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Investigation of Optimization-Based Reconstruction with an Image-Total-Variation Constraint in PET

Zheng Zhang¹, Jinghan Ye², Buxin Chen¹, Amy E. Perkins², Sean Rose¹, Emil Y. Sidky¹, Chien-Min Kao¹, Dan Xia¹, Chi-Hua Tang², and Xiaochuan Pan^{1,3}

¹Department of Radiology, The University of Chicago

²Philips Healthcare, USA

³Department of Radiation and Cellular Oncology, The University of Chicago

Abstract

Interest remains in reconstruction-algorithm research and development for possible improvement of image quality in current PET imaging and for enabling innovative PET systems to enhance existing, and facilitate new, preclinical and clinical applications. Optimization-based image reconstruction has been demonstrated in recent years to have potential utility for CT imaging applications. In this work, we investigate tailoring the optimization-based techniques to image reconstruction for PET systems with standard and non-standard scan configurations. Specifically, given an image-total-variation (TV) constraint, we investigated how the selection of different data divergences and associated parameters impacts the optimization-based reconstruction of PET images. The reconstruction robustness was explored also with respect to different data conditions and activity up-takes of practical relevance. A study was conducted particularly for image reconstruction from data collected by use of a PET configuration with sparsely populated detectors. Overall, the study demonstrates the robustness of the TV-constrained, optimization-based reconstruction for considerably different data conditions in PET imaging, as well as its potential to enable PET configurations with reduced numbers of detectors. Insights gained in the study may be exploited for developing algorithms for PET-image reconstruction and for enabling PET-configuration design of practical usefulness in preclinical and clinical applications.

1. Introduction

Image reconstruction is a key component in the development and application of advanced positron-emission tomography (PET) imaging. A great body of work exists on PET-image reconstruction algorithms of theoretical significance and practical value, including those developed based upon expectation-maximization (EM), row-action maximum-likelihood (RAMLA), maximum a posteriori (MAP), and penalized maximum-likelihood (PML) algorithms [1–12]. The algorithms have also been extended to list-mode [13–16]/time-of-flight (TOF) [17–20], and 4D spatial-temporal/parametric image reconstructions [21–30]. Strong interests remain, however, in research on reconstruction algorithm possibly for further improvement on image quality in PET imaging, and for enabling, together with empowering detection and electronic technologies, innovative PET systems tailored possibly to enhance existing, and facilitate new, applications [31–38]. In this work, we investigate tailoring optimization-based reconstructions with an image-total-variation (TV) constraint,

which have been demonstrated to have potential utility for CT imaging in recent years [39–44], for PET scans with standard and non-standard scan configurations.

The focus of the work is to investigate technical characteristics of TV-constrained, optimization reconstructions of PET images, instead of showing whether it is superior to existing reconstruction techniques. Specifically, we study how different designs of the optimization programs with different data divergences affect PET-image properties, and we identify clearly parameters involved in, and demonstrate their impact on, the optimization-based reconstructions considered. The investigation is facilitated by a specific set of primal-dual algorithms developed by Chambolle and Pock (CP) [45]. With the understanding of the characteristics of TV-constrained optimization programs and their associated parameters, insights may be gained for development of practically useful algorithms for PET-image reconstruction in preclinical and clinical applications.

The robustness of the TV-constrained, optimization-based reconstruction was investigated also by applying to objects with considerably distinct uptake maps, and to data with different levels of quality/quantity. Moreover, a study was conducted for image reconstruction from data collected by use of a PET configuration with sparsely populated detectors, amounting to only half the number of detectors in a digital prototype PET/CT system. The study demonstrates the potential to an algorithm-enabled design of a PET configuration with a reduced number of detectors. The saved detectors can be exploited, e.g., for extending the longitudinal coverage of a PET scanner.

We describe in Sec. 2 the design of, and approach to, the study. Following a study in Sec. 3 that validates the algorithms under the conditions for which they are developed, we carried out, respectively, a Monte-Carlo-simulation study in Sec. 4, a physical IEC-phantom study in Sec. 5, and a human-data study in Sec. 6. Sections 4, 5, and 6 follow a similar structure in which a brief description of the data is provided followed by investigations of TV-constraint, different optimization programs, convergent reconstructions from full and sparse data, and reconstructions as a function of iteration. Remarks about the work are given in Sec. 7.

2. Methods

We describe briefly in this section the PET configurations in a digital prototype PET/CT system and optimization-based reconstructions considered in the work.

2.1. PET configurations

As shown in Fig. 1, a full-scan configuration is formed by tightly assembling P identical detector modules, each of which itself is a flat panel containing $L \times K$ identical tiles of square shape, on a cylindrical surface of radius R . In particular, the centers of the modules are placed on the same circular ring on the cylindrical surface, while the horizontal central lines of the modules are parallel to the central axis of a cylindrical surface. Therefore, the full-scan configuration consists effectively of L rings of $P \times K$ tiles. Each of the tightly congregated tiles itself within a module is composed of $J \times J$ identical crystal bins of square shape and length d . A straight line connecting the centers of two crystal detector-bins forms a line-of-response (LOR) along which a data sample is collected. A PET configuration with

M distinct LORs thus yields a data set of at most M samples. In the work, this configuration is referred to as the full-scan configuration, and data collected with the configuration the *full data*.

In the Monte-Carlo-simulation and real-data studies performed below, PET data were collected by use of a PET configuration, in a digital prototype PET/CT system, identical to that described above, with $P = 18$, $L = 5$, $K = 4$, $J = 8$, and $d = 4$ mm, yielding a total of $M = 153, 446, 400$ distinct LORs. We also investigated image reconstruction from data collected from a sparse-scan configuration that is formed by removing odd- or even-numbered tiles in odd- or even-numbered rings in the full-scan configuration. The sparse-scan configuration thus consists of L rings each of which is composed only of $P/2 \times K$ uniformly, but sparsely, distributed tiles, as depicted in Fig. 1, and we refer to data collected with the sparse-scan configuration as *sparse data*. It can be shown that the sparse-scan configuration has a total of $M = 38, 361, 600$ distinct LORs. The investigation of the sparse-scan configuration is motivated by an interest in PET imaging with a considerably reduced number of crystals and associated electronics (relative to the full-scan configuration.)

2.2. PET-data model

In this work, we consider a discrete image defined on a three-dimensional (3D) array containing $N = N_x \times N_y \times N_z$ identical voxels of cubic shape, where N_x , N_y , and N_z denote the numbers of voxels along the Cartesian x -, y -, and z -axis, respectively. The z -axis coincides with the central axis of the PET configuration shown in Fig. 1. We use vector $\mathbf{u}$ of size N to denote the image in a concatenated form. The PET-data model used in the study is given by

$$\mathbf{g} = \mathcal{H}\mathbf{u} + \mathbf{g}_s + \mathbf{g}_r, \quad (1)$$

where vector $\mathbf{g}$ of size M denotes the model data, $\mathcal{H}$ is an $M \times N$ system matrix in which element h_{ij} is the intersection length of LOR i with voxel j , vectors $\mathbf{g}_s$ and $\mathbf{g}_r$ of size M denote scatter and random events, which are assumed to be known in the work. We use vector $\mathbf{g}_m$ of size M to denote the measured data. In this work, $\mathbf{g}_m$, $\mathbf{g}_s$, and $\mathbf{g}_r$ are corrected for the effect of photon attenuation. The goal of PET-image reconstruction is to determine (i.e., reconstruct) $\mathbf{u}$ from knowledge of $\mathbf{g}_m$, $\mathcal{H}$, $\mathbf{g}_s$, and $\mathbf{g}_r$.

2.3. Optimization programs

Using Eq. (1), we form an optimization program in the form of

$$\mathbf{u}^* = \underset{\mathbf{u}}{\operatorname{argmin}} D(\mathbf{g}_m, \mathbf{g}) \text{ s.t. } \|\mathbf{f}\|_{TV} \leq t_0 \text{ and } f_j \geq 0, \quad (2)$$

where $D(\mathbf{g}_m, \mathbf{g})$ denotes a measure of data fidelity between measured data $\mathbf{g}_m$ and model data $\mathbf{g}$, image $\mathbf{u}$ to be reconstructed is related to latent image vector $\mathbf{f}$ of size N through [46]

$$\mathbf{u} = \mathcal{G}\mathbf{f}, \quad (3)$$

$\mathcal{G}$ a matrix of size $N \times N$, $\|\mathbf{f}\|_{TV}$ image TV (see Eq. (2) in Ref. [39]), t_0 TV constraint parameter, f_j the j th element of vector $\mathbf{u}$, and $j = 1, 2, \dots, N$.

