Event-based exponential synchronization of complex networks

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Abstract In this paper, we consider exponential synchronization of complex networks. The information diffusions between nodes are driven by properly defined events. By employing the M -matrix theory, algebraic graph theory and the Lyapunov method, two kinds of distributed event-triggering laws are designed, which avoid continuous communications between nodes. Then, several criteria that ensure the event-based exponential synchronization are presented, and the exponential convergence rates are obtained as well. Furthermore, we prove that Zeno behavior of the event-triggering laws can be excluded before synchronization being achieved, that is, the lower bounds of inter-event times are strictly positive. Finally, a simulation example is provided to illustrate the effectiveness of theoretical analysis.

Keywords Event-triggered synchronization · Complex networks · Strongly connected network · Directed spanning tree · Convergence rate

Introduction

During the past few years, complex networks have become an interesting research topic and appeal to have more attention in different fields from mathematics, biology, engineering sciences (Osborn 2010; Kamel and Xia 2009; Zhang et al. 2013e, 2014b; Watts and Strogatz 1998;

Barabasi and Albert 1999; Zhou et al. 2006; Strogatz 2001; Lü and Chen 2005). A complex network is a large set of interconnected nodes, where the nodes and connections can be anything, examples are internet, transportation networks, coupled biological and chemical engineering systems, neural networks in human brains and so on. The synchronization is one of the most important dynamical properties of complex networks (Zhang et al. 2011, 2013a, b, c, d, 2014a, 2015a, b; Balasubramaniam and Jarina 2014; Alofi et al. 2015; Reyes and Tanner 2015; Das and Ghose 2015; Olfati-Saber and Murray 2004; An et al. 2014; Dörfler and Bullo 2014; Yu et al. 2009a, b, 2013a; Lu et al. 2006; Lu and Chen 2006). In essence, synchronization is a form of self-organization. It has been demonstrated that many real-world problems have close relationships with network synchronization (Strogatz 2001; Lü and Chen 2005).

It is noteworthy that the communications between nodes in Zhang et al. (c, d 2015a, b), Reyes and Tanner (2015), Olfati-Saber and Murray (2004), Das and Ghose (2015), An et al. (2014), Dörfler and Bullo (2014), Yu et al. (2009a, b, 2013a); Lu et al. (2006) and Lu and Chen (2006) are continuous. The continuous communications have an obvious insurmountable deficiency: the designed control law requires real-time updates, which promotes network nodes to be equipped with high performance processors and high-speed communication channels. However, in practice, autonomous nodes such as mobile robots are often equipped with digital microprocessors which coordinate the data acquisition, communication with other nodes, and control actuation. Thus, it is necessary to implement discrete-time controllers. The impulsive controllers (Yang 2001; Lu et al. 2013, 2010; Tan et al. 2015; Qi et al. 2014; Pu et al. 2015) and the sampled data controllers (Chen and Francis 1995; Yu et al. 2013b, 2011a) are two typical types

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of controllers with discrete time updates. In Åström and Bernhardsson (2002) and Åström (2008), the authors proposed the event-triggered controllers. Different from the traditional discrete-time updating controllers, the event-triggered controller updates are determined by certain events that are triggered depending on the nodes' behavior. Based on the seminal works (Åström and Bernhardsson 2002; Åström 2008), the event-triggered controllers have been adopted widely for control science and engineering applications (Wang and Lemmon 2011; Mazo and Tabuada 2011; Tabuada 2007; Dimarogonas et al. 2012; Almeida et al. 2014; Liuzza et al. 2013; Adaldo et al. 2014; Zhu et al. 2014; Fan et al. 2013; Li et al. 2015; Gao et al. 2014; Zhou et al. 2015a). In Dimarogonas et al. (2012), the authors studied the event-triggered consensus of multi-agent systems with undirected communication topology. A limitation of the event-control law proposed in Dimarogonas et al. (2012) is that it requires continuous communication between neighboring nodes to constantly monitor whether the designed events occur or not. In Almeida et al. (2014), Liuzza et al. (2013), Adaldo et al. (2014, 2015), the authors studied the event-triggered consensus of multi-agent systems. A limitation of the event-control law proposed in Almeida et al. (2014), Liuzza et al. (2013) and Adaldo et al. (2014, 2015) is that their triggering threshold functions only depend on continuous-time nonincreasing threshold function, which is independent of the states of nodes. In Zhu et al. (2014), the authors studied event-based consensus of general linear multi-agent systems. The event-triggering law proposed in Zhu et al. (2014) depends on the state errors, which requires decoupling of actual states of the agents. In Fan et al. (2013), the authors studied event-triggered consensus of multi-agent systems using combinational measurements, which avoids decoupling of actual states of the agents. A limitation of the event-control law proposed in Fan et al. (2013) is that the triggering function is constructed based on continuous communications between nodes. On the other hand, the considered model in Fan et al. (2013) is a simple one, without being aware of nodes' inherent nonlinear dynamics. In Li et al. (2015), the authors studied event-triggered synchronization of complex dynamical systems, in which inherent nonlinear dynamics of nodes have been considered. However, the triggering function designed in Li et al. (2015) still need to decouple the actual states of the nodes. Hence, we are eager to consider the synchronization problem of complex networks via distributed event-triggered diffusions, which has rarely been considered (Liuzza et al. 2013; Adaldo et al. 2014; Li et al. 2015; Gao et al. 2014; Zhou et al. 2015a).

Motivated by the above discussions, we consider exponential synchronization of complex networks for both irreducible and reducible topologies. The controller updates of each nodes are driven by properly defined events, which

depend only on the combinational measurements. The event-triggering law proposed in this paper avoids continuous communications between nodes, which is a generalization of event-triggering laws in Fan et al. (2013). By employing the M -matrix theory, algebraic graph theory and the Lyapunov method, two kinds of distributed event-triggering laws are designed. Then, several criteria that ensure the event-based exponential synchronization are presented, and the exponential convergence rates are obtained as well. The main results in this paper are that if the diffusion topology is strongly connected, then the nodes exponentially synchronize to the weighted average of all nodes; if the diffusion topology is not strongly connected, but has a directed spanning tree, then the nodes exponentially synchronize to the weighted average of the nodes without incoming edge. Furthermore, we prove that Zeno behavior of the event-triggering laws can be excluded, that is, the lower bounds of inter-event times are strictly positive. Finally, a simulation example is provided to illustrate the effectiveness of theoretical analysis. The contribution in this paper is mainly threefolds:

1. In this paper, the considered model is more complicated, which contains both the coupling of the agents and the nonlinear dynamics governing the evolution of each isolated agent, which is an extension of the works in Zhu et al. (2014), Fan et al. (2013) and Zhou et al. (2015b). Moreover, the communication topology is assumed to be directed, which is also an extension of the work in Fan et al. (2013).
2. In this paper, the event-triggering laws avoid continuous communications between nodes, which is a generalization of the event-triggering laws in Fan et al. (2013) and Zhou et al. (2015a). On the other hand, the event-triggering laws avoid decoupling the actual states of nodes, which is a generalization of the event-triggering laws in Li et al. (2015).
3. In this paper, we consider the synchronization of complex networks with both irreducible and reducible topologies, while in Gao et al. (2014) and Zhou et al. (2015a), the authors only considered pinning synchronization of complex networks.

