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Effects of Dielectric Substrates and Ground Planes on Resonance Frequency of Archimedean Spirals

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Abstract

Superconducting self-resonant spiral structures are of current interest for applications both in metamaterials and as probe coils for nuclear magnetic resonance (NMR) spectroscopy for high-sensitivity chemical analysis. Accurate spiral models are available in the literature for behavior of a spiral below and up to self-resonance. However, knowledge of the higher modes is also important. We present the relationships between the spiral parameters and the multiple mode frequencies of single sided spirals on dielectric substrates as modeled by method of moments simulation. In the absence of a ground plane, we find that the mode frequency has a linear though not necessarily harmonic dependence on the mode number. The effect of a thick substrate can be approximated by an effective dielectric constant. But when the thickness is less than 20% of the spiral trace width ($r_{\text{outer}} - r_{\text{inner}}$) this approximation is no longer accurate. We have developed a simple empirical formula to predict the higher modes.

Keywords

High-temperature superconducting (HTS) spiral resonators; nuclear magnetic resonance (NMR)

I. Introduction

Planar spirals are used in a variety of RF electronic applications such as filters [1], planar inductors [2], wireless power transfer [3], metamaterials [4], and NMR probe coils [5].

When used as HTS NMR probe coils specifically, and in some other applications, it is

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typical for the spiral to be mounted on a dielectric substrate [6]. In many of the cases the effect of the substrate is modeled through the use of an effective dielectric constant [7], [8]. This approach uses a substitute value to model the effect of a dielectric instead of the actual dielectric constant. This value represents the uniform dielectric medium in which the device would exhibit the same behavior. However, the concept of an effective dielectric constant is sometimes too limited for devices that are utilized above their self-resonant frequency since this approach is based on the assumption that all modes are affected equally which in many cases is not accurate. Archimedean spirals are one class of device where an effective dielectric constant can be insufficient to fully model the device behavior. The purpose of this study was to investigate the effect of the substrate thickness and proximity to ground plane on the resonances of various Archimedean spirals.

II. Background

In an earlier study, the effect of geometric parameters on the resonance frequencies of spiral resonators was investigated by simulating a large collection of freestanding Archimedean spirals and analyzing the results [9]. In [9] and in this paper, several parameters are used in the description of these spirals and are defined in Table I. The independent parameters defining an Archimedean spiral are annotated in Fig. 1. In [9] it was shown that the resonance mode frequencies of the spirals are a linear function of the mode number. Further it was shown that the y-intercept of that function does not in general fall on the origin. In other words, the spiral resonance behavior is not harmonic. Rather, the resonance frequencies are all shifted downward by some amount. It was determined that the strongest determining factor in this shift is the ratio of the inner and outer radii. The quantity D was introduced to measure the deviation of the spectrum from a harmonic series. D is calculated by fitting a line to the mode frequencies. It is then given by the y-intercept, normalized by the slope. If the value of D is zero it indicates that the spectrum is harmonic. The observed values for D in freestanding Archimedean spirals range from -0.6 to -0.2 . In [9] an empirical expression (1) was given to express D in terms of the spiral's parameters.

$$D = - \left((0.29 + 0.043F) \left(\frac{r_i}{r_o} \right) + 0.22 \right) \left((F - 0.5) \left(2.65 \left(\frac{P}{r_o} \right) + 0.03 \right) + 1 \right) \quad (1)$$

Further, D was used within the empirical formula (2), developed to calculate the fundamental and higher order resonance frequencies of freestanding spirals.

$$f_n (Hz) = \left(\frac{v}{2L} \right) \left(0.24N^{-0.46} + 0.95 \right) (n + D) \quad (2)$$

There have been many investigations into the prediction of spiral resonant frequencies. Some methods like that presented in [10] do not take the effect of the dielectric into account. Other methods that use circuit models address it indirectly in the calculation of capacitances [11]. In order to be able to predict the mode frequencies of most actual devices, the effect of the substrate also needs to be taken into account. For spirals on thick substrates, an effective

dielectric constant given by $(1+\epsilon_r)/2$ has been found to produce accurate results [7, 8]. However, the resonances of spirals on thin substrates are not consistently predicted with accuracy this way since the modes are not uniformly affected by the substrate. The higher modes are affected more strongly by a thin substrate than the lower modes.

In Fig. 2, we can observe how the modes of an Archimedean spiral are not equally affected by substrates of varying thickness. We begin with a freestanding spiral and as the substrate thickness is increased the resonance frequencies of the fundamental mode as well as the higher order modes decrease. This particular resonator has an inner radius of 5.5 mm and an outer radius of 15.5 mm. The conductor width and the gap between turns are both 0.5 mm and it has a total of 10 turns. All mode frequencies are normalized by the equivalent mode in the absence of any substrate. In this particular example the dielectric constant is 10 and the spiral dimensions are constant, only the thickness of the substrate is changed. The substrate thickness t is normalized by the spiral trace width T . In the context of this paper the term trace width refers to the combined width of the spiral turns, specifically the outer radius minus the inner radius. The frequencies of all modes converge to a lower limit of about 42% of the free-space frequency when the dielectric substrate thickness is increased relative to the trace width. This result corresponds well to the approximation $(1+\epsilon_r)/2$ mentioned earlier which predicts that the resonance frequencies of a spiral on a thick substrate should be roughly 42% of the free space value for this dielectric. The ratio of the substrate thickness to the trace width (t/T) is the primary factor in determining how the mode frequencies will shift relative to each other. The rate at which the frequencies converge toward the approximation is lower for lower order modes and higher for higher order modes. The cause for this unequal rate of shifting of the modes is the varying amount of fringing electric field in the substrate. While for the lower order modes, the E-field fringes away from the spiral, for the higher order modes, the E-field is confined very close to the spiral.