The program in the form of Eq. (2) has been used widely in CT imaging in which $\mathcal{G}$ is generally an identity matrix. As the selection of data divergence $D(\mathbf{g}_m, \mathbf{g})$ can significantly impact PET reconstructions, we consider below three different data divergences of potential research and practical interest. In PET imaging, data generally have a signal-to-noise ratio (SNR) substantially lower than that of typical CT data, and the gradient-magnitude image of a typical CT image of practical interest is generally sparser than that of a typical PET image (i.e., $\mathbf{u}$). Therefore, we formulate PET image $\mathbf{u}$ desired as latent image $\mathbf{f}$ blurred by matrix $\mathcal{G}$, and apply the image-TV constraint to $\mathbf{f}$ instead of $\mathbf{u}$. This formulation can potentially avoid yielding an image with significant patchy textures when data have a low level of SNR [43].

In the work, the blurring matrix is obtained as follows: for a 3D, isotropic Gaussian function centered at a given voxel in the image array, we calculate its values at the center locations of N voxels within the image array, and use the calculated values in a concatenated form identical to that of vector $\mathbf{u}$ to create a vector of size N . Repeating the calculation for each of the N voxels in the image array, N such vectors can be formed; and matrix $\mathcal{G}$ of size $N \times N$ can subsequently be built in which a row is the transpose of one of the N vectors, and the N rows are in an order consistent with the concatenated order of entries of $\mathbf{u}$. The unit of standard deviation of the Gaussian function is defined in terms of voxel size. With standard deviation taking zero value, $\mathcal{G}$ becomes identity matrix $\mathcal{Q}$.

(1) Optimization program DKL-fTV—Using the data-Kullback-Leibler (KL) divergence to replace $D(\mathbf{g}_m, \mathbf{g})$ in Eq. (2), we obtain the program

$$\mathbf{u}^* = \underset{\mathbf{u}}{\operatorname{argmin}} D_{KL}(\mathbf{g}_m, \mathbf{g}) \text{ s.t. } \|\mathbf{f}\|_{TV} \leq t_0 \text{ and } f_j \geq 0, \quad (4)$$

where $D_{KL}(\mathbf{g}_m, \mathbf{g})$ denotes the data-KL divergence given by

$$D_{KL}(\mathbf{g}_m, \mathbf{g}) = \sum_i^M [\mathbf{g} - \mathbf{g}_m + \mathbf{g}_m \ln \mathbf{g}_m - \mathbf{g}_m \ln \mathbf{g}]_i, \quad (4a)$$

$[\cdot]_i$ denotes the i th element of vector $[\cdot]$. When computing $D_{KL}(\mathbf{g}_m, \mathbf{g})$ in the work, the entries in $\mathbf{g}$ that are smaller than $\epsilon = 10^{-20}$ are replaced with ϵ . For the convenience of devising convergence conditions below, a normalized data-KL divergence is defined as

$D'_{KL}(\mathbf{g}_m, \mathbf{g}) = D_{KL}(\mathbf{g}_m, \mathbf{g}) / D_{KL}(\mathbf{g}_m, \mathbf{g}_\epsilon)$, where $\mathbf{g}_\epsilon$ is obtained by replacing all of the entries in $\mathbf{g}$ with ϵ . We refer to the optimization program in Eq. (4) as DKL-fTV.

(2) Optimization program DL2-fTV—We also consider the program

$$\mathbf{u}^* = \underset{\mathbf{u}}{\operatorname{argmin}} D_{\ell_2}(\mathbf{g}_m, \mathbf{g}) \text{ s.t. } \|\mathbf{f}\|_{TV} \leq t_0 \text{ and } f_j \geq 0, \quad (5)$$

which is obtained by the replacement of $D(\mathbf{g}_m, \mathbf{g})$ in Eq. (2) with

$$D_{\ell_2}(\mathbf{g}_m, \mathbf{g}) = \|\mathbf{g}_m - \mathbf{g}\|_2. \quad (5a)$$

This fidelity metric takes the ℓ_2 -norm of the difference between the measured data and model data. Also, for the convenience of specifying convergence conditions below, a normalized data- ℓ_2 is defined as $D'_{\ell_2}(\mathbf{g}_m, \mathbf{g}) = D_{\ell_2}(\mathbf{g}_m, \mathbf{g}) / D_{\ell_2}(\mathbf{g}_m, 0)$. We refer to the optimization program in Eq. (5) as *program DL2-fTV*.

(3) Optimization program DL1-fTV—In addition to $D_{KL}(\mathbf{g}_m, \mathbf{g})$ and $D_{\ell_2}(\mathbf{g}_m, \mathbf{g})$ that have been used widely for formulating optimization programs in image reconstruction, we consider an optimization program below

$$\mathbf{u}^* = \underset{\mathbf{u}}{\operatorname{argmin}} D_{\ell_1}(\mathbf{g}_m, \mathbf{g}) \text{ s.t. } \|\mathbf{f}\|_{TV} \leq t_0 \text{ and } f_j \geq 0, \quad (6)$$

which is obtained by the replacement of $D(\mathbf{g}_m, \mathbf{g})$ in Eq. (2) with

$$D_{\ell_1}(\mathbf{g}_m, \mathbf{g}) = \|\mathbf{g}_m - \mathbf{g}\|_1, \quad (6a)$$

denoting the ℓ_1 -norm of difference between measured data and model data. Again, for the convenience of describing convergence conditions below, a normalized data- ℓ_1 is defined as $D'_{\ell_1}(\mathbf{g}_m, \mathbf{g}) = D_{\ell_1}(\mathbf{g}_m, \mathbf{g}) / D_{\ell_1}(\mathbf{g}_m, 0)$. We refer to the optimization program in Eq. (5) as *program DL1-fTV*.

2.4. Program parameters

In an optimization-based reconstruction, numerous parameters are employed necessarily to specify explicitly the program, i.e., the solution set, which are referred to as *program parameters*. We consider *algorithm parameters* to be those that affect only algorithm's convergence path leading to the solution set, and thus have no impact on the theoretical specification (or design) of the solution set. Clearly, the solution set depends upon the specific form of the optimization program, and thus the optimization program itself constitutes a program parameter. This is the reason that we study programs with different data divergences in Eqs. (4)–(6). Moreover, an optimization program of a given form itself still involves additional program parameters. For the optimization programs considered in Eqs. (4)–(6), additional program parameters include system matrix $\mathcal{H}$, scatter and random components $\mathbf{g}_s$ and $\mathbf{g}_r$, the voxel size, TV-constraint parameter t_0 , and blurring matrix $\mathcal{G}$.

The CP algorithms used in the work can solve the convex optimization programs in Eqs. (4)–(6), but like any other iterative algorithms, they only converge to a solution in the limit of infinite iterations. In a practical study, due to the limitation of computer precision and computation time, one can obtain reconstructions only at finite iterations. Therefore, it is necessary to specify practical convergence conditions under which reconstructions can be achieved within a finite number of iterations; and the practical convergence conditions thus play a role in defining an actual solution set achievable within a finite number of iterations. In this work, when the practical conditions are satisfied, the reconstruction stops and is referred to as the *convergent reconstruction* $\hat{\mathbf{u}}^*$; and we also denote the corresponding latent image as $\hat{\mathbf{f}}^*$, where $\hat{\mathbf{u}}^* = \mathcal{G}\hat{\mathbf{f}}^*$.

