Published in final edited form as:

Phys Med Biol. 2016 October 21; 61(20): 7300–7333. doi:10.1088/0031-9155/61/20/7300.

Optimization-Based Image Reconstruction with Artifact Reduction in C-Arm CBCT

Dan Xia¹, David A. Langan², Stephen B. Solomon³, Zheng Zhang¹, Buxin Chen¹, Hao Lai², Emil Y. Sidky¹, and Xiaochuan Pan^{1,4}

¹Department of Radiology, The University of Chicago, USA

²GE Global Research, USA

³Memorial Sloan Kettering Cancer Center, USA

⁴Department of Radiation and Cellular Oncology, The University of Chicago, USA

Abstract

We investigate an optimization-based reconstruction, with an emphasis on image-artifact reduction, from data collected in C-arm cone-beam computed tomography (CBCT) employed in image-guided interventional procedures. In the study, an image to be reconstructed is formulated as a solution to a convex optimization program in which a weighted data divergence is minimized subject to a constraint on the image total variation (TV); a data-derivative fidelity is introduced in the program specifically for effectively suppressing dominant, low-frequency data artifact caused by, e.g., data truncation; and the Chambolle-Pock (CP) algorithm is tailored to reconstruct an image through solving the program. Like any other reconstructions, the optimization-based reconstruction considered depends upon numerous parameters. We elucidate the parameters, illustrate their determination, and demonstrate their impact on the reconstruction. The optimization-based reconstruction, when applied to data collected from swine and patient subjects, yields images with visibly reduced artifacts in contrast to the reference reconstruction, and it also appears to exhibit a high degree of robustness against distinctively different anatomies of imaged subjects and scanning conditions of clinical significance. Knowledge and insights gained in the study may be exploited for aiding in the design of practical reconstructions of truly clinical-application utility.

1. Introduction

C-arm cone-beam computed tomography (CBCT) is used increasingly as an imaging tool for treatment guidance, monitoring, and assessment in interventional, surgical, and radiotherapeutic procedures [1–9]. In the treatment and management of liver cancers, for example, C-arm CBCT can provide a three-dimensional (3D) image about the cancer and its feeding arteries for assisting interventional radiologist to navigate a catheter to the tumor feeders for tumor embolization and for assessing the treatment following the completion of catheter directed therapy [10–16]. Current clinical C-arm CBCT in interventional applications is plagued often by a number of issues, including (1) truncation artifacts caused by its limited field of view (FOV) and (2) artifacts engendered from metal/high-contrast objects involved in the procedure.

Methods have been proposed for addressing some of the issues. For example, an extrapolation is applied often to measured data for possible reduction of truncation artifacts [17–21]. While it may help reduce data truncation in some cases, it is known to have a limited utility for a subject size considerably larger than the FOV [21]. Segmentation and interpolation are used for minimizing metal/high-contrast-object artifacts [22–26]. They may work well for subjects containing very few, spatially isolated metal/high-contrast objects of small sizes, but their effectiveness generally diminishes rapidly for subjects containing less well-separated metal/high-contrast objects [25]. Moreover, the extrapolation, segmentation, and interpolation themselves, referred to as *pre-reconstruction data processing* in the work, may introduce additional artifacts as a result of their inadequate data estimation.

In this work, we investigate an optimization-based reconstruction, with an emphasis on image-artifact reduction, from C-arm CBCT data in an image-guided interventional procedure for treatment of liver diseases. It is designed to be applied directly to the data, collected from subjects and devices with a high degree of structural variability, thus avoiding the need of pre-reconstruction data processing and the associated issues. Furthermore, we investigate optimization-based reconstruction from data acquired with non-conventional scanning configurations with sparse-projection views, limited-angular range, and/or heightened data truncation of potentially high clinical workflow merit, with lowered imaging dose and time, and/or minimized likelihood of C-arm collision with the patient, interventional radiologist, and supporting staff engaged in the interventional procedure. The investigation was motivated by increasing evidence that optimization-based reconstruction may offer a degree of flexibility considerably higher than that of analytic reconstruction such as the FDK in compensating for data-degrading factors and in accommodating scanning conditions of clinical workflow significance [27–37].

We formulate in the study images to be reconstructed as solutions to a convex optimization program in which a weighted data divergence is minimized subject to a constraint on the image total variation (TV). In particular, a data-derivative fidelity is included in the weighted data fidelity for effectively suppressing dominant, low-frequency components of data truncation, X-ray scatter, and inaccurate estimation of X-ray incident flux [38–40]. The optimization program considered can be solved mathematically by use of the primal-dual algorithms. In the work, the preconditioned Chambolle-Pock (CP) algorithm, a primal-dual algorithm, is tailored to reconstruct images through solving the optimization program [39, 41–43].

Like any other reconstruction, the optimization-based reconstruction considered includes necessarily a number of parameters, which can significantly impact the reconstruction property. A focus of the work is to illustrate succinctly the parameters and to demonstrate visually their impact on the optimization-based reconstruction and especially on image-artifact reduction in C-arm CBCT employed in image-guided interventional procedures. We point out that the work is not intended to develop a reconstruction tool as a product for use in routine clinical applications because the design of such a tool necessarily involves technical and clinical specifications that are neither defined nor considered in the work. Instead, it is hoped that the investigation may yield insights that may be exploited, along

with other technical and clinical task considerations, for aiding in the development of truly useful, practical tools for C-arm CBCT image reconstruction in interventional procedures.

We organize the paper as follow. In Sec. 2, we describe data acquisition from swine and patient subjects by use of a clinical C-arm CBCT system. In Sec. 3, an image-TV-constrained, data-fidelity-minimization program is formulated, along with the derivation of the CP algorithm for solving the program. Following an inverse-crime study carried out in Sec. 4 for verifying the CP algorithm and its computer implementation, we discuss in Sec. 5 the determination of leading reconstruction parameters in real data studies. In Sec. 6, we present image reconstructions from swine and patient data acquired with current clinical imaging conditions. In an attempt to further investigate the effectiveness of the optimization-based reconstruction, we conduct image reconstructions under restrictive, yet clinically highly meritorious, scanning conditions in Secs. 7 and 8. Finally, discussions in Sec. 9 summarize the work and comment on its possible extension.

2. Data Acquisition

In this work, CBCT data were acquired with a C-arm imaging system (GE Healthcare, Innova 4100 Angiographic Imaging System), as shown in Fig. 1a, with an X-ray source and flat-panel detector mounted onto the opposing ends of the C-arm. Considering a circular scanning configuration, we introduce two coordinate systems: coordinate system $\{x, y, z\}$ is fixed on the imaged subject in which the z -axis coincides with the rotation axis; whereas coordinate system $\{u, v\}$ on the detector plane has u - and v -axis parallel and perpendicular, respectively, to the circular trajectory plane, and its origin at the perpendicular intersection of an X-ray with the detector plane, as shown in Fig. 1b. The distances from X-ray source to the rotation center (SOR) and the source to the detector (SOD) are 720 mm and 1165 mm, respectively.

The detecting bins on the detector plane form a 2D array of size $M_u \times M_v$, where M_u and M_v denote the numbers of detecting bins along the u - and v -axis on the detector plane. Let M_ϕ denote the number of views at which data are collected, and thus the data array is of size $M = M_u \times M_v \times M_\phi$. The detector considered in the work consists of $M_u = M_v = 500$ of identical detecting bins each of which has an effective size of $0.8 \times 0.8 \text{ mm}^2$, thus forming a FOV of size 24 cm, which may be insufficient to cover the entire transverse cross section of a patient subject, thus resulting in data truncation. Moreover, an interventional procedure is likely to involve injection-devices and/or contrast objects some of which are placed outside the body contour, thus leading to streak artifacts in clinical images.

In an attempt to investigate the performance of the optimization-based reconstruction for different imaged subjects, data were acquired from multiple subjects, including physical phantoms and swine and patient subjects, while no physical phantom result was presented because it does not reveal information more than that obtained in the swine and patient studies. In the work, we present results from swine data with aortogram and hepatic injections, respectively, and from patient data selected retrospectively in a clinical scan performed in an interventional procedure. The swine and patient data were collected at $M_\phi = 148$ and 147 views, respectively, over an short-scan angular range of $\sim 194^\circ$ for the CBCT

system. Therefore, the swine and patient data are of sizes $M = 500 \times 500 \times 148$ and $M = 500 \times 500 \times 147$, respectively. Due to mechanical flexion and other possible instability factors of the C-arm system, the actual scanning trajectory may deviate from a prescribed scanning trajectory. Therefore, a geometric matrix relating precisely detector and image spaces [44, 45] was obtained in a calibration scan and then incorporated into the CP algorithm for image reconstructions.

The clinical reconstructions considered were obtained from data with pre-reconstruction processing such as extrapolation, whereas the optimization-based reconstructions carried out in the work invoke no data processing, except for the study in Sec. 8 that is designed specifically to demonstrate the impact of data extrapolation on the optimization-based reconstruction. In the work, we consider an image array containing $N = N_x \times N_y \times N_z$ identical cubic voxels, where N_x , N_y , and N_z denote the numbers of voxels along the x -, y -, and z -axis, respectively, in the 3D Cartesian coordinate system defined above. The clinical reconstructions of the swine and patient subjects were presented on image arrays of $N = 512 \times 512 \times 478$ and $N = 512 \times 512 \times 498$, respectively, in which the voxel sizes are 0.464 mm and 0.453 mm. As discussed below, we refer to the clinical reconstruction as the *reference reconstruction*, which is used for ensuring that the parameter selection in the optimization-based reconstruction is done within a domain of clinical relevance in a multi-dimensional parameter space.

3. Optimization-Based Reconstruction

We use vector $\mathbf{f}$ of size N to denote the 3D discrete image of size $N = N_x \times N_y \times N_z$ in a concatenated form in the order of N_x , N_y , and N_z , and also vector $\mathbf{g}_m$ of size M to depict the 3D discrete data set of size $M = M_u \times M_v \times M_\phi$ in a concatenated form in the order of M_u , M_v , and M_ϕ .

3.1. Optimization program

The constrained optimization program below is considered:

$$\mathbf{f}^* = \underset{\mathbf{f}}{\operatorname{argmin}} D_W(\mathbf{g}_m, \mathbf{g}) \quad \text{s. t.} \quad \|\mathbf{f}\|_{TV} \leq t_1 \quad \text{and} \quad f_j \geq 0, \quad (1)$$

where $D_W(\mathbf{g}_m, \mathbf{g})$ denotes a data divergence between measured data $\mathbf{g}_m$ and model data $\mathbf{g}$ of size M , $\|\mathbf{f}\|_{TV}$ the image-TV, which is the ℓ_1 -norm of the gradient magnitude image, $t_1 > 0$ a constraint parameter on the image TV, and f_j entry j in image vector $\mathbf{f}$. The model data is related to image $\mathbf{f}$ through a linear, discrete-to-discrete model

$$\mathbf{g} = \mathcal{H} \mathbf{f}, \quad (2)$$

where $\mathcal{H}$ denotes the system matrix of size $M \times N$ into which the geometric projection matrix is incorporated for accommodating an actual scanning configuration.

