Published in final edited form as:

IEEE Trans Med Imaging. 2004 July ; 23(7): 896–902. doi:10.1109/TMI.2004.828356.

A Modified Uniform Cramer–Rao Bound for Multiple Pinhole Aperture Design

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Abstract

This paper presents a modified Uniform Cramer–Rao bound (UCRB) for studying estimator spatial resolution and variance tradeoffs. We proposed to use a resolution constraint that is imposed on mean gradient vectors of achieved estimators and derived the minimum achievable variance for any estimator satisfies this resolution constraint. This approach partially overcomes the limitations of the former UCRB approach based on a bias-gradient norm constraint. We applied this method in a feasibility study of using multiple pinhole apertures for small animal SPECT imaging applications. The SPECT system studied was based on an existing gamma camera. The achievable spatial resolution and variance tradeoffs for systems with different design parameters, such as number of pinholes and pinhole size, were studied.

Keywords

Multiple pinhole aperture; variance-resolution tradeoffs; UCRB

I. Introduction

PINHOLE cameras are widely used for small animal imaging [1], [2]. They offer a high spatial resolution that is necessary for small animal single photon emission computed tomography (SPECT) applications. Comparing with dedicated small animal PET, they also benefit from a wide range of readily available radiotracers and the use of longer-lived isotopes. However, the biggest disadvantage of small animal pinhole SPECT system is the very low detection efficiency. The use of multiple pinholes aperture has been studied as an alternative to the pinhole cameras to improve the detection sensitivity [3]–[5]. This leads to an improved raw sensitivity at the price of increased data multiplexing and reduced information content per detected photon. As a result, the multiple openings in apertures may sometimes lead to a worse signal to noise ratio in reconstructed images. In the context of three-dimensional (3-D) tomographic imaging applications, it is important to note that even a single pinhole aperture results in multiplexed data sets. Projections from different points in

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the object space to the detector space may be severely overlapped. Many studies have shown that the use of multiple pinhole apertures lead to improved imaging performances for certain tasks [6], [7]. The intention of this work is to develop and examine an analytical approach for quantifying system performances. This may serve as a step toward systematic optimization of multiple pinhole apertures.

In this paper, we present an extension of the UCRB approach formerly proposed by Hero [8], [9]. One of the key improvements is that we enforced a constraint on the actual point-spread function (PSF) rather than the bias gradient vector. This allows a more meaningful constraint on the achievable spatial resolution property. Using this approach, we compared different system designs based on comparing the minimum achievable variances at given spatial resolutions.

II. Variance-Resolution Tradeoff With UCRB

A variety of methods have been proposed for finding the “optimum” choice of detector design parameters. One possibility is to evaluate detector designs based on their performances for a certain task or a collection of tasks, using either machine or human observers [10]–[12]. When imaging systems are designed for these well defined tasks, this becomes a natural approach. Another way of analyzing detector performance is to use the Cramer–Rao bound. Hero has proposed a Uniform Cramer–Rao bound (UCRB) that gives the minimum achievable variance of all biased estimators that satisfy a constraint on the norm of the corresponding bias-gradient vector [8]. Although the bias-gradient norm is related to the reconstructed spatial resolution, their nonlinear relationship has led to some counter-intuitive results, concerning the “optimum” pinhole size as a function of target spatial resolution [13]. For example, one intuition would be that if one can accept worse spatial resolution in the reconstructed image, a larger pinhole should result in a lower variance. This was, however, not always apparent in these studies. Furthermore, since the reconstructed point-spread-function (PSF) is a multivariate variable, constraint on PSF based on any scalar function would lead to the same concern. Yet another problem regarding the original UCRB approach is the achievability of the bound. Although a penalized weighted least square (PWLS) estimator with an appropriate penalty function was shown to achieve this bound, the use of this penalty function is equivalent to incorporating *a priori* information that clearly lacks practical justification. Therefore, we do not directly know how far this UCRB deviates from the actual lower bound on estimator variance when more appropriate image priors are used.

