A Computer-Aided Type-II Fuzzy Image Processing for Diagnosis of Meniscus Tear

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Abstract Meniscal tear is one of the prevalent knee disorders among young athletes and the aging population, and requires correct diagnosis and surgical intervention, if necessary. Not only the errors followed by human intervention but also the obstacles of manual meniscal tear detection highlight the need for automatic detection techniques. This paper presents a type-2 fuzzy expert system for meniscal tear diagnosis using PD magnetic resonance images (MRI). The scheme of the proposed type-2 fuzzy image processing model is composed of three distinct modules: Pre-processing, Segmentation, and Classification. λ -enhancement algorithm is used to perform the pre-processing step. For the segmentation step, first, Interval Type-2 Fuzzy C-Means (IT2FCM) is applied to the images, outputs of which are then employed by Interval Type-2 Possibilistic C-Means (IT2PCM) to perform post-processes. Second stage concludes with re-estimation of “ η ” value to enhance IT2PCM. Finally, a Perceptron neural network with two hidden layers is used for Classification stage. The results of the proposed type-2 expert system have been compared with a well-known segmentation algorithm, approving the superiority of the proposed system in meniscal tear recognition.

Keywords Expert system · Computer-aided diagnosis (CAD) · Interval type-2 fuzzy set theory · Knee · Meniscus tear · Medical image processing

Introduction

Meniscal tears are very common knee joint conditions resulting in cartilage and bone degeneration which also cause pain and loss of mobility [1]. These crescent-shaped fibrocartilaginous washers, named menisci of the knee, protect articular cartilage. Their main function is to maximize contact area and minimize contact stress within the tibiofemoral joint. The contact mechanics of this joint could be easily altered due to meniscal tears. Moreover, the homeostasis of articular cartilage metabolism might be disrupted, which will place the knee at risk for early osteoarthritis [2]. Therefore, diagnosing the meniscal tears in an appropriate time is very essential for further treatments.

In recent years, medical imaging techniques have been significantly advanced and have become vital in human internal organ examination. Among these diverse imaging modalities, magnetic resonance imaging has proven to be a powerful tool for medical diagnosis. In particular, they also enable noninvasive assessment of tissue morphology and function. MRI has provided significant insights into the normal and diseased anatomy for medical research, and is known as a critical component in diagnosis and treatment planning [3]. Also, in the context of knee diagnosis, evaluation of the knee is usually performed with MRI; however, arthroscopy is the gold standard for in vivo evaluations [4]. In this paper, PD magnetic resonance images are used to diagnose meniscal tears.

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Fuzzy sets, systems, and relations are very useful to deal with environments where boundaries lack sharpness in the sets of symptoms, diagnosis, and diseases. Medical images like MRI suffer from uncertainties and vagueness, which makes it difficult to handle them with type-I fuzzy sets. Type-2 fuzzy sets are able to model such uncertainties, as their membership functions are themselves fuzzy, as compared to that of type-I fuzzy sets, where their membership functions are crisp [5]. This paper pursues two objectives. First, a hybridized approach based on type-2 fuzzy sets is developed to overcome the shortcomings of IT2FCM, besides improving the performance of IT2PCM, since type-2 fuzzy logic systems have potential to provide better performance [6]. Second, these techniques are employed to design, implement, and evaluate a type-2 fuzzy expert system to diagnose meniscal tears.

The rest of this paper is organized as follows: “**Knee Joint and Meniscus**” section explains knee joint briefly. “**Tear Diagnosis**” section addresses meniscal tear diagnosis. “**Type-2 Fuzzy Logic**” section introduces the fundamentals of type-2 fuzzy logic. “**Proposed Image Processing Method**” section outlines the proposed image processing approach. “**Material**” section provides material section. Experimental results and discussions are presented in “**Experiment Results and Discussion**”

Fig. 1 Top: structure of the knee joint, and the medial and lateral menisci [7]. Bottom: sagittal view (left) and coronal view (right) of the knee

section. Finally, “**Conclusion and Future Works**” section provides conclusions and future works.

Knee Joint and Meniscus

The knee is not only one of the largest but also one of the most complex joints in the human body. In many physical and sport activities, it encounters extreme levels of stresses leading to injury. The meniscus is essential to support the knee mechanism. It is basically a cushion-like structure between the bones of the knee joint (femur and tibia). It includes two large C-shaped (medial side) and O-shaped (lateral side) structures (Fig. 1, top) [8], and acts as a shock absorber by

Table 1 Common categories of meniscal tears [12–15]

Class of the tear	Main features	Prevalence and statistics
Horizontal tears	Parallel to the tibial plateau, dividing meniscus into upper and lower segments	Detection sensitivity, specificity, and accuracy in MRI mode: 59 to 80 %, 90 %, 78 to 86 %, respectively
Vertical tears	Between circumferential collagen fibers, parallel to long axis of meniscus, perpendicular to tibial plateau	Prevalence of vertical radial tears: 14 to 15 %, with 79 % in the posterior horns, detectable in 37 % up to 89 % of the cases Parrot-beak tears: difficult to detect with sensitivity of between 0 and 60 %
Bucket-handle tears	Inner meniscal segment of a longitudinal or oblique tear is flipped into the intercondylar notch	Occurring in approximately 10 to 26 % of the patients, with detection sensitivity of 64 to 94 % in MRI mode
Flap tears	Short segment, horizontal meniscal tears with fragments displaced into the notch or into the superior or inferior gutters	Coronal images are the most useful imaging mode for detection of these tears
Root tears	As an only meniscal tear observed in posterior horn of meniscus	In the setting of an anterior cruciate ligament (ACL) tear, the lateral meniscal root is torn in 75 % of the times, three times as often as the medial root
Complex tears	Joint characteristics derived from two or more different tear configurations	–

Fig. 2 a Interval-valued type-2, b generalized type-2 [28]

distributing the forces of weight-bearing on the joint surfaces or between the femur and the tibia. The meniscus also supports a lubricating effect on the knee joint, provides some degree of stability, and plays an essential role for the normal performance of the knee joint [9].

