2} &= X_1(2:N_x), \\ X_{2,1} &= X_2(1:N_x - 1), \\ X_{2,2} &= X_2(2:N_x), \end{aligned} \quad [\text{A3}]$$

then X' can be calculated by (1) initialize X' to be zero, and (2) sequentially implement the following update rules:

$$\begin{aligned} X'_{1,1} &\leftarrow X'_{1,1} + X_{1,2} \odot D_1 + X_{2,2} \odot D_2, \\ X'_{1,2} &\leftarrow X'_{1,2} + X_{1,1} \odot D_1 + X_{2,1} \odot D_3, \\ X'_{2,1} &\leftarrow X'_{2,1} + X_{1,2} \odot D_3 + X_{2,2} \odot D_4, \\ X'_{2,2} &\leftarrow X'_{2,2} + X_{1,1} \odot D_2 + X_{2,1} \odot D_4, \end{aligned} \quad [\text{A4}]$$

where $\circ$ represents the Hadamard product (entry-wise matrix multiplication). This allows us to obtain $X' = VX$ within linear time.

The preceding paradigm can be easily extended to larger-dimensional datasets. When a larger neighboring pixel set is employed, more difference matrices $\{D_i\}$ need to be included in [A4]; in fact, for the neighborhood $\mathcal{N}_r = \{s : d(r, s) \leq 5\}$ (56 pixels in 3D) applied in this work, 112 such difference matrices are included. As demonstrated in [A4], the computation of $X \leftarrow VX$ can be completed in time $\mathcal{O}(N_r N)$, where N_r is the neighborhood size and N the total number of image pixels.

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Figure 1.

Example of the formulation of the objective function in Eq. [6] with 2 image pixels. For simplicity, the spatial weighting and signal intensity weighting in Eq. [5] are ignored in this figure. There are 4 different possible phasor choices for the image pixel pair. For each possible phasor pair, the corresponding smoothness penalty ($v_1, \dots, v_4$) is illustrated in the figure. Depending on the phasor selection indicated by the binary vector X , a large and sparse penalty matrix V can be formulated.

Figure 2.

Demonstration of the neighboring pixel set applied in this work. For 3D datasets, 56 neighboring image pixels in space around the center image pixel ($\mathcal{N}_r = \{s : d(r, s) \leq Q5\}$) are included. For dynamic 3D datasets, two additional neighboring pixels in time are included. Neighboring pixels with different Euclidean distances are illustrated with different colors.

Figure 3.

Water/fat separation results of the pelvis dataset using the projected power method with different initialization scenarios and simulated signal phase offset. Only the water images are shown. For better viewing purposes, water and fat images have been manually switched back in the case of a global water/fat swap, and the corresponding water images are labeled with “G” at the top left of the image. When local swaps appear, the corresponding images are labeled with “L”. In the first scenario (first row) with the 1-peak fat model ($c_1 = 1$ and $c_2 = -1$), Eq. [6] is completely ambiguous, and therefore the proposed method relies on specific initialization conditions to converge. The problem is also sensitive to the additional signal phase offset. In the second scenario (second row) with the 1-peak fat model ($c_1 = 0.9999 - 0.0132i$ and $c_2 = -0.9998 + 0.0197i$), Eq. [6] has a unique solution (though very ill-conditioned), and the proposed method does not seem to heavily depend on the initialization condition. In the third scenario (last row) with the 6-peak fat model ($c_1 = 0.7975 - 0.2352i$ and $c_2 = -0.6088 + 0.2007i$), Eq. [6] also has a unique solution, and the proposed method does not rely on the initialization condition to converge.

Figure 4.

Water/fat separation results of the breast and abdomen datasets using the projected power method with three different neighboring pixel sets demonstrated in Fig. 2: $|\mathcal{N}_d| = 6$ ($d = 1$), $|\mathcal{N}_d| = 26$ ($d = 3$), and $|\mathcal{N}_d| = 56$ ($d = 5$). Only the water images and the estimated phase errors (field map) are shown. When local water/fat swaps appear, the corresponding water images are labeled with “L” at the top left of the image. For the breast dataset, similar water images are achieved by the proposed method for different neighboring pixel sets. For the abdomen dataset, the proposed method with the 1-peak fat model requires a large neighboring pixel set to avoid local water/fat swaps, possibly due to the ill conditioning of Eq. [6] using the 1-peak fat model.

Figure 5.

Water/fat separation results of the (a) chest, (b) foot, and (c) leg datasets using the projected power method with both the 1-peak fat model and 6-peak fat model (left: water; right: fat). No global or local water/fat swap was observed in any case. Significantly better water/fat separation was achieved in (b) and (c) when the 6-peak fat model was applied. Soft tissue tumor (arrows) is well delineated in the upper left chest in (a). Synovial inflammation is well seen in the water images in (b).

Figure 6.

Water/fat separation results of the breast DCE dataset for all contrast phases using the 6-peak fat model (left: water; right: fat). Excellent water/fat separation was achieved using the proposed method. No global or local water/fat swaps were observed at any contrast phase. Contrast dynamics of the tumor in the left breast is well captured.

Figure 7.

Water/fat separation results of the contrast-enhanced cardiac cine dataset for all cardiac phases using the 6-peak fat model (left: water; right: fat). Robust water/fat separation was achieved with flexible echo times of 1.6 and 2.2 ms. No global or local water/fat swaps were observed at any cardiac phase.

Figure 8.

Representative water images in the clinical evaluation. (a) Similar water images are achieved using both the region growing method and the projected power method. Two readers did not have preference between the two reconstructions. (b) Although region growing method suffered from a local water/fat swap (arrow), two readers did not have preference between the two reconstructions because the soft tissue tumor (left extremity) was well delineated in both reconstructions. (c) Due to the severe local swaps in the region growing reconstruction (arrow), both readers preferred the projected power method.

Table 1
Patient demographics, acquisition parameters and reconstruction times for high-resolution 3D datasets.

	Pelvis	Breast	Abdomen	Chest	Foot	Leg
Gender	F	F	F	F	F	M
Age (years)	2	49	34	13	7	0.7
TE1/TE2 (ms)	2.2/3.3	2.2/3.3	1.1/2.2	1.3/2.2	2.2/3.3	2.2/3.3
TR (ms)	6.8	6.3	4.1	4.1	6.7	5.9
FOV (cm ³)	18×14×15	27×27×19	32×29×28	40×40×23	20×20×10	30×30×23
In-plane matrix *	354×256	416×320	260×202	320×320	354×224	354×320
Number of slices	188	192	92	152	200	460
Receive coil	32ch Cardiac	16ch Breast	32ch Torso	32ch Cardiac	8ch Foot	32ch Cardiac
Phasor selection time (s)	1	4	7	9	1	6
Total recon time (s)	58	39	24	48	59	126

* All in-plane image matrices were zero-padded to 512×512 prior to the water/fat separation reconstruction.

Table 2

Patient demographics, acquisition parameters and reconstruction times for dynamic 3D datasets.

	Breast DCE	Cardiac CINE	Adbomen DCE
Gender	F	F	M
Age (years)	68	3	46
TE1/TE2 (ms)	2.2/3.3	1.6/2.2	1.1/2.2
TR (ms)	6.2	4.1	4.0
FOV (cm ³)	30×36×19	22×15×17	38×30×30
In-plane matrix	416×386	224×224	260×180
Number of slices	192	120	100
Receive coil	16ch Breast	32ch Cardiac	32ch Torso
Number of phases (contrast or cardiac)	19	20	9
Phasor selection time per phase (s)	33	8	53
Total recon time per phase (s)	37	9	54