2.5. Determination of program parameters

As mentioned above, system matrix $\mathcal{H}$ contains M row vectors of size N in which each entry depicts the intersection of an LOR with a voxel in the image array. In the validation and Monte-Carlo-simulation studies in Secs. 3 and 4, scatter and random events $\mathbf{g}_s$ and $\mathbf{g}_r$ are not considered; whereas in the IEC and human studies performed in Secs. 5 and 6, the single-scatter simulation method [47] and the delayed coincidence method [48] were employed for estimating $\mathbf{g}_s$ and $\mathbf{g}_r$ respectively. A voxel size of 4 mm was selected for the studies because it is used often in clinical studies. Results of previously relevant [46] and current studies indicate that matrix $\mathcal{G}$ generated by use of a 3D isotropic Gaussian function with a standard deviation of 2.4 mm, which is 0.6 times the image-voxel size, appears to yield appropriate reconstructions for data conditions considered.

For designing the practical convergence conditions, we introduce two unitless metrics as

$$\text{CTV}(\mathbf{f}_n) = \left| \|\mathbf{f}_n\|_{TV} - t_0 \right| / t_0 \quad (7)$$

and

$$D'(\mathbf{u}_n) = D(\mathbf{g}_m, \mathbf{g}_n) / D(\mathbf{g}_m, 0), \quad (8)$$

where $\mathbf{u}_n$ and $\mathbf{f}_n$ denote reconstructions at iteration n , $\mathbf{g}_n = \mathcal{H}\mathbf{u}_n + \mathbf{g}_s + \mathbf{g}_r$ the model data estimated at the n th iteration, obtained by replacing $\mathbf{u}$ with $\mathbf{u}_n$ in Eq. (1). Practical convergence conditions are devised with $\text{CTV}(\mathbf{f}_n)$ and $D'(\mathbf{u}_n)$ for the studies performed in Secs. 3–6 below.

With the program parameters and practical convergence conditions specified above, only image-TV-constraint parameter t_0 remains to be determined. We discuss the determination of t_0 in each specific study carried out below, because different data conditions in the studies can have considerably different impacts on the appropriate selection of t_0 .

2.6. Reconstruction algorithms

Optimization programs DKL-fTV, DL2-fTV, and DL1-fTV in Eqs. (4)–(6) are convex, and can be solved with a number of existing algorithms. We utilize algorithms belonging to a class of algorithms known as the primal-dual algorithms [49].

Primal-dual algorithms—A specific set of first-order, primal-dual algorithms developed by Chambolle and Pock (CP) [45] has been demonstrated to be an effective investigative tool for solving a variety of convex optimization programs in CT imaging [50], including the convex programs described in Eqs. (4)–(6). The explicit forms of the CP algorithms for the programs considered can readily be derived by taking the approach in Ref. [50], and their pseudo codes are given below, which are similar to those in Ref. [50] with only slight modifications discussed below.

Algorithm

Pseudo-code for $\mathcal{N}$ steps of the CP algorithm for solving Eq. (4)–(6)

```

1:  INPUT: measured events  $\mathbf{g}_m$ , scatter events  $\mathbf{g}_s$ , random events  $\mathbf{g}_r$ , TV constraint
    parameter  $t_0$ , Gaussian blurring operator  $\mathcal{G}$ 
2:  INPUT: parameter  $\lambda$ 
3:   $\mathbf{v} = \|\mathcal{H}\mathcal{G}\|_{SV} / \|\nabla\|_{SV}$ ,  $c = \mathbf{v}t_0$ 
4:   $L \leftarrow \|\mathcal{H}\mathcal{G}, \nabla\|_{SV}$ 
5:   $\tau \leftarrow 1/\lambda L$ ;  $\sigma \leftarrow \lambda/L$ ;  $\theta \leftarrow 1$ ;  $n \leftarrow 0$ 
6:  INITIALIZE:  $\mathbf{f}_0$ ,  $\mathbf{u}_0$ ,  $\mathbf{y}_0$ , and  $\tilde{\mathbf{z}}_0$  to zero
7:   $\tilde{\mathbf{f}}_0 \leftarrow \mathbf{f}_0$ 
8:  while Not done do
9:     $\mathbf{y}_{n+1} = \Phi(\mathbf{y}_n, \tilde{\mathbf{g}}_n, \mathbf{g}_m)$ 
10:    $\tilde{\mathbf{t}} = \tilde{\mathbf{z}}_n + \sigma \mathbf{v} \nabla \tilde{\mathbf{f}}_n$ 
11:    $\tilde{\mathbf{z}}_{n+1} = (\mathbf{1} - \sigma \text{ProjectOnto}_{\ell_1 \text{Ball}_c}(\tilde{\mathbf{t}}/(\sigma)/|\tilde{\mathbf{t}}|)) \tilde{\mathbf{t}}$ 
12:    $\mathbf{f}_{n+1} = \text{pos}(\mathbf{f}_n - \tau (\mathcal{G}^T \mathcal{H}^T \mathbf{y}_{n+1} + \mathbf{v} \nabla^T \tilde{\mathbf{z}}_{n+1}))$ 
13:    $\mathbf{u}_{n+1} = \mathcal{G} \mathbf{f}_{n+1}$ 
14:    $\tilde{\mathbf{f}}_{n+1} = \mathbf{f}_{n+1} + \theta(\mathbf{f}_{n+1} - \mathbf{f}_n)$ 
15:    $n \leftarrow n + 1$ 
16: end while
17:  OUTPUT: image  $\hat{\mathbf{u}}^* = \mathbf{u}_{\mathcal{N}}$ 

```

In the algorithm pseudo codes [50], ∇ depicts a matrix representing a finite differencing approximation of the image gradient, yielding vectors $\nabla \mathbf{f}$, $\tilde{\mathbf{t}}$, and $\tilde{\mathbf{z}}_n$ N -element vectors with each entry a vector of size 3, norm $\|\cdot\|_{SV}$ of a linear operator computes the largest singular value of that linear operator, $\tilde{\mathbf{g}}_n = \mathcal{H}\hat{\mathbf{f}}_n + \mathbf{g}_s + \mathbf{g}_r$, $|\tilde{\mathbf{t}}|$ denotes an N -element vector by taking the magnitude of each entry of $\tilde{\mathbf{t}}$, operator $[\text{ProjOnto}_{\ell_1 \text{Ball}_c}]$ yields a vector of size N by projection of vector $|\tilde{\mathbf{t}}|/\sigma$ onto the ℓ_1 -ball of scale c [51], the multiplication of $\tilde{\mathbf{t}}$ and $(\mathbf{1} - \sigma \text{ProjectOnto}_{\ell_1 \text{Ball}_c}(\tilde{\mathbf{t}}/(\sigma)/|\tilde{\mathbf{t}}|))$ in Line 11 is performed component-wise, and superscript “ T ” transpose operation. Operator $\text{pos}(\cdot)$ enforces non-negativity of the all entries of the input vector. Algorithm parameter λ can affect the convergence path and rate of the CP algorithms especially at early iterations. For the studies carried out in the work, $\lambda = 0.01$ was used

because it has been shown in the study to yield a reasonable convergence rate at early iterations.

We note that all of the lines in the pseudo code are identical for the three optimization programs in Eqs. (4)–(6), except for vector Φ of size M in line 9, which may vary depending upon the specific data divergence considered [50]. For the program in Eq. (4),

$$\Phi(\mathbf{y}_n, \bar{\mathbf{g}}_n, \mathbf{g}_m) = \frac{1}{2}[\mathbf{1}_D + \mathbf{y}_n + \sigma \bar{\mathbf{g}}_n] - \sqrt{(\mathbf{1}_D - \mathbf{y}_n - \bar{\mathbf{g}}_n)^2 + 4\sigma \mathbf{g}_m}, \quad (9)$$

where $\mathbf{1}_D$ is a vector of size M filled with 1s. For the optimization program in Eq. (5),

$$\Phi(\mathbf{y}_n, \bar{\mathbf{g}}_n, \mathbf{g}_m) = [\mathbf{y}_n + \sigma(\bar{\mathbf{g}}_n - \mathbf{g}_m)] / (1 + \sigma); \quad (10)$$

and for optimization program in Eq. (6),

$$\Phi(\mathbf{y}_n, \bar{\mathbf{g}}_n, \mathbf{g}_m) = \frac{\mathbf{y}_n + \sigma(\bar{\mathbf{g}}_n - \mathbf{g}_m)}{\max(\mathbf{1}_D, |\mathbf{y}_n + \sigma(\bar{\mathbf{g}}_n - \mathbf{g}_m)|)}, \quad (11)$$

where $\max(\cdot)$ is performed element-wise.