The specific structural properties of meniscus allow the knee to perform a variety of functions, including the distribution of stresses over the articular cartilage, absorbing the shocks during axial loading, stabilizing the joint in both flexion and extension, lubricating the joint, etc. They also contribute toward secondary stabilization

of the knee after infliction of cruciate ligament injuries [10]. The essence of the menisci is more highlighted when the body is imposed to different conditions. For instance, they balance the forces across the knee joint during walking and running, when the forces imposed to the knee are as high as two to four and up to six to eight times body weight, respectively [11].

Meniscal tears are one of the prevalent knee injuries. Athletes, especially those playing contact sports, are at higher risk of infliction of meniscal tears than others [9]. Generally, the cause of meniscal tears is divided

Fig. 3 Schema of the proposed model

Fig. 4 The original input MRI to the system (*left*), the resulted image from pre-processing module (*right*)

into two categories: increased force on a normal meniscus, which usually results in longitudinal or radial meniscus; and normal forces on degenerative meniscus, which usually produces horizontal tears. Medial meniscus is more susceptible to these tears, possibly because this meniscus is less mobile, bearing more force than lateral meniscus. These tears are more common among

the older sector, which also suffers from degenerative tears [7, 10].

Meniscal tears could come in different ways. They are usually noted by how they look, as well as the location of the tear in meniscus [9]. Table 1 [16] represents the different types of meniscal tears, including some specific features if necessary.

Fig. 5 Pseudo-code of the type-2 λ -enhancement algorithm

ALGORITHM 1. Type-2 λ -enhancement algorithm

input Gray image $M \times N$

1. Calculate histogram ($h(g)$)
2. **Find** g_{max} **and** g_{min}
Comment: Initialize the membership values
3. $\mu(g) = \frac{g - g_{min}}{g_{max} - g_{min}}$
4. $k \leftarrow 1$
5. $\tilde{Y}_0 \leftarrow 0$
6. $\lambda \leftarrow -0.9$
7. $\alpha \leftarrow 10$
8. **repeat**
9. $\mu_{\lambda}^*(g) \leftarrow \frac{\mu(g)(1+\lambda)}{1+\lambda\mu(g)}$
10. $\mu_{U_k}(g) \leftarrow (\mu_{\lambda}^*(g))^{1/\alpha}$
11. $\mu_{L_k}(g) \leftarrow (\mu_{\lambda}^*(g))^{\alpha}$
12. $\tilde{Y}_k(\tilde{A}) \leftarrow \frac{1}{MN} \sum_{g=0}^{L-1} h(g) \times [\mu_{U_k}(g) - \mu_{L_k}(g)]$
13. **if** $\tilde{Y}_{k-1} > \tilde{Y}_k$ **then**
14. $k \leftarrow k + 1$
15. $\lambda \leftarrow \lambda + 0.05$
16. **end if**
17. **until** $\tilde{Y}_{k-1} < \tilde{Y}_k$
- output** λ_{opt}
18. $\lambda_{opt} \leftarrow \lambda - 0.005$
Comment: Calculate new membership
19. $\mu_{new}^* \leftarrow \frac{\mu(g)(1+\lambda_{opt})}{1+\lambda_{opt}\mu(g)}$
Comment: Generate new gray level
20. $\hat{g} \leftarrow (1 - L) \times \mu_{new}^*(g)$

* $M \times N$: the size of the input image

g : gray levels with values in following interval $[0, L - 1]$

g_{max} : the maximum gray level

g_{min} : the minimum gray level

$h(g)$: intensity histogram

$\mu(g)$: gray levels membership functions

$\mu_{\lambda}^*(g)$: Sugeno complex membership function

$\mu_{U_k}(g)$: upper bound of interval type - 2 membership functions

$\mu_{L_k}(g)$: lower bound of interval type - 2 membership functions

α : fuzzifier

λ : Sugeno multiplier

$\tilde{Y}$: fuzzy index

Fig. 6 Schema of proposed algorithm for segmentation module

Magnetic resonance imaging (MRI) provides superior soft-tissue contrast, which assists radiologists to differentiate and extract pathologic abnormalities from normal tissues. MRI is an advantageous tool, which provides specific features in diagnosis of internal derangements of the knee, including meniscal tears. Generally, MR images depict a normal meniscus as uniform, dark triangles (i.e., low intensity) or polygons (sagittal view). However, this normal dark triangular appearance is transformed when the meniscus is torn [13]. Figure 1 (bottom) shows the sagittal and coronal view of a MR image of the knee joint.

By knowing these tear specifications and other general diagnosing factors (such as history, physical examination, and radiology findings), physicians and also systems could diagnose a variety of meniscal tears using various methods, which are shortly outlined in the next section.

Tear Diagnosis

In the past decade, a variety of methodologies have been developed for meniscal tear diagnosis. These methodologies include locating the meniscus on medical images; developing systems for tear diagnosis based on pattern recognition tools such as Support Vector Machine (SVM) classifiers, Fuzzy methods, boundary tracing methods, etc.; and, finally, analyzing different imaging modalities to determine which imaging setting will provide better screening.

Mainly, two approaches are used in the context of meniscus locating: histogram-based and segmentation-based approaches. In the histogram-based method, the morphology of the femur and the location of femoral notch play the key role in meniscus locating. This methodology involves generating two histograms (horizontal and vertical histograms) of the knee images from the original MR scan. After smoothing and normalizing these histograms, their maximum and minimum values are used to approximately locate meniscal regions [14].