In this study, we proposed a new resolution constraint that is imposed on the gradient of the mean estimators (or the mean gradient), denoted by $\mathbf{g}$. Although the mean gradient is not directly equivalent to the local impulse response function (LIR), they are very closely related and would become identical given certain conditions are satisfied [14]. If the actual local impulse response is symmetric, it is equivalent to the mean gradient. Suppose the desired mean gradient vector, corresponding to the j th pixel, takes the form $\mathbf{f}$, a natural question to ask is “which system design leads to the lowest variance when the actual mean gradient is close to $\mathbf{f}$?”. To answer this question, we defined a similarity measure and let the resulted similarity satisfy the following constraint

$$\| \mathbf{g} - \mathbf{f} \|_C \leq \delta \quad (1)$$

where δ is the threshold that defines the degree of similarity between $\mathbf{f}$ and $\mathbf{g}$. $\| \cdot \|$ is the Euclidean norm of a vector so that

$$\| \mathbf{g} - \mathbf{f} \|_C^2 = (\mathbf{g} - \mathbf{f})^T \cdot \mathbf{C} \cdot (\mathbf{g} - \mathbf{f}) \quad (2)$$

where $\mathbf{C}$ is a symmetric positively definite matrix. Now we can derive the minimum achievable variance for any biased estimators that satisfy this resolution constraint.

Let $\mathbf{x} = [x_1, x_2, \dots, x_N]^T \in \mathbb{R}^N$ denote the set of unknown parameters, e.g., object intensities underlying the projection data $\mathbf{y}$. The transformation from $\mathbf{x}$ to $\mathbf{y}$ is described by a conditional probability density function $p(\mathbf{y}; \mathbf{x})$. Let $\hat{x}_j$ be an estimator for a pixel of interest, the bias on $\hat{x}_j$ is given by

$$b(\hat{x}_j) = E[\hat{x}_j] - x_j. \quad (3)$$

The gradient of the bias, corresponding to the j th pixel in the image, is defined as an N -element vector of partial derivatives

$$\begin{aligned} \mathbf{d} &= \nabla_{\mathbf{x}} \mathbf{b}(\hat{x}_j) \\ &= [\partial b(\hat{x}_j) / \partial x_1, \partial b(\hat{x}_j) / \partial x_2, \dots, \partial b(\hat{x}_j) / \partial x_N] \\ &= \nabla_{\mathbf{x}} E(\hat{x}_j) - \mathbf{e}_j \\ &= \mathbf{g} - \mathbf{e}_j \end{aligned} \quad (4)$$

where $\mathbf{g}$ is the mean gradient vector that describes how perturbations on all image pixels are coupled to the target pixel through the reconstruction process. $\mathbf{e}_j$ is the j th unit vector. The CRB for any biased estimator with bias gradient $\mathbf{d}$, is given by [15]

$$\text{Var}(\hat{x}_j) \geq (\mathbf{e}_j + \mathbf{d})^T \cdot \mathbf{F}^{-1} \cdot (\mathbf{e}_j + \mathbf{d}) = \mathbf{g}^T \cdot \mathbf{F}^{-1} \cdot \mathbf{g} \quad (5)$$

where $\mathbf{F}$ is the Fisher information matrix (FIM) defined as

$$\mathbf{F} = E \left\{ \left[\frac{\partial}{\partial \mathbf{x}} \log p(\mathbf{y}; \mathbf{x}) \right] \cdot \left[\frac{\partial}{\partial \mathbf{x}} \log p(\mathbf{y}; \mathbf{x}) \right]^T \right\}. \quad (6)$$

For simplicity, we assumed that $\mathbf{F}$ is a nonsingular matrix. The optimum mean gradient that gives the minimum achievable variance and satisfies constraint (1) is found as

$$\mathbf{g}_{\text{optimum}} = \arg \min_{\|\mathbf{g} - \mathbf{f}\| \leq \delta} \left\{ \mathbf{g}^T \cdot \mathbf{F}^{-1} \cdot \mathbf{g} \right\}. \quad (7)$$