Notations Let $\|x\|$ and $\|A\|$ be the Euclidean norm of a vector x and a matrix A , respectively. Let I_n be the $n \times n$ identity matrix. Denote $\mathbf{1}_n$ be the vector whose elements are 1. Denote $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ be the smallest and largest eigenvalues of a symmetric matrix A . For a matrix A , $A > 0$ (≤ 0) means all elements in A is positive (non-positive). For a symmetric matrix A , $A \succ 0$ ($\prec 0$) means A is positive definite (negative definite). $\text{diag}\{\dots\}$ stands for a block-diagonal matrix. The superscript " T " represents the vector and matrix transpose. Let $\otimes$ be the Kronecker product.

Mathematical preliminaries

Algebraic graph theory

Several definitions and notations in the graph theory are introduced in the following which will be used in the later analysis. The interested readers please refer to some textbooks of graph theory (Godsil and Royle 2011) for more details.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted digraph of order N with the set of nodes $\mathcal{V} = \{1, 2, \dots, N\}$, set of directed edges $\mathcal{E} = \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij})_{N \times N}$. An edge in network $\mathcal{G}$ is denoted by (j, i) where i and j are called the terminal and initial node, respectively, which means that node i can receive information from node j . The neighboring set of agent i is denoted by $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. The adjacency elements associated with the edges of the graph are positive and the others are zero, i.e., $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Moreover, we assume that the graph contains no self-loops, i.e., $a_{ii} = 0$ for all $i = 1, 2, \dots, N$. Let $\mathcal{D} = \text{diag}\{d_1, d_2, \dots, d_N\}$, where $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian of the weighted digraph $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$, whose elements are presented as follows:

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{j=1, j \neq i}^N a_{ij}, & i = j. \end{cases}$$

A directed path from node j to node i is a sequence of edges of the form $(i, i_1), (i_1, i_2), \dots, (i_k, j)$ in the directed network with distinct nodes $i_l \in \mathcal{V}$, $l = 1, 2, \dots, k$. We say that a digraph $\mathcal{G}$ has a spanning tree if there exists a vertex called root which can access all other vertices. Denote the root set by root. We say a digraph $\mathcal{G}$ is strongly connected, if for any vertex pair (i, j) there exist a directed path from vertex j to vertex i and another directed path from vertex i to vertex j .

Some useful lemmas

Denote $L \in \mathbb{R}^{N \times N}$ be the Laplacian matrix of a weighted graph. Let $\xi = [\xi_1, \xi_2, \dots, \xi_N]^T$ be the normalized left eigenvector of L corresponding to eigenvalue 0. i.e., $\xi^T L = 0$ and $\sum_{i=1}^N \xi_i = 1$. Let $\Xi = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$, $W = \Xi - \xi \xi^T$, $\tilde{L} = \frac{\Xi L + L^T \Xi}{2}$. Note that W is a zero row sum matrix and satisfies $WL = \Xi L - \xi \xi^T L = \Xi L$.

The following definition on the general algebraic connectivity.

Definition 1 (Yu et al. 2011b) For a strongly connected network with Laplacian matrix L , the general algebraic connectivity is defined to be a real number

$$a_{\xi}(L) = \min_{x^T \xi = 0, x \neq 0} \frac{x^T \tilde{L} x}{x^T \Xi x},$$

where $\Xi = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$, $\xi = [\xi_1, \xi_2, \dots, \xi_N]^T$ with $\xi_i > 0$ for all $i = 1, 2, \dots, N$ and $\sum_{i=1}^N \xi_i = 1$.

Remark 1 As it is pointed in Yu et al. (2010, 2011b), the general algebraic connectivity of a strongly connected network can be computed by the following:

$$\begin{aligned} & \max \delta \\ & \text{subject to } Q^T (\tilde{L} - \delta \Xi) Q \succeq 0 \end{aligned}$$

where $Q = \begin{bmatrix} I_{N-1} \\ -\frac{\xi^T}{\xi_N} \\ \xi_N \end{bmatrix} \in \mathbb{R}^{N \times (N-1)}$ and $\hat{\xi} = [\xi_1, \xi_2, \dots, \xi_{N-1}]^T$.

In the following, we give the Perron-Frobenius Theorem.

Lemma 1 (Horn and Johnson 1991) For ξ , L and $\tilde{L}$ defined above. We have the following proposition.

1. If L is irreducible, then 0 is eigenvalue of L with multiplicity 1 and $\text{rank}(L) = N - 1$;
2. If L is irreducible, then ξ is a positive vector, i.e., $\xi > 0$ for all $i = 1, 2, \dots, N$; if L is reducible, then ξ is a nonnegative vector, i.e., $\xi \geq 0$ for all $i = 1, 2, \dots, N$;
3. If L is irreducible, then the eigenvalues of $\tilde{L}$ can be arranged as $0 = \lambda_1(\tilde{L}) < \lambda_2(\tilde{L}) < \dots < \lambda_N(\tilde{L})$.

Lemma 2 (Lu et al. 2010) For any vector $x, y \in \mathbb{R}^n$, we have $x^T y \leq \frac{1}{2} x^T x + \frac{1}{2} y^T y$.

Lemma 3 (Horn and Johnson 1991) The Kronecker product, denoted by $\otimes$, facilitates the manipulation of matrices with appropriate dimensions by the following properties:

1. $(A \otimes B)^T = A^T \otimes B^T$;
2. $(A + B) \otimes C = A \otimes C + B \otimes C$;
3. $(A \otimes B)(C \otimes D) = AC \otimes BD$.

M-matrices

In this section, some results on M -matrices are provided to investigate the synchronization of complex networks.

Definition 2 Let $A = (a_{ij})_{N \times N}$ be nonsingular and $a_{ij} \leq 0$ if $i \neq j$, $i, j = 1, 2, \dots, N$. Then A is called an M-matrix if and all the eigenvalues of A have positive real parts.

Lemma 4 (Horn and Johnson 1991) If A is a nonsingular M-matrix, then the following statements are equivalent.

1. Matrix A can be expressed by $A = \gamma I_N - B$, where $B \geq 0$ and $\gamma > \rho(B)$.
2. A^{-1} exists, and $A^{-1} \geq 0$.
3. $\text{Re}(\lambda_i(A)) > 0$ for all $i = 1, 2, \dots, N$.
4. There exists a positive definite diagonal matrix $\Xi = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$ such that $\Xi A + A^T \Xi \succ 0$.
5. There is a positive vector $x \in \mathbb{R}^N$ such that $Ax > 0$.

Complex networks via event-triggered communication

Consider a linearly coupled complex network composed of N identical nodes, in which each node is an n -dimensional dynamical system described by:

$$\dot{x}_i(t) = f(t, x_i(t)) + c \sum_{j=1}^N a_{ij} \Gamma(x_j(t) - x_i(t)), \quad i = 1, 2, \dots, N, \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ is the state of node i , and $f(t, x_i(t)) \in \mathbb{R}^n$ is a continuous vector-valued function. The positive constant c represents the coupling strength, the inner coupling matrix $\Gamma \in \mathbb{R}^{n \times n}$ is positive definite and describes a inner-coupling between the subsystems, the matrix $\mathcal{A} = (a_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$ represents the coupling configuration. We consider complex network in an induced graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, the vertices are labeled by $\mathcal{V} = \{1, 2, \dots, N\}$, the edge is defined as $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$; $(j, i) \notin \mathcal{E}$ if and only if $a_{ij} = 0$. Let L be the Laplacian matrix of the induced graph $\mathcal{G}$, ξ be the normalized left eigenvector of L corresponding to eigenvalue 0, $W = \Xi - \xi \xi^T$, and $\tilde{L} = \frac{\Xi L + L^T \Xi}{2}$. Note that W is a symmetric matrix with zero row sum. Then, from Lemma 1, we have W has an eigenvalue 0 with multiplicity 1, and all the other eigenvalues are positive, that is, $\lambda_{\max}(W) > 0$.

