Published in final edited form as:

Appl Radiat Isot. 2016 March ; 109: 335–340. doi:10.1016/j.apradiso.2015.11.108.

Corrections for the Combined Effects of Decay and Dead Time in Live-Timed Counting of Short-Lived Radionuclides

R. Fitzgerald¹

National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, MD 20899 USA

Abstract

Studies and calibrations of short-lived radionuclides, for example ^{15}O , are of particular interest in nuclear medicine. Yet counting experiments on such species are vulnerable to an error due to the combined effect of decay and dead time. Separate decay corrections and dead-time corrections do not account for this issue. Usually counting data are decay-corrected to the start time of the count period, or else instead of correcting the count rate, the mid-time of the measurement is used as the reference time. Correction factors are derived for both those methods, considering both extending and non-extending dead time. Series approximations are derived here and the accuracy of those approximations are discussed.

1. Introduction

Studies and calibrations of short-lived radionuclides are of particular interest in nuclear medicine. For instance, positron emission tomography (PET) imaging by ^{15}O , with a half-life of 122.46 s (Chisté and Bé, 2015), is the “gold standard” for assessment of blood flow in studies of tumor vascularity (Aboagye, et al., 2012).

Yet, counting measurements on short-lived nuclear states (either excited states or radionuclides) using live-timed counting systems are vulnerable to an error due to the combined effect of decay and dead time. The effect causes a bias in the apparent count rate due to the changing live-time fraction throughout the measurement. Since the count rate is decreasing during the measurement, the fractional live-time is increasing and the latter parts of the measurement contribute relatively more to the recorded count rate. If decay corrections do not account for this, then there remains a bias in the count rate at the reference time, which could be important when producing calibration sources or standards for short-lived radionuclides.

Counting experiments are subject to dead time, τ , which is the period of time following an event during which any part of the apparatus is unable to count another event. Moreover, a dead time is *extending* if events that are lost during that time cause further dead time, or *non-extending* if lost events do not cause dead time. A system could have *mixed* dead time, which is not considered further here, beyond noting that by designing a counting system with an imposed, extending dead-time that is longer than any intrinsic dead time of the

¹Corresponding author. Tel.: 1 301 975 5579; ryan.fitzgerald@nist.gov.

system, one can approximate the pure extending dead-time case. The uncertainties from various dead-time, pileup, and live-time corrections are the subject of a recent review article (Pommé et al, 2015), which indeed cites the final result of this paper.

Axton and Ryves (1963) derived an approximate correction formula with leading-order expansion for the count rate at the start of the counting period for non-extending dead times. Müller (1981) derived expressions for non-extending and extending systems without live-timing. For modern live-timed systems, which may have imposed dead times, it is desirable to use the live-timed count rate determined by the instrument, rather than calculating the dead-time correction.

Derivations are presented for the actual and apparent live count rate for a decaying source over a finite measurement duration. Furthermore, correction factors are defined for both the decay-to-start method and midpoint approximation. These expressions are meant to enable a simple implementation of the corrections and aid in experimental design, thereby limiting errors caused by this combination dead-time and decay effect. Tabulated results are presented for various conditions and as well as simple approximations to the correction formulas.

Finally, the equivalence between the correction factors derived in this work and the approximations given in previous reports is demonstrated for certain cases.

2. Derivation

From the start, let us note that the background count rate, B , will be ignored in this work because it would hardly affect the final correction factors f_m and f_d that conclude this paper. This fact was verified by deriving the results with B included and noting the equivalence to the present results for extending dead time. For the non-extending case, the change in the correction factor due to B for initial net count rate due to the radionuclide, $\rho(0)$, is of the order $B/\rho(0)$. As the corrections derived here are needed only at high count rates, this small correction on the correction is not significant.