In order to derive the optimum mean gradient vector, we form the Lagrangian

$$L = \mathbf{g}^T \cdot \mathbf{F}^{-1} \cdot \mathbf{g} + \lambda \cdot \left[(\mathbf{g} - \mathbf{f})^T \cdot \mathbf{C} \cdot (\mathbf{g} - \mathbf{f}) - \delta^2 \right]. \quad (8)$$

From Karush–Kuhn–Tucker condition [16], we must have $\lambda \geq 0$. Since L is strictly convex, it has a unique minimum. Differentiating L respecting to $\mathbf{g}$ and equating to 0

$$\mathbf{F}^{-1} \cdot \mathbf{g} + \lambda \cdot \mathbf{C} \cdot (\mathbf{f} - \mathbf{g}) = 0 \quad (9)$$

we have that the minimum of L is achieved by $\mathbf{g} = \mathbf{g}_{\text{optimum}}$ with

$$\begin{aligned} \mathbf{g}_{\text{optimum}} &= [\mathbf{F}^{-1} + \lambda \cdot \mathbf{C}]^{-1} \cdot \lambda \cdot \mathbf{C} \cdot \mathbf{f} \\ &= [(\lambda \cdot \mathbf{C})^{-1} \cdot \mathbf{F}^{-1} + \mathbf{I}]^{-1} \cdot \mathbf{f} \\ &= \mathbf{F} \cdot [\mathbf{F} + (\lambda \cdot \mathbf{C})^{-1}]^{-1} \cdot \mathbf{f} \end{aligned} \quad (10)$$

and the associated UCRB is given by

$$\text{Var}(\hat{x}_j) \geq \mathbf{f}^T \cdot \left\{ [\mathbf{F} + (\lambda \cdot \mathbf{C})^{-1}] \right\}^{-1} \cdot \mathbf{F} \cdot \left\{ [\mathbf{F} + (\lambda \cdot \mathbf{C})^{-1}] \right\}^{-1} \cdot \mathbf{f}. \quad (11)$$

Instead of solving (10) directly for each given value of δ , one can trace out the values for the bound corresponding to a wide range of δ , by continuously varying λ . Reference [9] gives a recipe for calculating the bound based on Conjugate Gradient methods. It calculates

$$\mathbf{z} = \left\{ [\mathbf{F} + (\lambda \cdot \mathbf{C})^{-1}] \right\}^{-1} \cdot \mathbf{f} \text{ by solving the simultaneous equations for } [\mathbf{F} + (\lambda \cdot \mathbf{C})^{-1}] \cdot \mathbf{z} = \mathbf{f} \text{ for } \mathbf{z}. \text{ Below are some discussions regarding this bound.}$$

- *Comparison with the original UCRB:* The mean gradient constraint (1) becomes the original bias-gradient norm constraint if the target mean-gradient vector is a delta function ($\mathbf{f} = \mathbf{e}_j$) [8], in which

$$\|\mathbf{d}\|_C^2 = \mathbf{d}^T \cdot \mathbf{C} \cdot \mathbf{d} \leq \delta^2. \quad (12)$$

Therefore, the bias-gradient norm constraint sets the maximum “distance” allowed between the actual mean gradient vector and a delta function. In

practice, since most reconstructions lead to a mean gradient that is significantly different from the delta function, there exist a rather large set of possible estimators satisfying this constraint. The particular estimator identified to achieve the bound is not what we normally use for reconstruction [14]. In contrast, the proposed resolution constraint (1) imposes a stronger limitation on the actual shape of the mean gradient vector (or LIR). When δ is small, the resulting mean gradient vector can only deviate slightly from the target f . Therefore, the UCRB based on such a constraint is convenient for comparing the noise in images with similar spatial resolution prosperities.