Notice that the Laplacian matrix L is irreducible if and only if the induced graph $\mathcal{G}$ is strongly connected. L is reducible if $\mathcal{G}$ is not strongly connected, but has a directed spanning tree (Horn 2005; Yu et al. 2010).

Definition 3 The coupled complex networks (1) is said to be exponentially synchronized, if there exists positive constants $\kappa > 0$, $\gamma > 0$ and $T > 0$ such that

$$\|x_i(t) - x_j(t)\| \leq \kappa e^{-\gamma t}, \quad i, j = 1, 2, \dots, N,$$

for any initial conditions and all $t > T$. γ is called the convergence rate.

Assumption 1 For nonlinear function $f(t, x)$, we assume that $f(t, 0) = 0$, and there exists a positive constant κ such that

$$\|f(t, x_1) - f(t, x_2)\| \leq \kappa \|x_1 - x_2\|,$$

for all $x_1, x_2 \in \mathbb{R}^n$ and $x_1 \neq x_2$.

It is noted that Assumption 1 is very mild, all linear and piecewise linear functions satisfy it. In addition, if $\frac{\partial f_i}{\partial x_j}$, $i, j = 1, 2, \dots, n$ are bounded, then this assumption is satisfied. So, many well known systems satisfy Assumption 1, including the Lorenz system, Chen system, Lü system, various neural networks, Chua's circuit (Yu et al. 2011b).

Main result

Our goal is to synchronize complex network (1). The previous most common models are investigated (An et al. 2014; Dörfler and Bullo 2014; Yu et al. 2009a, 2013a, 2009b; Zhang et al. 2013d; Lu et al. 2006; Lu and Chen 2006), however, their insurmountable deficiency is very obvious: each node's controller implementation must realize its all neighbors, current states at every time instant, that means continuous information communication between autonomous nodes is needed, which promotes nodes have to be equipped with high performance processors and communication facilities. Therefore, these traditional synchronization strategies undoubtedly will consume much more energy and be largely restricted in practical applications with limited computing resources and network bandwidth.

To overcome this conservation, we consider the nodes in the network communicates in discrete time instants that are determined by the event triggers. We assume that each node i samples its neighbors' information and updates the controller at its own triggering time instants. Then it broadcasts its information to its neighbors. Hence, the complex networks communicate in discrete time instants is described by

$$\begin{aligned} \dot{x}_i(t) &= f(t, x_i(t)) + c \sum_{j=1}^N a_{ij} \Gamma \left(x_j(t_{k_j}^i) - x_i(t_{k_i}^i) \right), \\ t &\in [t_{k_i}^i, t_{k_i+1}^i), \quad i = 1, 2, \dots, N, \end{aligned}$$

where $\{t_k^i\}_{k \geq 0}$ is the triggering time instants for node i , which will be determined later.

For $t \in [t_{k_i}^i, t_{k_i+1}^i)$, $i = 1, 2, \dots, N$, let $e_i(t) = x_i(t_{k_i}^i) - x_i(t)$. Then, we have

$$\begin{aligned}\dot{x}_i(t) &= f(t, x_i(t)) + c \sum_{j=1}^N a_{ij} \Gamma((e_j(t) + x_j(t)) \\ &\quad - (e_i(t) + x_i(t))) \\ &= f(t, x_i(t)) - c \sum_{j=1}^N l_{ij} \Gamma x_j(t) - c \sum_{j=1}^N l_{ij} \Gamma e_j(t),\end{aligned}\quad (2)$$

for $t \in [t_k^i, t_{k+1}^i)$, $i = 1, 2, \dots, N$. Let $e(t) = [e_1^T(t), \dots, e_N^T(t)]^T$, $x(t) = [x_1^T(t), \dots, x_N^T(t)]^T$, $F(t, x(t)) = [f^T(t, x_1(t)), \dots, f^T(t, x_N(t))]^T$, then, (2) can be written as

$$\dot{x}(t) = F(t, x(t)) - c(L \otimes \Gamma)x(t) - c(L \otimes \Gamma)e(t). \quad (3)$$

Now, we are in the position to present our main results.

$\mathcal{G}$ is strongly connected

In this subsection, we consider the case that $\mathcal{G}$ is strongly connected. By Lemma 1, we know that L is irreducible, and ξ is a positive vector corresponding to eigenvalue 0.

For agent i , we design the event triggering law as follows:

$$t_{k+1}^i = \inf \left\{ t > t_k^i : \|e_i(t)\| > \alpha_i \left\| \sum_{j \in N_i} a_{ij} (x_j(t_{k_i}^j) - x_i(t_{k_i}^i)) \right\| \right\}, \quad (4)$$

where $\alpha_i > 0$ are positive constants for all $i = 1, 2, \dots, N$.

Theorem 1 Suppose Assumption 1 hold. Then, exponential synchronization of complex network (1) with strongly connected communication topology under event-triggering diffusion law (4) can be achieved, if

$$\eta + \gamma + \varrho < 0$$

$$\text{where } \eta = 1 + \kappa^2, \quad \gamma = -\frac{2ca_\xi(L)\xi_{\min}\|F\|}{\lambda_{\max}(W)},$$

$$\varrho = \frac{2cl_{\max}^3 N^2 \alpha \lambda_{\max}(W) \|F\|}{(1 - l_{\max}^2 N^2 \alpha) \lambda_{\min}(\Xi)}, \quad \kappa_i, \quad i = 1, 2, \dots, N \text{ are defined in}$$

$$\text{Assumption 1, } \xi_{\min} = \min_{i=1,2,\dots,N} \{\xi_i\}, \quad l_{\max} = \max_{i,j=1,2,\dots,N} \{l_{ij}\}, \quad \alpha = \max_{i=1,2,\dots,N} \{\alpha_i\}, \quad 0 < \alpha < \frac{1}{l_{\max}^2 N^2},$$

$\xi = [\xi_1, \xi_2, \dots, \xi_N]^T$ satisfies $\xi^T L = 0$, $\xi_i > 0$, $\sum_{i=1}^N \xi_i = 1$, and $\Xi = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$. Furthermore, the Zeno behavior can be excluded before synchronization being achieved.

Proof Denote $\bar{x} = \sum_{i=1}^N \xi_i x_i(t)$. Consider the following Lyapunov function

$$\begin{aligned}V(t) &= \sum_{i=1}^N \xi_i (x_i(t) - \bar{x}(t))^T (x_i(t) - \bar{x}(t)) \\ &= x^T(t) (W \otimes I_n) x(t).\end{aligned}$$

Let w_{ij} be the element in the i th row and j th column of W . Then, by calculation, we have

$$V(t) = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{-w_{ij}}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)).$$

Differentiating $V(t)$ along the trajectories of (3), we have

$$\begin{aligned}\dot{V}(t) &= 2x^T(t) (W \otimes I_n) F(t, x(t)) - 2cx^T(t) (WL \otimes \Gamma)x(t) \\ &\quad - 2cx^T(t) (W \otimes \Gamma) (L \otimes I_n) e(t) \\ &= - \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_{ij} \left\{ (x_i(t) - x_j(t))^T (f(t, x_i(t)) - f(t, x_j(t))) \right\} \\ &\quad - 2cx^T(t) (WL \otimes \Gamma)x(t) - 2cx^T(t) (W \otimes \Gamma) (L \otimes I_n) e(t).\end{aligned}\quad (5)$$

By Lemma 2 and Assumption 1, we have

$$\begin{aligned}(x_i(t) - x_j(t))^T (f(t, x_i(t)) - f(t, x_j(t))) \\ \leq \frac{1}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)) \\ + \frac{1}{2} (f(t, x_i(t)) - f(t, x_j(t)))^T (f(t, x_i(t)) - f(t, x_j(t))) \\ \leq \frac{1}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)) \\ + \frac{1}{2} (x_i(t) - x_j(t))^T \kappa^2 (x_i(t) - x_j(t)) \\ \leq \frac{1}{2} (1 + \kappa^2) (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)).\end{aligned}$$

Notice that $w_{ij} \leq 0$ for all $i \neq j$. We have

$$\begin{aligned}2x^T(t) (W \otimes I_n) F(t, x(t)) \\ = - \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_{ij} \left\{ (x_i(t) - x_j(t))^T (f(t, x_i(t)) - f(t, x_j(t))) \right\} \\ \leq - \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_{ij} \left\{ \frac{1}{2} (x_i(t) - x_j(t))^T (1 + \kappa^2) (x_i(t) - x_j(t)) \right\} \\ \leq \eta \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{-w_{ij}}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)),\end{aligned}\quad (6)$$

where $\eta = 1 + \kappa^2$, with κ defined in Assumption 1. It is easy to see that $\eta > 0$.