Data divergences with different forms can be designed for addressing issues that arise in specific applications. In this work, the data divergence considered takes a form of [39, 42]

$$D_w(\mathbf{g}_m, \mathcal{H}\mathbf{f}) = c^2 \|\mathbf{g}_m - \mathcal{H}\mathbf{f}\|_2^2 + (1-c)^2 \|\mathcal{D}_u(\mathbf{g}_m - \mathcal{H}\mathbf{f})\|_2^2, \quad (3)$$

where the first term indicates the well-known data ℓ_2 -norm,

$$\mathcal{D}_u = \begin{bmatrix} \mathcal{D}' & & & & \\ & \mathcal{D}' & & & \\ & \cdots & & & \\ & & \cdots & & \\ & & & \mathcal{D}' & \\ & & & & \mathcal{D}' \end{bmatrix}, \quad (4)$$

an antisymmetric matrix of size $M \times M$, which comprises $M_v \times M_\phi$ identical matrices $\mathcal{D}'$ of size $M_u \times M_u$ in a concatenated form in the order of M_v and M_ϕ and matrix $\mathcal{D}'$ is given by

$$\mathcal{D}' = \begin{bmatrix} 0 & 1 & & & \\ -1 & 0 & 1 & & \\ & -1 & 0 & 1 & \\ & & \cdots & & \\ & & & \cdots & \\ & & & -1 & 0 & 1 \\ & & & & -1 & 0 & 1 \\ & & & & & -1 & 0 \end{bmatrix}, \quad (5)$$

representing a two-point difference along the u -axis on the detector plane at each of the M_ϕ views. Parameter $c \in [0, 1]$ modulates the contribution of the data-derivative ℓ_2 -norm to the overall data divergence. Because $\mathcal{D}_u$ de-emphasizes the low-frequency components in data inconsistency [46], it is introduced for suppressing artifacts often observed in an analytic reconstruction from truncated data. Since it may be difficult, if not impossible, to recover accurately gray-level information in a reconstruction by use only of the data-derivative ℓ_2 -norm, the standard data ℓ_2 -norm, $\|\mathcal{H}\mathbf{f} - \mathbf{g}_m\|_2^2$, is added to aid in the gray-level recovery. Using Eqs. (4) and (5) in Eq. (3), the data divergence can be rewritten as

$$D_w(\mathbf{g}_m, \mathcal{H}\mathbf{f}) = \|W_c(\mathcal{H}\mathbf{f} - \mathbf{g}_m)\|_2^2, \quad (6)$$

where weighting matrix W_c is defined as

$$W_c = c\mathcal{I} + (1-c)\mathcal{D}_u, \quad (7)$$

and $\mathcal{Q}$ denotes the identity matrix of size $M \times M$. It can be observed that data divergence $D_W(\mathbf{g}_m, \mathcal{H}(\mathbf{f}))$ is a weighted ℓ_2 -norm of the difference between measured and model data.

3.2. Reconstruction algorithm and its pseudo codes

A preconditioned CP-algorithm instance can be derived from its basic form [41, 47] specifically for solving the convex optimization program in Eq. (1). We show a detailed derivation of the CP algorithm [39,42] in Appendix A and its pseudo codes in Table 1.

Parameter $\nu_{\mathcal{H}} = \frac{\|W_c \mathcal{H}\|_{SV}}{\|\nabla\|_{SV}}$ in which norm $\|\cdot\|_{SV}$ denotes the largest singular value of a matrix.

∇ depicts a gradient matrix of size $3N \times N$ with its transpose given by $\nabla^T = (\nabla_x^T, \nabla_y^T, \nabla_z^T)$; superscript “T” indicates a transpose operation; ∇_x , ∇_y , and ∇_z are matrices of size $N \times N$ representing two-point differences along x -, y -, and z -axis, respectively, yielding vectors $\nabla_x \mathbf{f}$, $\nabla_y \mathbf{f}$, and $\nabla_z \mathbf{f}$ of size N , which in turn form vector $\nabla \mathbf{f}$ of size $3N$ in a concatenated form in the order of x , y , and z ; $|\nabla \mathbf{f}|_{\text{MAG}}$ denotes a vector of size N in which entry i is given by

$(|\nabla \mathbf{f}|_{\text{MAG}})_i = \sqrt{(\nabla \mathbf{f})_i^2 + (\nabla \mathbf{f})_{i+N}^2 + (\nabla \mathbf{f})_{i+2N}^2}$. Operator **ProjectOnto ℓ_1 Ball** $_{\nu t_1}(2\nu \mathbf{q}|_{\text{MAG}})$ yields vector $\mathbf{s}$ of size N by projecting vector $2\nu \mathbf{q}|_{\text{MAG}}$ of size N onto the ℓ_1 -ball of scale νt_1 [39, 48], whereas operator $\text{pos}(\mathbf{f}) = 0$ and f_j for $f_j \leq 0$ and $f_j > 0$, respectively, enforcing the non-negativity constraint. Finally, $\mathbf{f}_{\text{conv}}$ denotes the reconstruction satisfying the practical convergence conditions. In the pseudo codes, the multiplication and division between two vectors are defined in Appendix A.

3.3. Reconstruction parameters

The optimization program in Eq. (1) includes numerous parameters such as system matrix $\mathcal{H}$, image array (i.e., voxel shape and size), modulation parameter c , and TV-constraint parameter t_1 , which are referred to as *program parameters* in the work. Different choices of the program-parameter values lead to different reconstructions. On the other hand, the CP algorithm derived also involves parameters λ and ν , which are referred to *algorithm parameters*. Unlike program parameters that specify feasible solutions, algorithm parameters have no impact on the solution specification. Instead, they impact the algorithm’s convergence path and rate to the feasible solution set.

While the CP algorithm is shown mathematically to solve a convex optimization program, it is unlikely that the mathematical convergence conditions can be met in numerical studies because a solution only at a finite number of iterations can be obtained due to the limited computer precision and computation time. Therefore, numerical studies require practical convergence conditions. Let $\text{cPD}(\mathbf{f}_n)$ denote the conditional primal-dual gap at iteration n [41]

$$\text{cPD}(\mathbf{f}_n) = \frac{1}{2} \lambda \|W_c(\mathcal{H} \mathbf{f}_n - \mathbf{g}_m)\|_2^2 - \left[-\frac{1}{2\lambda} \|\mathbf{p}_n\|_2^2 - \mathbf{g}_m^T W_c^T \mathbf{p}_n - \nu t_1 \|(|\mathbf{q}_n|_{\text{MAG}})\|_\infty \right],$$

where $\mathbf{p}_n$ and $\mathbf{q}_n$ are defined in sublines 2 and 4 of Line 6 in the pseudo codes. We introduce metrics that are used for designing practical convergence conditions in the work:

$$\begin{aligned}\overline{D}_W(\mathbf{f}_n) &= \|W_c(\mathcal{H}\mathbf{f}_n - \mathbf{g}_m)\|_2^2 / \|W_c(\mathcal{H}\mathbf{f}_1 - \mathbf{g}_m)\|_2^2 \\ \overline{\text{TV}}(\mathbf{f}_n) &= \|\mathbf{f}_n\|_{TV} - t_1 / t_1 \\ \overline{\text{cPD}}(\mathbf{f}_n) &= |\text{cPD}(\mathbf{f}_n) / \text{cPD}(\mathbf{f}_1)|\end{aligned}\quad (8)$$

where $\overline{D}_W(\mathbf{f}_n)$, $\overline{\text{TV}}(\mathbf{f}_n)$, and $\overline{\text{cPD}}(\mathbf{f}_n)$ denote normalized data divergence, image-TV, and conditional primal dual gap at iteration n .

The practical convergence conditions to be designed by use of the metrics in Eq. (8) can be interpreted as program parameters because they play a role in specifying practically feasible solutions to the optimization program. In the inverse-crime and real-data studies below, we discuss the determination of their respective program and algorithm parameters.

4. Algorithm Verification - An Inverse-Crime Study

Prior to the real-data studies, we first perform an inverse-crime study quantitatively to verify that the derived CP algorithm and its implementation can, under ideal, sufficient data conditions, solve numerically accurately the optimization program. While an inverse-crime study is used for numerically verifying the algorithm and its implementation, it is not an assessment of the practical utility of an optimization-based reconstruction in real-data studies, which can be achieved only through a carefully designed evaluation using task-utility metrics.

Without loss of generality, the inverse-crime study use image and data arrays of sizes smaller than those used in real-data studies for the sake of saving computational effort. Specifically, we use a 3D discrete swine image as truth image $\mathbf{f}_{\text{true}}$, which contain $N = 128 \times 128 \times 6$ identical cubic voxels of size 1.82 mm, and display in the left panel of Fig. 3 a transverse slice in the truth image; a detector containing only 500×20 detecting bins of size 0.8 mm is used to generate the model data at 582 views uniformly distributed in a circular trajectory segment spanning an angular range of 194° from the truth image and thus $M = 500 \times 20 \times 500$. System matrix $\mathcal{H}$ mimicking a circular trajectory of the clinical C-arm CBCT system is used.

As discussed above, program and algorithm parameters need to be determined for the study. In an inverse-crime study, program parameters M , N , and $\mathcal{H}$ are known because they are used for generating the model data, and because $t_1 = 4854.97$ can be calculated from the known, truth image; and the practical convergence conditions are designed as

$$\begin{aligned}\overline{D}_W(\mathbf{f}_n) &< 10^{-5} \\ \overline{\text{TV}}(\mathbf{f}_n) &< 10^{-5} \\ \overline{\text{cPD}}(\mathbf{f}_n) &< 10^{-5}.\end{aligned}\quad (9)$$

The program parameters determined above, along with a selected c value, completely specify the practically achievable solutions in the inverse-crime study. A reconstruction satisfying the practical convergence conditions is denoted as $\mathbf{f}_{\text{conv}}$.

While algorithm parameters λ and ν have no effect on the solution specification, they can impact the path and convergence rate leading to a solution specified. We demonstrate a determination of λ and ν that yield a high convergence rate numerically. Using program parameters determined above, along with a selected $c = 0.05$, multiple combinations of λ and ν were used in the CP algorithm to reconstruct images from which metrics in Eq. (8) were calculated. Without loss of generality, we display in Fig. 2 $\bar{D}_W(\mathbf{f}_n)$ and $\overline{\text{cPD}}(\mathbf{f}_n)$, obtained with $\lambda = 0.001, 0.01, \& 0.1$ and $\nu = \nu_{\mathcal{H}}$ (row 1), and with $\lambda = 0.01$ and $\nu = 0.1 \nu_{\mathcal{H}}, \nu_{\mathcal{H}}, \& 10 \nu_{\mathcal{H}}$ (row 2), respectively. It can be observed that $\lambda = 0.01$ and $\nu = \nu_{\mathcal{H}}$ appear to yield $\bar{D}_W(\mathbf{f}_n)$ and $\overline{\text{cPD}}(\mathbf{f}_n)$ generally lower than do other choices of λ and ν at a given iteration. Similar result is obtained also for real-data studies. Therefore, $\lambda = 0.01$ and $\nu = \nu_{\mathcal{H}}$ were selected for the inverse-crime study summarized below and for the real-data studies in Secs. 6–8.