Consider a counting experiment with initial true, count rate at time $t = 0$ of $\rho(0)$. If the measurement is live-timed with an extending or non-extending dead time, τ , then the observed average live-timed count rate, $\bar{R}$, is the ratio of the total live counts, N_L , to the total live time, T_L . That is,

$$\bar{R} = \frac{N_L}{T_L}. \quad (1)$$

When $\rho(t)$ is decreasing exponentially with decay constant, λ , as,

$$\rho(t) = \rho(0)e^{-\lambda t}, \quad (2)$$

then one of two methods is typically used to report the experimental value for the instantaneous true count rate from the observed average rate. For a total count time, T_{tot} , that is much lower than λ^{-1} , the decay is nearly linear during the measurement. In that case, the midpoint method is sometimes employed. That is, $\bar{R}$ is assumed to approximate the true rate at the mid-time,

$$\rho\left(\frac{1}{2}T_{\text{tot}}\right) \approx \bar{R}. \quad (3)$$

A more accurate method is to average Equation (2) over the domain from $t = 0$ to $t = T_{\text{tot}}$ and then solve for $\rho(0)$ in terms of the average rate, $\bar{\rho}$, as

$$\rho(0) = \bar{\rho} \frac{\lambda T_{\text{tot}}}{1 - e^{-\lambda T_{\text{tot}}}}. \quad (4)$$

Ignoring the decay-dead-time effect in this case is equivalent to the assumption that,

$$\rho(0) \approx \bar{R} \frac{\lambda T_{\text{tot}}}{1 - e^{-\lambda T_{\text{tot}}}}. \quad (5)$$

We next calculate $\bar{R}$ from $\rho(t)$ and the resulting correction factors for the two assumptions.

For a counting system with a fixed, extending or non-extending dead time, τ , the instantaneous recorded count rate, $R'(t)$, is given by the well-known formulae (ICRU, 1994):

$$\begin{aligned} &\textbf{Extending} \\ R'(t) &= \rho(t) e^{-\rho(t)\tau} \end{aligned} \quad (6a)$$

$$\begin{aligned} &\textbf{Non - Extending} \\ R'(t) &= \frac{\rho(t)}{1 + \rho(t)\tau} \end{aligned} \quad (6b)$$

For measurement duration T_{tot} , the recorded live counts N_L and live time T_L can be calculated as:

$$N_L = \int_0^{T_{\text{tot}}} R'(t) dt \quad (7)$$

$$\begin{aligned} &\textbf{Extending} \\ T_L &= \int_0^{T_{\text{tot}}} e^{-\rho(t)\tau} dt \end{aligned} \quad (8a)$$

Non – Extending

$$T_L = \int_0^{T_{\text{tot}}} \frac{1}{1+\rho(t)\tau} dt \quad (8b)$$

The calculated value of $\bar{R}$, called $\bar{R}_c$, can now be written explicitly by combining equations 2, 6, 7 and 8 as:

Extending

$$\bar{R}_c = \frac{\int_0^{T_{\text{tot}}} \rho(0)e^{-\lambda t} e^{-(\rho(0)e^{-\lambda t})\tau} dt}{\int_0^{T_{\text{tot}}} e^{-(\rho(0)e^{-\lambda t})\tau} dt} \quad (9a)$$

Non – Extending

$$\bar{R}_c = \frac{\int_0^{T_{\text{tot}}} \frac{\rho(0)e^{-\lambda t}}{1+\rho(0)e^{-\lambda t}\tau} dt}{\int_0^{T_{\text{tot}}} \frac{1}{1+\rho(0)e^{-\lambda t}\tau} dt} \quad (9b)$$

Let us define $x = \lambda T_{\text{tot}}$ and $k = \rho(0)\tau$ as convenient parameters. The integrals in Equations 9 can be solved to be:

Extending

$$\bar{R}_c = \frac{\exp(ke^{-x}) - e^{-k}}{\tau[\text{Ei}(-k) - \text{Ei}(-ke^{-x})]} \quad (10a)$$

Non – Extending

$$\bar{R}_c = \frac{\ln(1+k) - \ln(1+ke^{-x})}{\tau[\ln(e^x+k) - \ln(1+k)]} \quad (10b)$$

Here, $\text{Ei}(x)$ is the exponential integral of x , which can be evaluated numerically.