By Definition 1 and $WL = \Xi L$, we have

$$\begin{aligned}-2cx^T(t) (WL \otimes \Gamma)x(t) &= -2cx^T(t) (\Xi L \otimes \Gamma)x(t) \\ &= -2cx^T(t) (\tilde{L} \otimes \Gamma)x(t) \\ &\leq -2ca_\xi(L)x^T(t) (\Xi \otimes \Gamma)x(t) \\ &\leq \gamma x^T(t) (W \otimes I_n)x(t),\end{aligned}\quad (7)$$

where $\xi_{\min} = \min_{i=1,2,\dots,N} \{\xi_i\}$, and $\gamma = -\frac{2ca_\xi(L)\xi_{\min}\|F\|}{\lambda_{\max}(W)}$. It is easy to see that $\gamma < 0$.

At sampling time instant $t_{k_i}^i$, the measurement $e_i(t)$ of agent i will be set to zero. Let $\hat{x}_i(t) = x_i(t) - \bar{x}(t)$ and

$\hat{x}(t) = [\hat{x}_1^T(t), \hat{x}_2^T(t), \dots, \hat{x}_N^T(t)]^T$. The event-based sampling triggering condition enforces:

$$\begin{aligned} \|e_i(t)\| &\leq \alpha_i \left\| \sum_{j \in N_i} a_{ij} \left(x_j(t_{k_j}^i) - x_i(t_{k_i}^i) \right) \right\| \\ &\leq \alpha_i \left\| \sum_{j \in N_i} a_{ij} (x_i(t) + e_i(t) - x_j(t) - e_j(t)) \right\| \\ &\leq \alpha_i \left\| \sum_{j=1}^N l_{ij} (x_j(t) - \bar{x}(t)) \right\| + \alpha_i \left\| \sum_{j=1}^N l_{ij} e_j(t) \right\| \quad (8) \\ &\leq l_{\max} \alpha_i \sum_{j=1}^N \|\tilde{x}_j(t)\| + \alpha_i \left\| \sum_{j=1}^N l_{ij} e_j(t) \right\| \\ &\leq l_{\max} \sqrt{N} \alpha_i \|\tilde{x}(t)\| + \sqrt{N} \alpha_i \|(L \otimes I_n) e(t)\|, \end{aligned}$$

for $i = 1, 2, \dots, N$, where $\tilde{x}(t) = x(t) - 1_N \otimes \bar{x}(t)$, $l_{\max} = \max_{i,j=1,2,\dots,N} \{ |l_{ij}| \}$ and the inequality

$$\frac{\sum_{j=1}^N \|\tilde{x}_j(t)\|}{N} \leq \sqrt{\frac{\sum_{j=1}^N \|\tilde{x}_j(t)\|^2}{N}} = \frac{\|x(t)\|}{\sqrt{N}}$$

is utilized. Then, it follows from (8) that

$$\begin{aligned} \|(L \otimes I_n) e(t)\|^2 &= \left\| \sum_{j=1}^N l_{1j} e_j(t) \right\|^2 + \left\| \sum_{j=1}^N l_{2j} e_j(t) \right\|^2 + \dots \\ &\quad + \left\| \sum_{j=1}^N l_{Nj} e_j(t) \right\|^2 \leq l_{\max}^2 \left\| \sum_{j=1}^N e_j(t) \right\|^2 \\ &\leq l_{\max}^2 \left(\sum_{j=1}^N \|e_j(t)\| \right)^2 \\ &\leq l_{\max}^4 \alpha_i^2 (l_{\max} \|\tilde{x}(t)\| + \|(L \otimes I_n) e(t)\|)^2. \end{aligned}$$

Thus, we have

$$\|(L \otimes I_n) e(t)\| \leq \frac{l_{\max}^3 N^2 \alpha}{1 - l_{\max}^2 N^2 \alpha} \|\tilde{x}(t)\|, \quad (9)$$

where $\alpha = \max_{i=1,2,\dots,N} \{ \alpha_i \}$, and $0 < \alpha < \frac{1}{l_{\max}^2 N^2}$.

Notice that W is a symmetric positive semi-definite matrix. Then, combining (9), we have

$$\begin{aligned} -2c x^T(t) (W \otimes \Gamma) (L \otimes I_n) e(t) &\leq 2c \lambda_{\max}(W) \|\Gamma\| \|\tilde{x}(t)\| \|(L \otimes I_n) e(t)\| \\ &\leq \frac{2c l_{\max}^2 N^2 \alpha \lambda_{\max}(W) \|\Gamma\|}{(1 - l_{\max}^2 N^2 \alpha)} \tilde{x}^T(t) \tilde{x}(t) \\ &\leq \frac{2c l_{\max}^2 N^2 \alpha \lambda_{\max}(W) \|\Gamma\|}{(1 - l_{\max}^2 N^2 \alpha) \lambda_{\min}(\Xi)} \tilde{x}^T(t) (\Xi \otimes I_n) \tilde{x}(t) \\ &\leq \varrho \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{-w_{ij}}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)), \end{aligned} \quad (10)$$

where $\varrho = \frac{2c l_{\max}^2 N^2 \alpha \lambda_{\max}(W) \|\Gamma\|}{(1 - l_{\max}^2 N^2 \alpha) \lambda_{\min}(\Xi)}$. It is easy to see that $\varrho > 0$. Substituting (6), (7) and (10) into (5), we have

$$\dot{V}(t) \leq (\eta + \gamma + \varrho) \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{-w_{ij}}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)). \quad (11)$$

Since $\eta + \gamma + \varrho < 0$, and $w_{ij} < 0$ for all $i \neq j$, it holds that $\dot{V}(t) \leq 0$, and $\dot{V}(t) = 0$ if and only if $x_i(t) = x_j(t)$ for all $i \neq j$, which implies that synchronization of complex network (1) under event-triggering law (4) can be achieved.

For $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, by (11), we have

$$\dot{V}(t) \leq (\eta + \gamma + \varrho) V(t).$$

Thus,

$$V(t) \leq V(t_{k_i}^i) e^{(\eta + \gamma + \varrho)(t - t_{k_i}^i)},$$

which implies that

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{-w_{ij}}{2} (x_i(t) - x_j(t))^T (x_i(t) - x_j(t)) \leq V(t_{k_i}^i) e^{(\eta + \gamma + \varrho)(t - t_{k_i}^i)}.$$

Thus,

$$\|x_i(t) - x_j(t)\| \leq \sqrt{\frac{2V(t_{k_i}^i)}{-w_{ij}}} e^{\frac{\eta + \gamma + \varrho}{2}(t - t_{k_i}^i)}.$$

Hence, the exponential convergence rate of complex network (1) under event-triggering law (4) is $-\frac{\eta + \gamma + \varrho}{2}$.

