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Hybrid radial-cones trajectory for accelerated magnetic resonance imaging

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Abstract

Purpose—To design and develop ultra-short echo time k-space sampling schemes, radial-cones, which enable high sampling efficiency while maintaining compatibility with parallel imaging and compressed sensing reconstructions.

Theory and Methods—Radial-cones is a trajectory which samples 3D k-space utilizing a single base cone distributed along radial dimensions through a cost function based optimization. Trajectories were generated for highly undersampled, short readout sampling and compared to 3D radial sampling in point spread function (PSF) analysis, digital and physical phantoms, and initial human volunteers. Parallel imaging reconstructions were evaluated with and without the use of compressed sensing based regularization.

Results—Compared to 3D radial sampling, radial-cones reduced the peak value and energy of PSF aliasing. In both digital and physical phantoms, this improved sampling behavior corresponded to a reduction in the root-mean square error with a further reduction utilizing compressed sensing. A slight increase in noise and corresponding increase in apparent resolution was observed with radial-cones. In in-vivo feasibility testing, radial-cones reconstructed images has markedly lower apparent artifacts. Ultimate gains in imaging performance were limited by off-resonance blurring.

Conclusion—Radial-cones is an efficient Non-Cartesian sampling scheme enabling short echo readout with a high level of compatibility with parallel imaging and compressed sensing.

Keywords

pseudo-random; sampling; spiral MRI; ultra-short echo times; radial MRI

Introduction

Magnetic Resonance Imaging (MRI) with Non-Cartesian trajectories can substantially reduce sensitivity to artifacts. For example, center out 2D radial sampling provides insensitivity to T2 signal decay (1), flow (2), motion (3) and data inconsistencies from contrast modulation between shots (4,5). This is especially true when combined with ultra-

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short echo time techniques (1). However, center out 2D radial sampling, required for ultra-short echo time imaging, results in a degradation of sampling and signal to noise (SNR) efficiency, the k-space coverage and SNR per unit time respectively. For Nyquist sampling, it requires twice as many views as compared to Cartesian. The incorporation of angular twisting into radial to form spiral trajectories creates an opportunity to harness readout time for improved sampling and SNR efficiency. In fact, constant density spiral sampling with a long readout is the most sampling and SNR efficient method of covering 2D k-space. Such sampling with long readouts is unfortunately also highly sensitive to artifacts from off-resonance (6), flow (7), concomitant gradients (8), and inter-shot data inconsistencies (9). Further, constant density spiral sampling leads to structured undersampling artifacts which make it less amenable to acceleration utilizing compressed sensing without additional randomization (10). Hybrid sampling with short readout, variable density spirals provides an attractive alternative which can be tailored to balance sampling efficiency with artifacts; for example, to reduce sensitivity to motion(9).

The conflict between sampling efficiency and sensitivity to artifacts remains when considering fully three dimensional Non-Cartesian sampling patterns. Fully 3D trajectories can be divided into three basic types: radial, twisting, and optimization driven, all of which have certain strengths. 3D radial trajectories can be insensitive to signal decay (11) and purposeful or incidental undersampling (12). However, this comes at the cost of low sampling ($\sim 1/4$ of 3D Cartesian) and SNR ($\sim 74\%$ of 3D Cartesian) efficiency. Twisting sampling strategies include spiral projection imaging (13), stack of cones (13), FLORET (14), and distributed spirals (15). All can provide higher sampling and SNR efficiency than Cartesian and do so by structured organization of spiral-like trajectories. Thus they maintain similar or moderately improved sensitivity to artifacts as compared to 2D spirals. A third class of 3D trajectories has been proposed based on constrained optimization (16,17). Such techniques use optimization principles to create trajectories which satisfy user defined metrics. In principle these techniques hold potential to create, as implied by optimization, the best possible sampling pattern for a given task. However, there is a high degree of freedom when filling a 3D space with 1D lines and accurate task performance metrics are expensive to evaluate forcing the use of simple geometric properties. As such, the optimization can be computationally impractical or utilize a metric that incompletely captures imaging performance.

In this work, we develop a hybrid trajectory that draws benefits from 3D cone trajectories, 3D radial sampling, and systematic optimization. This hybrid trajectory, dubbed radial-cones, allows and even promotes incoherent sampling, which allows image recovery with compressed sensing reconstruction. We compare this new sampling scheme to 3D radial sampling in digital simulations, physical phantoms, and in-vivo.

Theory

Radial-cones is a heuristically designed trajectory; the generation of which is outlined in Figure 1. The aim of the trajectory is to maximize k-space sampling efficiency for a given readout length while maintaining an incoherent point spread function and center-out sampling properties. First, a base cone is designed to maximize k-space sampling for a given

readout duration and maximum radial extent in 3D k-space, $k_{\max}$. This base cone is a center-out, cone-like trajectory. Subsequently, a set of radial-cones are generated by replicating and rotating the base cone in k-space such that the cone axes ($\hat{r}$) are oriented along a set of isotropic radial directions ($\hat{r}'$). A rotation of each radially distributed cone about its cone axis is performed utilizing a pseudo-random cone rotation angle. The end result is a center-out, 3D, Non-Cartesian trajectory, dubbed radial-cones, which progresses from radial at the center of k-space to pseudo-random sampling at the edge of k-space. The design of the trajectory is split into the design of the base cone and the orientation of the base cones to cover 3D k-space.