III. Observations

A similar approach to that used in [9] for freestanding spirals, was applied here to model the effect of thin substrates on planar Archimedean spirals. The resonances of a collection of spirals on substrates of varying thicknesses and dielectric constants were simulated. Then an empirical formula to calculate the resonance frequencies of Archimedean spirals on dielectric substrates was obtained by adding correction factors to (2).

The effect of varying the substrate thickness on D for four different spirals is shown in Fig. 3. In this set of spirals represented in Fig. 3, all substrates have a dielectric constant of 10 which approximately matches the value for sapphire substrates. The dimensions of each spiral are given in Table II. The geometry of these spirals is quite similar. They all possess the same pitch, filling factor, and outer radius. The filling factor is 50%, indicating that the turn spacing and conductor width are equal. The inner radius and number of turns varies between them. The spirals with fewer turns can all be generated by removing some number of turns from the inside of spiral a25. As the substrate thickness was varied a non-monotonic effect on D became evident. For every spiral the value of D increased with the substrate thickness until a certain point was reached, after which a monotonic decrease in D was observed. Only small variations in the height of the peak and slight variations in its location

were observed for this set of spirals. Changing other parameters of the spirals changed the width and height of the peak but had little or no effect on its location, but are not shown here. The results from the empirical formula that was developed as a part of this study are also included in the plot though the formula itself is not discussed until later in this section.

As can be observed in Fig. 3, the thickness of the dielectric can alter the spectrum of the resonator substantially. When the substrate is very thick all modes are affected equally as half of the total electric field of each mode is contained within the substrate, and D is nearly the same as a freestanding spiral. In between the two extreme cases of a free standing spiral and a thick substrate, some modes are affected more strongly by the substrate than others resulting in a significant rise in D . When the thickness is less than 20% of the trace width, the effect of the substrate on mode frequencies of a spiral is not as simple as calculating an effective dielectric constant and applying it to adjust the wave velocity.

The plot in Fig. 4 shows the frequencies of the first six modes of a spiral on substrates of several different thicknesses. There is a significant variation in the observed frequencies even though the spiral geometry and dielectric constant remain unchanged. Determining an effective dielectric constant only affects the slope of the mode spectrum fit line. Methods that only address the effective dielectric constant fall short because a significant change in the y-intercept of the best-fit line can also be observed.

To account for the effect of the dielectric substrate, three correction factors are introduced to (2). The first correction factor, d_1 adjusts D based on the substrate dielectric constant and thickness as well as the spiral dimensions. The second correction factor, c_1 is introduced to adjust the slope of the mode plot. It can be interpreted as an adjustment leading to an effective dielectric constant of the substrate. Lastly the factor c_2 accounts for the unequal effects on each mode. These three correction factors were all determined empirically based on the simulated data that were collected.

The expression used to calculate mode frequencies of freestanding spirals is given by (2). With the additional correction factors, the expression appears as below in (3).

$$f_n (Hz) = \left(\frac{v}{2L} \right) \left(\frac{c_1}{c_2} \right) \left(0.24N^{-0.46} + 0.95 \right) (n + D + d_1) \quad (3)$$

The three correction factors are defined by (4), (5), and (6)

$$d_1 = 0.000007 (0.5 \ln(\epsilon_r) - 0.15) \left(\frac{t}{T} \right) \left(\frac{t}{T} + 0.3 \right)^{-14.35} \quad (4)$$

$$c_1 = \left(1 - \sqrt{\frac{2}{(1+\epsilon_r)}} \right) e^{-177 \frac{t}{T}} + \sqrt{\frac{2}{(1+\epsilon_r)}} \quad (5)$$

$$c_2=0.09 \left[\frac{1600n^{-2} \left(e^{-\frac{45n^{-0.5}t}{T}} + e^{-\frac{44n^{-0.5}t}{T}} \right)}{-(n^{-0.5}t/T+1)^{-1}} \right] + n^{0.06} \quad (6)$$

IV. Accuracy

In order to test the accuracy of the empirical formula including the correction factors, we calculated the error in predicting the over 400 simulated mode frequencies generated as a part of this study. Over the collection of spirals that was simulated the empirical formula results in an average error of approximately 2.6%. This calculation was based on a dielectric constant of 10, however dielectric constants ranging from 1 to 20 were tested and showed similar error. Experimental values are not a part of this particular calculation. The majority of the error was seen in cases involving very thin substrates. As shown in Fig. 3, the value of D changes very rapidly as the substrate thickness increases from zero. Even though the function models this change very well, very small errors are amplified substantially because of the rapid change. For this reason substrates with thickness that was less than ~1% of the trace width were not modeled as well.