Next, we will show that the Zeno behavior can be excluded, i.e., inter-event times $\{t_{k_{i+1}}^i - t_{k_i}^i : k \geq 0\}$ have a positive lower bound for all $i = 1, 2, \dots, N$ before synchronization being achieved.

For $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, we have

$$\begin{aligned} \frac{d}{dt} \|e_i(t)\| &\leq \|\dot{e}_i(t)\| \\ &\leq \|f(t, x_i(t))\| + u_i(t_{k_i}^i, t_{k_j}^j) \\ &\leq \kappa \|e_i(t)\| + \bar{u}_i(t_{k_i}^i). \end{aligned} \quad (12)$$

where

$$\begin{aligned} \bar{u}_i(t_{k_i}^i) &= \kappa \|x_i(t_{k_i}^i)\| + \sup_{t_{k_i}^i \leq t_{k_j}^j < t_{k_{i+1}}^i, j=1,2,\dots,N} \{u_i(t_{k_i}^i, t_{k_j}^j)\}, \\ u_i(t_{k_i}^i, t_{k_j}^j) &= c \sum_{j=1}^N |a_{ij}| \|\Gamma\| \|x_j(t_{k_j}^j) - x_i(t_{k_i}^i)\|. \end{aligned}$$

Thus, we have

$$\|e_i(t)\| \leq \frac{\bar{u}_i(t_{k_i}^i)}{\kappa} \left(e^{\kappa(t-t_{k_i}^i)} - 1 \right). \quad (13)$$

By the triggering law (4), when the event of agent i is triggered, we have

$$\|e_i(t_{k_{i+1}}^i)\| \geq \alpha_i \left\| \sum_{j \in N_i} a_{ij} (x_j(t_{k_j}^j) - x_i(t_{k_i}^i)) \right\|. \quad (14)$$

Combining (13) and (14), we have

$$\alpha_i \left\| \sum_{j \in N_i} a_{ij} (x_j(t_{k_j}^j) - x_i(t_{k_i}^i)) \right\| \leq \frac{\bar{u}_i(t_{k_i}^i)}{\kappa} \left(e^{\kappa(t_{k_{i+1}}^i - t_{k_i}^i)} - 1 \right).$$

By simple calculation, we have

$$t_{k_{i+1}}^i - t_{k_i}^i \geq \frac{1}{\kappa} \ln \left(1 + \frac{\alpha_i \kappa \left\| \sum_{j \in N_i} a_{ij} (x_j(t_{k_j}^j) - x_i(t_{k_i}^i)) \right\|}{\bar{u}_i(t_{k_i}^i)} \right) > 0,$$

which implies that agent i does not exhibit Zeno-behavior before synchronization being achieved. $\square$

$\mathcal{G}$ is not strongly connected, but has a directed spanning tree

In many cases, the induced communication graph $\mathcal{G}$ is not strongly connected, it may have a directed spanning tree, for example, the master-slave system. If $\mathcal{G}$ is not strongly connected, but has directed spanning tree, then L is reducible. For such networks, we can relabel the nodes such that its Laplacian matrix is lower block triangular.

Proposition 1 *If a directed graph $\hat{\mathcal{G}}$ is not strongly connected, but has a directed spanning tree, then it can be relabeled such that its Laplacian matrix is lower block triangular.*

Proof We can relabel all the nodes by the following procedure.

- Step 1. Find all the strongly connected components in graph $\hat{\mathcal{G}}$.
- Step 2. Find a strongly connected component that has no incoming edge. Label it as SCC_1 , and label the nodes in SCC_1 as $1, 2, \dots, n_1$ randomly. Note that such SCC_1 can always be found, because $\hat{\mathcal{G}}$ has a directed spanning tree.
- Step 3. Find the strongly connected component that has no incoming edge from un-relabeled nodes. Label it as SCC_2 , and label the nodes in SCC_2 as $n_1 + 1, n_1 + 2, \dots, n_1 + n_2$ randomly. Note

that such SCC_2 can always be found, because $\hat{\mathcal{G}}$ is not strongly connected.

- Step s. Find the strongly connected component that has no incoming edge from un-relabeled nodes. Label it as SCC_s , and label the nodes in SCC_s as $\sum_{j=1}^{s-1} n_j + 1, \sum_{j=1}^{s-1} n_j + 2, \dots, \sum_{j=1}^s n_j$ randomly.

By Proposition 1, we can relabel the nodes in the complex networks. Suppose the weighted adjacency matrix of the relabeled communication graph be $\hat{\mathcal{A}} = (\hat{a}_{ij})_{N \times N}$ and the Laplacian matrix of the relabeled graph be

$$\bar{L} = \begin{pmatrix} \bar{L}_{11} & 0 & \cdots & 0 \\ \bar{L}_{21} & \bar{L}_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{L}_{m1} & \bar{L}_{m2} & \cdots & \bar{L}_{mm} \end{pmatrix},$$

which means there are m strongly connected components in $\mathcal{G}$, which are denoted by $SCC_1, \dots, SCC_m$ corresponding to each block rows in $\bar{L}$. We assume that there are n_p nodes in SCC_p , for $p = 1, 2, \dots, m$, respectively, and $\sum_{p=1}^m n_p = N$.

Lemma 5 *Let*

$$\hat{L} = \begin{pmatrix} \bar{L}_{22} & 0 & \cdots & 0 \\ \bar{L}_{32} & \bar{L}_{33} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{L}_{m1} & \bar{L}_{m2} & \cdots & \bar{L}_{mm} \end{pmatrix}.$$

There exist positive definite diagonal matrices $\Sigma = \text{diag}\{\Sigma_2, \Sigma_3, \dots, \Sigma_m\}$ and $\Delta = \text{diag}\{\Delta_2 I_{n_2}, \Delta_3 I_{n_3}, \dots, \Delta_m I_{n_m}\}$, such that

$$\Delta \Sigma \hat{L} + \hat{L}^T \Sigma \Delta \succ 0.$$

Proof For each $i = 2, 3, \dots, m$, let $\bar{L}_{ii} = \bar{L}_{ii} + D_i$, where $\bar{L}_{ii}$ is a zero-row-sum matrix whose off-diagonal elements are non-positive and D_i is a diagonal matrix. By the relabeling procedure, we have $D_i \geq 0$ and at least one diagonal element in D_i is positive. By Hu and Hong (2007), we have $\bar{L}_{ii}$ is positively stable i.e., all the eigenvalues of $\bar{L}_{ii}$ have positive real parts. Hence, $\bar{L}_{ii}$ is an M -matrix. By Lemma 4, we have there exists a diagonal matrix $\Sigma_i = \text{diag}\{\zeta_{i1}, \zeta_{i2}, \dots, \zeta_{in_i}\}$, such that $\Sigma_i \bar{L}_{ii} + \bar{L}_{ii}^T \Sigma_i > 0$. Let $\Sigma = \text{diag}\{\Sigma_2, \Sigma_3, \dots, \Sigma_m\}$ and $\Delta = \text{diag}\{\Delta_2 I_{n_2}, \Delta_3 I_{n_3}, \dots, \Delta_m I_{n_m}\}$, where $\Delta_i, i = 2, 3, \dots, m$, are appropriate positive numbers to be chosen such that $\Delta \Sigma \hat{L} + \hat{L}^T \Sigma \Delta \succ 0$ by the following way.