Base Cone Design

The radial-cones trajectory generation guarantees that k-space will not be uniformly sampled. As such, the design of each basic shot, a base cone, is not strictly constrained to provide exact, uniform spacing. The base cone k-space trajectory, $\vec{k}_{base}$, is described as a function of α , an arbitrary scalar parameter, that ranges from 0 to $k_{\max}$:

$$\vec{k}_{base}(\alpha) = \hat{r}\alpha \cos \phi + \frac{\alpha^2}{k_{\max}} \sin \phi \left(\hat{t}_x \cos \theta(\alpha) + \hat{t}_y \sin \theta(\alpha) \right) \quad \text{Eq 1}$$

where $\hat{r}$ is a unit vector, the cone axis, and $\hat{t}_x$ and $\hat{t}_y$ are unit vectors orthogonal to each other and to $\hat{r}$. For convenience during design, these axes can be set to the Cartesian space vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$. ϕ is the cone angle at the edge of k-space, as shown in Figure 1. The cone angle is a fixed parameter which dictates the cone envelope and transitions from pure radial ($\phi=0$) to spiral ($\phi=\frac{\pi}{2}$). For intermediate values, the envelope of this trajectory about the cone axis maintains a constant solid-angle in k-space. The angular position, or more specifically the rate of change in angular position, about the cone axis, $\theta(\alpha)$, controls the amount of k-space coverage at each radial position. For a set of 3D radially oriented trajectories, the k-space surface which must be covered for uniform sampling increases with the square of k-space radius, k_r^2 . This is due to the spreading of trajectories and is the reason why center-out 3D radial sampling requires 4 times as many shots as Cartesian sampling with corner cutting. To counteract this effect, we aim to design base cone trajectories with k-space coverage, defined as the derivative of the arc length, that increases with k_r^2 . To solve for such an $\theta(\alpha)$, given a desired readout length, we utilize $\theta(\alpha)$ of the form:

$$\theta(\alpha) = A\alpha^p \quad \text{Eq 2}$$

where p controls the shape and A controls the overall length of the readout. This form of angular change is continuously differentiable which prevents discontinuities in curvature of the final base cone trajectory that would require reducing the gradient to zero. A can be fully determined from the values p , the target readout length, the target resolution, the cone angle, and gradient system parameters. To determine p , we minimize the fitting residue with a k_r^2 function:

$$\underset{p}{\operatorname{argmin}} \left\| \operatorname{arclength}_{k_r} \left(\vec{k}_{base}(\alpha) \right) - S k_r^2 \right\|_2^2 \quad \text{Eq 3}$$

where $\operatorname{arclength}_{k_r}$ is the arc length as a function of k-space radius and S is a nuisance parameter, calculated by a closed form expression, that minimizes the residue. Given the error function in Eq. 3 and a fixed readout length, a multi-resolution search was performed to determine p and A . For a candidate p , trajectories are calculated for increasing values of A until the readout length is greater than or equal to the desired readout length. The step size is reduced and A is incremented again starting at the last value in which the readout length was less than the desired readout length. This process is repeated until the readout length is within tolerances. For this work, the gradient temporal resolution and tolerance was assumed to be $4\mu\text{s}$, p was initialized to 0.1 with an initial step size of 0.3, and A was initialized to 0 with an initial step size of 1. The value of Eq. 3 is recorded for the given p . Candidate values of p are evaluated, increasing p by a step size, until the cost function (Eq. 3) no longer reduces. The step size is then reduced and a search about the current value of p is performed. For calculations in this work, p was initialized to 0 and was refined with a final step size of 0.001. Typical final values of A and p are 1 and 3 respectively. Gradients and subsequent readout times are calculated using previously described, rapid, discrete arc length optimization (18). To avoid undersampling along the base cone readout, the maximum gradient strength, $g_{\max}$, is set to:

$$g_{\max} = \max \left(\frac{BW}{\bar{\gamma} FOV}, g_{\max\text{-system}} \right) \quad \text{Eq 4}$$

where BW is the desired data sampling bandwidth, FOV is the field of view, $\bar{\gamma}$ is the gyromagnetic ratio divided by 2π , and $g_{\max\text{-system}}$ is the maximum gradient strength of the system. Given this optimization at a fixed readout length, the base cone is a function of the cone angle (ϕ), gradient system parameters, FOV, and $k_{\max}$.

Cone Orientation

In order to cover 3D k-space, many base cones are required with different cone orientations (i.e. the resulting directions of $\hat{r}$, $\hat{t}_x$ and $\hat{t}_y$). The number of base cones (shots) required to fully sample k-space is at minimum equal to that required by 3D radial divided by the relative arc length of the base cone trajectory. However, more shots will be required when significant overlap occurs between shots. Each shot is a rotation of the base cone, described by two rotation matrices:

$$\vec{k}_{shot} = R_{\Omega} R_r, \vec{k}_{base} \quad \text{Eq 5}$$

where $\vec{k}_{base}$ is the k-space trajectory of the base cone, R_r is the rotation matrix which rotates the cone axis of the base cone ($\hat{r}$) to a target cone axis ($\hat{r}'$), R_{Ω} is a rotation about $\hat{r}'$

by a cone rotation angle (Ω), and k_{shot} is the resulting k-space trajectory of a given shot. The target cone axes, $\hat{r}'$, are fixed to set of isotropically distributed radial directions as designed by isohedral minimization (19). The radial-cones trajectory is identical to a 3D radial trajectory if the cone angle is equal to 0° (i.e. the base cone is a line). Due to the rotationally asymmetric nature of the base cone about the cone axis, a rotation of each radially distributed cone about its axis is required to avoid structured overlap and resulting gaps in k-space coverage. The cone rotation angle of each shot about its cone axes (i.e. the resulting directions of $\hat{t}_x$ and $\hat{t}_y$) is initially set at random. This creates incoherent sampling but does not minimize large gaps in k-space which may lead to substantial noise amplification when utilizing parallel imaging and increases the peak aliasing energy. To safeguard against such gaps, cone rotation angles are refined utilizing a greedy optimization scheme outlined in Figure 2. Starting with a given trajectory, a sampling density estimate is evaluated for each point in k-space. Subsequently, a simple search is performed to find the shot, $\hat{i}$, which has the maximum overlap with all other shots defined as the sum of squared density estimates:

$$\hat{i} = \arg \min_i \sum_{j=0}^N \rho_{j,i}^2 \quad \text{Eq 6}$$

where ρ is the density estimate, i is the shot index, j is the position along the readout, and N the number of points per readout. For the $\hat{i}$ th shot, a set of test cone rotation angles covering a full 2π rotation about the cone axis ($\hat{r}'$) is established. For each candidate cone rotation angle, the density is re-estimated and the cone rotation angle which minimizes the density estimate is chosen for the updated angle. This process is repeated until convergence, defined as the point when the global density does not change between iterations (i.e. the optimal candidate angle is the current angle). This process requires many iterations, at least as many as the number of shots, and is thus highly limited by computation time for density estimation. To do this in an efficient fashion, a grid and interpolation scheme is utilized to minimize the number of updates to the overall density estimate, as described in the Appendix.