Studies with different values of $c \in [0, 1]$ were also carried out, yielding results numerically identical to each other. Again, without loss of generality, we show in the middle panel of Fig. 3 reconstruction obtained with $c = 0.05$ by use of the CP algorithm with $\lambda = 0.01$ and $\nu = \nu_{\mathcal{H}}$, in company with its absolute difference than the truth image displayed in the right panel of Fig. 3. It can be observed that the practically convergent reconstruction is visually and numerically virtually identical to the truth image. In Fig. 4, we display the convergence metrics as functions of iterations, including $\bar{\epsilon}(\mathbf{f}_n) = \|\mathbf{f}_n - \mathbf{f}_{\text{true}}\|_2 / \|\mathbf{f}_{\text{true}}\|_2$, a normalized root-mean-square-error between the reconstructions and the known, truth image.

5. Determination of Parameters in Real-Data Studies

In the real-data studies below, without loss of generality, algorithm parameters $\lambda = 0.01$ and $\nu = \nu_{\mathcal{H}}$ used in the inverse-crime study were selected, with the understanding that $\nu_{\mathcal{H}}$ is calculated for system matrix $\mathcal{H}$ corresponding to the real-data scans. With the algorithm parameters selected, we then focus on the determination of the program parameters identified in Sec. 3.3.

5.1. Determination of program parameters except for c and t_1

In real-data studies below, scanning geometry information was incorporated into system matrix $\mathcal{H}$ in which an element was defined as the intersection length of X-ray measured and a voxel in the image array. We used detecting-bin size, voxel size, and data size M identical to clinical parameters discussed in Sec. 2, for the purpose of comparison with the reference reconstructions, while the image arrays of sizes $N = 600 \times 800 \times 660$ and $N = 600 \times 1200 \times 600$ were used, respectively, in swine and patient reconstructions.

The practical convergence conditions are designed as

$$\begin{aligned} \left| \frac{\partial \bar{D}_W(\mathbf{f}_n)}{\partial n} \right| &< 10^{-3} \\ \frac{\text{TV}(\mathbf{f}_n)}{\text{CPD}(\mathbf{f}_n)} &< 10^{-3} \\ \frac{\text{TV}(\mathbf{f}_n)}{\text{CPD}(\mathbf{f}_n)} &< 10^{-3}. \end{aligned} \quad (10)$$

Because inconsistency exists between measured and model data in a real-data study, it is likely that $\bar{D}_W(\mathbf{f}_n)$ converges to a non-zero number that is unknown. Therefore, it is unclear how an upper bound on $\bar{D}_W(\mathbf{f}_n)$ can be devised. On the other hand, we expect that when the algorithm converges, the absolute difference in $\bar{D}_W(\mathbf{f}_n)$ at two consecutive iterations becomes small. Also, because the data and image sizes in the real-data studies are substantially larger than those in the inverse-crime study, it is challenging in terms of computation time and computer precision required for meeting the tight convergence conditions in Eq. (9). Our study also suggests that the convergence conditions in Eq. (10) yield reconstructions visually identical to those obtained with tighter convergence conditions. Therefore, practical convergence conditions in Eq. (10) were used in real-data studies. Again, we refer to a reconstruction satisfying the practical convergence conditions as $\mathbf{f}_{\text{conv}}$.

With the determination of program parameters above, c and t_1 are the only two remaining program parameters to be determined. Without loss of generality, we demonstrate their determination in Secs. 5.2 and 5.3 below, in terms of visualization of artifact reduction, by using the swine data with aortogram injection.

5.2. Determination of parameter c

Without loss of generality, we first selected $t_1 = 12000$ based upon the result in Sec. 5.3 below so that the study can focus on the determination of c . For a given c value in $[0, 1]$, we applied the CP algorithm derived to reconstructing an image by solving Eq. (1) under the practical convergence conditions in Eq. (10). Repeating the study for a number of c values, we obtained multiple reconstructions and display in Fig. 5 those reconstructions obtained with $c = 0.000, 0.025, 0.050, 0.075, 0.100, 0.200, 0.300$, and 1.000 , respectively, along with the reference reconstruction. The reconstructions obtained with $c > 0$ in Fig. 5 appear to reduce artifacts observed in the reference reconstruction. However, reconstruction obtained with $c = 0$, while picking up some edge information, fails recovering the overall image-gray level. Furthermore, even though the image-gray level recovers as c value increases, a noticeably elevated intensity shows up near the edge of the FOV especially when $c > 0.2$ as a result of transverse data truncation. Overall, reconstructions with $c \in [0.025, 0.1]$ appear visually with reduced artifacts as well as a reasonable recovery of image-gray level. This observation was made also for reconstructions from the swine data with hepatic injection and from the patient data, and thus we chose $c = 0.05$ in real-data studies throughout the work.

The selected $c = 0.05$ can be appreciated by considering the impact of c on the weighted system matrix $W_c \mathcal{H}$ (see Eqs. (6) and (7)). While it is well-known that the condition number can provide a characterization of the ill-conditioning level of a matrix, it is generally

challenging numerically to compute the condition number of the matrix of a large size considered. On the other hand, the largest singular value can readily be calculated, and because a condition number is the ratio between the largest and lowest singular values, the largest singular value itself may be used as an index for loosely characterizing the ill-conditioning level of the matrix [40]. Therefore, we have calculated the largest singular values of matrices $W_c \mathcal{H}$ specified with multiple values of $c \in [0, 1]$ as shown in Fig. 6. The result shows that the largest singular value remains relatively low for $0 \leq c < 0.16$, decreases slightly from $c = 0$ to $c \approx 0.16$, and increases rapidly for $c > 0.16$, suggesting that a c -value between 0 and 0.16 would be preferred, which is consistent with the selection of $c = 0.05$ based upon visual inspection of the reconstructions with different c values above.

5.3. Determination of parameter t_1

Reconstruction properties are dependent on TV-constraint parameter t_1 . Investigation exists on the determination of t_1 by use of quantitative metrics of specific interests [49,50]. In this work, we focus on the determination of t_1 based upon visual inspection of artifact reduction in the absence of quantitative metrics appropriate for measuring artifact reduction in C-arm CBCT employed in an interventional procedure. Using the program parameters selected in Secs. 5.1 and 5.2, we carried out multiple reconstructions for different values of t_1 , and show in Fig. 7 reconstructions obtained with $t_1 = 4000, 8000, 12000, 20000$, and 40000 , respectively, along with the reference reconstruction. As t_1 decreases, reconstructions become smoothed in which, while streak artifacts are reduced, some structural details are wiped out. Conversely, when t_1 increases, more image structures appear to be picked up, but image artifacts also become conspicuous. Based upon visual inspection, we selected parameter $t_1 = 12000$ that yields a reconstruction with a reasonable balance between a recovery of detailed structures and reduction of image artifacts.

6. Reconstructions for Conventional, Clinical Scanning Configurations

In this section, we present results of the optimization-based reconstruction from real data of swine and patient subjects collected by use of the conventional, clinical scanning configuration described in Sec. 2.

6.1. Results of animal study

In the two swine studies with aortogram and hepatic injections, respectively, data information was described in Sec. 2, whereas program parameters, including $c = 0.05$ and practical convergence conditions, were determined in Secs. 5.1 and 5.2. Furthermore, using the approach described in Sec. 5.3, we have selected $t_1 = 12000$ and 15000 for the two data sets, respectively.

For the swine study with aortogram injection, we display in Figs. 8 and 9 the optimization-based reconstruction, along with the reference reconstruction. Conspicuous streak artifacts in the reference reconstruction can be visually distracting and possibly obscure information of clinical significance; whereas the optimization-based reconstruction suppresses the artifacts, despite the fact that it invokes no pre-reconstruction data processing. While some residual artifacts remain especially within transverse slices, dominant artifacts that plague

images within coronal and sagittal slices are substantially reduced. This can be clinically meritorious because image-guidance tasks in the interventional liver-cancer treatment involve frequently coronal and sagittal images.

We have also performed reconstruction from the swine data with hepatic injection and show in Figs. 10 and 11 the reconstructed images, along with the reference reconstruction. Observations similar to those for the swine study with aortogram injection above can also be made for the swine study with hepatic injection.

In Figs. 12 and 13, we display the optimization-based reconstructions from the two swine-data sets within transverse slices specified by $z = -110.20$ mm and 108.81 mm near the top and bottom of the longitudinal FOV, along with the reference reconstructions. The results show that in addition to artifact reduction, the optimization-based reconstruction may yield 3D images with slightly extended longitudinal volume. We also display in Fig. 14 reconstructions $\mathbf{f}_n$ at iterations $n = 20, 50, 100, 200$, and 300, along with $\mathbf{f}_{\text{conv}}$ (i.e., $n = 1000$) from the swine data with aortogram injection. Reconstruction at early iteration, e.g., at $n = 300$, visually resembles the convergent reconstruction obtained at iteration $n = 1000$. Again, similar observations can be made for the swine study with hepatic injection.

6.2. Results of patient study

In the patient study, data information was described in Sec. 2, whereas program parameters, including $c = 0.05$ and practical convergence conditions, were determined in Secs. 5.1 and 5.2. We also selected $t_1 = 15000$ by using the approach described in Sec. 5.3 based upon the patient data. In Figs. 15 and 16, we display the optimization-based reconstruction, which reduces artifacts observed in the reference reconstruction.

In Fig. 17, reconstruction is shown within transverse slices specified by $z = -111.21$ mm and 106.23 mm near the top and bottom of the longitudinal FOV. Again, it can be observed that the optimization-based reconstruction yields a longitudinally slightly extended volume. We also show in Fig. 18 reconstructions $\mathbf{f}_n$ at iterations $n = 20, 50, 100, 200$, and 300, along with $\mathbf{f}_{\text{conv}}$ (i.e., $n = 1000$). Again, reconstruction at early iteration, e.g., at $n = 300$, visually resembles the convergent reconstruction. Overall, observations similar to those for the swine studies can be made for the patient study.

7. Reconstructions for Non-Conventional Scanning Configurations

We have thus far focused on reconstructions from data acquired with a clinical scanning configuration with ~ 150 projection views over an angular range of 194° , i.e., the short-scan angular range of the C-arm CBCT system, as described in Sec. 2. The number of projection views and short-scan angular range represent the sampling conditions minimally sufficient for an analytic algorithm such as the FDK to yield reconstructions of some practical utility in an interventional procedure. From the point view of clinical workflow, it is desirable that the number of projection views and/or the scanning angular range can be reduced, thus leading to lowered imaging dose and time, and/or to minimized likelihood of C-arm collision with the patient, interventional radiologist, and supporting staff engaged in the

procedure. Also, when a patient of large size is scanned with the current C-arm system, its limited FOV can result in a heightened data truncation.