Row-action maximum-likelihood algorithm (RAMLA)—Reconstruction techniques based upon the RAMLA algorithm [8] are used frequently in PET research and application for yielding reconstructions of practical utility. It can be viewed as a relaxed ordered-subset algorithm in which the step size is subset-independent and gradually decreases to zero. Under certain conditions, the RAMLA algorithm is mathematically equivalent to the EM algorithm [8], but they involve different implementation procedures and can lead to different solutions when a finite number of iterations are used as in all practical reconstructions.

In the work we applied the RAMLA algorithm to reconstructing from *full data* PET images $\mathbf{u}_{\text{ref}}$, and use them as *reference reconstructions* to gauge the relevance of the study considered. Specifically, the RAMLA implementation in the study consists of subsets with the number of LORs varying from 291 to 291×40 , and yields the reconstruction after two full iterations, as is done typically in practical research and clinical applications. RAMLA reconstructions from sparse data were also carried out in each of the studies described below, and were observed to have quality substantially lower than that of the reference reconstructions. Therefore, they are not included in the work.

3. Algorithm Validation

We first performed a study to validate that under ideal, sufficient data conditions, the CP algorithms and their computer implementation numerically recover the truth image by solving the optimization programs. In the study, which is referred to as an inverse-crime study, a known truth image represented on a discrete array of identical voxels is used for

generating model data by use of a known, system matrix, which is used also in reconstruction from the model data on the same image array. It should be pointed out that an inverse-crime study reveals no information about the practical utility of an optimization reconstruction in real-data studies. Such information can be obtained only through carefully designed evaluation studies of task-specific significance.

3.1. Study design

For the purpose of validation and computation-efficiency consideration, we use the full-scan configuration described in Sec. 2.1 but with only one ring of tiles, and a 3D image array consisting of $50 \times 50 \times 8$ identical cubic voxels of size 4 mm. With the configuration and image array given, system matrix $\mathcal{H}$, and Gaussian matrix $\mathcal{G}$ can be formed, as described in Sec. 2.5 above. According to Eq. (3), we create truth image $\mathbf{u}_{\text{true}}$ shown in Fig. 2a by application of matrix $\mathcal{G}$ to a known, numerical IEC phantom $\mathbf{f}_{\text{true}}$, i.e., the truth latent image. Subsequently, model data $\mathbf{g}$ is generated from $\mathbf{u}_{\text{true}}$ by use of system matrix $\mathcal{H}$. Without loss of generality, random and scatter are not considered (i.e., $\mathbf{g}_s = 0$ and $\mathbf{g}_r = 0$) in the study.

The mathematical convergence conditions for the CP algorithms include $\text{CTV}(\mathbf{f}_n) \rightarrow 0$, $D'(\mathbf{u}_n) \rightarrow 0$, and $\text{cPD}(\mathbf{u}_n) \rightarrow 0$, as $n \rightarrow \infty$, where $\text{cPD}(\mathbf{u}_n)$ denotes the conditional primal-dual (cPD) gap [45, 49]. They are unachievable, however, due to limited computer precision and computation time involved in any practical, numerical study. Therefore, for the inverse-crime study considered, we design practical convergence conditions

$$\text{CTV}(\mathbf{f}_n) < 10^{-5},$$

$$D'(\mathbf{u}_n) < 10^{-5}, \quad (12)$$

$$\text{cPD}(\mathbf{u}_n) < 10^{-3},$$

and require that the convergence metrics maintain their decaying trends even after that the conditions are satisfied, as n increases, where $D'(\mathbf{u}_n) = D'_{KL}(\mathbf{u}_n)$, $D'_{\ell_2}(\mathbf{u}_n)$, and $D'_{\ell_1}(\mathbf{u}_n)$, respectively, for optimization programs in Eqs. (4), (5), and (6). Obviously, practical convergence conditions tighter or looser than those in Eq. (12) can readily be designed, depending upon the amount of computation resources to be invested for the verification study.

3.2. Study results

We have performed inverse-crime studies on reconstructions based upon the three optimization programs in Eqs. (4)–(6) in which $t_0 = \|\mathbf{f}_{\text{true}}\|_{\text{TV}}$ is computed from truth-latent-image $\mathbf{f}_{\text{true}}$. For brevity, we show results obtained only for program DKL-fTV in Eq. (4), as similar results are obtained also for programs in Eqs. (5) and (6). It can be observed in Fig. (2) that convergent reconstruction $\hat{\mathbf{u}}^*$ is visually virtually identical to truth image $\mathbf{u}_{\text{true}}$. Also,

Fig. 3 shows how convergence metrics evolve as functions of the iteration number, demonstrating that the practical convergence conditions in Eq. (12) are satisfied. Because

$\mathbf{u}_{\text{true}}$ is known, we also calculated $\overline{D}(\mathbf{u}_n) = \frac{\|\mathbf{u}_n - \mathbf{u}_{\text{true}}\|_2}{\|\mathbf{u}_{\text{true}}\|_2}$, and display it in Fig. 3, which indicates that the reconstruction converges to the truth image. Furthermore, it can be observed that the convergent metrics maintain their decaying trends beyond the convergence conditions in Eq. (12). Therefore, the results of the inverse-crime study numerically assure the correct computer implementation of the CP algorithms.

4. Monte-Carlo-Simulation Study

For studies in Secs. 4–6 below, we design practical convergence conditions

$$\text{CTV}(\mathbf{f}_n) < 10^{-5}, \quad (13)$$

$$D'(\mathbf{u}_n) \rightarrow \text{plateau},$$

as n increases. The convergence conditions are less restrictive than those in Eq. (12) for the inverse-crime study, and they are designed based upon the practical considerations: (a) in a real-data study, because inconsistency exists between measured data $\mathbf{g}_m$ and model data $\mathbf{g}$, $D'(\mathbf{u}) > 0$ is generally non-zero; and because full knowledge is unavailable about model data $\mathbf{g}$, the value of $D'(\mathbf{u})$ is generally unknown. Therefore, condition $D'(\mathbf{u}_n) \rightarrow \text{plateau}$, instead of $D'(\mathbf{u}_n) \rightarrow 0$, is considered. Unlike metrics $\text{CTV}(\mathbf{f})$ and $D'(\mathbf{u})$ that provide directly meaningful measures of physical properties of reconstructions in a practical study, metric $\text{cPD}(\mathbf{u}_n)$ yields a mathematical check on the correctness of the algorithm implementation. Consequently, once the implementation correctness is verified in the inverse-crime study, metric $\text{cPD}(\mathbf{u}_n)$ is not used in real-data studies in which practical convergence conditions in Eq. (13) appear to yield reconstructions of practical relevance, as the study results below show.

Prior to physical-phantom and human studies, we conducted a Monte-Carlo-simulation study in which full data of ~200 million total counts were generated from the digital Jaszczak phantom by using the GATE simulation package [52] for the full-scan configuration discussed in Sec. 2.1. From the full data, we also extracted sparse data and carried out reconstructions for the sparse-scan configuration. Without loss of generality, the simulation study includes only truth events. Images are reconstructed on a 3D array of $70 \times 70 \times 41$ identical cubic voxels of size 4 mm. Digital phantom $\mathbf{u}_{\text{true}}$ consists of cold- and hot-rod sections each of which contains six types of cylindrical-shaped rods of diameters 4.8, 6.4, 7.9, 9.5, 11.1, and 12.7 mm. Although GATE data contain only true coincidence events, they are inconsistent with the model data in Eq. (1) due to noise and other physical factors included by GATE, but not in the data model. Using GATE data and knowledge of truth image $\mathbf{u}_{\text{true}}$, we characterize how optimization-based reconstruction responds to data

inconsistency prior to its application to real data in which knowledge of the truth image is unavailable.