- *Achievability of the bound:* For measurements that follow the standard linear model with additive noise, where

$$\mathbf{y} = \mathbf{A} \cdot \mathbf{x} + \text{noise} \quad (13)$$

the following penalized weighted least squares (PWLS) object function is often used for estimation:

$$\Phi(\mathbf{x}, \mathbf{y}) = \frac{1}{2} \cdot (\mathbf{y} - \mathbf{A} \cdot \mathbf{x})^T \cdot \mathbf{\Pi} \cdot (\mathbf{y} - \mathbf{A} \cdot \mathbf{x}) - \beta \cdot R(\mathbf{x}) \quad (14)$$

where $\mathbf{\Pi}$ is a nonnegative definite weighting matrix and $R(\mathbf{x})$ is the penalty. Matrix $\mathbf{\Pi}$ is normally defined as $\mathbf{\Sigma}^{-1}$, where $\mathbf{\Sigma}$ is diagonal with the i th entry σ_i^2 , an estimate of the variance of measurement y_i . When the penalty term takes a quadratic form

$$R(\mathbf{x}) = \mathbf{x}^T \cdot \mathbf{R} \cdot \mathbf{x} \quad (15)$$

where $\mathbf{R}$ is a symmetric positive definite matrix, the mean of the estimator can be explicitly given as [17]

$$\hat{\mathbf{x}}(\mathbf{y}) = [\mathbf{A} \cdot \mathbf{\Pi} \cdot \mathbf{A} + \beta \cdot \mathbf{R}]^{-1} \cdot \mathbf{A} \cdot \mathbf{\Pi} \cdot \mathbf{y}. \quad (16)$$

The mean gradient associated with the j th pixel is then given as

$$\begin{aligned} \mathbf{g}_{\text{PWLS}} &= \mathbf{A}^T \cdot \mathbf{\Pi} \cdot \mathbf{A} \cdot \left[\mathbf{A}^T \cdot \mathbf{\Pi} \cdot \mathbf{A} + \beta \cdot \mathbf{R} \right]^{-1} \cdot \mathbf{e}_j \\ &= \mathbf{F} \cdot [\mathbf{F} + \beta \cdot \mathbf{R}]^{-1} \cdot \mathbf{e}_j. \end{aligned}$$

(17)

It is easy to verify that if one filters the PWLS solution shown in (17) with a filter function $\mathbf{h}$, the resulting mean gradient is

$$\mathbf{g}_{\text{PWLS+ filtering}} = \mathbf{F} \cdot [\mathbf{F} + \beta \cdot \mathbf{R}]^{-1} \cdot \mathbf{h}. \quad (18)$$

Comparing (18) with (10), we can see that the optimum mean gradient vector, which results in the minimum achievable variance under constraint (1), can be obtained exactly by postfiltering the PWLS estimator that uses a quadratic penalty function

$$R(\mathbf{x}) = \mathbf{x}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{x}. \quad (19)$$

- *Effect of regularization:* From (10), (17), and (18), an estimator, with a mean gradient vector $\mathbf{g}$, identical to the target $\mathbf{f}$ can be achieved if term $(\lambda\mathbf{C})^{-1} = 0$. This corresponds to running the weighted least squares algorithm with no regularization to converge and then convolve the above solution with the filter $\mathbf{f}$. Although this allows the control of the exact shape of the LIR, a very large number of iterations are normally required to ensure the convergence. In practice, the convergence rate can be improved by including a small regularization term in (17). This makes the actual mean gradient slightly different from the target. As will be shown in the result section, studying the performances of the optimum estimators with a small δ (by including a small amount of regularization) provides a possible indicator for system performance.