For convenience, we refer to data collected with the clinical scanning configuration as *full data*, and to scanning configurations with sparse projection views, reduced angular range, and/or heightened data truncation as *non-conventional* scanning configurations. Because of their potentially high level of clinical significance, we investigate below optimization-based reconstructions from data acquired with non-conventional scanning configurations. The investigation focus on reconstructions from data “collected” with non-conventional scanning configurations from swine subject with aortogram injection as described below. Similar results have also been obtained from the swine data with hepatic injection and from the patient data.

Using the swine data with aortogram injection at every other view, we first formed a sparse-view data set consisting of a total $M_\phi = 74$ projections. Using the program parameters described in Secs. 5.1 and 5.2, we determined $t_1 = 15000$ for the sparse-view data by using the approach described in Sec. 5.3. Using the parameters determined we performed the optimization-based reconstruction and show it in Fig. 19. It can be observed that the reconstruction visually resembles the optimization-based reconstruction from the full data (see Fig. 8), and reduces artifacts observed in the reference reconstruction. For comparison, an FDK reconstruction from the sparse data was also shown in Fig. 19, which exhibits a considerably increased level of artifacts.

We also created a set of limited-angular-range data by using the projections at the last 115 views (i.e., $M_\phi = 115$) over an angular range of 150° in the full swine data. Again, using the program parameters described in Secs. 5.1 and 5.2, we determined $t_1 = 12000$ for the limited-angular-range data by using the approach in Sec. 5.3. In Fig. 20, we display the optimization-based and FDK reconstructions from the data. Again, it can be observed that the optimization-based reconstruction reduces considerably artifacts observed in the FDK reconstruction. Moreover, the optimization-based reconstruction within coronal and sagittal slices appears with much reduced artifacts even when compared to the reference reconstruction in Fig. 8.

In an attempt to construct data with a heightened level of transverse truncation, we removed the first and last 50 columns of projections on the detector plane at each view, thus forming severely truncated data of size $M = 400 \times 500 \times 147$. Again, using the program parameters described in Secs. 5.1 and 5.2, we determined $t_1 = 12000$ from the data by using the approach described in Sec. 5.3. In Fig. 21, we display the optimization-based reconstruction, along with the FDK reconstruction. As expected, the FDK reconstruction suffers from significant cupping artifact (i.e., intensity elevation) near the transverse edge of the FOV. On the other hand, the optimization-based reconstruction appears virtually free of such an artifact, suggesting that the latter is considerably less susceptible to severe data truncation than the former. The reconstruction within the FOV visually resembles that of the optimization-based reconstruction from the full data, and contains reduced artifacts as compared to the reference image in Fig. 8.

8. Reconstructions from Data with Pre-Reconstruction Processing

A leading motivation of the work is to investigate optimization-based reconstruction without invoking pre-reconstruction data processing, as demonstrated in Secs. 5–7. Even so, for some special situations, it may be of practical interest in performing some pre-reconstruction data processing. Using the swine study with aortogram injection as an example, we demonstrate below that an appropriate data processing may further increase the reconstruction coverage and reduce the residual artifacts in the reconstruction. It should be reminded, however, that the example is used only for demonstrating the potential impact of a specific pre-reconstruction data processing for a specific case, and that it is not intended to investigate and evaluate thoroughly the effectiveness of a pre-reconstruction data processing in C-arm CBCT applications, which is clearly beyond the scope of the work.

In an attempt to extend the longitudinal FOV, we performed a longitudinal extrapolation by adding 50 rows of “projection data” adjacent to the top and bottom of the detector plane, where the added top and bottom “projection data” are identical to, respectively, the projections in the top and bottom rows of the detector plane. Using the program parameters described in Secs. 5.1 and 5.2, we determined $t_1 = 15000$ from the longitudinally extrapolated data by taking the approach in Sec. 5.3. We display the reconstruction in Fig. 22, along with the optimization-based reconstruction from the non-processed swine data and reference reconstruction. The result suggests that, with an appropriate longitudinal data extrapolation, the optimization-based reconstruction can yield a longitudinally extended image of the swine subject.

We have also performed a transverse extrapolation to the swine data by including 50 columns of “projection data” to the first and last columns on the detector plane. The values of the added “projection data” fall off to zero smoothly from the values of projections in the first and last columns of the detector plane. With the program parameters determined in the way similar to that for the longitudinal extrapolation case, we reconstructed images from the transversely extrapolated data, and display them in Fig. 23, along with the optimization-based reconstruction from the non-processed swine data and reference reconstruction. The result indicates that, with an appropriate transverse extrapolation, the optimization-based reconstruction may yield a transversely extended image.

It can be observed in Fig. 8 that, while image artifacts are considerably reduced in the optimization-based reconstruction, some residual artifacts remain especially in the transverse images. In an attempt to minimize the residual artifacts, we estimated high contrast objects/devices based upon an initial reconstruction and then their projections onto the detector plane at each view. Using the projection information, we identified the detection bins that collect data from X-rays intersecting with high contrast objects/devices and then estimated new projection values on the bins by using a linear interpolation of projection data on the respective neighboring detecting bins that contain no contribution from the high contrast objects/devices. Subsequently, using the program parameters described in Secs. 5.1 and 5.2, we determined $t_1 = 12000$ from the interpolated data set by using the approach described in Sec. 5.3. As reconstruction result in Fig. 24 shows, appropriately designed data processing

can further reduce image artifacts in the optimization-based reconstruction for the swine study with aortogram injection.

9. Discussions

We investigated an optimization-based reconstruction, with an emphasis on artifact reduction, in C-arm CBCT employed in image-guided interventional procedures. In the investigation, a convex optimization program was formulated in which a data-derivative fidelity was introduced specifically for suppression of dominant, low-frequency artifacts caused by, e.g., data truncation, X-ray scatters, and inaccurate estimation of the incident X-ray flux, whereas the CP algorithm was tailored to reconstruct images through solving the program. The optimization-based reconstruction is designed to be applied directly to C-arm CBCT data, thus avoiding pre-reconstruction data processing and the associated issues.

The optimization-based reconstruction was characterized in studies with real data collected from swine and patient subjects. Visual inspection of the reconstructions shows that, overall, the optimization-based reconstruction yields images with visibly reduced artifacts in contrast to the reference reconstruction. The results also suggest a reasonably high degree of robustness of the optimization-based reconstruction, as the studies involve imaged subjects and surgical devices with distinctively different anatomies and structures.

In addition to artifact reduction, we also investigated non-conventional scanning configurations of potentially high clinical workflow merit, with lowered imaging dose and time, and/or minimized likelihood of C-arm collision with the patient, interventional radiologist, and supporting staff engaged in an interventional procedure. Using clinical, full data acquired, we generated data mimicking those acquired with non-conventional scanning configurations. Study results indicate that the optimization-based reconstruction can yield images with considerably reduced artifacts observed otherwise in the corresponding FDK reconstructions; and they also reveal a high degree of robustness of the optimization-based reconstruction when applied to challenging, non-conventional scanning conditions.

The complete specification of the optimization-based reconstruction investigated, like any other reconstructions, requires a number of parameters, which in turn impact significantly the reconstruction characteristics. We have identified precisely the parameters involved and discussed their quantitative determination given data conditions. Among them, parameter c modulates the impact of the data-derivative fidelity on the program properties: $c = 1$ yields a program with the conventional data ℓ_2 -norm, whereas $c = 0$ a program with a pure data-derivative ℓ_2 -norm. The former has been studied extensively, e.g., for image reconstruction from sparse-view data, whereas the latter can produce a reconstruction analogous to a local tomographic reconstruction with anatomic boundary information. For data collected we have investigated the parameter determination based upon visualization of artifact reduction.

In the work, while visualization inspection used for examining image-artifact reduction is arguably qualitative, it indeed proves to be useful in revealing artifact reduction and in determining parameters c and t_1 especially in the current absence of meaningful utility metrics for quantitatively characterizing reconstruction quality in an interventional

procedure. Conversely, while beyond the scope of the work, it can be of practical significance to design, based upon well-specified, clinically relevant imaging tasks, utility metrics for quantitatively assessing the reconstruction performance in the tasks.

Visual inspection of artifact reduction in (or a task-specific utility metric calculated based upon) the reconstruction is in essence a function in a multi-dimensional space formed by the parameters involved. Based upon qualitative, visual inspection of artifact reduction, we have demonstrated the determination of c and t_1 through searching their respective dimensions, while other parameters were fixed in the space, as discussed in Sec. 5. It is possible, however, that different values of other parameters can lead to different selections of c and t_1 for effective artifact reduction. Therefore, it is interesting to investigate in future work the reconstruction characteristics (or a utility metric) along dimensions formed by other parameters.

We remind that the study is not meant to produce a practical procedure that needs otherwise to meet necessarily numerous, additional task-specific requirements in clinical interventional procedures. Instead, it is intended to investigate and characterize an optimization-based reconstruction directly from C-arm-CBCT data of subjects possessing clinically-relevant, diverse anatomies by use of scanning configurations of clinical-workflow significance in interventional procedures. Knowledge and insights gained in the study, along with other important clinical task-specific considerations, can be exploited for aiding in the design of practical procedures of truly clinical-task utility. Finally, while the work focused on optimization-based reconstruction for C-arm CBCT in interventional procedures, it can readily be generalized for investigating image reconstruction in other CBCT applications.

Acknowledgments

This work was supported in part by NIH R01 Grants Nos. CA158446, CA182264, and EB018102. The contents of this article are solely the responsibility of the authors and do not necessarily represent the official NIH views.