4.1. Determination of TV-constraint parameter t_0

In the study, given the convergence conditions in Eq. (13), all of the program parameters are determined, as described in Sec. 2.5, except for TV-constraint parameter t_0 , which is determined by use of the root-mean-square error (RMSE),

$$\text{RMSE} = \frac{1}{\sqrt{N}} \|\mathbf{u}_{\text{true}} - \hat{\mathbf{u}}^*\|_2, \quad (14)$$

between truth image $\mathbf{u}_{\text{true}}$ and convergent reconstruction $\hat{\mathbf{u}}^*$. For each of a set of t_0 values, we solve program DKL-fTV in Eq. (4) to obtain convergent reconstruction $\hat{\mathbf{u}}^*$ from full data and calculate the RMSE. Repeating the reconstruction and calculation for all values of t_0 , we obtain an RMSE of t_0 , as depicted in Fig. 4. Based upon the RMSE result, we select $t_0 = 13488$, which yields a minimum RMSE, for obtaining reconstruction results in subsequent sections. The RMSE metric was used for selecting t_0 in the study simply because knowledge of truth image $\mathbf{u}_{\text{true}}$ is available, and the recovery of the truth image is the “utility” goal in the study.

4.2. Reconstructions based upon different optimization programs

Using the selected t_0 , we obtain reconstructions from full data by solving programs DKL-fTV, DL2-fTV, and DL1-fTV in Eqs. (4)–(6), and display them in Fig. 5. It can be observed that all of the convergent reconstructions $\hat{\mathbf{u}}^*$ for the cold-rod section appear visually comparable, only with slightly different noise textures, while program DKL-fTV yields a reconstruction of the hot-rod section with spatial resolution slightly superior to those obtained with programs DL2-fTV and DL1-fTV. However, the DL1-fTV reconstruction of the hot-rod section appears to contain prominent zero-valued artifacts. In fact, the DL1-fTV reconstructions from data of the IEC phantom and human subject also comprise significant artifacts, as shown in Figs. 11 and 18 below.

In an attempt to illustrate the artifact source, we define

$$\mathbf{g}'_m = \mathbf{g}_m - \mathbf{g}_s - \mathbf{g}_r \quad \text{and} \quad \mathbf{g}' = \mathcal{H} \hat{\mathbf{u}}^*, \quad (15)$$

where $\mathbf{g}'_m$ denotes the measured data with scatter/random corrected for, and $\mathbf{g}'$ the model data estimated from the convergent reconstruction $\hat{\mathbf{u}}^*$, also with scatter/random corrected for. In particular, we use $\mathbf{g}'_{KL}$, $\mathbf{g}'_{\ell_2}$, and $\mathbf{g}'_{\ell_1}$ to specify explicitly the model data estimated, respectively, by use of programs DKL-fTV, DL2-fTV, and DL1-fTV, and display in Fig. 6 their example profiles for the Jaszczak and IEC phantoms and the human subject. The profile results suggest that the minimization of data ℓ_1 -norm in program DL1-fTV yields the estimated model data “biased” toward zero due to the prevalence of zero or small-valued measurements, thus producing artifacts observed in the DL1-fTV-based reconstruction of the

hot-rod section in Fig. 5. It is possible that artifacts in DL1-fTV reconstructions may be different when program parameters different than those used in the work.

4.3. Reconstructions based upon optimization program DKL-fTV

The results above indicate that overall, program DKL-fTV appears to produce reconstructions of reasonable visual textures for both cold- and hot-rod sections. Program DKL-fTV was chosen for further investigation based on visualization of converged patient data reconstructions — the data set used in section 6 — at different values of t_0 for each optimization program. As can be seen from the example in Figure 18, the DKL-fTV reconstructions tended to have better delineated boundaries than the DL2-fTV reconstructions, while DL1-fTV yielded zero-valued reconstructions. The results of patient data reconstructions at different values of t_0 are not included here due to spatial constraints, but the same trends were observed at all values of t_0 considered. Convergent reconstructions $\hat{\mathbf{u}}^*$ obtained from both full and sparse data by use of the CP algorithm to solve DKL-fTV with $t_0 = 13488$ are displayed Fig. 7, along with the reference reconstruction. The results indicate that, in general, reconstructions from full data appear visually to possess better spatial resolution and lower noise level than do those from sparse data, and that reconstructions obtained with program DKL-fTV seem to reveal enhanced spatial and contrast resolution in which hot/cold rods of size 6.4 mm appear to remain resolved. As shown in the pseudo code for the CP algorithm, latent image $\mathbf{f}$ defined in Eq. (3) can also be reconstructed. It is interesting to display convergent reconstructions $\hat{\mathbf{u}}^*$ and $\hat{\mathbf{f}}^*$ of desired and latent images $\mathbf{u}$ and $\mathbf{f}$. As shown in Fig. 8, it can be observed that $\hat{\mathbf{f}}^*$ tends to have noisier textures than $\hat{\mathbf{u}}^*$.

4.4. Reconstruction as a function of iterations

The reconstructions above were obtained when the convergence conditions in Eq. (13) were satisfied. In an attempt to inspect how they evolve as functions of iterations, we display in Fig. 9 those obtained from full data with program DKL-fTV in Eq. (4) at iterations $n = 50, 300, \text{ and } 800$, as well as convergent reconstruction $\hat{\mathbf{u}}^*$. Furthermore, convergence metrics $\text{CTV}(\mathbf{f}_n)$ and $D'_{KL}(\mathbf{u}_n)$, computed from the reconstructions are shown in Fig. 9 as functions of iterations. Similar reconstructions and plots were obtained also for the sparse-data study, but are not shown here. These results reveal that the reconstruction at, e.g., about iterations 300 is visually similar to the convergent reconstruction.

5. Physical-Phantom-Data Study

The IEC phantom is adopted widely for quantitative characterization of PET-image reconstructions. We collected full data of ~ 100 million total counts from the phantom by using a full-scan configuration, as depicted in Sec. 2.1, in a digital prototype PET/CT system. From the full data, we extracted sparse data to mimic data collected with the sparse-scan configuration. As shown in the Appendix, the IEC phantom is composed of 6 fillable spheres of diameters 10, 13, 17, 22, 28, and 37 mm, respectively, in which the two largest spheres have zero activity, while the other four are filled with positron-emitter activity at a concentration level 4 times the background-activity level. Scatter and random events were

measured, and used as known components in the study. Images are reconstructed on a 3D array of $100 \times 100 \times 41$ identical cubic voxels of size 4 mm.

5.1. Determination of image-constraint parameter t_0

In the study, given the practical convergence conditions in Eq. (13), all program parameters but image-constraint parameter t_0 were determined as described in Sec. 2.5. Percent contrasts of hot and cold spheres and percent background variability, described in the Appendix, are standard metrics designed for evaluation of reconstruction quality of the IEC-phantom. In this study, full knowledge of the truth image is unknown. Therefore, combining the metrics, we form in the Appendix a single quality metric, referred to as the QNR, for determination of t_0 . For a set of t_0 values, convergent reconstructions from full data of the IEC phantom were obtained by use of the CP algorithm to solve program DKL-fTV in Eq. (4); and the QNRs calculated from the reconstructions are plotted in Fig. 10, along with the reconstructions with QNRs highlighted by black diamonds. Based upon the QNR result, we select $t_0 = 977$, which appears to yield a maximum QNR, for reconstructions from full and sparse data of the IEC phantom below.

5.2. Reconstructions based upon different optimization programs

Using the selected t_0 , we reconstruct IEC images from full data by solving programs DKL-fTV, DL2-fTV, and DL1-fTV in Eqs. (4)–(6), and display them in Fig. 11. Again, the observation can be made that convergent reconstructions obtained with programs DKL-fTV and DL2-fTV appear visually comparable, while the latter has slightly noisier textures than the former, and that program DL1-fTV yields reconstructions with prominent cupping artifacts. Similar to that in the Jaszczak-phantom study, the data- ℓ_1 -norm minimization yields the estimated model data of the IEC phantom “biased” toward zero, as shown in Fig. 6b, and such a data-estimate “bias” thus gives rise to the artifacts observed in the DL1-fTV-based reconstruction of the IEC phantom in Fig. 11. Again, artifacts in the DL1-fTV reconstruction may vary with program parameters different than those used in the work.