III. Application to Aperture Design

A. A Small Animal Spect System

We used this UCRB approach to analyze the performance of a small animal SPECT system, which is based on a pre-existing Gamma camera and pinhole or multiple pinhole apertures. Detector performances with different aperture configurations were compared based on the pixel-wise variance at given spatial resolutions. The simulated gamma camera has an active area of $\sim 40 \times 40 \text{ cm}^2$. The pinhole or multiple pinhole apertures were made of tungsten sheet of 3 mm thick. The designed field-of-view is a cylinder of $\sim 4\text{-cm}$ diameter and $\sim 8 \text{ cm}$ long that is pixelated into $32 \times 32 \times 64$ pixels of $1.2 \times 1.2 \times 1.2 \text{ mm}^3$. The 8-cm axis of the object cylinder was parallel to the surface of the Gamma camera. During the data

acquisition, the object is rotated with 32 steps of 11.25 degrees each. The axis of this rotation is 8 cm from the aperture and parallel to the aperture plate. The detector to aperture distance was 22 cm and the axis of the object was 7.5 cm from the aperture. We used pinholes having round cross sections and knife-edges with an acceptance angle of 45 degrees. For the following study, we focused on imaging performance for 35 keV gamma rays. The system model included effects of pinhole geometry and photon penetration [6]. In reality, 35 keV gamma rays suffer significant attenuation in tissues. This effect was, however, not modeled in our study for speeding up the data generation process. The gamma camera active area was modeled with $256 \times 256 \times 1.5 \times 1.5 \text{ mm}^2$ pixels and no depth-of-interaction effect. The detector model also included a Gaussian blur of 3-mm full-width at half-maximum (FWHM) to account for the limited accuracy of position calculations using Anger logic. We assumed the detector has a perfect energy resolution and an energy threshold of 10 keV. Therefore, almost all gamma rays passing through the collimator are detected by the detector. These configurations were used in our simulations and UCRB calculations.

B. Comparing Pinhole and Multiple-Pinhole Apertures

We applied the proposed UCRB approach to compare the imaging performances of similar systems that use the same photon detectors but different collimation apertures. Four different collimator designs using 1, 3, 6, and 12 pinholes were compared. The pinhole diameters were fixed to 1 mm. The multiple-pinhole apertures were constructed based on compact nonredundant array (NRA) patterns [18]. These NRA patterns have the property that the displacement between any two pinholes occurs once and only once. The autocorrelation function of a NRA pattern has a central peak of amplitude N equal to the number of pinholes, and a sidelobe of amplitude 1 goes out to a certain extent. The 3-, 6-, and 12-pinhole patterns are shown in Fig. 1. To demonstrate the degree of multiplexing, we simulated a phantom that entirely filled with uniform background activity. A 2.5-mm diameter and 8-cm-long cylinder was superimposed on the background along the 8-cm axis of the phantom. An uptake ratio of 1:10 was assumed between the background and the “hot” cylinder. It is easily seen that the use of multiple pinhole apertures resulted in a significant multiplexing in the projection data, as shown in Fig. 2.

We started from a fixed target mean gradient that follows a 3-D Gaussian distribution with 1.5-mm FWHM. Within the object, only the central 2-cm-diameter cylindrical volume along the 8-cm length was filled with a uniform activity, with a concentration of $\sim 15 \mu\text{Ci/cc}$. We derived the UCRB for δ within a wide range. For these calculations, we used the identity matrix for the penalty matrix C . We also repeated these calculations for an object entirely filled with activity, for checking the robustness of the detector ranking based on this method. The bound values for different aperture designs are shown in Figs. 3 and 4. We compared the variance minimizing mean gradient vectors (or local impulse function) as a function of the threshold δ . The results are shown in Fig. 5. For this, the 12-pinhole NRA was used and the estimators chosen are corresponding to the operating points shown in Fig. 3. When using a small amount of regularization, say $1/\lambda < 10^{-10}$, resulted mean gradient vectors are very similar to the target f . Study the performance of the optimum estimators (9) with a small amount of regularization provides a useful way for comparing different detector designs. It

is also seen that to achieve the minimum variance in reconstructions, one would use a tiny amount of penalty (or regularization), which is only for improving the convergence rate. The actual form of the penalty of the matrix $\mathbf{C}$ is less important, as long as $\mathbf{C}^{-1}$ has a null space disjoint from that for the Fisher information matrix $\mathbf{F}$.