In the segmentation-based meniscal region locating methods, the anatomy of the knee incorporates the locating procedure. Thus, the accuracy of the diagnosis increases, and the computation time is reduced as compared to the histogram-based methods. Köse et al. [14] proposed a statistical segmentation method to segment the meniscal regions. This method employs the statistical properties of a texture and then segments the bones of the knee. They additionally proposed to calculate a characteristic image in order to minimize the threshold error during the segmentation procedure. Then, the statistical properties of this reference characteristic image are compared with the sample image. If the difference is less than the desired threshold value, the label of the pixel will be assigned as bone and this process is carried out on all the other pixels of the image. Finally, an adaptive region growing technique, proposed by Pohle

ALGORITHM 2. Interval Type-2 Possibilistic C-Means (IT2PCM)

input Image $M \times N$

Comment: Initialize primary values

1. $C \leftarrow$ Optimal number of clusters driven from Kwon validity index
2. $m \leftarrow$ Optimal value of fuzzifier driven from Kwon validity index

3. $k \leftarrow 1$

4. $l \leftarrow 1$

5. $H \leftarrow 1$

6. $m_1 \leftarrow m - 0.3$

7. $m_2 \leftarrow m + 0.2$

8. $u_{ij} \leftarrow \frac{\bar{u}_{ij} + \underline{u}_{ij}}{2}$

9. $\eta_j \leftarrow k \frac{\sum_{i=1}^N u_{ij}^m d_{ij}^2}{\sum_{i=1}^N u_{ij}^m}$

10. **repeat**

11. $d_{ij}^2 \leftarrow (x_i - C_j)^T A^{-1} (x_i - C_j)$

12. **if** $\frac{d_{ij}^2}{1 + (d_{ij}^2/\eta_j)^{1/(m_1-1)}} > \frac{d_{ij}^2}{1 + (d_{ij}^2/\eta_j)^{1/(m_2-1)}}$ **then**

13. $\bar{u}_{ij}(x_i) \leftarrow \frac{1}{1 + (d_{ij}^2/\eta_j)^{1/(m_1-1)}}$

14. $\underline{u}_{ij}(x_i) \leftarrow \frac{1}{1 + (d_{ij}^2/\eta_j)^{1/(m_2-1)}}$

15. **else**

16. $\bar{u}_{ij}(x_i) \leftarrow \frac{1}{1 + (d_{ij}^2/\eta_j)^{1/(m_2-1)}}$

17. $\underline{u}_{ij}(x_i) \leftarrow \frac{1}{1 + (d_{ij}^2/\eta_j)^{1/(m_1-1)}}$

18. **end if**

19. $u_{ij} \leftarrow \frac{\bar{u}_{ij}(x_i) + \underline{u}_{ij}(x_i)}{2}$

20. **if** $\max |u_{ij}^l - u_{ij}^{l-1}| > \varepsilon$ **then**

21. **calculate cluster centers by algorithm 5**

22. **else**

23. **re-estimate η by algorithm 4**

24. **if** $H < 2$

25. $H \leftarrow 2$

26. **end if**

27. **end if**

28. **until** $H \geq 2$

output cluster centers

* $M \times N$: the size of the input image

u_{ij} : membership value for i^{th} data in the j^{th} cluster

$\bar{u}_{ij}$: membership upper bound

$\underline{u}_{ij}$: membership lower bound

m, m_1, m_2 : fuzzifier parameters

d_{ij} : distance measure between i^{th} data and the j^{th} cluster, Mahalanobis distance

η_j : a positive value

Fig. 7 Pseudo-code of IT2PCM algorithm

Fig. 8 Pseudo-code of Kwon validity index algorithm for finding the optimal number of clusters

ALGORITHM 3. Kwon validity index algorithm for finding optimal number of clusters

input Image $M \times N$

Comment: Initialize primary values

1. $k \leftarrow 1$
2. $m \leftarrow 2$
3. $C^k \leftarrow 3$
4. **run** FCM
5. $v_k(U, V; X) = \frac{(\sum_{j=1}^N \sum_{i=1}^C u_{ij}^2 \|x_j - v_i\|^2 + \frac{1}{C} \sum_{i=1}^C \|v_i - \bar{v}\|^2)}{\min_{l \neq k} (\|v_l - v_k\|^2)}$
6. **if** $C^k \geq 7$ **then**
7. **find** c that v_k be Min
8. **else**
9. $C^{k+1} \leftarrow C^k + 1$
10. $k \leftarrow k + 1$
11. **end if**

* $M \times N$: the size of the input image

m : degree of fuzziness

C : number of clusters

v_k : Kwon validity index

U : input matrix to IT2FCM

v_i : i^{th} cluster center

u_{ij} : type-2 possibilistic membership values for the i^{th} data in the j^{th} cluster

$\bar{v}$: mean value of cluster centers

et al. [15], is employed to enlarge the segmented bones (femur and tibia) to obtain the final segmented image.

Systems developed for meniscal tear diagnosis come in various contexts. Wang et al. [17] proposed a two-stage computerized system for meniscal tear diagnosis. In the first stage, an active contour with level sets model was applied to calculate the location and shape of the meniscus area. In the second stage, the features were extracted and then Sequential Floating Forward Selection (SFFS) was applied for relevant feature selection. The feature vectors were then input to a Support

Vector Machine (SVM) classifier to detect meniscus tear.

Hata et al. [18] employed the fuzzy if-then rules to diagnose the meniscal tears from 3D knee T1-weighted and T2-weighted MR images. They could successfully identify all voxels in the menisci through three steps of their proposed model. First, a 3D image is manually registered between both images on the computer display. Next, the candidate region of the menisci from T2-weighted MR image is determined using fuzzy if-then rules regarding the location and intensity. Finally, the

Fig. 9 Pseudo-code of re-estimation of value

ALGORITHM 4. Proposed algorithm for re-estimation of η value

input type-2 membership function distances

Comment: Initialize primary values

1. $\alpha \leftarrow 0.7$
2. $\beta \leftarrow 0.8$
3. $\eta_j^1 \leftarrow \frac{\sum_{\bar{u}_{ij} > \alpha} d_{ij}^2}{\sum_{\bar{u}_{ij} > \alpha} 1}$
4. $\eta_j^2 \leftarrow \frac{\sum_{\underline{u}_{ij} > \alpha} d_{ij}^2}{\sum_{\underline{u}_{ij} > \alpha} 1}$
5. **if** $\eta_j^1 \geq \eta_j^2$ **then**
6. $\bar{\eta}_j \leftarrow \eta_j^1$
7. $\underline{\eta}_j \leftarrow \eta_j^2$
8. **else**
9. $\bar{\eta}_j \leftarrow \eta_j^2$
10. $\underline{\eta}_j \leftarrow \eta_j^1$
11. **end if**
12. $\eta_j \leftarrow \beta \bar{\eta}_j + (1 - \beta) \underline{\eta}_j$

output η_j

* $M \times N$: the size of the input image

η : bandwidth parameter

$\bar{\eta}$: bandwidth parameter upper bound

$\underline{\eta}$: bandwidth parameter lower bound

α : minimum memberships incorporated to calculate η

β : association coefficient

Fig. 10 *Top*: original meniscus region image; *middle*: result of running original IT2PCM; *bottom*: result of running IT2PCM after re-estimation of proposed algorithm

voxels in the menisci are determined from the candidate regions.