5.3. Reconstructions based upon optimization program DKL-fTV

For reasons described in section 4.3, we select program DKL-fTV for further study on image reconstruction from full and sparse data of the IEC phantom. In Fig. 12, we show convergent reconstructions $\hat{\mathbf{u}}^*$ obtained with program DKL-fTV from full and sparse data of the IEC phantom. As expected, the reconstruction from full data appears to have a level of spatial and contrast resolution slightly higher than the reconstruction from sparse data, while both convergent reconstructions have a relatively low level of background noise in which the hot spot of the smallest size (i.e., diameter of 10 mm) remains visible. We subsequently computed from the reconstructions quantitative metrics described in the Appendix, and display them in Fig. 13. These results suggest that convergent reconstructions from sparse data are slightly inferior to those from full data, as expected, and that they appear to be largely comparable to, or better than, the reference reconstructions from full data. Again, we display in Fig. 14 convergent reconstructions $\hat{\mathbf{u}}^*$ and $\hat{\mathbf{f}}^*$ of desired and latent images $\mathbf{u}$ and $\mathbf{f}$, and observe that the latter is slightly noisier than the former.

5.4. Reconstruction as a function of iterations

Convergent reconstructions $\hat{\mathbf{u}}^*$ of the IEC phantom above were obtained again when the convergence conditions in Eq. (13) are satisfied. In an attempt to reveal the reconstruction evolution, we display in row 1 of Fig. 15 images of the IEC phantom reconstructed from full data at iterations $n=10, 50$, and 200 as well as convergent reconstruction $\hat{\mathbf{u}}^*$. It can be observed that the reconstruction at, e.g., about iteration 200 visually resembles convergent reconstruction $\hat{\mathbf{u}}^*$. Similar observations can be made for reconstructions obtained from sparse data, which are not shown here. We also calculated quality metrics described in the Appendix from full- and sparse-data reconstructions at different iterations, and show in Fig. 16 $Q_{H,2}$ of the hot sphere 2 with a diameter of 13 mm, $Q_{C,5}$ of the cold sphere 5 with a diameter of 28 mm, and N_3 within the ROIs of size 17 mm of sphere 3. Results in Fig. 16 confirm that reconstructions at about iteration 200 resemble their corresponding convergent reconstructions.

6. Human-Data Study

In the human-data study, full data were acquired from the subject by use of a full-scan configuration, as depicted in Sec. 2.1, in a digital prototype PET/CT system at a single patient-bed position. In an effort to increase the longitudinal coverage of the human subject, 5 full scans were performed at 5 bed positions along the central axis of the system with a separation of 70 mm between the centers of two consecutive bed positions. The 5 sets of full data acquired contain approximately 23.9, 25.0, 26.7, 24.3, and 22.1 million total counts. Again, we also obtained five sparse data sets each of which was extracted from one of the five sets of full data by use of the method described in Sec. 2.1. Scatter and random measurements were included in the study. We reconstruct the human image on a 3D array consisting of $N=144 \times 144 \times 41$ identical cubic voxels of size 4 mm.

6.1. Determination of image-constraint parameter t_0

In the study, again, given the practical convergence conditions in Eq. (13), all of the program parameters, but image-constraint parameter t_0 , are determined as described in Sec. 2.5. Unlike the studies on the digital Jaszczak phantom in Sec. 4 and the physical IEC phantom in Sec. 5 in which quantitative metrics were used for selecting t_0 , we selected t_0 for the human study based simply upon visual inspection. (It is not entirely clear how a technical efficacy metric analogous to those in the phantom studies can be devised meaningfully for the human study. Although clinical efficacy metrics for a specific clinical task may be devised and used meaningfully for the determination of t_0 optimal for the clinical task, it is beyond the scope of the current work.) For full data collected at bed position 3, we reconstructed human images by solving program DKL-fTV in Eq. (4) for a set of t_0 values, and display in Fig. 17 convergent reconstructions, obtained with $t_0 = 230, 287, 345, 460, 575$, and 690, respectively. As expected, high t_0 values yield reconstructions with noisy texture while revealing some structural details; conversely, low t_0 values lead to reconstructions with smoothed texture but missing structural details. Based upon the visual inspection of the reconstructions, we select $t_0 = 460$ because it appears to yield reconstructions with a reasonable balance between structural details and image-noise

texture. Using the same method, t_0 's were determined for reconstructions from other full- and sparse-data sets.

6.2. Reconstructions based upon different optimization programs

In an attempt to investigate the impact of optimization programs, we carried out a study on reconstructions by using the CP algorithms to solve programs DKL-fTV, DL2-fTV, and DL1-fTV in Eqs. (4)–(6). For full data collected at a single bed position, we selected t_0 as described above and then reconstruct human images by solving the three programs. We show in Fig. 18 the reconstructions from full data collected at bed position 3. Reconstruction obtained with program DKL-fTV appears to possess better delineated boundaries, more structural details, and lower texture noise than do that obtained with program DL2-fTV. However, program DL1-fTV yields a convergent reconstruction with strong artifacts, i.e., with numerically zero values, as depicted in panel 3 of Fig. 18. Again, the estimated model data become negatively “biased” as a result of the data- ℓ_1 -norm minimization and scatter correction. Consequently, zero-valued estimated model data revealed in Fig. 6c and, the zero-valued $\hat{\mathbf{u}}^*$ (or, equivalently, zero-valued $\hat{\mathbf{f}}^*$) shown in Fig. 18, were obtained with the positivity constraint. Clearly, when different program parameters such as t_0 are used, program DL1-fTV may yield reconstructions different than a zero-valued image. Similar results were obtained for reconstructions from full and sparse data collected at this and other bed positions.

6.3. Reconstructions based upon optimization program DKL-fTV

Based upon the observation above, we select program DKL-fTV for further study on image reconstruction from human data. For a single bed position, using t_0 selected in Sec. 6.1 we reconstructed images from full data collected by using the CP algorithm to solve program DKL-fTV. Repeating this for all of the five bed positions, we obtained five convergent reconstructions, and then summed them up to form a *final-convergent* reconstruction with an extended longitudinal coverage. Following the same procedure, we also obtained the final-convergent reconstruction of the human subject from the five sets of sparse data. In Figs. 19–21, we display the final-convergent reconstructions within transverse, coronal, and sagittal slices of the human subject. It can be observed that program DKL-fTV appears to yield reconstructions from full data with well-delineated boundaries and suppressed noise in the background region. Similar observation can be drawn for reconstructions from sparse data as well, despite the fact that reconstructions from sparse data are visually somewhat noisier than those from full data, as expected. We also display in Fig. 22 final-convergent reconstructions $\hat{\mathbf{u}}^*$ and $\hat{\mathbf{f}}^*$ of desired and latent images $\mathbf{u}$ and $\mathbf{f}$.

6.4. Reconstruction as a function of iterations

Reconstructions of the human images above were obtained again when the practical convergence conditions in Eq. (13) are satisfied. We also investigated how the summed reconstruction of the human subject evolves as a function of the iteration number. We show in Fig. 23 summed reconstructions within a coronal slice of the human subject at iteration numbers $n = 10, 100, 300, 500, 600, 800$, and 1000, together with final-convergent reconstruction $\hat{\mathbf{u}}^*$. It can be observed that the reconstruction at, e.g., about iteration 600,

visually resembles the convergent reconstruction. Similar observations can also be made for reconstructions obtained from sparse data of the human subject.

7. Discussions

We have performed an investigation on tailoring of TV-constrained optimization techniques developed for CT imaging to reconstructing PET images. In particular, we have studied whether they can be exploited for enabling a PET configuration with sparsely populated detectors by extracting the sparse data from the full data. We point out that, in a realistic sparse-scan configuration, the sensitivity of detector blocks and scatter/random behavior may be considerably different than that of the simulated sparse-scan configuration considered. The work is not intended to demonstrate whether the optimization-based reconstruction investigated is superior to existing reconstruction techniques, because such a demonstration, which is beyond the scope of the work, can meaningfully be carried out only in carefully designed evaluation studies in which imaging tasks and the associated utility metrics must be clearly defined. Instead, the goal of the work is to investigate technical characteristics in the application of TV-constrained optimization reconstructions to PET imaging.

Optimization programs with different data divergences were investigated in the work, because evidence exists indicating their potential theoretical and practical utility in CT imaging. In each of the programs considered, an image-TV constraint was used, because accumulating results in CT imaging appear to suggest that the TV-constrained programs may yield reconstructions superior, or at least comparable, to those obtained with other optimization-based reconstructions. It is meritorious to investigate additional optimization programs with different data divergences and image constraints.