Let $\Theta_2 = \Delta_2 (\Sigma_2 \bar{L}_{22} + \bar{L}_{22}^T \Sigma_2)$ and $\Theta_i, i = 2, 3, \dots, m$, is defined as

$$\Theta_i = \begin{pmatrix} A_2(\Sigma_2 \bar{L}_{22} + \bar{L}_{22}^T \Sigma_2) & A_3 \Sigma_3 \bar{L}_{32}^T & \cdots & A_i \Sigma_i \bar{L}_{i2}^T \\ A_3 \Sigma_3 \bar{L}_{32} & A_3(\Sigma_3 \bar{L}_{33} + \bar{L}_{33}^T \Sigma_3) & \cdots & A_i \Sigma_i \bar{L}_{i3}^T \\ \vdots & \vdots & \ddots & \vdots \\ A_i \Sigma_i \bar{L}_{i2} & A_i \Sigma_i \bar{L}_{i3} & \cdots & A_i(\Sigma_i \bar{L}_{ii} + \bar{L}_{ii}^T \Sigma_i) \end{pmatrix}.$$

Then, Δ_i , $i = 2, 3, \dots, m$, can be chosen step by step. By (15), Δ_1 can be chosen as any positive number. Suppose that $\Delta_1, \Delta_2, \dots, \Delta_i$ are chosen. Noting that $\Sigma_{i+1} \bar{L}_{i+1,i+1} + \bar{L}_{i+1,i+1}^T \Sigma_{i+1} > 0$, by the Shur complement, we can choose $\Delta_{i+1} > 0$ satisfies the following matrix inequality

$$\Theta_i - \Delta_{i+1} \Gamma_{i+1}^T (\Sigma_{i+1} \bar{L}_{i+1,i+1} + \bar{L}_{i+1,i+1}^T \Sigma_{i+1})^{-1} \Gamma_{i+1} > 0,$$

where $\Gamma_{i+1} = [\Sigma_{i+1} \bar{L}_{i+1,2}, \Sigma_{i+1} \bar{L}_{i+1,3}, \dots, \Sigma_{i+1} \bar{L}_{i+1,i}]$. By induction, we can choose Σ_i and Δ_i , $i = 2, 3, \dots, m$.

Since the nodes in SCC_1 are strongly connected and have no incoming edge, $\bar{L}_{11}$ is irreducible. In view of Theorem 1, synchronization of nodes in SCC_1 can be achieved. Suppose that the synchronized state in SCC_1 be $x_s(t)$. Then, it satisfies the following dynamical system.

$$\dot{x}_s(t) = f(t, x_s(t)), \quad (15)$$

where $x_s(t) \in \mathbb{R}^n$. Moreover, the dynamics of the nodes in SCC_p , $p = 2, 3, \dots, m$, can be shown as

$$\dot{x}_i(t) = f(t, x_i(t)) + c \sum_{j=1}^N \hat{a}_{ij} \Gamma (x_j(t_{k_j}^i) - x_i(t_{k_i}^i)), \quad (16)$$

for $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, where $i \in SCC_p$, $p = 2, 3, \dots, m$. Let $\hat{x}_i(t) = x_i(t) - x_s(t)$. By (15) and (3.2), we have

$$\dot{\hat{x}}_i(t) = f(t, x_i(t)) - f(t, x_s(t)) + c \sum_{j=1}^N \hat{a}_{ij} \Gamma (\hat{x}_j(t_{k_j}^i) - \hat{x}_i(t_{k_i}^i)), \quad (17)$$

for $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, where $i \in SCC_p$, $p = 2, 3, \dots, m$.

Let the measure error $\hat{e}_i(t)$ be $\hat{e}_i(t) = \hat{x}_i(t_{k_i}^i) - \hat{x}_i(t)$, then (17) can be rewritten as

$$\begin{aligned} \dot{\hat{x}}_i(t) &= f(t, x_i(t)) - f(t, x_s(t)) \\ &\quad + c \sum_{j=1}^N \hat{a}_{ij} \Gamma ((\hat{e}_j(t) + \hat{x}_j(t)) - ((\hat{e}_i(t) + \hat{x}_i(t))) \\ &= f(t, x_i(t)) - f(t, x_s(t)) - c \sum_{j=1}^N \hat{a}_{ij} \Gamma \hat{x}_j(t) - c \sum_{j=1}^N \hat{a}_{ij} \Gamma \hat{e}_j(t), \end{aligned} \quad (18)$$

for $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, where $i \in SCC_p$, $p = 2, 3, \dots, m$.

Let $\hat{e}(t) = [\hat{e}_{n_1+1}^T(t), \dots, \hat{e}_N^T(t)]^T$, $\hat{x}(t) = [\hat{x}_{n_1+1}^T(t), \dots, \hat{x}_N^T(t)]^T$, $F(t, x(t)) = [f^T(t, x_{n_1+1}(t)), \dots, f^T(t, x_N(t))]^T$. Then, then vector form of (18) is that

$$\begin{aligned} \dot{\hat{x}}(t) &= F(t, x(t)) - 1_N \otimes f(t, x_s(t)) - c(\hat{L} \otimes \Gamma) \hat{x}(t) \\ &\quad - c(\hat{L} \otimes \Gamma) \hat{e}(t), \end{aligned} \quad (19)$$

for $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, where $i \in SCC_p$, $p = 2, 3, \dots, m$.

For node i , we design the triggering law as follows:

$$t_{k+1}^i = \inf \left\{ t > t_k^i : \|\hat{e}_i(t)\| > \hat{\alpha}_i \left\| \sum_{j \in N_i} \hat{a}_{ij} (\hat{x}_j(t_{k_j}^i) - \hat{x}_i(t_{k_i}^i)) \right\| \right\}, \quad (20)$$

where $\hat{\alpha}_i > 0$, $i \in SCC_p$, $p = 2, 3, \dots, m$ are positive constant parameters.

Theorem 2 Suppose Assumption 1 hold. Suppose the communication graph of complex network (1) is not strongly connected, but has a directed spanning tree. After relabeling, the strongly connected components are $SCC_1, SCC_2, \dots, SCC_m$. Then, exponential synchronization of complex network (1) can be reached under event-triggering law (4) and (20), if the parameters of SCC_1 satisfy Theorem 1, and the parameters of $SCC_2, \dots, SCC_m$ satisfy

$$\eta + \hat{\gamma} + \hat{Q} < 0,$$

$$\text{where } \eta = 1 + \kappa^2, \hat{\gamma} = -\frac{c \lambda_{\min}(\Phi) \|\Gamma\|}{\lambda_{\max}(\Sigma \Delta)}, \hat{Q} = \frac{2c \hat{L}_{\max}^2 N^2 \hat{\alpha}_{\max}(\Sigma \Delta) \|\Gamma\|}{\left(1 - \frac{1}{\hat{L}_{\max}^2 N^2 \hat{\alpha}_{\min}(\Sigma \Delta)}\right) \lambda_{\min}(\Sigma \Delta)},$$

with $\Phi = \Delta \Sigma \hat{L} + \hat{L}^T \Sigma \Delta$, $0 < \hat{\alpha} < \frac{1}{\hat{L}_{\max}^2 N^2}$, $\hat{\alpha} = \max_{i=1,2,\dots,N} \{\hat{\alpha}_i\}$. Furthermore, the Zeno behavior can be excluded before synchronization being achieved.

Proof For SCC_1 , if the parameters satisfy the conditions in Theorem 1, then by Theorem 1, synchronization of complex network (1) can be reached under event-triggering law (4). In the following, we consider the nodes in $SCC_2, \dots, SCC_m$.