Correction factors f_d and f_m are now defined as the ratio of the $\bar{R}$ values assumed by the decay-to-start time approximation in Equation (5) and mid-time approximation in Equation (3), respectively to the true $\bar{R}_c$,

$$f_d = \frac{\bar{p}}{\bar{R}_c} = \frac{\rho(0) \frac{1-e^{-\lambda T_{\text{tot}}}}{\lambda T_{\text{tot}}}}{\bar{R}_c}, \quad (11)$$

and,

$$f_m = \frac{\rho(\frac{1}{2}T_{\text{tot}})}{\bar{R}_c}. \quad (12)$$

In this way, the instantaneous rates determined by the usual simple methods can be corrected to give the true rates at $t = 0$ and $t = \frac{1}{2}T_{\text{tot}}$ as,

$$\rho(0) = f_d \bar{R} \frac{\lambda T_{\text{tot}}}{1 - e^{-\lambda T_{\text{tot}}}}, \quad (13)$$

and

$$\rho\left(\frac{1}{2}T_{\text{tot}}\right) = f_m \bar{R}. \quad (14)$$

Exact forms of f_d and f_m are given by solving Equation (13) and Equation (14):

$$\begin{aligned} &\textbf{Extending} \\ f_d &= \frac{k[1 - e^{-x}][\text{Ei}(-k) - \text{Ei}(-e^{-x}k)]}{x[e^{-ke^{-x}} - e^{-k}]} \end{aligned} \quad (15a)$$

$$\begin{aligned} &\textbf{Non-Extending} \\ f_d &= \frac{k[\ln(e^x + k) - \ln(1 + k)](1 - e^{-x})}{x[\ln(1 + k) - \ln(1 + ke^{-x})]} \end{aligned} \quad (15b)$$

$$\begin{aligned} &\textbf{Extending} \\ f_m &= \frac{ke^{-\frac{x}{2}}[\text{Ei}(-k) - \text{Ei}(-e^{-x}k)]}{[e^{-ke^{-x}} - e^{-k}]} \end{aligned} \quad (16a)$$

$$\begin{aligned} &\textbf{Non-Extending} \\ f_m &= \frac{k[\ln(e^x + k) - \ln(1 + k)]e^{-x/2}}{\ln(1 + k) - \ln(1 + ke^{-x})} \end{aligned} \quad (16b)$$

Series expansions, calculated to 4th power of x , are given by the following:

$$\textbf{Extending } f_d \approx 1 + \frac{kx^2}{12} - \frac{kx^3}{24} - \frac{k^3 - 2k^2 - 9k}{720}x^4 \quad (17a)$$

$$\textbf{Extending } f_m \approx 1 + \frac{1}{24}(2k - 1)x^2 - \frac{kx^3}{24} - \frac{8k^3 - 16k^2 - 52k - 7}{5760}x^4 \quad (18a)$$

$$\text{Non-Extending } f_d \approx 1 + \frac{kx^2}{12(1+k)} - \frac{kx^3}{24(1+k)^2} + \frac{2k^3 - 8k^2 + 9k}{720(1+k)^3} x^4 \quad (17b)$$

$$\text{Non-Extending } f_m \approx 1 + \frac{(k+1)x^2}{24(1+k)} - \frac{kx^3}{24(1+k)^2} + \frac{3k^3 - 83k^2 + 73k + 7}{5760(1+k)^3} x^4 \quad (18b)$$

Limiting the expansions to second order yields results that are accurate to about $5 \cdot 10^{-3}$ for most cases of interest, as illustrated in Table 1, and should be particularly useful for experimental design, due to their simplicity.