Example source code for both base cone generation and rotation matrix optimization is available online (<https://bitbucket.org/kmjohnson3/radial-cones>).

Methods

Experiments were performed to characterize the influence of cone angle and readout length on image quality. In each case, radial-cones was compared to center out 3D radial sampling. Given uncertainty in determining the number of shots for radial-cones due to trajectory overlap, and potential artifact differences due to inter-shot data inconsistencies and contrast, the number of shots and repetition time was equalized for radial and radial-cone trajectories. Cone angle experiments were investigated in digital phantoms, physical phantoms, and initial human subjects imaging. Due to the complexity of modeling off-resonance, readout lengths were only investigated in PSF analysis, physical phantoms, and initial human subjects testing. All experiments assumed design for 0.86 mm isotropic resolution, 256 cubic matrix, maximum slew rate of 150 T/m/s, and maximum system gradient strength of

50 mT/m. The target maximum gradient strength was 25 mT/m corresponding to a sampling rate of 234 kHz for a 22cm FOV. To reduce errors due to the receiver sampling chain and signal outside the prescribed FOV, data sampling was performed at 500 kHz, 2 μ s per sample. Trajectory design, simulations, and reconstructions were performed utilizing code written in Matlab (Natick, Massachusetts, USA) and C++ on a Linux workstation (16 Virtualized AMD 6134 Cores, 64Gb ram). All studies involving human subjects were performed in compliance with the health insurance portability and accountability act (HIPAA) and all subjects gave written informed consent under a protocol approved by our institutional review board (IRB).

Cone Angle – Trajectories and Digital Simulations

Radial-cones trajectories were designed with cone angles ranging from 0° to 60° in 10° increments. Cone angle were only considered up to 60° as the packing efficiency of base cones is expected to reduce with increasing cone angle. Pure radial trajectories ($\phi=0^\circ$) were designed utilizing SNR optimized gradient design (18,20,21). For all trajectories the number of shots was set to 5,000 and the readout length was 2.5ms. For 3D radial sampling, this corresponds to an undersampling factor relative to Nyquist of 3.4 fold in each direction; a relatively high level compared to prior work. Given this readout length and limited overlap of base cone trajectories, radial-cones will reduce the undersampling factor to a more manageable undersampling factor of 2.1-2.5 in each direction, depending on the cone angle. This experiment thus tests the ability of radial-cones with varying base cone trajectories to reduce undersampling factors and hence reduce penalties from parallel imaging and compressed sensing for fixed imaging parameters. Cone rotation angle (Ω) optimization of radial-cone trajectories was performed utilizing a maximum of 100,000 iterations. For each iteration, 1000 test cone rotation angles spanning 360° were utilized resulting in angular spacing of 0.36°.

Point spread function (PSF) analysis was performed to examine the effect of cone angle (ϕ) on aliasing. PSFs were evaluated utilizing resolution preserving density correction obtained by iterative density estimates with a large kernel size (22) and optimized overgridding (23) with factor of 2. To reduce interpolation errors, k-space was zero filled to a 384 cubic matrix. As both 3D radial and radial-cones trajectories are approximately rotationally invariant, reconstructed PSFs were resampled into spherical shells to enable analysis as a function of radial distance. At each radius, the sum of squared values (the aliasing energy) and peak value was calculated. Noise performance estimates were determined from the sampling density correction (9) and normalized to SNR optimal sampling, where the density weights are uniform.

The same trajectories were utilized to simulate the acquisition of radio frequency (RF) spoiled gradient echo (SPGR/FLASH) imaging of a digital head phantom. Fat and water tissue component images were generated from fuzzy segmentation of high resolution head images utilizing SPGR signal equations for assumed tissue properties at 3T (24). Simulated imaging parameters include flip=17°, TE/TR=180 μ s/20ms. Generated images were multiplied by coil sensitivity maps, Fourier transformed and interpolated to the acquired k-space trajectory. Coil sensitivities were derived from Biot-Savart calculations of a 16

channel coil consisting of two rows of partial overlapped coils on the surface of a cylinder. Fat was modulated to an off-resonance of 440 Hz before complex addition to the on-resonant, water k-space data. No field map was incorporated in simulations due to impractical forward signal model calculations. Images were reconstructed utilizing calibrationless parallel imaging (CLEAR) (25,26) with and without an L1 wavelet penalty. In cases of L1 wavelet regularization, thresholds were set automatically based on the SureShrink technique(27). Wavelet thresholding was applied with cycle spinning (28), a technique in which thresholding is averaged over shifted versions of the input image to counteract the translational dependence of the discrete wavelet transform. In each case, iterations were performed until the current residue was 10^{-9} times lower than the initial residue or a maximum of 100 iterations were performed. Root mean square error (RMSE) was subsequently calculated as compared to a ground truth calculated by multiplication of source water and fat images with the net coil sensitivity. Root mean square error was normalized to the square root of the average squared value of the reference image (i.e. the RSME of an image of zeros).