A variety of boundary-tracing techniques, developed to extract boundaries of objects, are also exploited to segment the cartilage or bones around the knee joint. These methodologies include seed point and region growing [19], active shape model [20], or snake models [21, 22]. These techniques could also be applicable for meniscus detection and extraction. However, Li et al. [23] pointed out that these models could result in

Fig. 11 Pseudo-code of KM algorithm for type reduction or defuzzification

ALGORITHM 5. KM algorithm for type reduction or defuzzification

input Image $M \times N$ and $\bar{u}_i(x_k), \underline{u}_i(x_k)$

Comment: Initialize primary values

1. $m \leftarrow 2$ or $m \leftarrow 1, 5$
2. $u_i(x_k) \leftarrow \frac{\bar{u}_i(x_k) + \underline{u}_i(x_k)}{2}$
3. $v'_i \leftarrow \frac{\sum_{k=1}^{M \times N} x_k u_i(x_k)}{\sum u_i(x_k)}$
4. **sort** x
5. **repeat**
6. **find** k that $x_k \leq v'_i \leq x_{k+1}$
7. **for** $j \leq k$
8. $u_i(x_j) \leftarrow \underline{u}_i(x_j)$
9. **end for**
10. **for** $j > k$
11. $u_i(x_j) \leftarrow \bar{u}_i(x_j)$
12. **end for**
13. $v''_i \leftarrow \frac{\sum x_k u_i(x_k)^m}{\sum u_i(x_k)^m}$
14. **if** $v''_i \neq v'_i$ **then**
15. $v''_i \leftarrow v'_i$
16. **end if**
17. **until** $v''_i = v'_i$
18. $v_R \leftarrow v''_i$

output v_R

* $M \times N$: the size of the input image

u_i : membership grade

$\bar{u}_i$: membership upper bound

$\underline{u}_i$: membership lower bound

m : fuzzifier parameter

v'_i, v''_i : type – reduced sets

Table 2 Corresponding cluster codes

Cluster	Binary codes
1	00
2	01
3	10
4	11

different segmentation outcomes because of human intervention through initialization.

Different characteristics of knee tissues, including intensity of each tissue, regions of menisci, and the like, create barriers to assign an exact degree of belonging to one cluster. Hence, using type-I fuzzy logic will not be adequate. Type-2 membership functions have been proved to provide better performance than type-I fuzzy membership functions in medical diagnosis based on image processing approaches, as their membership functions are themselves fuzzy. The concepts of type-2 fuzzy logic are shortly addressed in the following section.

Type-2 Fuzzy Logic

Fuzzy logic has been successful in a variety of image processing areas, including image enhancement, image segmentation, image classification, and thresholding value selection [24]. In this context, several uncertainties

Table 3 Types of tears and their codes

Meniscus tear and location	Binary codes
Medial anterior horn and posterior horn are normal	00001
Lateral anterior horn and posterior horn are normal	00010
Medial anterior horn is normal and posterior horn is torn	00100
Lateral anterior horn is torn and posterior horn is torn	01000
Lateral anterior horn is torn and posterior horn is normal	10000

take place, which may occur by projecting a 3D object into a 2D image, digitizing analog images, uncertain boundaries, etc. Type-2 fuzzy sets allow us to model such uncertainties and provide better results than type-I fuzzy sets [5].

Type-2 fuzzy sets were first introduced by Zadeh as an extension of type-I fuzzy sets [25]. The membership function of a fuzzy set is itself a fuzzy set, i.e., the membership grade for each element is a fuzzy set in interval $[0, 1]$ [26]. Such sets can be applicable where membership values are contaminated with uncertainties [27].

Generally, two types of type-2 fuzziness are regarded: interval-valued type-2 and generalized type-2 fuzzy. Interval-valued type-2 fuzzy sets are characterized by their crisp upper and lower membership bounds, spread of membership distribution of which are ignored with the assumption that membership values between upper and lower bounds are uniformly distributed or scattered with a membership value of 1 on the $\mu(\mu(x))$ (Fig. 2a). For generalized type-2 fuzzy sets, the upper and lower bounds (fuzzily or probabilistically) as well as the spread of membership values (probabilistically or possibilistically) are defined (Fig. 2b) [3, 29].

As mentioned before, MR images are contaminated with uncertainties, caused by non-uniformity, chemical shift, motion and flow, and even inhomogeneity [30, 31]. Thus, type-2 fuzzy logic will assist the medical diagnosis based on MR imaging analysis. The next section is dedicated to the proposed image processing technique.

Table 4 Cluster centers for the healthy lateral meniscus

Cluster	Prototype of IT2PCM	Prototype of IT2FCM
C_1	11.2592	11.7991
C_2	61.8816	59.9389
C_3	91.8457	92.8118
C_4	125.7601	128.5658
C_5	183.5627	203.6740

Proposed image processing method

This paper pursues an efficient image processing approach which could assist physicians to diagnose meniscal tears in appropriate time. The proposed model is composed of three main modules: Pre-processing, Segmentation, and Classification.

Module 1—Pre-processing: MR images are contaminated with noises and artifacts. The type-2 λ -enhancement algorithm, proposed by Tizhoosh [32], is used in this module to enhance the input image. This algorithm pursues two objectives: removing the noises and improving the contrast of the image. Using this pre-processing module, the noises in the pre-processed image are reduced and the edges are sharpened (“Pre-processing” section).

Module 2—Segmentation: In this module, the region of interest (ROI) is extracted from enhanced images by expert (a rectangle area), and the meniscus area is segmented into four classes. This ROI remains the same from image to image for each patient and also from patient to patient. The proposed segmentation algorithm combines the advantages of both Interval Type-2 Fuzzy C-Means (IT2FCM) and Interval Type-2 Possibilistic C-Means (IT2PCM) clustering algorithms. The Kwon validity index [33] is used to obtain the number of clusters (C) and the degrees of fuzziness (m), outputs of which are subsequently employed to compute the membership degrees of IT2PCM, using the algorithm proposed by FazelZarandi et al. [28, 34]. In the proposed model, the initial value of m is assumed to be 2 (“Segmentation” section).