3. Examples

For example, a scintillation detector system may have a fixed dead time of $\tau = 5.0 \cdot 10^{-5}$ s. For a decaying source with a half-life of $T_{1/2} = 10$ s, an initial true count rate of $\rho(0) = 1 \cdot 10^5$ s⁻¹, and a real count period of $T_{\text{tot}} = 5.0$ s, we would have $x = 0.347$ and $k = 5.0$. Using Equations (15) and (16), then $f_d = 1.041$ and $f_m = 1.036$ for the extending case, and $f_d = 1.0081$ and $f_m = 1.0031$ for the non-extending case.

Table 1 lists some examples of f_d and f_m calculated using Equations (15) and (16) as well as the error from using either the 4th-order and only the 2nd-order expansions of Equations (17) and (18). Figure 1 and Figure 2 shows 3D plots created using the exact solutions to f_d and f_m , respectively.

3. Tabulations

Table 2 through Table 5 display values for f_d and f_m for pairs of x and k values. For reference, the measurement time, T_{tot} , and true count rate, $\rho(0)$, corresponding to x and k respectively in the case of ¹⁵O with $\tau = 50$ μ s are listed.

4. Comparisons to previous derivations

Michotte *et al.* (2007) have cited an internal report by B. Chauvenet giving an equation accounting for the decay-to-start-time correction for extending dead-time. His correction term (Michotte, 2010) is equivalent to our f_d^{-1} written in the form,

$$f_d^{-1} = 1 - \rho_0 \tau (\lambda T_{\text{tot}})^2 \left(\frac{1}{12} - \frac{\lambda T_{\text{tot}}}{24} + \frac{\lambda^2 T_{\text{tot}}^2}{80} \right) + \rho_0^2 \tau^2 \frac{(\lambda T_{\text{tot}})^4}{240} + \rho_0^3 \tau^3 \frac{(\lambda T_{\text{tot}})^4}{720} \dots \quad (19)$$

Also, our exact equation for f_d [Equation (17a)] for the extending case agrees with Baerg's term in square brackets in his Equation (10) (Baerg et al., 1976) to better than 2 % for $x < 2$ and $k < 2$ ($f_d = 1.29$). Baerg's equation can be expressed as,

$$f_d \approx 1 + 0.0842x^{2.004} \left(\ln \left(\frac{x}{\lambda T_L} \right) - B\tau \right). \quad (20)$$

Here, we have omitted his additional decay correction to an arbitrary reference time.

For non-extending dead time, Axton and Ryves (1963) derived their equation (4) for their error “ k ”, which is a measure of our f_d^{-1} .

$$f_d - 1 \approx \frac{\tilde{n}\tau(\lambda T_{\text{tot}})^2}{12}, \quad (21)$$

where their $\tilde{n}$ is the observed average count rate, without dead-time correction, that is, N_L/T_{tot} . To zeroth order in x this is,

$$\tilde{n} \approx \frac{\rho(0)}{1+k}. \quad (22)$$

Therefore, Axton and Ryves’ formula would be equivalent to,

$$f_d - 1 \approx \frac{kx^2}{12(1+k)}, \quad (23)$$

in agreement with the 2nd –order expansion given in our Equation(17b).

5. Summary

Correction factors have been derived for the combined effect of decay and dead-time for live-timed systems with extending and non-extending dead times. The uncertainty in these correction factors can be determined from the uncertainty of the input parameters, k and x . In addition, if an approximate expression is used, then the uncertainty in the approximation can be included, as indicated for representative values in Table 1 or by examining the next-order term.

Two useful parameters have been defined:

$$x = \lambda T_{\text{tot}} \text{ (radioactive decay constant times total real count time)}$$

$$k = \rho(0)\tau \text{ (initial true count rate times dead time)}$$

The corrections (f_d and f_m) to live-timed count rates that have been already corrected for decay (f_d) or assuming the mid-time approximation (f_m), are given by 2nd order expansions in x as follows:

Extending	Non-Extending
$f_d \approx 1 + \frac{kx^2}{12} \quad (24a)$	$f_d \approx 1 + \frac{kx^2}{12(1+k)} \quad (24b)$
$f_m \approx 1 + \frac{(2k-1)x^2}{24} \quad (25a)$	$f_m \approx 1 + \frac{(k-1)x^2}{24(1+k)} \quad (25b)$

Acknowledgments

The author is grateful to Dr. Carine Michotte of the BIPM who, as chair of the CCRI(II) Transfer Instrument working group, encouraged this work and provided valuable feedback on an early draft of the manuscript.