Cone Angle - Physical Phantoms and Preliminary In-Vivo Results

A SPGR radial-cones sequence was implemented on a clinical 3T scanner (MR 750, GE Healthcare, Waukesha, WI, USA). A pulse sequence diagram for a cone angle of 40° is provided in supporting material (Supporting Figure S1). At the end of each shot, gradients were ramped to zero and trapezoidal rewinder/spoilers were applied to provide a consistent gradient dephasing of $2 \cdot k_{\max}$ in the slab select direction. Images were collected utilizing 5,000 shot, 2.5 ms readout 3D radial and radial-cone trajectories described above of a quality assurance phantom (American College of Radiology, Large Phantom, ACR-PH1, J.M. Specialty Parts, San Diego, CA, USA) utilizing an 8 channel phased array coil (8 ch Brain, In-vivo, Gainesville, FL, USA). For each trajectory, images were collected twice. Relevant imaging parameters include: flip= 10° , TE/TR= $180\mu\text{s}/10\text{ms}$, FOV= 22cm , acquired resolution= 0.86mm isotropic, imaging time= 50s . No gradient calibration or corrections, beyond those specified by the vendor's gradient subsystem, were applied to any of the data. In addition, a highly sampled dataset was collected with 100,000 projections utilizing the SNR optimized 3D radial readout with otherwise identical imaging parameters; imaging time= $16:40$. Prior to reconstruction, k-space data were pre-whitened utilizing noise samples collected at the end of each acquisition. Images were reconstructed offline utilizing CLEAR and L1-CLEAR similar to that performed in digital phantoms. RMSE was calculated for each trajectory relative to images reconstructed with the fully sampled 3D radial dataset. Additionally, a pseudo noise measurement was performed utilizing the standard deviation of a region of interest (ROI) spanning the entire phantom in the difference between the two separate acquisitions of each trajectory(29). This noise analysis utilized images reconstructed with CLEAR only to eliminate non-linearity introduced by L1 wavelet regularization.

In-vivo feasibility images were collected in volunteers (N=4) on the same 3T scanner utilizing a 32 channel phased array head coil (NMSC075-32, Nova Medical, Wilmington, MA, USA) with 5,000 shot, 2.5ms readout acquisitions and cone angles of 0° , 30° , and 60° . To reduce Gibbs ringing artifacts from the heterogeneous coil sensitivity, fat saturation was

applied. Relevant imaging parameters include: flip=17°, TE/TR=180 μ s/20ms, reconstructed FOV=22cm, acquired resolution=0.86mm isotropic, imaging time = 100s. Images were reconstructed with CLEAR and L1-CLEAR. Coil sensitivity variations were approximately corrected with a vendor supplied post processing algorithm (Surface Coil Intensity Corrections (SCIC), GE Healthcare, Waukesha, WI, USA). Resulting images were visually evaluated for subjective quality and appearance of artifacts.

Readout Length – Trajectories and Digital Simulations

Further experiments were performed to investigate the effect of readout length on image quality for a fixed number of shots and cone angle. Radial-cone trajectories were designed with a cone angle of 35° (based on the results of cone angle optimization experiments) and readout lengths spanning from 1024 μ s to 4096 μ s in 512 μ s increments. This covers the range of typically utilized readout lengths for structural imaging (e.g. Cartesian readouts with imaging bandwidths of $\pm$ 15.625 to $\pm$ 62.5 kHz). For comparison, pure radial trajectories ($\phi=0^\circ$) of the same readout length were designed utilizing optimized gradient design. To enable better separation of aliasing and off-resonance artifact, 10,000 shots were utilized for all trajectories resulting in a net acceleration of 20.5 for 3D radial, roughly 2.7 in each direction. Optimization of cone rotation angles was performed utilizing a maximum of 200,000 iterations utilizing 1000 test angles resulting in a cone rotation angle resolution of 0.36°. PSFs were evaluated utilizing same reconstruction as in cone angle experiments. PSFs were generated before and after cone rotation angle optimization and were resampled into spherical shells to calculate the sum of square values (aliasing energy) and peak value as a function of radial distance. Noise performance estimates were determined from the sampling density correction (9) and normalized to SNR optimal sampling. Additional analysis of the off-resonance PSF is detailed in supporting material (Supporting Figures S2 and S3).

Readout Length - Physical Phantoms and Preliminary In-Vivo Results

Utilizing the same pulse sequence as in cone angle experiments, images were collected of a quality assurance phantom (GE DQA Phantom, GE Healthcare, Waukesha, WI). To accentuate undersampling artifacts, images were collected with a single channel receive coil (Quadrature Head, GE Healthcare, Waukesha, WI). Images were collected utilizing 10,000 shot 3D radial and radial-cone trajectories described above. Relevant imaging parameters include: flip=15°, TE/TR=180 μ s/10ms, imaging time=100s, reconstructed FOV=22cm, acquired resolution=0.86mm isotropic. Images were reconstructed with L1 wavelet penalized least squares estimation. This reconstruction was identical to the L1-CLEAR reconstruction described above but excludes the regularization based parallel imaging (i.e. CLEAR). Mean background aliasing artifact in gridding images was quantified utilizing a 3D ROI generated utilizing a threshold segmented mask of a nearly fully sampled data set, eroded with a 4 $\times$ 4 $\times$ 4 mask. The near fully sampled data set was acquired with 3D radial sampling with a 1024 μ s readout and 100,000 projections. Thresholds were set manually to remove the structures of the phantom. Prior to measurement the peak signal across the entire FOV was normalized to 1. ROI averaged values were computed using root mean square deviation from zero. Additional evaluation of the off-resonance response is detailed in supporting material (Supporting Figure S2).