Module 3—Classification: This step is vital to final diagnosis and drawing conclusions. For classification, a multi-layer Perceptron with two hidden layers is designed. This network is trained specifically to detect the meniscal tear in MR images, which is exposed as a gray line because of synovial fluid invasion to the meniscus area. The architecture of the designed network consists of 256, 128, and 5 neurons for the first hidden layer, second hidden layer, and output layer, respectively. The output layer is dedicated to pre-defined binary codes, which correspond to specific type of meniscal tear with its location (“Classification” section).

The schema of the proposed model is presented in Fig. 3.

Pre-processing

Image enhancement is an indispensable part of digital image processing context. Hence, enhancement can be

Fig. 12 Four-millimeter cut-offs of the PD MRI of medial meniscus in sagittal plane

interpreted as noise removing, smoothing, deblurring, and sharpening the edges or, in most cases, gray level modification for contrast enhancement [35].

Generally, contrast enhancement is a point operation, which is applied to gray levels of the image histogram to achieve a different (better) gray-level distribution. In this context, histogram equalization is a common task to enhance the image contrast. However, selection of a suitable function for gray-level modification is a difficult and time-consuming task. Fuzzy techniques have proven to be useful tools for knowledge-based image processing [36].

Type-2 λ -enhancement was developed to enhance the quality, remove the noises, and improve the contrast of the image. It is designed on the basis of type-2 fuzzy sets, so it not only provides better performance in ambiguous environments but also has shown superior results as compared to other applicable methodologies. Moreover, the processing time of the type-2 λ -enhancement is approximately half compared to the type-1 algorithm. Hence, it could be favorable for the pre-processing module [37].

In order to convert the membership functions to interval form, a power parameter α is used. After the

optimal value of λ was achieved, the new gray levels are computed. Figure 4 (left) shows the input MRI to the system, and Fig. 4 (right) shows the resulted image from pre-processing module. Figure 5 provides the corresponding pseudo-code of the type-2 λ -enhancement algorithm [32].

Segmentation

Segmentation is a substantive issue in the area of image processing context. Currently, in many clinical studies or experiments, segmentation still remains on a manual basis or is heavily dependent on a human expert. It is inevitable to avoid the deteriorating effect of human intervention. Hence, manual segmentation hinders an effective diagnosis. That is why we will need automated MRI segmentation tools [37].

Segmentation methods are generally categorized into four main groups, including thresholding, edge detection, clustering, and region extraction. In clustering, the observation space is usually partitioned into exclusive clusters or groups of objects with respect to attributes or variables. However, it seems inadequate to explain the related features of attributes in terms of such a

Fig. 13 Four-millimeter cut-offs of the PD MRI of lateral meniscus in sagittal plane

Fig. 14 The plot of function for a sample image

partitioning scheme. Fuzzy clustering compensates the shortcomings of crisp clustering since it could obtain the degrees of belongingness of objects to fuzzy clusters. In other words, fuzzy clustering detects objects that belong to several fuzzy clusters simultaneously with certain degrees [38].

A miscellany of fuzzy clustering techniques exist in literature. Fuzzy C-Means (FCM) and Possibilistic C-Means (PCM) are among the prevalent methods, leading to promising results. On the other hand, these algorithms cannot model all the ambiguities in images since their membership functions are type-1 fuzzy sets and images have perception uncertainty. The proposed model is extended form of FCM and PCM, developed by Hwang et al. [39, 40]. The proposed algorithm exploits the advantages of both Interval Type-2 Fuzzy C-Means (IT2FCM) and Interval Type-2 Possibilistic C-Means (IT2PCM), while minimizing their disadvantages. That is, first IT2FCM is applied to the image, outputs of which are then used as initial type-2 membership degrees for IT2PCM. The inputs to the IT2FCM include each pixel coordinate in addition to its intensity value of the images. Moreover, a new algorithm is proposed to enhance the IT2PCM performance, which will improve the diagnosis precision. The schema of the segmentation module is shown in Fig. 6. Figure 7 represents the outline of the IT2PCM algorithm.

Fig. 15 *Left*: original focal meniscal region; *right*: enhanced focal meniscal region

Fig. 16 Optimal number of clusters for the processed image with whole area. The minimum value of validity index determines the optimal number of clusters

FCM uses probabilistic constraint that the memberships of a data point across classes must sum to 1. However, since the memberships generated by this constraint are relative numbers, they are not suitable for applications in which the memberships are supposed to represent “typicality,” or “compatibility” with an elastic constraint [41, 42]. The memberships resulting from FCM and its derivative, however, do not always correspond to the intuitive concept of degree of belonging or compatibility. On the other hand, the results of the possibilistic algorithms depend on initialization, just as any clustering technique. In possibilistic algorithms, the clusters do not have a lot of mobility. Therefore, a reasonably good initialization is required for the algorithms to converge to the global minimum. Any suitable clustering algorithm may be used to obtain the initial partition. For this reason, we use membership outputs from IT2FCM as initial memberships of IT2PCM.

The number of clusters chosen by expert was validated by Kwon validity index, and the obtained result was $C=4$ (C stands for number of clusters) [33]. Membership function used for validity index is driven outputs from IT2FCM. Figure 8 represents the pseudo-code of Kwon validity index for finding the optimal number of clusters.

In IT2PCM (Fig. 7), the value of η_i determines the distance at which the membership becomes 0.5. Thus, η_i incorporates the “zone of influence” of a point. A point x_j will have little influence on the estimates of the prototype parameters of a cluster, if $d^2(x_j, C_i)$ is

Fig. 17 Optimal number of clusters for the processed image of meniscal region. The minimum value of validity index determines the optimal number of clusters

large when compared with η_i . Therefore, a ball-park value for η_i is the variance (average intra-cluster distance) of cluster i . So, good estimates of the scale parameters η_i are needed. For this purpose, outputs of IT2FCM are defuzzified in terms of type reduction, and then η_i is estimated using Eq. 1 [41]:

$$\eta_i = k \frac{\sum_{j=1}^N u_{ij}^m d_{ij}^2}{\sum_{j=1}^N u_{ij}^m} \quad (1)$$

where u_{ij} is type-1 membership function, d_{ij}^2 is Mahalanobis distance, and m is a fuzzifier. Typically, K is chosen to be 1. Then, we run IT2PCM. We have chosen $m_1 = m - 0.3$ and $m_2 = m + 0.2$ [43], where m is found by Kwon validity index, proposed by FazelZarandi et al. [31, 44], pseudo-code of which is shown in Fig. 8.