References

- Aboagye EO, Gilbert FJ, Fleming IN, et al. Recommendations for measurement of tumour vascularity with positron emission tomography in early phase clinical trials. *Eur. Radio.* 2012; 22:1465–1478.
- Axton EJ, Ryves TB. Dead-time corrections in the measurement of short-lived radionuclides. *Int. J. Appl. Radiat. Isot.* 1963; 14:159–161. [PubMed: 13965713]
- Baerg AP, Munzenmayer K, Bowes GC. Live-timed anti-coincidence counting with extending Dead-time circuitry. *Metrologia.* 1976; 12:77–80.
- Chisté, V.; Bé, MM. Decay Data Evaluation Project; 2015. Table de Radionucléides ¹⁵O. http://www.nucleide.org/DDEP_WG/DDEPdata.htm [accessed April 2015]
- ICRU. Particle Counting in Radioactivity Measurements, ICRU Report, 52. Bethesda, MD: International Commission on Radiation Units and Measurements; 1994. p. 56-57.
- Michotte C. Report of the CCRI(II) Transfer Instrument Working Group 2005–2007. CCRI(II)/07-14. 2007 [http://www.bipm.org/cc/CCRI\(II\)/Allowed/19/CCRI\(II\)07-14.pdf](http://www.bipm.org/cc/CCRI(II)/Allowed/19/CCRI(II)07-14.pdf).
- Michotte C. Bureau International des Poids et Mesures, Sèvres, personal communication. 2010
- Müller JW. Counting statistics of short-lived nuclides. *Journal of Radioanalytical Chemistry.* 1981; 61:345–359.
- Pommé S, Fitzgerald R, Keightley J. Uncertainty of nuclear counting. *Metrologia.* 2015; 52:S3–S17.

Figure 1.
Plots of f_d as a function of k and x for extending (left) and non-extending (right) dead-times.

Figure 2.
Plots of f_m as a function of k and x for extending (left) and non-extending (right) dead times.

Table 1

Some calculated values for f_d and f_m (Equations 15a, 15b, 16a, and 16b) as a function of x and k . Numbers in parentheses are the errors in the last reported digits that would occur from using the series expansion to 4th-order (bold) and 2nd-order respectively.

x	k	Extending		Non-Extending	
		f_d	f_m	f_d	f_m
0.5	0.5	1.0082 (1) (22)	0.9978 (0) (22)	1.0059 (0) (11)	0.9955 (0) (12)
1.0	0.2	1.0104 (5) (63)	0.9695 (3) (55)	1.0092 (1) (47)	0.9683 (1) (39)
0.5	1.0	1.0164 (1) (44)	1.0059 (1) (45)	1.0091 (0) (13)	0.9988 (1) (12)
1.0	3.0	1.1561 (61) (940)	1.1093 (102) (990)	1.0546 (3) (79)	1.0119 (1) (89)

Table 2

Calculated values of f_d for the extending case using Equation (15a) as a function of x and k . Also listed are the corresponding values of the total measurement time T_{tot} , in seconds, and of the real initial count rate of the source p_0 , in s^{-1} , for ^{15}O ($\lambda=5.66 \cdot 10^{-3} \text{ s}^{-1}$) and $\tau = 5 \cdot 10^{-5} \text{ s}$.