To evaluate potential for imaging vascular disease, SPGR images were collected in the pseudo steady state phase after the administration of 0.1mmol/kg gadobenate dimeglumine. Feasibility images comparing the above described radial-cones trajectories with readout lengths of 1024 μ s and 2048 μ s were collected in a volunteer on the same 3T scanner utilizing a 32 channel phase array head coil (NMSC075-32, Nova Medical, Wilmington, MA, USA). Short readouts were utilized to avoid artifacts due to signal evolution during the readout, which is amplified in vessels due to contrast media and blood flow. Images were collected ~120s after contrast administration in rapid succession. Relevant imaging parameters include: flip=15°, TE/TR=180 μ s/3.5ms, reconstructed FOV=22cm, acquired resolution=0.86mm isotropic, imaging time 35s. Images were reconstructed with standard gridding and L1-CLEAR.

Results

Cone Angle –Digital Simulations

Figure 3a shows point spread functions for 3D radial and radial-cone trajectories with 10° and 30° cone angles. Quantitative evaluation of PSF images is detailed in Table 1. With increasing cone angle, the relative k-space coverage per shot increases with a corresponding reduction in total aliasing energy. The peak aliasing intensity is initially reduced with increasing cone angle. This is visualized in Figure 3b which plots the aliasing energy and peak intensity as a function of radius. Radial-cones supports a smaller aliasing free FOV but has less overall aliasing within the FOV. As the cone-angle increases the peak aliasing intensity eventually increases. This corresponds to a decrease in the estimated noise performance which is generally lower than 3D radial sampling by ~3-5%. Note, PSF analysis does not incorporate the effect of parallel imaging or constrained reconstruction.

Figure 4 shows representative digital head phantom images for radial-cone trajectories with cone angles of 0° (3D radial), 30°, and 60°, reconstructed with and without compressed sensing. Images are cropped to enable better visualization. Images acquired with 3D radial sampling tended to have much higher levels of artifact with both CLEAR and L1-CLEAR reconstructions. This manifested as higher apparent blurring and loss of high resolution features in 3D radial images reconstructed with compressed sensing. Images acquired with 3D radial-cones and reconstructed with CLEAR and L1-CLEAR had higher apparent resolution and improved depiction of high resolution features; however, some high resolution features are incompletely recovered by the reconstruction. This is appreciated when comparing to the truth image. Quantified values are shown in Table 1. Compared to radial-cones with a 40° cone angle, 3D radial sampling had 65% and 121% higher error levels with CLEAR and L1-CLEAR reconstructions, respectively. In digital phantoms with CLEAR, the RMSE was minimized with a cone angle of 40°. This angle decreased to 30° with the use of L1-CLEAR. In each case, the observed error was relatively insensitive to the choice of cone angle about the RMSE minimizing angle.

Cone Angle - Physical Phantoms and Preliminary In-Vivo Results

Physical phantom experiments mirrored results in digital phantoms. Figure 5 shows resulting axial images from radial-cone and 3D radial trajectories. CLEAR based reconstruction of 3D

radial data tended to produce images with low apparent spatial resolution, evident in difference images. Initial gridding images (not shown) demonstrated substantial streaking emanating from the edges of the phantom. Radial-cones tended to produce images with better preservation of high resolution structures, lower RMSE, and a diffuse aliasing pattern. The addition of L1 wavelet regularization did little to improve the image quality of 3D radial images but substantially reduced RMSE and background artifact in radial-cones. Results for all cone angles are summarized in Table 1. Similar to digital phantom results, the RMSE minimizing cone angle is lower with L1-CLEAR than CLEAR alone (40° vs 60°). Note these experiments were performed with an 8ch coil and the net undersampling for 3D radial sampling was $\sim 41\times$. Measured SNR was $\sim 50\%$ higher in 3D radial acquisitions; however, this corresponded with a more than $2\times$ increase in RMSE and could be attributed to resolution loss in the 3D radial images.

Figure 6 demonstrates a comparison of coronal and axial reformats of 3D radial, and radial-cone trajectories with 30° and 60° cone angles. Registration of images acquired with differing trajectories was not performed. 3D radial images have substantial artifact with parallel imaging alone, which is partially removed with the use of L1 wavelet regularization. However, some structured artifact remains in L1 regularized images, readily observed in areas of flat intensity such as the vitreous humor of the eyes. Radial-cone images acquired with both 30° and 60° cone angles demonstrate higher overall image quality compared to the 3D radial images. A moderate reduction in artifact is observed with addition of L1 wavelet regularization. The effect of L1 wavelet regularization is pronounced on axial reformatted images, bottom two rows in Figure 6. In this case, L1 wavelet regularization leads to a substantial and obvious reduction in the apparent noise level in all images. This is most pronounced in central areas of the brain which are inherently areas of lower SNR due to lower net coil sensitivity.

Readout Length – Digital Simulations

Quantitative analyses of readout length experiments are given in Table 2. As expected, increasing the readout length leads to a decrease in the net undersampling and a slight reduction in the noise performance. Note the noise performance is given for the cone rotation angle optimized trajectory. Noise performance is slightly reduced with longer readout lengths due to an increase in overlap between shots. Peak aliasing was substantially affected by optimization of cone rotation angles, especially in central regions. At a distance well inside the expected aliasing-free FOV of 3D radial imaging ($\text{FOV}/5$), the average change in peak aliasing intensity, averaged over all readout lengths, was -22.2% compared to -0.4% at the edge of the FOV. The average aliasing intensity followed a similar pattern (not shown).

Readout Length - Physical Phantoms and Preliminary In-Vivo Results

Figure 7, demonstrates representative phantom images from the comparison of radial-cones to radial sampling at readout lengths of 1024, 2560, and 4096 μs . 3D radial images have significant streak artifact emanating from high contrast edges, regardless of the readout length. Radial-cones images, even with short readouts and gridding reconstruction, do not show the same level of artifact as radial images. Further, images reconstructed with an L1-

CLEAR demonstrate the failure to remove streak artifacts in 3D radial images. L1-CLEAR images are shown with a display gamma of 0.5 (i.e. the square root of the image is shown) to enhance the visualization of background artifact. As the readout length increases, blurring is apparent in the top bar and logo in upper right quadrant of the shown images (red arrows). This area corresponds to local off-resonance consistently observed in this phantom. This blur is notably worse 3D radial images, likely due to the slower traversal through the center of k-space. Further analysis, including off-resonance PSF and phantom data is included in supporting material (Supporting Figure S4).