Examples from literature can be used to evaluate the effectiveness of the formula. In Table III, spirals SR1 and SR2 from [7] are evaluated using the method presented above. The spiral from [12] is also used to evaluate the effectiveness of this formula. The error associated with using this method is also presented in Table III. It was determined by comparing the resonance frequencies reported in the above mentioned references to the results of (3). The negative error values indicate cases where the prediction was lower than the experimental result reported. There were not enough examples from literature to determine which spiral characteristics caused predictions to be too high as opposed to too low with any level of certainty. The error associated with the first two modes in the examples from literature tended to be slightly higher than the error associated with other modes. It is suspected that this is a result of nonlinearities in the mode spectrum. While (3) has been designed to account for nonlinearities there are parameters that have not been taken into account. For example, anisotropy in the substrate could cause dielectric constants to vary from the assumed values.

V. Effect of Ground Planes

The location of a nearby ground plane can also have a substantial effect on the observed resonance frequencies of a planar Archimedean spiral as shown in Fig. 5. The resonant frequencies of the first four modes of an example spiral were obtained through simulations. The frequencies were normalized to the case where the ground plane is very far. As seen in Fig. 5, the effect of the ground plane can be up to a 30% increase in the fundamental resonance frequency, but much lower on the higher order modes. This effect can be explained using the method of images, which states that the effect of the ground plane can be represented by placing a mirror image spiral carrying the opposite current at twice the distance of the ground plane. This cancels any magnetic field perpendicular to the ground

plane on its surface. The mutual inductance between the two coils is larger for the fundamental mode, and smaller for the higher modes. The mutual inductance between the coils is given by $M = kL$. The level of coupling is strongly tied to the distance from the ground plane. The cancellation of flux results in a reduced overall inductance and a higher fundamental resonant frequency.

When the distance to the ground plane is equal to the outer radius of the spiral, the observed shift in frequency is only about 1%. The proximity of the ground plane may also account for some of the increased errors observed in Table III for the lowest mode since the distance between the spiral and ground plane in the measurement set up is not clearly known.

VI. Conclusion

The formula that was first presented in [9] and modified here is effective in modeling the resonance frequencies of planar Archimedean spirals on substrates. Error levels around or below 2% are maintained over a large range of substrate thicknesses. The lower limit for application of this method is not known at this time, however based on the simulations performed results should be within about 2% accuracy as long as the substrate thickness is above 1% of the trace width. Many practical applications including superconductive NMR detectors utilize spirals on dielectric substrates, and the formula will be helpful for calculating the spectra of these devices.

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Fig. 1. Illustration of a spiral with relevant independent parameters annotated. These parameters include the wire width w , the turn spacing s , the pitch P , the inner radius r_i and the outer radius r_o .

Fig. 2. The change in resonance frequencies as a function of the substrate thickness. The normalized modes of one spiral are plotted while the substrate thickness was varied. The simulated resonance frequencies are normalized to the corresponding mode frequency in free space.

Fig. 3.

The relationship between D and the relative thickness of the substrate for 4 different spirals. These spirals have the same pitch, wire width, and outer radius. The parameters that vary are the number of turns, inner radius, and thus the trace width.

Fig. 4.

The relationship between the mode frequencies and the relative thickness of the substrate. A significant variation in the mode frequencies is observed when the substrate thickness is varied. The two trend lines included display the difference in mode frequencies created by a very thin substrate and a thick substrate.

Fig. 5.

The resonance frequencies of a spiral at various distances from the ground plane. The values are normalized to the case where the ground plane is absent. In this particular case the fundamental mode can be shifted by as much as 30% by the presence of the ground plane.

TABLE I

Variable and Parameter definitions

Parameter	Definition
N	Number of turns
P	pitch
r_i	Inner radius
r_o	Outer radius
s	Spacing between turns
w	Width of the conductor
F	Filling factor
T	Trace width
t	Substrate thickness
n	Mode number
v	Velocity of waves along the spiral
L	Length of the spiral conductor

TABLE II

Coil Parameters

	Coil a10	Coil a15	Coil a20	Coil a25
Number of turns	10	15	20	25
Inner radius	9 mm	7 mm	5 mm	3 mm
Outer radius	13 mm	13 mm	13 mm	13 mm
Pitch	0.4 mm	0.4 mm	0.4 mm	0.4 mm

TABLE III

Results of empirical formula compared to literature examples

Mode	Maleeva SR1 (MHz)	SR1 Error	Maleeva SR2 (MHz)	SR2 Error	Kurter (MHz)	Kurter Error
1	80	-15%	128	12%	74	9%
2	180	-8%	299	4%	219	-1%
3	268	-3%	465	1%	355	-1%
4	353	-1%	635	-1%	498	-4%
5	437	-2%	801	-1%	636	-4%
6	522	-3%	971	-1%		