Data collected in PET imaging generally have SNR considerably lower than that of data in typical CT imaging, and transitions among different uptake regions in a PET-activity map is observed to be generally not as sharp as that among some anatomic regions in a CT-attenuation map. Therefore, a desired PET image is formulated as a product of a latent image and Gaussian blurring matrix, and the TV constraint is applied to the latent image instead of the desired PET image. This formulation thus allows for a latent image with sparser gradient magnitude image than the desired image, and avoids yielding an image with significant patchy textures when the data have low SNR [46].

A central focus of this work is to clarify clearly parameters involved in, and to demonstrate their impact on, optimization-based reconstructions. While it is natural to appreciate the impact of parameters in a given optimization program, the form of the optimization program itself is also a parameter that can significantly affect PET-image reconstruction. This is the motivation of the study carried out on optimization programs with different data divergences. Based upon the study results, we observe that, for data conditions considered, program DKL-fTV appears to yield reconstructions superior to those obtained with the other two programs investigated.

In addition to the program form, numerous parameters used for specification of a program can have a significant impact on the final reconstruction. Numerous parameters were identified explicitly and investigated, as discussed in Sec. 2.5. Among the parameters, image-TV-constraint parameter t_0 remains a leading factor affecting reconstruction properties. Therefore, we have specifically investigated the effect of parameter t_0 in real-data studies, and on devising the selection of t_0 specifically tailored to a set of given study conditions, involving different scanning configurations, imaged objects, and data properties. Although some evidence was given in the work that justifies the selection of other parameters, including voxel size, matrix $\mathcal{G}$, scatter/random events, and convergence conditions, it remains interesting and necessary to investigate more thoroughly their optimal selections for specific applications.

For example, scatter component $\mathbf{g}_s$ with attenuation-effect corrected is considered a program parameter that is estimated from experimental measurements in the work. Clearly, the degree of estimation variability of $\mathbf{g}_s$ can impact the reconstruction. In an attempt to inspect the impact, we repeated the DKL-fTV reconstruction from full data of the human subject at bed position 3. Using $\mathbf{g}_s$ obtained experimentally at bed position 3, we created hypothetically “under”- and “over”-estimated scatter events by scaling $\mathbf{g}_s$ with a factor ranging from 0 to 2. In Fig. 24, we show convergent reconstructions from the data corrected with scatter events 0, $\mathbf{g}_s$, and $1.8\mathbf{g}_s$, respectively. The visual difference among these images indicates the effect of different estimates of the scatter events on the reconstruction. The study can conversely be exploited for fine-tuning the estimate of the scatter events for yielding desired reconstructions.

We have carried out image reconstructions in different studies involving objects with considerably distinct activity-uptake distributions of practical relevance and data with different quality/quantity conditions of interest. The results show that the reconstruction based upon program DKL-fTV appears to be robust for the different activity up-takes and data sets under consideration. Moreover, a study was conducted for image reconstruction from data collected with a PET configuration containing only half of the detectors in a digital prototype PET/CT scanner. The study reveals, on one hand, the robustness of DKL-fTV reconstruction with respect to significantly different data conditions, and, on the other hand, its potential for enabling PET-scanner configuration with a sparse detector distribution. It is of merit to investigate possible enabling of additional designs of PET configurations, which are of application-, workflow-, and/or cost-reduction-interest, by use of optimization-based reconstructions. Such an investigation can also provide a further assessment of the robustness of optimization-based reconstructions under additional, different imaging and data conditions.

In this study, the investigation was based upon a data model in which list-mode and/or TOF information is not considered because an element of the system matrix is designed as the intersection of an LOR with a voxel in the image array. Therefore, the data model and optimization programs considered cannot be applied to list-mode and/or TOF-PET data. We are currently developing optimization-based reconstruction to incorporate list-mode and TOF information, and expect to report the results elsewhere in the future. It would be of high interest to exploit list-mode and TOF information for possibly enabling the design,

investigation, and implementation of innovative PET configurations of workflow-, cost-reduction-, and/or application-significance.

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Appendix

We display in Fig. 25 a transverse slice of the IEC phantom used widely for evaluating PET-image quality. Six spheres labeled as s_1 , s_2 , s_3 , s_4 , s_5 , and s_6 are embedded within the background and are with diameters of 10, 13, 17, 22, 28, and 37 mm, respectively. Spheres s_1 – s_4 , referred to hot spheres, have an identical concentration level of positron emitters, which is 4 times of that in the background, whereas spheres s_5 and s_6 referred to as cold spheres contain no positron emitters. The dark circle at the center displays a transverse cross section of the cylinder containing zero activity in the phantom. Additionally, 12 identical circular background ROIs of diameter 37 mm are drawn in the slice, as shown in Fig. 25, and also in each of its four nearest neighboring slices, thus amounting to a total of 60 background ROIs. Within each ROI, 6 sub-ROIs of sizes corresponding to those of the hot and cold spheres are also drawn, as indicated in the top ROI, and thus a total of $T = 60$ sub-ROIs for each of the sphere sizes is obtained, which are used below for calculating the average background activity for the corresponding sphere.

Figure 25: A transverse slice of the IEC phantom containing six spheres, labeled as s_1 , s_2 , s_3 , s_4 , s_5 , and s_6 . Hot spheres s_1 – s_4 hold an identical level of activity concentration, 4 times of that in the background, whereas cold spheres s_5 and s_6 contain no activity. The dark circle at the center of the phantom displays a cross section of the central cylinder containing zero activity. Also, 12 identical circular background ROIs of diameter 37 mm are drawn. Within

each ROI, 6 concentric sub-ROIs of sizes of the 6 spheres are also drawn, as indicated in the top ROI.

Using $C_{B,t,k}$ to denote the average background activity within sub-ROI t of the size of sphere k , we define an average background activity corresponding to sphere k as

$C_{B,k} = \sum_{t=1}^T C_{B,t,k}$. With this, percent contrasts $Q_{H,j}$ and $Q_{C,i}$ for hot sphere j , where $j = 1, 2, 3$, and 4, and cold sphere i , where $i = 5$ and 6, and background variability N_k for sphere k , where $k = 1, 2, 3, 4, 5$ and 6, defined in NEMA NU 2-2012 [53] are calculated as

$$Q_{H,j} = \frac{C_{H,j} - C_{B,j}}{a_H - a_B} \times 100\%,$$

$$Q_{C,i} = \left(1 - \frac{C_{C,i}}{C_{B,i}}\right) \times 100\%, \quad (16)$$

$$N_k = \frac{\sqrt{\sum_{t=1}^T (C_{B,t,k} - C_{B,k})^2 / (T - 1)}}{C_{B,k}} \times 100\%, \quad (17)$$

where a_H and a_B denote truth activity concentrations in a hot sphere and background, $a_H = 4 \times a_B$, and $C_{H,j}$ and $C_{C,i}$ are the average activities within hot sphere j and cold sphere i .

We define a metric, which takes into account the trade-off between contrast and background noise, as

$$\text{QNR} = \frac{Q_{H,2}}{N_2} \frac{Q_{C,5}}{N_5}, \quad (18)$$

where $Q_{H,2}$ and $Q_{C,5}$ denote percent contrasts for hot sphere s_2 of diameter 13 mm and for cold sphere s_5 of diameter 28 mm, and N_2 and N_5 percent background variabilities corresponding to the two spheres.

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Figure 1.

Full- and sparse-scan PET configurations considered. In the full-scan configuration, 18 modules, each of which consists of 5×4 identical tiles of square shape, are assembled tightly on a cylindrical surface, forming effectively 5 full rings of tiles. Each tile itself is composed of 8×8 identical, square-shaped crystal bins of size 4 mm. The sparse-scan configuration is obtained by the removal of odd- or even-numbered tiles in an odd- or even-numbered ring in the full-scan configuration.

Figure 2.

Truth image $\mathbf{u}_{\text{true}}$, convergent reconstruction $\hat{\mathbf{u}}^*$, and difference $\hat{\mathbf{u}}^* - \mathbf{u}_{\text{true}}$. Display window $[0.0, 0.1]$ for the truth image and reconstruction, and display window $[-0.0001, 0.0001]$ for their difference. Image values are in arbitrary units.