References

1. Daly MJ, Siewerdsen JH, Moseley DJ, Jaffray DA, Irish JC. Intraoperative cone-beam CT for guidance of head and neck surgery: assessment of dose and image quality using a C-arm prototype. *Med Phys.* 2006; 33:3767–3780. [PubMed: 17089842]
2. Chen GH, Zambelli J, Nett BE, Supanich M, Riddell C, Belanger B, Mistretta CA. Design and development of C-arm based cone-beam CT for image-guided interventions. *SPIE Medical Imaging.* 2006:614 210–614 210.
3. Louie JD, Kothary N, Kuo WT, Hwang GL, Hofmann LV, Goris ML, Iagaru AH, Sze DY. Incorporating cone-beam CT into the treatment planning for yttrium-90 radioembolization. *J Vasc Interv Radiol.* 2009; 20:606–613.
4. Solomon SB, Silverman SG. Imaging in interventional oncology. *Radiol.* 2010; 257:624–640.
5. Schafer S, Nithiananthan S, Mirota DJ, Uneri A, Stayman JW, Zbijewski W, Schmidgunst C, Kleinszig G, Khanna AJ, Siewerdsen JH. Mobile C-arm cone-beam CT for guidance of spine surgery: image quality, radiation dose, and integration with interventional guidance. *Med Phys.* 2011; 38:4563–4574. [PubMed: 21928628]
6. Floridi C, Radaelli A, Abi-Jaoudeh N, Grass M, Lin MD, Chiaradia M, Geschwind JF, Kobeiter H, Squillaci E, Maleux G, Giovagnoni A. C-arm cone-beam computed tomography in interventional oncology: technical aspects and clinical applications. *La Radiologia Medica.* 2014; 119:521–532. [PubMed: 25012472]

7. Tognolini A, Louie JD, Hwang GL, Hofmann LV, Sze DY, Kothary N. Utility of C-arm CT in patients with hepatocellular carcinoma undergoing transhepatic arterial chemoembolization. *J Vasc Interv Radiol*. 2010; 21:339–347.
8. Morimoto M, Numata K, Kondo M, Nozaki A, Hamaguchi S, Takebayashi S, Tanaka K. C-arm cone beam CT for hepatic tumor ablation under real-time 3D imaging. *Am J Roentgenol*. 2010; 194:w452–w454. [PubMed: 20410393]
9. Bapst B, Lagadec M, Breguet R, Vilgrain V, Ronot M. Cone beam computed tomography (CBCT) in the field of interventional oncology of the liver. *Cardiov Interv Radiol*. 2015; 39:1–13.
10. Meyer BC, Frericks BB, Voges M, Borchert M, Martus P, Justiz J, Wolf KJ, Wacker FK. Visualization of hypervascular liver lesions during TACE: comparison of angiographic C-arm CT and MDCT. *Am J Roentgenol*. 2008; 190:w263–w269. [PubMed: 18356419]
11. Deschamps F, Solomon SB, Thornton RH, Rao P, Hakime A, Kuoch V, de Baere T. Computed analysis of three-dimensional cone-beam computed tomography angiography for determination of tumor-feeding vessels during chemoembolization of liver tumor: a pilot study. *Cardiovascul Interv Radiol*. 2010; 33:1235–1242.
12. Loffroy R, Lin M, Yenokyan G, Rao PP, Bhagat N, Noordhoek N, Radaelli A, Blijd J, Liapi E, Geschwind JF. Intraprocedural C-arm dual-phase cone-beam CT: can it be used to predict short-term response to TACE with drug-eluting beads in patients with hepatocellular carcinoma? *Radiol*. 2013; 266:636–648.
13. Wang Z, Lin M, Lesage D, Chen R, Chapiro J, Gu T, Tacher V, Duran R, Geschwind JF. Three-dimensional evaluation of lipiodol retention in HCC after chemoembolization: a quantitative comparison between CBCT and MDCT. *Acad Radiol*. 2014; 21:393–399. [PubMed: 24507426]
14. Miyayama S, Yamashiro M, Hashimoto M, Hashimoto N, Ikuno M, Okumura K, Yoshida M, Matsui O. Comparison of local control in transcatheter arterial chemoembolization of hepatocellular carcinoma 6 cm with or without intraprocedural monitoring of the embolized area using cone-beam computed tomography. *Cardiovascul Interv Radiol*. 2014; 37:388–395.
15. Tacher V, Radaelli A, Lin M, Geschwind JF. How I do it: cone-beam CT during transarterial chemoembolization for liver cancer. *Radiol*. 2015; 274:320–334.
16. Scherthaner RE, Chapiro J, Sahu S, Withagen P, Duran R, Sohn JH, Radaelli A, van der Bom IM, Geschwind JFH, Lin M. Feasibility of a modified cone-beam CT rotation trajectory to improve liver periphery visualization during transarterial chemoembolization. *Radiol*. 2015; 277:833–841.
17. Ohnesorge B, Flohr T, Schwarz K, Heiken JP, Bae KT. Efficient correction for CT image artifacts caused by objects extending outside the scan field of view. *Med Phys*. 2000; 27:39–46. [PubMed: 10659736]
18. Hsieh J, Chao E, Thibault J, Grekowitz B, Horst A, McOlash S, Myers TJ. A novel reconstruction algorithm to extend the CT scan field-of-view. *Med Phys*. 2004; 31:23852391.
19. Sourbelle K, Kachelriebe M, Kalender WA. Reconstruction from truncated projections in CT using adaptive detruncation. *Euro Radiol*. 2005; 15:1008–1014.
20. Starman J, Pelc N, Strobel N, Fahrig R. Estimating 0th and 1st moments in C-arm CT data for extrapolating truncated projections. *SPIE Medical Imaging*. 2005:378–387.
21. Kolditz D, Meyer M, Kyriakou Y, Kalender WA. Comparison of extended field-of-view reconstructions in C-arm flat-detector CT using patient size, shape or attenuation information. *Phys Med Biol*. 2010; 56:39–56. [PubMed: 21119229]
22. Kalender WA, Hegel R, Ebersberger J. Reduction of CT artifacts caused by metallic implants. *Radiol*. 1987; 164:576–577.
23. Xia D, Roeske J, Yu L, Pelizzari C, Pan X. A hybrid approach to reducing CT metal artifact in intracavitary brachytherapy. *Brachytherapy*. 2005; 4:18–23. [PubMed: 15737902]
24. Lemmens C, Faul D, Nuyts J. Suppression of metal artifacts in CT using a reconstruction procedure that combines MAP and projection completion. *IEEE Trans Med Imaging*. 2009; 28:250–260. [PubMed: 19188112]
25. Meyer E, Raupach R, Lell M, Schmidt B, Kachelriebe M. Normalized metal artifact reduction (NMAR) in computed tomography. *Med Phys*. 2010; 37:5482–5493. [PubMed: 21089784]
26. Haase C, Schafer D, Kim M, Chen SJ, Carroll J, Eshuis P, Dossel O, Grass M. Automatic cable artifact removal for cardiac C-arm CT imaging. *SPIE Medical Imaging*. 2014:90 330K–90 330K.

27. Deak Z, Grimm JM, Treitl M, Geyer LL, Linsenmaier U, Korner M, Reiser MF, Wirth S. Filtered back projection, adaptive statistical iterative reconstruction, and a model-based iterative reconstruction in abdominal CT: an experimental clinical study. *Radiol.* 2006; 266:197–206.
28. Sidky EY, Pan X. Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization. *Phys Med Biol.* 2008; 53:4777–4807. [PubMed: 18701771]
29. Singh S, Kalra MK, Hsieh J, Licato PE, Do S, Pien HH, Blake MA. Abdominal CT: comparison of adaptive statistical iterative and filtered back projection reconstruction techniques. *Radiol.* 2010; 257:373–383.
30. Pontana F, Pagniez J, Flohr T, Faivre J, Duhamel A, Remy J, Remy-Jardin M. Chest computed tomography using iterative reconstruction vs filtered back projection (Part 1): evaluation of image noise reduction in 32 patients. *Euro Radiol.* 2011; 21:627–635.
31. Winklehner A, Christoph K, Puippe G, Schmidt B, Flohr T, Goetti R, Pfammatter T, Frauenfelder T, Alkadhi H. Raw data-based iterative reconstruction in body CTA: evaluation of radiation dose saving potential. *Euro Radiol.* 2011; 21:2521–2526.
32. Defrise M, Vanhove C, Liu X. An algorithm for total variation regularization in high-dimensional linear problems. *Inv Prob.* 2011; 27:065 002–065 002.
33. Ramani S, Fessler J. A splitting-based iterative algorithm for accelerated statistical X-ray CT reconstruction. *IEEE Trans Med Imag.* 2012; 31:677–688.
34. Han X, Bian J, Ritman EL, Sidky EY, Pan X. Optimization-based reconstruction of sparse images from few-view projections. *Phys Med Biol.* 2012; 57:5245–5274. [PubMed: 22850194]
35. Bian J, Wang J, Han X, Sidky EY, Shao L, Pan X. Optimization-based image reconstruction from sparse-view data in offset-detector CBCT. *Phys Med Biol.* 2013; 58:205–230. [PubMed: 23257068]
36. Muller K, Schwemmer C, Hornegger J, Zheng Y, Wang Y, Lauritsch G, Rohkohl C, Maier AK, Schultz C, Fahrig R. Evaluation of interpolation methods for surface-based motion compensated tomographic reconstruction for cardiac angiographic C-arm data. *Med Phys.* 2013; 40:031 107–031 107.
37. Langet H, Riddell C, Reshef A, Troussset Y, Tenenhaus A, Lahalle E, Fleury G, Paragios N. Compressed-sensing-based content-driven hierarchical reconstruction: theory and application to C-arm cone-beam tomography. *Med Phys.* 2015; 42:5222–5237. [PubMed: 26328972]
38. Frikel J, Quinto ET. Characterization and reduction of artifacts in limited angle tomography. *Inv Prob.* 2013; 29:125 007–125 007.
39. Sidky EY, Kraemer DN, Roth EG, Ullberg C, Reiser IS, Pan X. Analysis of iterative region-of-interest image reconstruction for x-ray computed tomography. *J Med Imaging.* 2014; 1:031 007–031 007.
40. Kraemer DN, Roth EG, Sidky EY, Reiser IS, Pan X. Robust iterative image reconstruction for breast CT by use of projection differentiation. *SPIE Medical Imaging.* 2015:94 123I–94 123I.
41. Chambolle A, Pock T. A first-order primal-dual algorithm for convex problems with applications to imaging. *J Math Imag Vis.* 2011; 40:120–145.
42. Sidky EY, Jørgensen JH, Pan X. Convex optimization problem prototyping for image reconstruction in computed tomography with the Chambolle-Pock algorithm. *Phys Med Biol.* 2012; 57:3065–3091. [PubMed: 22538474]
43. Sidky EY, Chartrand R, Boone JM, Pan X. Constrained minimization for enhanced exploitation of gradient sparsity: application to CT image reconstruction. *IEEE J Trans Eng Health Med.* 2014; 2:1–18.
44. Claus BE. Geometry calibration phantom design for 3D imaging. *Medical Imaging, Proc SPIE.* 2006; 6142:61 422E–61 422E.
45. Li X, Zhang D, Liu B. A generic geometric calibration method for tomographic imaging systems with flat-panel detectors-a detailed implementation guide. *Med Phys.* 2010; 37:3844–3854. [PubMed: 20831092]
46. Pan X, Sidky EY, Vannier M. Why do commercial CT scanners still employ traditional, filtered back-projection for image reconstruction? *Inv Prob.* 2009; 25:123009. Link: <http://iopscience.iop.org/article/10.1088/0266-5611/25/12/123009/meta>.