k/x	Extending	0.10	0.30	0.50	0.70	0.90	1.1	1.3	1.5	1.7	1.9
	p_0/T_{tot}	18	53	88	123	159	194	229	265	300	335
0.2	$4.0 \cdot 10^3$	1.000	1.001	1.003	1.006	1.009	1.012	1.015	1.019	1.022	1.026
0.4	$8.0 \cdot 10^3$	1.000	1.003	1.007	1.012	1.018	1.024	1.031	1.038	1.045	1.052
0.6	$1.2 \cdot 10^4$	1.001	1.004	1.010	1.018	1.027	1.036	1.047	1.057	1.068	1.078
0.8	$1.6 \cdot 10^4$	1.001	1.005	1.013	1.024	1.036	1.049	1.063	1.077	1.091	1.105
1.0	$2.0 \cdot 10^4$	1.001	1.007	1.016	1.029	1.044	1.061	1.078	1.096	1.114	1.132
1.2	$2.4 \cdot 10^4$	1.001	1.008	1.020	1.035	1.053	1.073	1.094	1.116	1.138	1.159
1.4	$2.8 \cdot 10^4$	1.001	1.009	1.023	1.041	1.062	1.086	1.110	1.136	1.161	1.187
1.6	$3.2 \cdot 10^4$	1.001	1.010	1.026	1.047	1.071	1.098	1.126	1.155	1.185	1.214
1.8	$3.6 \cdot 10^4$	1.001	1.012	1.030	1.053	1.080	1.110	1.142	1.175	1.208	1.241
2.0	$4.0 \cdot 10^4$	1.002	1.013	1.033	1.059	1.089	1.122	1.158	1.194	1.231	1.269

Table 3

Calculated values of f_m , for the extending case using Equation (16a) as a function of x and k . Also listed are the corresponding values of the total measurement time T_{tot} , in seconds, and of the real initial count rate of the source p_0 , in s^{-1} , for ^{15}O ($\lambda=5.66 \cdot 10^{-3} \text{ s}^{-1}$) and $\tau = 5 \cdot 10^{-5} \text{ s}$.

k/x	Extending	0.10	0.30	0.50	0.70	0.90	1.1	1.3	1.5	1.7	1.9
	p_0/T_{tot}	18	53	88	123	159	194	229	265	300	335
0.2	$4.0 \cdot 10^3$	1.000	0.998	0.993	0.986	0.976	0.963	0.947	0.929	0.909	0.886
0.4	$8.0 \cdot 10^3$	1.000	0.999	0.996	0.991	0.984	0.974	0.962	0.947	0.929	0.909
0.6	$1.2 \cdot 10^4$	1.000	1.000	0.999	0.997	0.993	0.986	0.977	0.964	0.949	0.932
0.8	$1.6 \cdot 10^4$	1.000	1.001	1.003	1.003	1.001	0.998	0.991	0.982	0.970	0.955
1.0	$2.0 \cdot 10^4$	1.000	1.003	1.006	1.009	1.010	1.009	1.006	1.000	0.991	0.978
1.2	$2.4 \cdot 10^4$	1.001	1.004	1.009	1.014	1.019	1.021	1.021	1.018	1.011	1.002
1.4	$2.8 \cdot 10^4$	1.001	1.005	1.012	1.020	1.027	1.033	1.036	1.036	1.032	1.025
1.6	$3.2 \cdot 10^4$	1.001	1.007	1.016	1.026	1.036	1.044	1.050	1.054	1.053	1.049
1.8	$3.6 \cdot 10^4$	1.001	1.008	1.019	1.032	1.045	1.056	1.065	1.071	1.074	1.073
2.0	$4.0 \cdot 10^4$	1.001	1.009	1.022	1.037	1.053	1.068	1.080	1.089	1.095	1.096

Calculated values of f_d for the non-extending case using Equation (15b) as a function of x and k . Also listed are the corresponding values of the total measurement time T_{tot} , in seconds, and of the real initial count rate of the source ρ_0 , in s^{-1} , for ^{15}O ($\lambda=5.66 \cdot 10^{-3} \text{ s}^{-1}$) and $\tau = 5 \cdot 10^{-5} \text{ s}$.