End of IT2PCM includes re-estimating η by proposed type-2 fuzzy algorithm. Figure 9 depicts the outline of the proposed model. Type-2 membership distances are

Fig. 18 Plot of Kwon validity index against different values of fuzzifier “ m ” within interval [1.1, 4]. The minimum value of validity index determines the optimal value of fuzzifier

Fig. 19 Plot of Kwon validity index against different values of fuzzifier “ m ” within interval [1.1, 2]. The minimum value of validity index determines the optimal value of fuzzifier

fed to the model. Then, Eq. 2 is employed to re-estimate η value, using upper and lower bounds (Eqs. 3 and 4, respectively) [41]:

$$\eta_i = \frac{\sum_{x_j \in (\Pi_i)_\alpha} d_{ij}^2}{|(\Pi_i)_\alpha|} = \frac{\sum_{u_{ij} \geq \alpha} d_{ij}^2}{\sum_{u_{ij} \geq \alpha} 1} \quad (2)$$

$$\eta_i^1 = \frac{\sum_{u_{ij} \geq \alpha} d_{ij}^2}{\sum_{u_{ij} \geq \alpha} 1} \quad (3)$$

$$\eta_i^2 = \frac{\sum_{u_{ij} \geq \alpha} d_{ij}^2}{\sum_{u_{ij} \geq \alpha} 1} \quad (4)$$

where $(\Pi_i)_\alpha$ is an appropriate α -cut of Π_i . The following rule is used to find the upper and lower bounds of η ;

$$\text{IF } \eta_i^1 > \eta_i^2 \text{ THEN } \bar{\eta}_i = \eta_i^1 \text{ and } \underline{\eta}_i = \eta_i^2 \text{ otherwise } \bar{\eta}_i = \eta_i^2 \text{ and } \underline{\eta}_i = \eta_i^1$$

After obtaining upper and lower bounds of η , crisp value of η is calculated as follows:

$$\eta_i = \beta \bar{\eta}_i + (1 - \beta) \underline{\eta}_i \quad (5)$$

where β is a coefficient, which is heuristically chosen equal to 0.8. Since the distances of some elements are calculated in both upper and lower bounds, to overcome this re-calculation, the value of β is defined. After the new value of η is obtained, running IT2PCM for a second time is proposed. Figure 10 (top) shows the original meniscus region, Fig. 10 (middle) shows the result of original IT2PCM without re-estimation of η , and Fig. 10 (bottom) shows the effect of the proposed algorithm which elevates IT2PCM performance.

In our proposed model, KM algorithm [45, 46] is used for type reduction (Fig. 11 corresponds to

Fig. 20 Membership functions of a selected column of pixels in each cluster

pseudo-code of used KM algorithm), which is incorporated to reduce the computational load. The KM algorithm is an iterative algorithm which estimates a left value (ν_L) and a right value (ν_R) of an interval cluster

center. From these values, we can obtain a type-reduced set (an interval type-1 fuzzy set). Once we have achieved an interval type-1 fuzzy set, a crisp center using defuzzification can be obtained as follows [43]:

$$\nu_j = \frac{\sum_{v \in J_{Y_j}} (u(v))v}{\sum_{v \in J_{Y_j}} (u(v))} = \frac{\nu_L + \nu_R}{2} \quad (6)$$

where ν_j is center of j th cluster. After the final centers are obtained, a pattern could be assigned to a cluster based on the fuzzy memberships. This is carried out for all patterns that have already been estimated to obtain type-reduced centers as follows [43]:

$$u_j(x_i) = \frac{\bar{u}_j(x_i) + \underline{u}_j(x_i)}{2}, \quad j = 1, 2, \dots, C \quad (7)$$

Finally, hard partitioning is performed for patterns using (7), as follows:

IF $u_j(x_i) > u_k(x_i)$, *FOR* $k = 1, 2, \dots, C$ and $j \neq k$ *THEN* x_i is assigned to cluster j .

For feature selection phase, binary codes have been defined to assign each pixel to its pertaining cluster. Since there exist four clusters in the meniscal area (meniscus,

Fig. 21 Gray-level IT2PCM segmentation of the knee

Fig. 22 *Left:* color level IT2FCM segmentation of the knee; *right:* color level IT2PCM segmentation of the knee

bone, cartilage, and synovial liquid), four different codes are defined. Table 2 provides these pre-defined cluster codes.

Classification

For diagnosis of meniscal tears, a Perceptron Neural Network (PNN) with two hidden layers, 256 neurons for the first hidden layer and 128 neurons for the second hidden layer, is used. Input layer has $M \times N \times 2$ nodes ($M \times N$ is size of input images) and output layer has five neurons in correspondence with Table 3. Since binary codes are defined to identify cluster labels (Table 4), the number of input nodes is twice the number of pixels of input images. Spatial information modeled by PNN is associated with cluster labels of each pixel out of four pre-defined classes in Table 4. Hence, the matrix which is fed into the network contains elements which get values ranging from 1 to 4 (Class labels in Table 4). The input nodes are fully connected to hidden layers.

A training set is a group of matched input and output patterns used for training the network, usually by suitable adaptation of the synaptic weights. Output of the

network is tear and recognized location of tears. Due to the fact that some locations of tears are not common, we did not have sufficient images for all tear locations. Table 3 shows all types of tears that we had sufficient images for and their pre-defined binary codes. The activation function used to train the network is unipolar sigmoid function. The learning rate is $\eta=0.7$, and momentum coefficient is defined as $\alpha=0.5$.

Material

Proton density-weighted magnetic resonance images (PD MRI) of knee in sagittal plane have been used to test the model. PD MRI provide better screening of the tears in comparison with T1 and T2 MRI. The images were obtained from Imaging and Radiology center of MohebMehr Hospital, located in Tehran. Details about the patients, except their demographic statistics, including age and sex, were anonymized. Images were gathered by stepping 4-mm cut-offs for each patient with the acquisition of $TR=2000$ ms, $TE=20$ ms, flip angle $=90^\circ$.