Figure 8 shows representative images from post-contrast radial-cones images collected with readout lengths of 1024 and 2048 μ s. Images reconstructed with gridding demonstrate artifact in the brain tissue. This artifact, while diffuse, appears to limit the visualization of small veins and arteries and is more prominent in the short readout radial-cones images. Images reconstructed with L1-CLEAR show substantially improved image quality and notable improvement in visualization of small arteries and veins. Long readout images demonstrate improved image quality, especially in small vein in the central posterior veins. Minor blurring was perceived, albeit barely, in structures superior to the nasal cavity in long readout images.

Discussion

The radial-cones sequence is a hybrid between 3D radial, 3D cones, and optimization based sampling strategies and utilizes pseudo-randomly oriented conical trajectories. PSF and trajectory analysis revealed that, compared to 3D radial trajectories of the same readout length, radial-cones samples more k-space per unit time (lower net aliasing/higher sampling efficiency) and in a more incoherent fashion (lower peak aliasing). Utilizing least squares reconstruction techniques, radial-cones allowed for improved recovery of high resolution features resulting in images with lower RMSE compared to ground truth images. Radial-cones performance was relatively insensitive to the choice of cone-angle with a maximum deviation in RMSE of 1% for angles ranging from 20° to 60° and images reconstructed with CLEAR (Table 1). With radial-cones, increasing the readout length led to lower aliasing energy in PSF analysis and lower background artifact in physical phantom experiments (Table 2). The use of radial-cones with increasingly long readouts is limited by the acceptable level of off-resonance blurring (Figure 7/8 and supporting material) for the specific imaging task. In-vivo images demonstrated higher subjective image quality in initial feasibility testing (Figures 6 and 8).

Numerous Non-Cartesian trajectories have been proposed, most of which have failed to achieve widespread clinical penetration. However, despite their lower sampling and SNR efficiency, recent commercial packages have adopted radial based trajectories to address challenges with acoustic noise, motion artifacts, and short T2 sensitivity. Radial-cones harnesses longer readout windows to improve sampling efficiency (e.g. k-space arc length per excitation) while preserving SNR efficiency (e.g. relatively noise performance, Table 1) and maintaining center-out and centrally oversampled k-space sampling. It is therefore expected to maintain, at least in part, the insensitivity of 3D radial sampling to artifacts from data inconsistencies arising from motion, flow, and magnetization modulations. Radial-

cones also reduces the coherent structure of undersampling artifacts which are detrimental to the performance of many compressed sensing and low rank reconstruction algorithms. This is one contributing factor to the observed lower RMSE error in radial-cones as compared to 3D radial. This influence of artifact coherence is expected to be more pronounced with L1 regularized reconstructions (e.g. L1-CLEAR). Low rank parallel imaging methods such as CLEAR also benefit from incoherency; as they can learn coil encoding behavior from irregularly spaced samples. In the cases investigated, the central k-space was well sampled up to a radius of ~20 points for all trajectories, which is sufficient to estimate coil information. These properties may make it more attractive than other twisting based 3D Non-Cartesian trajectories and more computationally practical than fully optimization driven trajectory design. Further task based clinical evaluations are needed to substantiate the clinical value of radial-cones trajectory.

Several modifications to radial-cones may offer improved performance or greater practicality. In this study, gradients were designed with a center out sampling with the gradients starting at zero. However, gradients need not start at zero which would allow higher sampling efficiency. This is also precisely the scheme required for zero-echo time imaging (ZTE) (30) which has higher efficiency and shorter echo times (31). Further improvements may be possible with alternative optimization routines of the k-space trajectory. In the current implementation, the base cone is a heuristic design and not linked to the orientation of the cones. As shown in table 1, choosing the cone angle requires a balance between the overall aliasing, which would suggest higher cone angles, and the coherency of aliasing artifacts, which would suggest lower cone angles. The optimal cone angle depends on the reconstruction as demonstrated by the downward shift in the optimal cone angle with the addition of L1-wavelet regularization. However, experiments do not indicate strong sensitivity to the choice of cone angle. This suggest that with a given cone angle and readout length, the base cone may be designed and the number of shots can be determined utilizing the approximate net undersampling. This ignores overlap of trajectories which may result in higher artifact levels. It is possible that more uniform sampling of k-space could be realized by jointly optimizing the base cone and the rotation matrices. It may also be advantageous to create scaled versions of the base cone as to provide more freedom to the optimization algorithm and vary the number of shots. Such schemes may reveal alternative strategies to design the base cone itself and may be required in applications requiring very few shots where randomization properties are greatly reduced. As design variables are increased, the computational complexity becomes limiting, converging to that of more unrestricted optimizations schemes detailed in prior work (16,17).

Radial-cones is more complex compared to Cartesian and pure radial sampling strategies. The twisting of the trajectory which leads in an increased sampling efficiency, also leads to a complimentary increase in sensitivity to gradient fidelity. As radial-cones uses a single trajectory, it may be ideally suited for gradient impulse response techniques such as those proposed using gradient field monitoring (32) and self-encoding strategies (33). In the presented experiments, no gradient correction was utilized and the effects of gradient imperfections were not explored. As such, more favorable results may be achieved utilizing gradient corrections as demonstrated with 3D stack of cones trajectories (33) or conversely, less favorable results on a system with a poor gradient impulse response. The complexity of

radial-cones also requires time for gradient calculation. Currently, the design of the base cone and initial placement is relatively quick (<10s with unoptimized code); however, the cone rotation angle optimization routine is prohibitively time intensive for real-time design (~20 minutes depending on specific imaging parameters). Further work is required to shorten this time or provide precomputed rotation tables.