Figure 3.

Convergence metrics $\text{CTV}(\mathbf{f}_n)$, $D'_{KL}(\mathbf{u}_n)$, $\text{cPD}(\mathbf{u}_n)$, and $\bar{D}(\mathbf{u}_n)$ as functions of iteration number n .

Figure 4.

Convergent reconstructions $\hat{\mathbf{u}}^*$ within transverse slices containing cold (row 1) and hot (row 2) rods in the Jaszczak phantom obtained from full data with program DKL-fTV for different t_0 values. The plot shows metrics RMSE calculated from $\hat{\mathbf{u}}^*$ (solid) and the reference reconstruction (dashed), as functions of t_0 .

Figure 5.

Convergent reconstructions $\hat{u}^*$ within transverse slices containing cold (row 1) and hot (row 2) rods in the Jaszczak phantom obtained from full data with programs DKL-fTV, DL2-fTV, and DL1-fTV in Eqs. (4)–(6), respectively. Display windows: [0, 40000] (row 1) and [0, 15000] (row 2). Note that the DL1-fTV reconstruction of the hot-rod section contains a large, zero-valued region.

Figure 6.

Profiles of measured data g'_m (thin solid), and model data g'_{KL} (thick solid), g'_{ℓ_2} (dotted), and g'_{ℓ_1} (dashed) obtained with programs DKL-fTV, DL2-fTV, and DL1-fTV for (a) the Jaszczak phantom, (b) the IEC phantom, and (c) the human subject.

Figure 7.

Reference reconstructions (column 1), and convergent reconstructions $\hat{\mathbf{u}}^*$ within transverse slices containing cold (row 1) and hot (row 2) rods in the Jaszczak phantom obtained from full (column 2) and sparse (column 3) data with program DKL-fTV. Display windows: [0, 40000] (row 1) and [0, 15000] (row 2).

Figure 8.

Convergent reconstructions $\hat{u}^*$ and $\hat{f}^*$ within transverse slices containing cold (row 1) and hot (row 2) rods in the Jaszczak phantom obtained from full (columns 1 & 2) and sparse (columns 3 & 4) data with program DKL-fTV. Display windows: $[0, 40000]$ (row 1) and $[0, 15000]$ (row 2).

Figure 9.

Reconstructions $\mathbf{u}_n$ at iteration $n = 50, 300$, and 800 within a transverse containing cold rods in the Jaszczak phantom from full data with program DKL-fTV, along with convergent reconstruction $\hat{\mathbf{u}}^*$ (iteration 71160). Display windows: $[0, 40000]$. The two plots display convergence metrics $\text{CTV}(\mathbf{f}_n)$ and $D'_{KL}(\mathbf{u}_n)$ as functions of iterations n .

Figure 10.

Convergent reconstructions $\hat{\mathbf{u}}^*$ within the central transverse slice of the IEC phantom obtained from full data with program DKL-fTV for different t_0 values. The plot shows metrics QNR calculated from $\hat{\mathbf{u}}^*$ (solid) and the reference reconstruction (dashed), as functions of t_0 .

Figure 11.

Convergent reconstructions $\hat{u}^*$ within the central transverse slice of the IEC phantom obtained from full data with programs DKL-fTV, DL2-fTV, and DL1-fTV, respectively. Display window: [0, 16000]. Note that prominent artifacts can be observed in the DL1-fTV reconstruction.

Figure 12.

Reference reconstructions (column 1), and convergent reconstructions $\hat{\mathbf{u}}^*$, within the central transverse (row 1) and central coronal (row 2) slices of the IEC phantom, obtained from full (column 2) and sparse (column 3) data with program DKL-fTV. Display window: [0, 16000].

Figure 13.

Percent contrast $Q_{H,j}$ of hot spheres, where $j=1, 2, 3$, and 4 , percent contrast $Q_{C,i}$ of cold spheres, where $i=5$ and 6 , and percent background variability N_k , where $k=1, 2, 3, 4, 5$, and 6 , respectively, calculated based upon reference reconstruction ($\circ$) and convergent reconstructions $\hat{\mathbf{u}}^*$ from full ($\times$) and sparse (∇) data of the IEC phantom shown in Fig. 12.

Figure 14.

Convergent reconstructions $\hat{\mathbf{u}}^*$ and $\hat{\mathbf{f}}^*$ within the central transverse (row 1) and central coronal (row 2) slices of the IEC phantom obtained from full (columns 1 & 2) and sparse (columns 3 & 4) data with program DKL-fTV. Display window: [0, 16000].

Figure 15.

Reconstructions $\mathbf{u}_n$ at iteration $n = 10, 50$, and 200 , along with the convergent reconstruction (iteration 2701), within the central transverse slice (row 1) and central coronal slice (row 2) of the IEC phantom obtained from full data with program DKL-fTV. Display window: $[0, 16000]$.

Figure 16.

Percent contrast $Q_{H,2}$ of hot sphere 2, percent contrast $Q_{C,5}$ of cold sphere 5, and percent background variability N_3 of an ROI of the size of sphere 3, respectively, as functions of iterations n , computed based upon u_n and upon converged reconstruction u^* , obtained from full (X) and sparse (∇) data with program DKL-fTV, along with results obtained from the reference reconstruction (dotted).

Figure 17.
 Negative convergent reconstructions $\hat{\mathbf{u}}^*$ within a transverse slice of the human subject
 obtained from full data at bed position 3 with program DKL-fTV for different t_0 values.
 Display window: $[-1600, 0]$.

$\hat{\mathbf{u}}^*$, DKL-fTV $\hat{\mathbf{u}}^*$, DL2-fTV $\hat{\mathbf{u}}^*$, DL1-fTV**Figure 18.**

Negative convergent reconstructions $\hat{\mathbf{u}}^*$ within a transverse slice of the human subject obtained from full data at bed position 3 with programs DKL-fTV, DL2-fTV, and DL1-fTV. Display window: $[-1600, 0]$. Note that program DL1-fTV yields a zero-valued reconstruction.

Figure 19.

Negative reference reconstructions (column 1), and negative final-convergent reconstructions $\hat{\mathbf{u}}^*$, within two transverse slices (rows 1 and 2) of the human subject obtained from full (column 2) and sparse (column 3) data with program DKL-fTV. Display window: $[-1600, 0]$.

Figure 20.

Negative reference reconstructions (column 1), and negative final-convergent reconstructions $\hat{\mathbf{u}}^*$, within two coronal slices (rows 1 and 2) of the human subject obtained from full (column 2) and sparse (column 3) data with program DKL-fTV. Display window: $[-1600, 0]$.

Figure 21.

Negative reference reconstructions (column 1), and negative final-convergent reconstructions $\hat{\mathbf{u}}^*$, within two sagittal slices (rows 1 and 2) of the human subject obtained from full (column 2) and sparse (column 3) data with program DKL-fTV. Display window: $[-1600, 0]$.

Figure 22.

Negative final-convergent reconstructions $\hat{\mathbf{u}}^*$ and $\hat{\mathbf{f}}^*$ within a transverse (row 1), coronal (row 2), and sagittal (row 3) slices of the human subject obtained from full (columns 1 & 2) and sparse (columns 3 & 4) data with program DKL-fTV. Display window: $[-1600, 0]$.

Figure 23.

Negative reconstructions $\mathbf{u}_n$ at iteration $n = 10, 100, 400, 500, 600, 800$, and 1000 , along with negative final-convergent reconstruction $\hat{\mathbf{u}}^*$, within a coronal slice of the human subject obtained from full data of all bed positions with program DKL-fTV. Display window: $[-1600, 0]$.

Figure 24.

Negative convergent reconstructions $\hat{\mathbf{u}}^*$ within a transverse (row 1), coronal (row 2), and sagittal (row 3) slices of the human subject obtained with program DKL-fTV from full data at bed position 3 corrected for scatter events by use of $0.0\mathbf{g}_s$ (column 1), $\mathbf{g}_s$, and $1.8\mathbf{g}_s$, respectively. Display window: $[-1600, 0]$.