In order to relate the minimum achievable variance to a common resolution measure, we derived the bound as a function of the FWHM of the local impulse response corresponding to the optimum estimator. The FWHM value was derived by fitting the LIRs into Gaussian shape. These results are shown in Figs. 6 and 7. Clearly, for the cases studied, the single-pinhole aperture resulted in the highest variance at the same resolutions. The use of multiple-pinhole apertures, in particular the 12-pinhole NRA, provided a significant reduction in imaging variance at the pixel of interest. This improvement is partially due to the increased number of view angles and partially due to the improved detection efficiency. When δ is getting smaller and smaller, the minimum achievable variance tends to change at a slower and slower rate.

The effect of using multiple-pinhole apertures on image quality can also be visualised through reconstructed images. We simulated a simple phantom with a uniform background that filled the entire 4-cm-diameter field of view. A “hot” rod (uptake ratio 2:1) and a “cold” rod (contains no activity) were inserted. Both are 1-mm diameter and 8 cm long, with their axes parallel to the aperture and 1.0 cm away from the axis of the object cylinder. The total activity simulated was corresponds to injecting 1.5 mCi activity into the phantom and observing for ~90 min (shared by observations at 32 view angles). Data were simulated for using both a single-pinhole aperture and a 6-pinhole aperture. In Figs. 8 and 9, we showed images reconstructed with the post filtered penalized maximum-likelihood (PML) algorithm. The penalty function was modeled with an identity matrix and the filter function was a 3-D Gaussian function of 1.5-mm FWHM. We controlled the reconstructed resolution at the centre of the field-of-view, to be 1.55-mm FWHM for both cases, by selecting appropriate λ . The same measuring time was assumed for systems with both apertures. At the same spatial resolution, the reconstructed images for the 6-pinhole aperture appear smoother than those for the single aperture. This agrees with the indication from comparing the minimum achievable variance.

C. Effect of Pinhole Size

The effect of pinhole size on imaging performance has been studied by many authors. In the context of detecting a known Gaussian signal in a stationary uniform background, Mayer *et al.* [19] showed that the optimum aperture size was fairly close to the Gaussian signal width. Fessler used an alternative approach that adapts the formulation of kernel based density estimation [20]. His study showed that the variance minimizing pinhole size is proportional to the desired target spatial resolution. However, such a relationship between the pinhole size and desired spatial resolution was not apparent in the original UCRB experiments with bias-gradient norm constraint [13]. Here, we studied this issue with the modified UCRB approach. We derived the minimum achievable variance for a single-pinhole aperture with 0.5-, 1, 1.5-, and 2-mm pinholes. These comparisons were performed for three target spatial resolutions 1, 1.5, and 2 mm, respectively. The results are shown in Figs. 10–15. It is evident

that as far as the local resolution/variance tradeoff is concerned, the optimum pinhole size varies with the desired spatial resolution. A larger pinhole diameter results in lower variance if a worse spatial resolution is acceptable. This result is similar to the conclusions made in [19] and [20]. One may expect a monotonic relationship between the desired spatial resolution and the variance minimizing pinhole size.

IV. Conclusion and Discussion

We proposed to use a modified UCRB to compare the minimum achievable variance in reconstructed images at given spatial resolutions. The constraint on spatial resolution is imposed on the mean gradient vector. It is, therefore, more direct and meaningful than the constraint on the bias gradient vectors, as formerly used by Hero *et al.* [8]. As we also showed, this bound can be achieved by postfiltering the PWLS estimators.