47. Pock T, Chambolle A. Diagonal preconditioning for first order primal-dual algorithms in convex optimization. International Conference on Computer Vision (ICCV 2011). 2011:1762–1769.
48. Duchi J, Shalev-Shwartz S, Singer Y, Chandra T. Efficient projections onto the ℓ_1 -ball for learning in high dimensions. Proceedings of the 25th International Conference on Machine Learning. 2008:272–279.
49. Bian J, Yang K, Boone JM, Han X, Sidky EY, Pan X. Investigation of iterative image reconstruction in low-dose breast CT. Phys Med Biol. 2014; 59:2659–2685. [PubMed: 24786683]
50. Han X, Pearson E, Pelizzari C, Al-Hallaq H, Sidky EY, Bian J, Pan X. Algorithm-enabled exploration of image-quality potential of cone-beam CT in image-guided radiation therapy. Phys Med Biol. 2015; 60:4601–4633. [PubMed: 26020490]
51. Rockafellar, RT. Convex Analysis. Princeton Univ. Press; 1970.
52. Moreau J. Fonctions convexes duales et points proximaux dans un espace hilbertien. CR Acad Sci Paris Ser A Math. 1962; 255:2897–2899.

Appendix A: Derivation of the CP Algorithm for Solving Eq. (1)

Using two positive parameters λ and ν [39, 42], we change the form of Eq. (1) to

$$\mathbf{f}^* = \underset{\mathbf{f}}{\operatorname{argmin}} \frac{1}{2} \lambda \|W_c(\mathcal{H}\mathbf{f} - \mathbf{g}_m)\|_2^2 \quad \text{s.t.} \quad \|\nu\mathbf{f}\|_{TV} \leq \nu t_1 \quad \text{and} \quad f_i \geq 0. \quad (\text{A.1})$$

Clearly, optimization programs in Eqs. (1) and (A.1) specify an identical feasible solution set. In this sense, parameters λ and ν thus have no impact on the designed solutions. However, they can affect the convergence behaviors, in terms of path and rate, of the CP algorithm, as shown in Fig. 2.

We show below the CP algorithm tailored specifically to solve the program in Eq. (A.1) or, equivalently, the program in Eq. (1), which can be reformulated into an unconstrained problem as [39, 42]

$$\mathbf{f}^* = \underset{\mathbf{f}}{\operatorname{argmin}} \left\{ \frac{1}{2} \lambda \|W_c(\mathcal{H}\mathbf{f} - \mathbf{g}_m)\|_2^2 + \delta_{\text{Diamond}(\nu t_1)}(|\nu\nabla\mathbf{f}|_{\text{MAG}}) + \delta_P(\mathbf{f}) \right\}, \quad (\text{A.2})$$

where ∇ depicts a gradient matrix of size $3N \times N$ with its transpose given by $\nabla^\top = (\nabla_x^\top, \nabla_y^\top, \nabla_z^\top)$; superscript “ $\top$ ” indicates a transpose operation; ∇_x , ∇_y , and ∇_z are matrices of size $N \times N$ representing two-point differences along x -, y -, and z -axis, respectively, yielding vectors $\nabla_x\mathbf{f}$, $\nabla_y\mathbf{f}$, and $\nabla_z\mathbf{f}$ of size N , which in turn form vector $\nabla\mathbf{f}$ of size $3N$ in a concatenated form in the order of x , y , and z ; $|\nabla\mathbf{f}|_{\text{MAG}}$ denotes a vector of size N in which entry i is given by $(|\nabla\mathbf{f}|_{\text{MAG}})_i = \sqrt{(\nabla\mathbf{f})_i^2 + (\nabla\mathbf{f})_{i+N}^2 + (\nabla\mathbf{f})_{i+2N}^2}$; and indicator functions $\delta_P(\mathbf{x})$ and $\delta_{\text{Diamond}(\nu t_1)}(\mathbf{x})$ are defined as

$$\begin{aligned}\delta_P(\mathbf{x}) &\equiv \begin{cases} 0 & \mathbf{x} \geq 0 \\ \infty & \text{Otherwise} \end{cases} \\ \delta_{\text{Diamond}(\nu t_1)}(\mathbf{x}) &\equiv \begin{cases} 0 & \|\mathbf{x}\|_1 \leq \nu t_1 \\ \infty & \|\mathbf{x}\|_1 > \nu t_1 \end{cases}.\end{aligned}\quad (\text{A.3})$$

For discussion convenience, we consider two vectors $\mathbf{a}$ and $\mathbf{b}$ of sizes kN and N , where $k \geq 1$ is a positive integer. Let a_j denote the j th entry of $\mathbf{a}$, $j = 1, 2, \dots, kN$, and b_j the j th entry of $\mathbf{b}$, $j = 1, 2, \dots, N$, respectively. We use vectors $\mathbf{a}\mathbf{b}$ and $\mathbf{a}/\mathbf{b}$ of sizes kN to denote the multiplication and division of $\mathbf{a}$ and $\mathbf{b}$, with entries j given by

$$\begin{aligned}(\mathbf{a}\mathbf{b})_j &= a_j b_{j_m} \\ (\mathbf{a}/\mathbf{b})_j &= a_j / b_{j_m},\end{aligned}$$

respectively, where $j = 1, 2, \dots, kN$, and $j_m = \text{mod}(j, N)$ indicates the remainder of j dividing by N , and $b_{j_m} \neq 0$. In particular, when $k = 1$, $\mathbf{a}$ and $\mathbf{b}$ are of the same size N , and vectors $\mathbf{a}\mathbf{b}$ and $\mathbf{a}/\mathbf{b}$ thus are of size N depicting element-wise multiplication and division. We also use $\mathbf{a}^\top \mathbf{b}$ to denote their inner product for vectors $\mathbf{a}$ and $\mathbf{b}$ of an identical dimension.

The CP algorithm [41], a primal-dual algorithm, solves simultaneously a primal minimization problem

$$\mathbf{x}^* = \underset{\mathbf{x}}{\text{argmin}} \{F(K\mathbf{x}) + G(\mathbf{x})\}, \quad (\text{A.4})$$

and its Fenchel dual maximization problem

$$\mathbf{y}^* = \underset{\mathbf{y}}{\text{argmax}} \left\{ -F^*(\mathbf{y}) - G^*(-K^\top \mathbf{y}) \right\}, \quad (\text{A.5})$$

where primal variable $\mathbf{x}$ and dual variable $\mathbf{y}$ are vectors in spaces X and Y , respectively, K a linear transform from X to Y , F and G convex functions mapping the X and Y spaces to non-negative real numbers, and the convex conjugate functions [51]

$$\begin{aligned}F^*(\mathbf{y}) &= \max_{\mathbf{y}'} \left\{ \mathbf{y}^\top \mathbf{y}' - F(\mathbf{y}') \right\} \\ G^*(-K^\top \mathbf{y}) &= \max_{\mathbf{x}'} \left\{ (-K^\top \mathbf{y})^\top \mathbf{x}' - G(\mathbf{x}') \right\}.\end{aligned}\quad (\text{A.6})$$

The pseudo codes of the preconditioned CP algorithm [41, 42, 47] that solves simultaneously the primal-dual problem in Eqs. (A.4) and (A.5) are given in Table 2, in

which $|\cdot|$ takes the absolute value of each entry of a matrix, vectors $\mathbf{1}_X$ and $\mathbf{1}_Y$ of the sizes identical to that of $\mathbf{x}$ and $\mathbf{y}$ have all components set to 1, and the proximal mappings are given by

$$\text{prox}_{\Sigma}[F^*](\mathbf{y}) = \underset{\mathbf{y}'}{\text{argmin}} \left\{ F^*(\mathbf{y}') + \frac{1}{2}(\mathbf{y} - \mathbf{y}')^\top \left(\frac{\mathbf{y} - \mathbf{y}'}{\Sigma} \right) \right\}, \quad (\text{A.7})$$

$$\text{prox}_{\mathcal{T}}[G](\mathbf{x}) = \underset{\mathbf{x}'}{\text{argmin}} \left\{ G(\mathbf{x}') + \frac{1}{2}(\mathbf{x} - \mathbf{x}')^\top \left(\frac{\mathbf{x} - \mathbf{x}'}{\mathcal{T}} \right) \right\}. \quad (\text{A.8})$$

We show below that Eq. (A.2) can be interpreted as a primal problem with the form in Eq. (A.4) and that its dual problem can be derived, by using the assignments:

$$\begin{aligned} \mathbf{x} &= \mathbf{f} \\ \mathbf{r} &= W_c \mathcal{H} \mathbf{f} \\ \mathbf{z} &= \nu \nabla \mathbf{f} \\ K &= \begin{pmatrix} W_c \mathcal{H} \\ \nu \nabla \end{pmatrix} \end{aligned} \quad (\text{A.9})$$

and

$$F(\mathbf{r}, \mathbf{z}) = F_1(\mathbf{r}) + F_2(\mathbf{z}), \quad (\text{A.10})$$

$$G(\mathbf{x}) = \delta_p(\mathbf{x}), \quad (\text{A.11})$$

where

$$F_1(\mathbf{r}) = \frac{1}{2} \lambda \|\mathbf{r} - W_c \mathbf{g}_m\|_2^2 \quad (\text{A.12})$$

$$F_2(\mathbf{z}) = \delta_{\text{Diamond}(\nu t_1)}(|\mathbf{z}|_{\text{MAG}}), \quad (\text{A.13})$$

and $\mathbf{z}|_{\text{MAG}}$ depicts a vector of size N in which entry i is given by

$(\mathbf{z}|_{\text{MAG}})_i = \sqrt{z_i^2 + z_{i+N}^2 + z_{i+2N}^2}$, and z_i indicates entry i of vector $\mathbf{z}$. Substitution of Eqs. (A.9)–(A.13) into Eq. (A.4) and noticing $\mathbf{x} = \mathbf{f}$ yields

$$\begin{aligned} \mathbf{f}^* &= \underset{\mathbf{f}}{\operatorname{argmin}} \{F_1(W_c \mathcal{H} \mathbf{f}) + F_2(\nu \nabla \mathbf{f}) + G(\mathbf{f})\} \\ &= \underset{\mathbf{f}}{\operatorname{argmin}} \left\{ \frac{1}{2} \lambda \|w_c(\mathcal{H} \mathbf{f} - \mathbf{g}_m)\|_2^2 + \delta_{\text{Diamond}(\nu t_1)}(|\nu \nabla \mathbf{f}|_{\text{MAG}}) + \delta_P(\mathbf{f}) \right\}, \end{aligned} \quad (\text{A.14})$$

suggesting that with the assignments in Eqs. (A.9)–(A.13), optimization program in Eq. (A.2) can be casted into a primal minimization problem.