Consider the following Lyapunov function

$$V(t) = \hat{x}^T(t) (\Sigma \Delta \otimes I_n) \hat{x}(t).$$

Differentiating $V(t)$ along the trajectories of (19), we have

$$\begin{aligned} \dot{V}(t) &= 2\hat{x}^T(t) (\Sigma \Delta \otimes I_n) (F(t, x(t)) - 1_N \otimes f(t, x_s(t))) \\ &\quad - c\hat{x}^T(t) (\Phi \otimes \Gamma) \hat{x}(t) - 2c\hat{x}^T(t) (\Sigma \Delta \otimes \Gamma) (\hat{L} \otimes I_n) \hat{e}(t) \end{aligned} \quad (21)$$

where $\Phi = \Delta \Sigma \hat{L} + \hat{L}^T \Sigma \Delta$.

By Lemma 2 and Assumption 1, we have

$$\begin{aligned} 2\hat{x}^T(t) (\Sigma \Delta \otimes I_n) (F(t, x(t)) - 1_N \otimes f(t, x_s(t))) \\ \leq (1 + \kappa^2) \hat{x}^T(t) (\Sigma \Delta \otimes I_n) \hat{x}(t) \\ \leq \eta \hat{x}^T(t) (\Sigma \Delta \otimes I_n) \hat{x}(t), \end{aligned} \quad (22)$$

where $\eta = \max_{i=1,2,\dots,N} \{1 + k_i^2\}$.

By Lemma 5, we have

$$-c\hat{x}^T(t)(\Phi \otimes \Gamma)\hat{x}(t) \leq \gamma\hat{x}^T(t)(\Sigma\Delta \otimes I_n)\hat{x}(t) \quad (23)$$

where $\hat{\gamma} = -\frac{c\lambda_{\min}(\Phi)\|\Gamma\|}{\lambda_{\max}(\Sigma\Delta)}$ with $\Phi = \Delta\Sigma\hat{L} + \hat{L}^T\Sigma\Delta$.

At sampling time instant $t_{k_i}^i$, the measurement $e_i(t)$ of agent i will be set to zero. The event-based sampling triggering condition enforces:

$$\begin{aligned} \|e_i(t)\| &\leq \hat{\alpha}_i \left\| \sum_{j \in N_i} \hat{a}_{ij} \left(\hat{x}_j(t_{k_i}^i) - \hat{x}_i(t_{k_i}^i) \right) \right\| \\ &\leq \hat{\alpha}_i \left\| \sum_{j=1}^N \hat{l}_{ij} \hat{x}_j(t) \right\| + \hat{\alpha}_i \left\| \sum_{j=1}^N \hat{l}_{ij} e_j(t) \right\| \\ &\leq \hat{l}_{\max} \hat{\alpha}_i \sum_{j=1}^N \|\hat{x}_j(t)\| + \hat{\alpha}_i \left\| \sum_{j=1}^N \hat{l}_{ij} e_j(t) \right\| \\ &\leq \hat{l}_{\max} \sqrt{N} \hat{\alpha}_i \|\hat{x}(t)\| + \sqrt{N} \hat{\alpha}_i \|(\hat{L} \otimes I_n)e(t)\|, \end{aligned} \quad (24)$$

for all $i = 1, 2, \dots, N$, where $\hat{l}_{\max} = \max_{i,j=1,2,\dots,N} \{\|\hat{l}_{ij}\|\}$

and the inequality $\frac{\sum_{j=1}^N \|\hat{x}_j(t)\|}{N} \leq \sqrt{\frac{\sum_{j=1}^N \|\hat{x}_j(t)\|^2}{N}} = \frac{\|\hat{x}(t)\|}{\sqrt{N}}$ is utilized. Then, it follows from (23) that

$$\begin{aligned} \|(\hat{L} \otimes I_n)e(t)\|^2 &= \left\| \sum_{j=1}^N \hat{l}_{1j} e_j(t) \right\|^2 + \left\| \sum_{j=1}^N \hat{l}_{2j} e_j(t) \right\|^2 + \dots \\ &\quad + \left\| \sum_{j=1}^N \hat{l}_{Nj} e_j(t) \right\|^2 \leq \hat{l}_{\max}^2 \sum_{j=1}^N \|e_j(t)\|^2 \\ &\leq \hat{l}_{\max}^2 N \left(\sum_{j=1}^N \|e_j(t)\| \right)^2 \\ &\leq \hat{l}_{\max}^2 N^4 \hat{\alpha}^2 (\hat{l}_{\max} \|\hat{x}(t)\| + \|(\hat{L} \otimes I_n)e(t)\|)^2 \end{aligned}$$

Thus, we have

$$\|(\hat{L} \otimes I_n)e(t)\| \leq \frac{\hat{l}_{\max}^3 N^2 \hat{\alpha}}{1 - \hat{l}_{\max}^2 N^2 \hat{\alpha}} \|\hat{x}(t)\|, \quad (25)$$

where $0 < \hat{\alpha} < \frac{1}{\hat{l}_{\max}^2 N^2}$, $\hat{\alpha} = \max_{i=1,2,\dots,N} \{\hat{\alpha}_i\}$. Then, combining (25), we have

$$\begin{aligned} -2c\hat{x}^T(t)(\Sigma\Delta \otimes \Gamma)(\hat{L} \otimes I_n)e(t) &\leq 2c\lambda_{\max}(\Sigma\Delta) \|\hat{x}(t)\| \|\Gamma\| \\ &\quad \|(\hat{L} \otimes I_n)e(t)\| \\ &\leq \varrho \hat{x}^T(t)(\Sigma\Delta \otimes I_n)\hat{x}(t), \end{aligned} \quad (26)$$

$$\text{where } \hat{\varrho} = \frac{2c\lambda_{\max}(\Sigma\Delta) \|\Gamma\| \hat{l}_{\max}^3 N^2 \hat{\alpha}}{(1 - \hat{l}_{\max}^2 N^2 \hat{\alpha}) \lambda_{\min}(\Sigma\Delta)}.$$

Substituting (22), (23) and (26) into (21), we have

$$\begin{aligned} \dot{V}(t) &\leq (\eta + \gamma + \varrho) \hat{x}^T(t)(\Sigma\Delta \otimes I_n)\hat{x}(t) \\ &= (\eta + \gamma + \varrho) V(t). \end{aligned}$$

Since $\eta + \gamma + \varrho < 0$, we have $\dot{V}(t) \leq 0$ and $\dot{V}(t) = 0$ if and only if $\|\hat{x}(t)\| = 0$, which implies that synchronization of complex network (1) under event-triggering law (20) can be achieved.

For $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, $i \in SCC_p$, $p = 2, 3, \dots, m$ we have

$$\dot{V}(t) \leq (\eta + \gamma + \varrho) V(t).$$

Thus,

$$V(t) \leq V(t_{k_i}^i) e^{(\eta + \gamma + \varrho)(t - t_{k_i}^i)},$$

which implies that

$$\|x_i(t) - x_s(t)\| \leq \sqrt{\frac{2V(t_{k_i}^i)}{\lambda_{\min}(\Sigma\Delta)}} e^{\frac{\eta + \gamma + \varrho}{2}(t - t_{k_i}^i)}.$$

Hence, the exponential convergence rate of complex network (1) under event-triggering law (20) is $-\frac{\eta + \gamma + \varrho}{2}$.

Next, we will show that the Zeno behavior can be excluded, i.e., the differences of inter-event times $\{t_{k_{i+1}}^i - t_{k_i}^i : k \geq 0\}$ have a positive lower bound for all $i = 1, 2, \dots, N$ before synchronization being achieved.