The performance of radial-cones in clinical application remains unproven. First and foremost, the achievable readout length will be highly dependent on off-resonance and T2 decay sensitivity of specific applications. As shown in supporting material and in prior works (20,21), the rapid k-space velocity through the center of k-space is more favorable with respect to the appearance of T2 decay and off-resonance evolution during the readout. Applications requiring the shortest possible readout length will benefit little from the radial-cones readout. Further, radial-cones may not achieve gains in sampling efficiency in cases with minimal TR overhead for excitation, magnetization preparation, and spoiling. For example, the sampling duty cycle of ZTE imaging with high bandwidth radial readouts is close to 100%. Utilizing a longer readout length radial-cones trajectory may result in higher SNR efficiency but will not reduce the total scan time. For each application, the sampling window will need to be adapted to balance potential gains from improved sampling, artifacts from off-resonance, and sampling duty cycle.

Conclusions

In this work, we introduce radial-cones, a novel trajectory that allows a synergistic combination of Non-Cartesian sampling and advanced reconstructions based on parallel imaging and compressed sensing. This sampling trajectory trades requirements for structured and consistent gaps in k-space for substantially higher sampling efficiency. The radial-cones trajectory allows a reduction in the overall acceleration factor compared to 3D radial sampling while avoiding increased coherent undersampling artifacts. In evaluations, including PSF analysis, digital and physical phantoms, and feasibility studies in healthy volunteers, radial-cones substantially out performed 3D radial sampling. This trajectory has broad implications for imaging applications which benefit from center out sampling.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Appendix A: Fast Density Estimate Approach

We adopt the non-iterative grid based density estimation scheme described in Jackson et al (34). In this approach, k-space samples are convolved and sampled on a Cartesian grid (gridding):

$$\rho_c = \delta_c \cdot (k \otimes G) \quad A1$$

where ρ_c is the density estimate in Cartesian coordinates, δ_c is the Cartesian sampling operator, k is the set of k-space sample positions, and G is the convolution kernel. The density is estimated from the interpolation of the Cartesian density estimate (inverse gridding):

$$\rho = \delta \cdot (\rho_c \otimes G) \quad A2$$

where δ is the sampling operator corresponding to the set of k-space sample positions. The global cost function is the sum of squared densities over all shots and k-space positions. This cost function is non-convex and computing the cost function is computationally intensive. In order to approximately minimize this cost function, we utilize a greedy optimization method which has computational advantages. The gridding (A1) and inverse gridding (A2) operations are of equal complexity; however, inverse gridding is often substantially faster in multi-threaded implementations due to absence of multiple write operations to a single point. Updating a single shot at a time allows a substantially lower complexity. Given ρ_c from all projections the density of single shot can be subtracted from this set

$$\rho_c^* = \rho_c - \delta_c \cdot (k_j \otimes G) \quad A3$$

Where k_j corresponds to the k-space positions of the j th shot, identified as the shot with the highest degree of overlap with all other shots (highest sum of densities squared). The cost of a set of test rotations can then be calculated as a function of cone rotation angle (Ω):

$$\rho_{test} = \delta_{test} \cdot (\rho_c^* \otimes G) \quad A4$$

where δ_{test} is sampling operator at the test rotation about the cone axis ($\hat{r}'$). Since this operation does not require altering ρ_c^* , ρ_{test} can be computed for a wide range of rotations in parallel. Employing a simple search a cone rotation can be found with minimizes the density squared for that shot. The global density can then be updated by adding the density back:

$$\rho_c^* = \rho_c + \delta_c \cdot (R_\Omega k_j \otimes G) \quad A5$$

This updates only the k-space density in the resampled Cartesian grid. In principle, the density of each shot is or could be modified by changing the rotation of a single shot. However, since the density of an individual shot is only needed to identify the shot with the highest overlap, it is sufficient to only update the per shot density periodically. In our case, this is only updated every 100 iterations such that 100 shots with the highest overlap are identified and optimized in order of overlap.

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Figure 1.

Graphical description of hybrid radial-cones trajectory based off a single base cone which is oriented in space utilizing 3D radial axes, $\hat{r}'$, and a rotation about $\hat{r}'$ by a pseudo-random cone rotation angle, Ω . See text for detailed description of trajectory design. The end result is a trajectory which progresses from 3D radial at the center of k-space to pseudo-random sampling at the edge.

Figure 2.

Cone rotation angle optimization. Starting with the current trajectory a sampling density estimate is calculated for each shot. Subsequently, the shot with the highest degree of overlap, defined as the sum of the sampling density squared, is identified from all shots (yellow line). The index j is the position along the readout. This cone is then rotated about its cone axis and a new density is calculated for all possible rotations. Picking the rotation in which density squared is minimized (dashed line), the current trajectory is updated for that shot. The process is repeated until a stopping criteria.

Figure 3.

Representative 2D slice of point spread functions for 3D radial and radial cones (a), peak PSF (b) and total energy (c) as a function of distance from center of the image. As quantified in Table 1, as the cone angle initially increases, the peak and total aliasing energy decrease. This comes at the cost of a smaller aliasing free field of view, approximately half that of radial, arising from unstructured gaps in k-space.

Figure 4.

Simulated images utilizing realistic digital phantoms of images as compared to a ground truth image. In this case, 3D radial imaging (0°) is subject to substantial noise enhancement with parallel imaging only and resolution loss with the addition of an L1 wavelet penalty. In both radial-cones cases shown, there is a reduction in both the apparent SNR loss and resolution loss with the addition of L1 wavelet regularization. This is most readily observed on difference images which are scaled four fold to improve visualization.