Table 5 Cluster centers for the healthy medial meniscus

Cluster	Prototype of IT2PCM	Prototype of IT2FCM
C_1	8.4931	8.8243
C_2	52.2843	51.2805
C_3	84.4737	83.7408
C_4	108.7112	109.6243
C_5	134.7079	139.0885

Table 6 number of pixels in each cluster

Cluster	Number of pixels in each cluster of IT2PCM	Number of pixels in each cluster of IT2FCM
C_1	52531	54495
C_2	12762	10798
C_3	10838	11517
C_4	9042	9977
C_5	3147	1533

Fig. 23 Segmented healthy meniscus in gray level by IT2PCM

Fig. 25 Segmented healthy meniscus in gray level by IT2PCM

These stepping 4-mm cut-offs of the knee resulted in approximately ten images for each patient, of which only four images had met the minimum requirements for the covering criteria of meniscal area. Figures 12 and 13 represent the medial and lateral meniscal tears, respectively. As shown, there exist dramatic differences between the medial and lateral meniscal tears. This would cause the neural network to result in different tear recognition results. Hence, two distinct procedures are considered in the tear recognition process.

Totally, 654 images (4-mm cut-off setting), including medial and lateral meniscal tears, were obtained from 28 patients. Eighteen of the patients were male with a range of 25 to 64 years old, and ten of them were female with a range of 44 to 78 years old. Eighty-three percent of the males and 60 % of females were reported to have unhealthy meniscus.

As discussed, only four best-covering images were selected by expert as region of interest (ROI) from each sample, based on the covering criteria. In total, 248 images were employed to build and evaluate the model.

Experiment Results and Discussion

Image Enhancement

All images were enhanced using λ -enhancement technique. Figure 14 shows the plot of $\gamma(\lambda)$ function for a sample image, selected from the 62×4 dataset. As shown, the optimal value of λ for this image is $\lambda_{optimal} = 9.8350$.

The optimal value for λ in the focal meniscal tear region is obtained as $\lambda_{optimal} = 1.7500$. The images are enhanced after the optimal value for λ is obtained. The result of performing enhancement on a sample image is shown in Fig. 4. The result of the latter on the meniscal region, with $\alpha = 10$, is shown in Fig. 15.

Segmentation

First, Kwon validity index is used to obtain the optimal number of clusters. For this purpose, first, the whole area of input images is used, and then the meniscus regions are employed to find the optimal number of clusters. Figures 16 and 17 represent the optimal number of clusters for the whole area and meniscal area, respectively.

The meniscal area is composed of four clusters, including meniscus, bone, cartilage, and synovial liquid. Kwon validity index is also used to find the optimal value of fuzzifier “ m ” for IT2PCM algorithm. For finding the optimal value of “ m ,” it is supposed to be investigated within the interval of $m \in [1.1, 4]$. Figure 18 shows the validity index for the different values of “ m ” within interval $[1.1, 4]$. Since the differences for the minimum values are intangible in the provided plot (Fig. 18), we have adjusted the range of “ m ” values to the interval of $[1.1, 2]$. As shown, Fig. 19 provides the optimal value of “ m ” equal to 1.6. After the initial parameters are obtained, the segmentation procedure could be initiated.

First, used membership functions are introduced, and then the proposed model is applied to the sample image of Fig. 4. These techniques have been evaluated through the cluster centers and the number of pixels in each cluster. All of these steps are first carried out for the healthy meniscus, and then the images with meniscal tears are investigated.

The algorithm is thoroughly based on the interval type-2 fuzzy functions. Figure 20 represents these membership functions for a selected column of pixels with the lowest number of pixels. These functions are generated by two distinct values of fuzzifier “ m .”

First, the proposed segmentation algorithm is applied to the enhanced image of Fig. 4 in gray level (Fig. 21). Then, the color levels of both IT2FCM and IT2PCM have been provided in Fig. 22, as applied to the same enhanced image (image of the knee with healthy lateral meniscus). The meniscal area is

Fig. 24 Segmented healthy meniscus in color level; *left*: IT2FCM; *right*: IT2PCM

Fig. 26 Segmented torn meniscus in color level; left: IT2FCM; right: IT2PCM

Table 7 Results of re-estimated values of η

Cluster	Estimation of η by FCM	Re-estimation of η by traditional algorithm (PCM)	Re-estimation of η by proposed algorithm
η_1	326.2121	142.6190	74.1326
η_2	208.7982	91.1823	51.2932
η_3	419.4199	238.8456	96.5138
η_4	811.1790	526.6211	204.6205

shown by red color. Moreover, Tables 4, 5, and 6 represent the cluster centers for healthy lateral meniscus, healthy medial meniscus, and the number of pixels in each cluster, respectively.

These tables facilitate the performance evaluation of the proposed algorithm in terms of cluster centers or

number of pixels in each cluster. The differences between cluster centers of medial meniscus and those of lateral meniscus are easily noticeable from Tables 4 and 5. Hence, the medial and lateral menisci have been separated to avoid the tear recognition faults by adopted neural network.

Afterward, the segmentation procedure is applied to the meniscal area in order to elevate performance and improve meniscal tear recognition. Healthy menisci are represented in Figs. 23 and 24 by red and white triangles for the gray level and color focal meniscal images, respectively. Figures 25 and 26 show torn menisci which are not triangular in shape, and other colors are noticeable in the meniscal area.

Our proposed model to re-estimate the value of η is then performed. Re-estimation of η transfers the cluster centers. Results of re-estimated values of η using FCM,

Fig. 27 The membership functions of a selected column of pixels (column 15) before and after re-estimation of η . Green: before; red: after. Top, left: C_1 ; top, right: C_2 ; bottom, left: C_3 ; bottom, right: C_4

Table 8 Classification rates of testing data for 0 and 1 mode

		Condition +	Condition –	
Test outcome	Test outcome positive	True positive 45	False positive 1	+ predictive value (Precision) 97.83 %
	Test outcome negative	False negative 4	True negative –	– predictive value 0 %
		Sensitivity 91.84 %		Accuracy 90.00 %

after performing PCM, and finally after performing the proposed model are represented in Table 7. We can observe that the proposed model substantially reduces the radiuses of clusters, which results in fuzziness reduction. Since the images do not provide tangible differences, a column of pixels has been selected to show these changes using the membership values, which are plotted in Fig. 27.