We used this bound to compare several multiple-pinhole aperture designs of gamma camera for small animal imaging applications. The results showed that the use of multiple-pinhole apertures offers clear advantages over single-pinhole for this application, from the standpoint of resolution/variance tradeoffs. There is a monotonic relationship between the desired spatial resolution and the variance minimizing pinhole size. One limitation of this approach is that the derivation of the UCRB does not take into account the nonnegativity constraint. For imaging in low count rate situations, algorithms that enforce the nonnegativity constraint may result in a variance lower than that predicted by this bound. The effect of this discrepancy on detector ranking may worth further study.

Due to limitations on the computing power currently available in our lab, the proposed method was applied only for comparing different systems rather than finding the “optimum” aperture design. The latter would require a search through a very large set of possible aperture configurations. However, the proposed UCRB may be calculated much more efficiently using FFT based methods for matrix inversion [21]. For example, the computing of a single point on the resolution/variance tradeoff curve can be finished on a single PC within several minutes using the locally shift-invariant approximation, whilst it was taking several hours using methods based on iterative Conjugate-gradient inversion of the Fisher Information matrix. Given the rapid advance in computing power, it would be possible to find an optimum aperture within a fairly large number of candidates.

Acknowledgment

The authors would like to thank Prof. J. Fessler, from the Department of EECS, University of Michigan, for valuable discussions.

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Fig. 1.
Multiple pinhole patterns used in simulations.

Fig. 2. Projections of the phantom through several apertures onto the detector at a single view angle.

Fig. 3.

Comparing different aperture designs based on the minimum achievable variance (UCRB) by any estimator, which satisfies the constraint $\|g - f\| < \delta$, as a function of δ . The phantom used contained a 20-mm diameter, 8-cm-long “hot” cylinder.

Fig. 4.

Comparing different aperture designs based on the minimum achievable variance (UCRB) by any estimator, which satisfies the constraint $\|g - f\| < \delta$, as a function of δ . The phantom used contained a 40-mm diameter, 8-cm-long “hot” cylinder.

Fig. 5. Optimum (circles and dashed curves) and target (solid curves) mean gradient vectors for the 12-pinhole aperture and different δ .

Fig. 6. Variance at the centre of the phantom as a function of spatial resolution achieved with the “optimum” estimator and different values of λ . The phantom contained a 20-mm “hot” cylinder and no background.

Fig. 7. Variance at the centre of the phantom as a function of spatial resolution achieved with the “optimum” estimator and different values of λ . The phantom contained a 40-mm “hot” cylinder and no background.

Fig. 8.

Comparing reconstructed images (filtered PML with 1.5-mm Gaussian filter) from the single and 6-pinhole apertures. The actual spatial resolutions at the centre of the images were identical (1.52-mm FWHM). Upper left: single-pinhole, unfiltered. Lower left: single-pinhole, filtered. Upper right: 6-pinhole, unfiltered; lower right: filtered. Approximated locations of pixels of interest are indicated by white circles.

Fig. 9. Central axial slices corresponding to the optimum reconstructions shown in Fig. 8. Both slices are parallel to the aperture. Upper: single-pinhole aperture. Lower: 6-pinhole aperture. Approximated locations of pixels of interest are indicated by white circles.

Fig. 10. Resolution variance tradeoff for a single-pinhole imager with different pinhole size and a target spatial resolution of 1-mm FWHM.

Fig. 11. Resolution variance tradeoff for a single-pinhole imager with different pinhole size and a target spatial resolution of 1-mm FWHM.

Fig. 12. Resolution variance tradeoff for a single-pinhole imager with different pinhole size and a target spatial resolution of 1.5-mm FWHM.

Fig. 13. Resolution variance tradeoff for a single-pinhole imager with different pinhole size and a target spatial resolution of 1.5-mm FWHM.

Fig. 14. Resolution variance tradeoff for a single-pinhole imager with different pinhole size and a target spatial resolution of 2-mm FWHM.

Fig. 15. Resolution variance tradeoff for a single-pinhole imager with different pinhole size and a target spatial resolution of 2-mm FWHM.