Table 4

k/x	Non-Extending	0.10	0.30	0.50	0.70	0.90	1.1	1.3	1.5	1.7	1.9
	ρ_0/T_{tot}	18	53	88	123	159	194	229	265	300	335
0.2	$4.0 \cdot 10^3$	1.000	1.001	1.003	1.005	1.008	1.011	1.014	1.017	1.020	1.023
0.4	$8.0 \cdot 10^3$	1.000	1.002	1.005	1.009	1.014	1.019	1.025	1.031	1.037	1.043
0.6	$1.2 \cdot 10^4$	1.000	1.003	1.007	1.012	1.019	1.026	1.034	1.043	1.051	1.060
0.8	$1.6 \cdot 10^4$	1.000	1.003	1.008	1.015	1.023	1.032	1.042	1.053	1.064	1.075
1.0	$2.0 \cdot 10^4$	1.000	1.004	1.009	1.017	1.027	1.038	1.049	1.062	1.075	1.088
1.2	$2.4 \cdot 10^4$	1.000	1.004	1.010	1.019	1.030	1.042	1.055	1.070	1.085	1.100
1.4	$2.8 \cdot 10^4$	1.001	1.004	1.011	1.020	1.032	1.046	1.061	1.077	1.094	1.111
1.6	$3.2 \cdot 10^4$	1.001	1.004	1.012	1.022	1.034	1.049	1.065	1.083	1.101	1.120
1.8	$3.6 \cdot 10^4$	1.001	1.005	1.012	1.023	1.036	1.052	1.070	1.089	1.109	1.129
2.0	$4.0 \cdot 10^4$	1.001	1.005	1.013	1.024	1.038	1.055	1.074	1.094	1.115	1.137

Table 5

Calculated values of f_m , for the non-extending case using Equation (16b) as a function of x and k . Also listed are the corresponding values of the total measurement time T_{tot} , in seconds, and of the real initial count rate of the source ρ_0 , in s^{-1} , for ^{15}O ($\lambda=5.66 \cdot 10^{-3} \text{ s}^{-1}$) and $\tau = 5 \cdot 10^{-5} \text{ s}$.

k/x	Non-Extending	0.10	0.30	0.50	0.70	0.90	1.1	1.3	1.5	1.7	1.9
	ρ_0/T_{tot}	18	53	88	123	159	194	229	265	300	335
0.2	$4.0 \cdot 10^3$	1.000	0.997	0.992	0.985	0.975	0.962	0.946	0.928	0.907	0.884
0.4	$8.0 \cdot 10^3$	1.000	0.998	0.995	0.989	0.981	0.970	0.956	0.940	0.922	0.901
0.6	$1.2 \cdot 10^4$	1.000	0.999	0.996	0.992	0.985	0.976	0.965	0.951	0.935	0.916
0.8	$1.6 \cdot 10^4$	1.000	0.999	0.998	0.994	0.989	0.982	0.973	0.960	0.946	0.929
1.0	$2.0 \cdot 10^4$	1.000	1.000	0.999	0.997	0.993	0.987	0.979	0.969	0.956	0.940
1.2	$2.4 \cdot 10^4$	1.000	1.000	1.000	0.998	0.996	0.991	0.985	0.976	0.964	0.950
1.4	$2.8 \cdot 10^4$	1.000	1.000	1.000	1.000	0.998	0.995	0.990	0.982	0.972	0.960
1.6	$3.2 \cdot 10^4$	1.000	1.001	1.001	1.001	1.000	0.998	0.994	0.988	0.979	0.968
1.8	$3.6 \cdot 10^4$	1.000	1.001	1.002	1.002	1.002	1.001	0.998	0.993	0.986	0.976
2.0	$4.0 \cdot 10^4$	1.000	1.001	1.002	1.003	1.004	1.004	1.002	0.998	0.991	0.983