We then obtain below the corresponding dual maximization problem so that the CP algorithm can be applied. Using Eqs. (A.9)–(A.13) in Eq. (A.6), the convex conjugate functions of F and G can be obtained as

$$\begin{aligned} F^*(\mathbf{y}) &= \max_{\mathbf{r}, \mathbf{z}} \left\{ \mathbf{p}^\top \mathbf{r} + \mathbf{q}^\top \mathbf{z} - F_1(\mathbf{r}) - F_2(\mathbf{z}) \right\} \\ &= F_1^*(\mathbf{p}) + F_2^*(\mathbf{q}) \end{aligned} \quad (\text{A.15})$$

$$\begin{aligned} G^*(-K^\top \mathbf{y}) &= \max_{\mathbf{x}} \left\{ (-K^\top \mathbf{y})^\top \mathbf{x}' - G(\mathbf{x}') \right\} \\ &= \delta_P(-K^\top \mathbf{y}), \end{aligned} \quad (\text{A.16})$$

where $\mathbf{y}$ denotes a vector of size $M+3N$ with a transpose given by $\mathbf{y}^\top = (\mathbf{p}^\top, \mathbf{q}^\top)$; $\mathbf{p}$ and $\mathbf{q}$ are vectors of sizes M and $3N$,

$$\begin{aligned} F_1^*(\mathbf{p}) &= \max_{\mathbf{r}} \left\{ \mathbf{p}^\top \mathbf{r} - F_1(\mathbf{r}) \right\} \\ &= \frac{1}{2\lambda} \|\mathbf{p}\|_2^2 + \mathbf{g}_m^\top W_c^\top \mathbf{p}, \end{aligned} \quad (\text{A.17})$$

$$\begin{aligned} F_2^*(\mathbf{q}) &= \max_{\mathbf{z}} \left\{ \mathbf{q}^\top \mathbf{z} - F_2(\mathbf{z}) \right\} \\ &= \nu t_1 \|(\mathbf{q}|_{\text{MAG}})\|_\infty; \end{aligned} \quad (\text{A.18})$$

$\|\cdot\|_\infty$ denotes the largest entry of the vector; $\mathbf{q}|_{\text{MAG}}$ depicts a vector of size N in which entry i is given by $(\mathbf{q}|_{\text{MAG}})_i = \sqrt{q_i^2 + q_{i+N}^2 + q_{i+2N}^2}$, and q_i indicates entry i of vector $\mathbf{q}$.

Substitution of Eqs. (A.15) and (A.16) into Eq. (A.5) and noticing $K^\top = (\mathcal{H}^\top W_c^\top, \nu \nabla^\top)$ yields the dual maximization problem:

$$(\mathbf{p}^*, \mathbf{q}^*) = \underset{\mathbf{p}, \mathbf{q}}{\operatorname{argmax}} \left\{ -\frac{1}{2\lambda} \|\mathbf{p}\|_2^2 - \mathbf{g}_m^\top W_c^\top \mathbf{p} - \nu t_1 \|(|\mathbf{q}|_{\text{MAG}})\|_\infty - \delta_P(-\mathcal{H}^\top W_c^\top \mathbf{p} - \nu \nabla^\top \mathbf{q}) \right\}. \quad (\text{A.19})$$

19)

The CP algorithm can thus be tailored to solve simultaneously the primal-dual problems in Eqs. (A.14) (or, equivalently, Eq. (A.1)) and (A.19). Observation of the algorithm-pseudo codes in Table 2 indicates that the key steps are the calculations of the proximal mappings in Eqs. (A.7) and (A.8).

The proximal mapping of F_1^ :* Substitution of Eq. (A.17) into Eq. (A.7) yields

$$\operatorname{prox}_{\Sigma_1}[F_1^*](\mathbf{p}) = \underset{\mathbf{p}'}{\operatorname{argmin}} \left\{ \frac{1}{2\lambda} \|\mathbf{p}'\|_2^2 + \mathbf{g}_m^\top W_c^\top \mathbf{p}' + \frac{1}{2} (\mathbf{p} - \mathbf{p}')^\top \left(\frac{\mathbf{p} - \mathbf{p}'}{\Sigma_1} \right) \right\}, \quad (\text{A.20})$$

where vector Σ_1 of size M is given by

$$\Sigma_1 = \mathbf{1}_D / (|W_c \mathcal{H}| \mathbf{1}_I),$$

and vectors $\mathbf{1}_D$ and $\mathbf{1}_I$ of sizes M and N with all components set to 1. The solution of Eq. (A.20) can be obtained by setting its first order derivative with respect to $\mathbf{p}'$ to zero, i.e.,

$$\left[\frac{1}{\lambda} \mathbf{p}' + W_c \mathbf{g}_m - \left(\frac{\mathbf{p} - \mathbf{p}'}{\Sigma_1} \right) \right] = 0. \quad (\text{A.21})$$

Therefore, using the solution of Eq. (A.21) in Eq. (A.20), we obtain

$$\operatorname{prox}_{\Sigma_1}[F_1^*](\mathbf{p}) = (\mathbf{p} - \sum_1 W_c \mathbf{g}_m) / (\mathbf{1}_D + \sum_1 / \lambda). \quad (\text{A.22})$$

The proximal mapping of F_2^ :* Using Eqs. (A.13) and (A.18) and also Moreau's decomposition [52], we obtain

$$\begin{aligned}
\text{prox}_{\Sigma_2}[F_2^*](\mathbf{q}) &= \underset{\mathbf{q}'}{\text{argmin}} \left\{ \nu t_1 \|(\mathbf{q}'|_{\text{MAG}})\|_{\infty} + \frac{1}{2}(\mathbf{q} - \mathbf{q}')^{\top} \left(\frac{\mathbf{q} - \mathbf{q}'}{\Sigma_2} \right) \right\} \\
&= \mathbf{q} - \Sigma_2 \text{prox}_{\frac{\mathbf{1}_V}{\Sigma_2}}[F_2]\left(\frac{\mathbf{q}}{\Sigma_2}\right) \\
&= \mathbf{q} - \Sigma_2 \underset{\mathbf{q}'}{\text{argmin}} \left\{ \delta_{\text{Diamond}(\nu t_1)}(|\mathbf{q}'|_{\text{MAG}}) + \frac{1}{2}(\mathbf{q}/\Sigma_2 - \mathbf{q}')^{\top} \left(\frac{\mathbf{q}/\Sigma_2 - \mathbf{q}'}{\mathbf{1}_V/\Sigma_2} \right) \right\}, \quad (\text{A.23})
\end{aligned}$$

where vector $\mathbf{1}_V$ of size $3N$ with entries set to 1, and vector Σ_2 of size $3N$ is given by

$$\Sigma_2 = \mathbf{1}_V / (\nu |\nabla| \mathbf{1}_I). \quad (\text{A.24})$$

When ∇_x , ∇_y , and ∇_z represent two-point differences along x -, y -, and z -axis, respectively, it can be shown that $|\nabla| \mathbf{1}_I = 2\mathbf{1}_V$. Using this result in Eq. (A.24), we obtain

$$\Sigma_2 = \mathbf{I}_V / 2\nu. \quad (\text{A.25})$$

Following the strategy in Appendix C of Ref. [43], we rewrite $\mathbf{q} = \hat{\mathbf{q}} |\mathbf{q}|_{\text{MAG}}$ and $\mathbf{q}' = \hat{\mathbf{q}}' |\mathbf{q}'|_{\text{MAG}}$, where $\hat{\mathbf{q}}$ and $\hat{\mathbf{q}}'$ of size $3N$ denote the directions of $\mathbf{q}$ and $\mathbf{q}'$. It can be observed that, because the indicator function in Eq. (A.23) is independent of $\hat{\mathbf{q}}'$ and because each entry of vector Σ_2 is identical, the second term in Eq. (A.23) is minimized for a given $|\mathbf{q}'|_{\text{MAG}}$ as $\hat{\mathbf{q}}'$ and $\hat{\mathbf{q}}$ are pointing to the same direction. Therefore, we can re-express Eq. (A.23) as

$$\begin{aligned}
\text{prox}_{\Sigma_2}[F_2^*](\mathbf{q}) &= \mathbf{q} - \Sigma_2 \underset{|\mathbf{q}'|_{\text{MAG}}}{\text{argmin}} \left\{ \delta_{\text{Diamond}(\nu t_1)}(|\mathbf{q}'|_{\text{MAG}}) + \frac{\|(2\nu|\mathbf{q}|_{\text{MAG}} - |\mathbf{q}'|_{\text{MAG}})\|_2^2}{4\nu} \right\} \\
&= \mathbf{q} - \Sigma_2 \frac{\mathbf{q}}{|\mathbf{q}|_{\text{MAG}}} \mathbf{s}, \\
&= \mathbf{q} (\mathbf{1}_V - \Sigma_2 \mathbf{s} / |\mathbf{q}|_{\text{MAG}}), \quad (\text{A.26})
\end{aligned}$$

where vector $\mathbf{s}$ of size N is given by

$$\begin{aligned}
\mathbf{s} &= \underset{|\mathbf{q}'|_{\text{MAG}}}{\text{argmin}} \left\{ \delta_{\text{Diamond}(\nu t_1)}(|\mathbf{q}'|_{\text{MAG}}) + \frac{\|(2\nu|\mathbf{q}|_{\text{MAG}} - |\mathbf{q}'|_{\text{MAG}})\|_2^2}{4\nu} \right\} \\
&= \text{ProjectOnto}_{\ell_1 \text{Ball}_{\nu t_1}}(2\nu|\mathbf{q}|_{\text{MAG}}), \quad (\text{A.27})
\end{aligned}$$

and operator $\text{ProjectOnto}_{\ell_1 \text{Ball}_{\nu t_1}}$ projects vector $2\nu|\mathbf{q}|_{\text{MAG}}$ of size N onto the ℓ_1 -ball of scale νt_1 [39, 48].

The proximal mapping of G : Substituting Eq. (A.11) into Eq. (A.8) and noticing $\mathbf{x} = \mathbf{f}$, we obtain

$$\begin{aligned} \text{prox}_{\mathcal{T}}[G](\mathbf{f}) = \underset{\mathbf{f}'}{\text{argmin}} \left\{ \delta_P(\mathbf{f}') + \frac{1}{2}(\mathbf{f} - \mathbf{f}')^\top \left(\frac{\mathbf{f} - \mathbf{f}'}{\mathcal{T}} \right) \right\} \\ = \text{pos}(\mathbf{f}), \end{aligned} \quad (\text{A.28})$$

where operator $\text{pos}(\mathbf{f}) = 0$ and f_j for $f_j = 0$ and $f_j > 0$, respectively, enforcing the non-negativity constraint, vector $\mathcal{T}$ of size N is given by

$$\mathcal{T} = \mathbf{1}_I / (|\mathcal{H}^\top W_c^\top| \mathbf{1}_D + \nu |\nabla^\top| \mathbf{1}_V). \quad (\text{A.29})$$

The derivation of the last step in Eq. (A.28) can be found in Sec. 3.1.1 of Ref. [42] by replacing $\boldsymbol{\tau}$ with $\mathcal{T}$.

Using $\text{prox}_{\sum_1}[F_1^*](\mathbf{p})$ and $\text{prox}_{\sum_2}[F_2^*](\mathbf{q})$ in Eqs. (A.22) and (A.26) to replace $\text{prox}_{\Sigma}[F^*](\mathbf{y})$ in Line 6, and $\text{prox}_{\mathcal{T}}[G](\mathbf{f})$ in Eq. (A.28) to substitute $\text{prox}_{\mathcal{T}}[G](\mathbf{x})$ in Line 7, in Table 2, we obtain the pseudo codes for the CP algorithm tailored specifically to solve the program in Eq. (A.2), or equivalently, the program in Eq. (1).