At the end of the IT2PCM segmentation, $2 \times C$ matrices are obtained, C of which represent the upper bounds for membership degrees, and the rest of C represent the lower bounds for membership degrees. Finally, in defuzzification stage, a pixel belongs to a cluster which has the maximum membership value. Hence, the matrices are transferred to zero–one matrices, which implies that a pixel belongs to a cluster or not.

Classification

First, 198 out of 248 images (approximately 80 %) are introduced to the network for training. The minimum error of the network, when the outputs are defined as 0 and 1 (five neurons in output layer), and when the outputs are defined in binary forms (three neurons in output layer), was obtained as $E_1 = 0.0288$ and $E_2 = 0.0583$, respectively.

After training the network, it was tested using the same images, and the accuracy of classification obtained was 97 and 88 % for 0 and 1 mode, and binary mode, respectively. Afterwards, the remaining 50 images (testing data) were introduced to the network, and this time

the results were 90 % (Table 8) and 78 % (Table 9) for 0 and 1 mode, and binary mode, respectively. Tables 8 and 9 represent the confusion matrices of classification results for testing in terms of precision, sensitivity, and accuracy.

Discussion

In this section, we discuss the impact of different parameter configurations and values on the classification results. We conclude this section by providing a comparison between the performances of our proposed model and the model proposed by Tizhoosh et al. [47].

One of the major issues which may arise is associated with the selection of values of α and β in Eqs. (2) to (5). Since the results of IT2PCM is directly correlated to the value of η , the selection of proper values of α and β is vital to draw eligible final results. Figure 28 represents the impact of different values of α and β on the classification results. As shown, the higher classification result is achieved when $\alpha = 0.7$ and $\beta \cong 0.8$. Analysis of different configurations of these parameters suggests that the more the value of α drops, the more the classification rate plunges. The results approved less sensitivity of classification rate to β values, in contrast with α values. The whole chain of the proposed model, including image enhancement, segmentation, and classification, takes 15 ± 1.1 seconds.

In the previous section, the overall classification results were evaluated in terms of precision, sensitivity, and accuracy. One of the insights through the performance of the model

Table 9 Classification rates of testing data for binary mode

		Condition +	Condition –	
Test outcome	Test outcome positive	True positive 39	False positive 3	+ predictive value (precision) 92.86 %
	Test outcome negative	False negative 8	True negative –	– predictive value 0 %
		Sensitivity 82.98 %		Accuracy 78.00 %

Fig. 28 *Top*: contour curve for different ranges of beta and alpha values. *Bottom*: classification rates for different values of beta, for three values of alpha

would be its classification rate for different classes, individually. Here, we have analyzed the performance of the model for different classes of meniscus. Table 10 represents the performance of the model for different settings of meniscus tear. As shown, the results suggest high performance of the model for classification, when the state of the meniscus is symmetric. In other words, when both the lateral anterior and posterior horns are normal or torn, the model provides better results.

Moreover, in order to evaluate the performance of the proposed model, FCM was used to segment the enhanced image (instead of proposed type-2 fuzzy segmentation), the result of which was then fed into the PNN. For 50 images of the test dataset, the model showed an accuracy of 72.52 %, which implies a 17.48 % drop in the classification success rate.

Finally, a comparison between the performances of our model and the model proposed by Tizhoosh et al. [47] is discussed. Fifty images were used to test the classifier, where 8 images were healthy and 42 were unhealthy with torn menisci. The previous model wrongly detected five of the healthy menisci as torn menisci, which resulted in 37.5 % of specificity and accuracy. The proposed model only detected one of the healthy menisci as torn meniscus, which resulted in 87.5 % specificity and accuracy. Moreover, the previous model

predicted only 32 out of 42 images in correct classes, resulting in 76% accuracy. The proposed model successfully predicted 38 out of 42 images in correct classes, with 90.5 % accuracy.

Figure 29 illustrates one of the cases in which the previous model wrongly detects a meniscus tear on a healthy meniscus (middle). Also, the segmented version of the same image, using our proposed model, is represented (bottom) in which we can easily observe the triangular meniscus. Moreover, in the case of

Table 10 Precision of classifier for different settings of tear

Meniscus tear and location	Classification rate (precision %)
Medial anterior horn and posterior horn are normal	88.82 %
Lateral anterior horn and posterior horn are normal	92.13 %
Medial anterior horn is normal and posterior horn is torn	84.24 %
Lateral anterior horn is torn and posterior horn is torn	91.96 %
Lateral anterior horn is torn and posterior horn is normal	87.64 %

Fig. 29 *Top*: original healthy meniscal MRI; *middle*: the output of algorithm proposed in [47], meniscal tear is speciously detectable; *bottom*: the output of proposed algorithm

meniscal tear presence, the algorithm proposed in [47] magnifies the tear which is noticeably intelligible from Fig. 30.

Conclusion and Future Works

In this paper, a type-2 fuzzy expert system for meniscal tear diagnosis has been developed. The proposed system assists the physicians in effective and efficient meniscal

Fig. 30 *Top*: original torn meniscal MRI; *middle*: the output of algorithm proposed in [47], meniscus tear is magnified; *bottom*: the output of proposed algorithm

tear diagnosis, supporting further treatment if necessary. The main contributions in this paper comprise an improved type-2 PCM through the synthesis of IT2FCM and IT2PCM, a novel algorithm to elevate overall performance of IT2PCM, and developing a Perceptron network for tear diagnosis. The proposed system has been encoded and its main modules have been tested and validated in real-world problems. The results were promising in terms of better tear diagnosis. Type-2 fuzzy sets dominate the proposed model since it overcomes type-1 fuzzy sets where the target environment is contaminated with a great deal of uncertainty, as there is in focal meniscal MRI too. 3D meniscal tear reconstruction and detecting tear location in terms of better and effective diagnosis could be a subject of future work.

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