Figure 5.

Physical phantom images collected with an 8 channel coil and 5,000 shot radial-cones trajectories. Difference images, scaled fivefold, are taken with respect to a fully sampled 3D radial acquisition with otherwise identical parameters. With the relatively low channel count, neither reconstruction restores the large gaps at the edge of k-space for 3D radial sampling (0°). This results in substantial blurring of the high resolution grid within the phantom. In the case of radial-cones, some structured artifact is present in the CLEAR images, especially visible on unsubtracted images. However, the image resolution is maintained. L1 wavelet

regularization is able to remove this artifact, suggesting incoherence of this artifact in the wavelet domain.

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Figure 6.

Representative coronal and axial reformats of in-vivo images collect with radial-cones and cone angles of 0° , 30° , and 60° and reconstructed using CLEAR with and without an L1 wavelet penalty. A substantial reduction in artifacts is apparent with the radial-cones approach in both reconstructions. Notably, only modest differences are noted between regularized and un-regularized acquisition with radial-cones. In axial images, high levels of noise in the central image region are more apparent. Noise levels appear flat in radial images (0°) suggesting aliasing artifact dominance, while, noise appears to increase in radial-cones images suggesting reduced aliasing artifact and increased noise dominance. In all cases, L1

wavelet regularization leads to an apparent reduction in noise and noise-like artifact. Note inter-scan motion led to small locational differences which was not corrected for.

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Figure 7.

Central axial reformats of phantom radial-cones and radial acquisitions with readout lengths of 1024, 2560, and 4096 μ s and reconstructed with simple gridding and a L1 wavelet regularized iterative reconstruction. Iterative reconstructions are window leveled with a display gamma of 0.5 (i.e. the square root of the image is shown) to enhance visualization of background artifact. Radial images demonstrate coherent artifacts (yellow arrow) which are not present in radial-cones images and not removed with the application of iterative

reconstruction. With long readout times, blurring manifests in the upper right quadrant of the phantom (red arrow) which is slightly worse in radial images.

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Figure 8.

Representative contrast enhanced radial-cones images collected with a cone angle of 35° and readout lengths of 1024 and 2048 μ s. Images are shown as 7mm limited maximum intensity projections. Images reconstructed with non-iterative gridding (top row) demonstrate artifact with is markedly reduced in the longer readout images. The applications of parallel imaging and compressed sensing (bottom row, L1-CLEAR) leads to substantial reduction in artifacts

and high conspicuity of small structures. Longer readout images show improved visualization of small vessels and fewer artifacts.

Table 1

Summary of quantitative metrics derived from PSF analysis and digital and physical phantoms as a function of cone angle for a 2.5ms readout.

Cone Angle	Trajectory		PSF Analysis			Digital Phantom		Physical Phantom		
	Relative K Space Arc Length	Approx. Net Undersampling factor [‡]	Estimated noise performance	Peak aliasing within FOV	Aliasing Energy at FOV/2	CLEAR RMSE[%]	L1 CLEAR RMSE[%]	CLEAR RMSE[%]	L1 CLEAR RMSE[%]	SNR Estimate [*]
0°	1.0	41.2	0.957	0.0216	0.823	16.2	14.2	11.9	11.59	19.4
10°	2.66	15.5	0.909	0.0054	0.363	11.8	8.4	7.1	3.50	12.7
20°	3.26	12.6	0.914	0.0045	0.295	10.4	6.9	6.8	2.94	12.4
30°	3.71	11.1	0.923	0.0038	0.275	10.0	6.4	6.1	2.68	12.1
40°	3.95	10.4	0.930	0.0037	0.266	9.8	6.4	5.6	2.58	12.4
50°	4.06	10.1	0.930	0.0035	0.263	10.3	6.8	5.3	2.59	13.0
60°	4.10	10.0	0.927	0.0038	0.263	12.0	7.4	5.2	2.62	12.8

Bolded values indicate optimal cone angle for each measure. The PSF was normalized to a peak value of 1.

^{*} Note the SNR estimate is derived from the subtraction of back to back acquisitions and thus does not account for resolution loss or structured noise.

[‡] Undersampling denotes the upper bound assuming no additional overlap of base cone trajectories.

Table 2

Summary of quantitative metrics derived from PSF analysis and physical phantoms as a function of readout length for a fixed cone angle of 35°.

Readout Length (μs)	Trajectory			PSF Analysis					Physical Phantom	
	Relative K-space Arc Length	Aprox. Net Undersampling factor [‡]	Estimated noise performance	Peak Aliasing at FOV/5			Peak Aliasing within FOV		Radial	Radial-cones
				Random	Optimized	Change (%)	Random	Optimized		
1024	1.77	11.6	0.89	0.0044	0.0035	-20.6	0.0024	0.0025	0.15	0.11
1536	2.65	7.8	0.89	0.0034	0.0026	-22.0	0.0020	0.0020	0.15	0.095
2048	3.53	5.8	0.88	0.0028	0.0024	-15.7	0.0017	0.0016	0.14	0.086
2560	4.40	4.7	0.87	0.0024	0.0017	-25.7	0.0014	0.0014	0.16	0.077
3072	5.28	3.9	0.86	0.0020	0.0016	-16.6	0.0014	0.0012	0.12	0.070
3584	6.16	3.3	0.85	0.0018	0.0013	-29.3	0.0011	0.0011	0.12	0.064
4096	7.03	2.9	0.84	0.0016	0.0012	-25.5	0.0010	0.0011	0.12	0.051

The PSF was normalized to a peak value of 1. Relative arc length is reported with respect to a kmax. Neglecting off-resonance and T2* decay, increasing the readout length leads to lower artifacts. Optimization most prominently affects the central region of the PSF.

[‡]Undersampling denotes the upper bound assuming no additional overlap of base cone trajectories. Mean background signal was normalized to the maximum signal.