Figure 1.

The C-arm CBCT system (left) and scanning configurations (right) considered in the work. The dashed curve depicts a circular source trajectory. Coordinate system $\{x, y, z\}$ is fixed on the imaged subject, and its z -axis coincides with the rotation axis, whereas coordinate system $\{u, v\}$ is attached onto the detector with u - and v -axis are parallel and perpendicular to the circular trajectory plane, respectively, and with its origin coinciding with the perpendicular intersection of an X-ray with the detector plane. Due to the C-arm mechanical instability, the actual scanning trajectory (solid curve) can deviate, e.g., from the circular trajectory (dashed curve) prescribed.

Figure 2.

Metrics $\bar{D}_W(\mathbf{f}_n)$ and $\overline{\text{cPD}}(\mathbf{f}_n)$ as functions of iteration number n , obtained with $\nu = \nu_{\mathcal{H}}$ and varying λ (row 1), and with $\lambda = 0.01$ and varying ν (row 2).

Figure 3.

Truth image f_{true} , convergent reconstruction f_{conv} , and their absolute difference $|f_{\text{true}} - f_{\text{conv}}|$.

Display window: $[0.15, 0.35]$ for the truth and reconstructed images, whereas display window: $[0, 10^{-12}]$ for their absolute difference.

Figure 4.

Metrics $\bar{D}_W(\mathbf{f}_n)$, $\overline{TV}(\mathbf{f}_n)$, $\overline{cPD}(\mathbf{f}_n)$, and $\bar{\Delta}(\mathbf{f}_n)$ as functions of iteration number n , obtained with algorithm parameters $\lambda = 0.01$ and $\nu = \nu_{\mathcal{H}}$.

Figure 5.

Optimization-based reconstructions $\mathbf{f}_{\text{conv}}$ from the swine data with aortogram injection within a coronal slice specified by $x = 0.23$ mm obtained with, respectively, parameter $c = 0.000, 0.025, 0.050, 0.075, 0.100, 0.200, 0.300$, and 1.000 , along with the reference reconstruction. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 6.

The largest singular value of matrix $W_c \mathcal{H}$ as a function of parameter c for the scanning geometry considered in the study on the swine subject with aortogram injection.

Figure 7.

Optimization-based reconstructions $\mathbf{f}_{\text{conv}}$ from the swine data with aortogram injection within a coronal slice specified by $x = 0.23$ mm obtained with, respectively, parameter $t_1 = 4000, 8000, 12000, 20000$, and 40000 , along with the reference reconstruction. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 8.

Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the swine data with aortogram injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -16.47$ mm, $y = 29.93$ mm, and $z = 83.29$ mm, respectively. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 9.

Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the swine data with aortogram injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = 69.37$ mm, $y = -53.59$ mm, and $z = 48.49$ mm, respectively. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 10.

Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the swine data with hepatic injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -55.91$ mm, $y = -62.87$ mm, and $z = -0.23$ mm, respectively. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 11.

Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the swine data with hepatic injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -14.15$ mm, $y = -2.55$ mm, and $z = 55.45$ mm, respectively. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 12. Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the swine data with aortogram injection within transverse slices specified by (a) $z = -110.20$ mm and (b) $z = 108.81$ mm. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 13. Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the swine data with hepatic injection within transverse slices specified by (a) $z = -110.20$ mm and (b) $z = 108.81$ mm. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 14.

Optimization-based reconstructions $\mathbf{f}_n$ from the swine data with aortogram injection within coronal (rows 1 and 3) and transverse (rows 2 and 4) slices specified by $x = -16.47$ mm and $z = 48.49$ mm at iterations $n=20, 50, 100, 200$, and 300 , respectively, along with $\mathbf{f}_{\text{conv}}$ (i.e., at $n = 1000$.) Different gray-scale windows are used for revealing reconstruction details at different iterations.

Figure 15.

Optimization-based reconstruction f_{conv} (row 1) and reference reconstruction (row 2) from the patient data within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -43.26$ mm, $y = -2.49$ mm, and $z = -86.30$ mm, respectively. Display window: $[0.13, 0.24]$ cm^{-1} .

Figure 16.

Optimization-based reconstruction f_{conv} (row 1) and reference reconstruction (row 2) from the patient data within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -46.43$ mm, $y = 15.63$ mm, and $z = 26.95$ mm, respectively. Display window: $[0.13, 0.24]$ cm^{-1} .

Figure 17.

Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and reference reconstruction (row 2) from the patient data within transverse slices specified by (a) $z = -111.21$ mm and (b) $z = 106.23$ mm. Display window: $[0.13, 0.24] \text{ cm}^{-1}$.

Figure 18.

Optimization-based reconstructions $\mathbf{f}_n$ from the patient data within coronal (rows 1 and 3) and transverse (rows 2 and 4) slices specified by $x = -2.49$ mm and $z = -2.49$ mm, at iterations $n=20, 50, 100, 200$, and 300 , respectively, along with $\mathbf{f}_{\text{conv}}$, the convergent reconstruction (i.e., at $n = 1000$.) Different gray-scale windows are used for revealing reconstruction details at different iterations.

Figure 19.

Optimization-based reconstruction $\mathbf{f}_{\text{conv}}$ (row 1) and FDK reconstruction (row 2) from the sparse-view swine data with aortogram injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -16.47$ mm, $y = 29.93$ mm, and $z = 83.29$ mm. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 20.

Optimization-based reconstruction f_{conv} (row 1) and FDK reconstruction (row 2) from the limited-angular-range swine data with aortogram injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -16.47$ mm, $y = 29.93$ mm, and $z = 83.29$ mm. Display windows are $[0.11, 0.23] \text{ cm}^{-1}$ for optimization-based reconstruction and $[0.06, 0.18] \text{ cm}^{-1}$ for FDK reconstruction, respectively.

Figure 21.

Optimization-based reconstruction f_{conv} (row 1) and FDK reconstruction (row 2) from the severely-truncated swine data with aortogram injection within (a) coronal, (b) sagittal, and (c) transverse slices specified by $x = -16.47$ mm, $y = 29.93$ mm, and $z = 83.29$ mm. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 22.

Optimization-based reconstructions $\mathbf{f}_{\text{conv}}$ from the swine data (a) with, and (b) without, a longitudinal extrapolation, along with the reference reconstruction (c), within slices specified by $x = -16.47$ mm (row 1) and $z = 108.81$ mm (row 2). Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 23. Optimization-based reconstructions $\mathbf{f}_{\text{conv}}$ from the swine data (a) with, and (b) without, a transverse extrapolation, along with the reference reconstruction (c), within slices specified by $x = -16.47$ mm (row 1) and $z = 83.29$ mm (row 2). Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Figure 24.

Optimization-based reconstructions $\mathbf{f}_{\text{conv}}$ from the swine data (a) with, and (b) without, an interpolation, along with the reference reconstruction (c), within slices specified by $x = -16.47$ mm (row 1) and $z = 83.29$ mm (row 2). Arrows indicate the residual artifacts. Display window: $[0.11, 0.23] \text{ cm}^{-1}$.

Table 1

Pseudo codes of the preconditioned CP algorithm for solving Eq. (1)

	INPUT: $\mathbf{g}_m$; W_c and $\mathcal{H}$; c and t_1 ; and λ
1:	$\mathbf{v} \leftarrow \mathbf{v}_{\mathcal{H}}, \Sigma_1 \leftarrow \mathbf{1}_D / (W_c \mathcal{H} \mathbf{1}_D), \Sigma_2 \leftarrow \mathbf{1}_V / (\mathbf{v} \nabla \mathbf{1}_D), \mathcal{T} \leftarrow \mathbf{1}_I / (\mathcal{H}^\top W_c^\top \mathbf{1}_D + \nu \nabla^\top \mathbf{1}_V)$
2:	$\theta \leftarrow 1, n \leftarrow 0$
3:	INITIALIZE $\mathbf{f}_0, \mathbf{p}_0$, and $\mathbf{q}_0$ to zero
4:	$\tilde{\mathbf{f}}_0 \leftarrow \mathbf{f}_0$
5:	repeat
6:	$\tilde{\mathbf{p}} \leftarrow \mathbf{p}_n + \Sigma_1 W_c \mathcal{H} \tilde{\mathbf{f}}_n$ $\mathbf{p}_{n+1} \leftarrow (\tilde{\mathbf{p}} - \Sigma_1 W_c \mathbf{g}_m) / (\mathbf{1}_D + \Sigma_1 / \lambda)$ $\tilde{\mathbf{q}} \leftarrow \mathbf{q}_n + \Sigma_2 \mathbf{v} \tilde{\mathbf{f}}_n$ $\mathbf{q}_{n+1} \leftarrow \tilde{\mathbf{q}} (\mathbf{1}_V - \Sigma_2 \mathbf{s} / \tilde{\mathbf{q}} _{\text{MAG}})$
7:	$\mathbf{f}_{n+1} \leftarrow \text{pos} [\mathbf{f}_n - \mathcal{T} (\mathcal{H}^\top W_c^\top \mathbf{p}_{n+1} + \nu \nabla^\top \mathbf{q}_{n+1})]$
8:	$\tilde{\mathbf{f}}_{n+1} \leftarrow \mathbf{f}_{n+1} + \theta (\mathbf{f}_{n+1} - \mathbf{f}_n)$
9:	$n \leftarrow n + 1$
10:	until the practical convergence conditions are satisfied
11:	OUTPUT: image $\mathbf{f}_{\text{conv}} \leftarrow \mathbf{f}_n$

Table 2

Pseudo codes of the preconditioned CP algorithm [41, 47]

1:	$\Sigma \leftarrow \mathbf{1}_{Y'/(K\mathbf{1}_X)}; \mathcal{T} \leftarrow \mathbf{1}_{X'/(K\mathbf{1}_Y)}$
2:	$\theta \leftarrow 1; n \leftarrow 0$
3:	INITIALIZE $\mathbf{x}_0$ and $\mathbf{y}_0$ to zero
4:	$\mathbf{x}_0 \leftarrow \mathbf{x}_0$
5:	repeat
6:	$\mathbf{y}_{n+1} \leftarrow \text{prox}_{\Sigma}[F^*](\mathbf{y}_n + \Sigma K \mathbf{x}_n)$
7:	$\mathbf{x}_{n+1} \leftarrow \text{prox}_{\mathcal{T}}[G](\mathbf{x}_n - \mathcal{T} K^T \mathbf{y}_{n+1})$
8:	$\mathbf{x}_{n+1} \leftarrow \mathbf{x}_{n+1} + \theta(\mathbf{x}_{n+1} - \mathbf{x}_n)$
9:	$n \leftarrow n + 1$
10:	until the practical convergence conditions are satisfied
11:	OUTPUT: image $\mathbf{f}_{\text{conv}} \leftarrow \mathbf{f}_{n+1}$