For $t \in [t_{k_i}^i, t_{k_{i+1}}^i)$, we have

$$\begin{aligned} \frac{d}{dt} \|\hat{e}_i(t)\| &\leq \|\dot{\hat{e}}_i(t)\| \\ &\leq \|f(t, x_i(t)) - f(t, x_s(t))\| + \hat{u}_i(t_{k_i}^i, t_{k_j}^j) \\ &\leq \kappa \|\hat{e}_i(t)\| + \bar{\hat{u}}_i(t_{k_i}^i). \end{aligned} \quad (27)$$

where $\bar{\hat{u}}_i(t_{k_i}^i) = \kappa \|\hat{x}(t_{k_i}^i)\| + \sup_{t_{k_i}^i \leq t_{k_j}^j < t_{k_{i+1}}^i, j=1,2,\dots,N} \{\hat{u}_i(t_{k_i}^i, t_{k_j}^j)\}$, $\hat{u}_i(t_{k_i}^i, t_{k_j}^j) = c \sum_{j=1}^N |\hat{a}_{ij}| \|\Gamma\| \|\hat{x}_j(t_{k_j}^j) - \hat{x}_i(t_{k_i}^i)\|$. Thus, we have

$$\|e_i(t)\| \leq \frac{\bar{\hat{u}}_i(t_{k_i}^i)}{\kappa} \left(e^{\kappa(t - t_{k_i}^i)} - 1 \right). \quad (28)$$

By the triggering law (20), when the event of agent i is triggered, we have

$$\|e_i(t_{k_{i+1}}^i)\| \geq \hat{\alpha}_i \left\| \sum_{j \in N_i} \hat{a}_{ij} \left(\hat{x}_j(t_{k_j}^j) - \hat{x}_i(t_{k_i}^i) \right) \right\|. \quad (29)$$

Combining (28) and (29), we have

$$\hat{\alpha}_i \left\| \sum_{j \in N_i} \hat{a}_{ij} (\hat{x}_j(t_{k_j}^i) - \hat{x}_i(t_{k_i}^i)) \right\| \leq \frac{\tilde{u}_i(t_{k_i}^i)}{\kappa} \left(e^{\kappa(t_{k_{i+1}}^i - t_{k_i}^i)} - 1 \right).$$

By simple calculation, we have

$$t_{k_{i+1}}^i - t_{k_i}^i \geq \frac{1}{\kappa} \ln \left(1 + \frac{\hat{\alpha}_i \kappa \left\| \sum_{j \in N_i} \hat{a}_{ij} (\hat{x}_j(t_{k_j}^i) - \hat{x}_i(t_{k_i}^i)) \right\|}{\tilde{u}_i(t_{k_i}^i)} \right) > 0,$$

which implies that agent i does not exhibit Zeno-behavior before synchronization being achieved.

Simulation example

In this section, we will provide two simulation examples to illustrate the proposed approaches. Consider the information interactive network with communication graph $\mathcal{G}$ given in Fig. 1. It can be seen from Fig. 1 that the network is not strongly connected but has a directed spanning tree, $SCC_1 = \{1, 2\}$ is the first strongly connected component, $SCC_2 = \{3, 4\}$ is the second strongly connected component, $SCC_3 = \{5, 6\}$ is the third strongly connected component. Let the weighted adjacency matrix and its Laplacian matrix L be

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 5 & 2 & 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & -3 & -1 & 4 & 0 & 0 \\ -4 & 0 & 0 & 0 & 7 & -3 \\ 0 & 0 & 0 & -5 & -2 & 7 \end{bmatrix}.$$

Then, we have

$$\bar{L}_{11} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}, \hat{L} = \begin{bmatrix} 2 & -2 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 0 & 0 & 7 & -3 \\ 0 & -5 & -2 & 7 \end{bmatrix}.$$

Each network node's kinematic under study is modeled by a chaotic Chua's circuit which is described by

Fig. 1 The interaction diagram of 6 nodes, in which $\{1, 2\}$ is the first strongly connected component (SCC_1); $\{3, 4\}$ is the second strongly connected component (SCC_2); $\{5, 6\}$ is the third strongly connected component (SCC_3)

$$\begin{cases} \dot{x}_{i1}(t) = 1.5(x_{i2} - g(x_{i1})), \\ \dot{x}_{i2}(t) = x_{i1} - x_{i2} + x_{i3}, \\ \dot{x}_{i3}(t) = -1.5x_{i2}, \end{cases}$$

for all $i = 1, 2, \dots, 6$, where $g(x_{i1}) = \frac{1}{4}x_{i1} - \frac{7}{24}(|x_{i1} + 1| - |x_{i1} - 1|)$. Suppose the inner-coupling matrix be $\Gamma = \text{diag}\{1, 1, 1\}$. Thus, $\|\Gamma\| = \sqrt{3}$. In view of Assumption 1, we can calculate that $\kappa = 3$. Then, we have $\eta = 10$.

For $SCC_1 = \{1, 2\}$, where $N = 2$. We can get that $\xi = (0.3333, 0.6667)^T$ is the normalized left eigenvector of $\bar{L}_{11}$ corresponding to eigenvalue 0. Thus, $\xi_{\min} = 0.3333$. It is easy to see that $\Xi = \begin{bmatrix} 0.3333 & 0 \\ 0 & 0.6667 \end{bmatrix}$, $W = \begin{bmatrix} 0.3361 & -0.2222 \\ -0.2222 & 0.4500 \end{bmatrix}$. Then, we have $\lambda_{\max}(W) = 0.6224$, and $l_{\max} = 2$. By Definition 1, we have $a_{\xi}(\bar{L}_{11}) = 4.2358$. Moreover, we choose $\alpha_i = 0.001$, $i = 1, 2$. Thus, $0 < \alpha < \frac{1}{l_{\max} N^2}$ is satisfied.

For $SCC_2 = \{3, 4\}$, $SCC_3 = \{5, 6\}$, where $N = 4$. By Lemma 5, we choose $\Sigma_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $\Sigma_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, such that

$\Phi = \Sigma \Delta \hat{L} + \hat{L}^T \Delta \Sigma \succ 0$. By simple calculation, we have $\hat{L}_{\max} = 7$, $\lambda_{\min}(\Phi) = 0.1684$, $\lambda_{\min}(\Sigma \Delta) = \lambda_{\max}(\Sigma \Delta) = 0.1$. Then, we choose $\alpha_i = 0.001$, $i = 3, 4, 5, 6$. Thus, $0 < \alpha < \frac{1}{\hat{L}_{\max}^2 N^2}$ is satisfied. Moreover, we set $c = 20$. Then all the conditions in Theorems 1 and 2 are satisfied, which means that complex network, whose communication topology shown in Fig. 1, can be synchronized under event-triggering law (4) and (20).

Set the initial values as $[x_1(0), x_2(0), x_3(0), x_4(0), x_5(0), x_6(0)] = \begin{bmatrix} 1 & -1 & 2 & -3 & 3 & -3 \\ 0 & -1 & -2 & 2 & 3 & 0 \\ 1 & 0 & -4 & -1 & -3 & 1 \end{bmatrix}$.

The total simulation time and the step length are set as 1 and 0.00005, which lead to 20000 iterations.

The simulation results are shown in Figs. 2, 3, 4, 5, 6, 7, 8 and 9. Figure 2 shows the evolution of the states of all the nodes, it can be observed that synchronization has been reached. Figure 3 shows the evolution of the synchronization errors, in which the synchronization errors converge to 0 over time. Figures 4, 5, 6, 7, 8 and 9 show the evolutions of the measurement errors $\|e_i(t)\| = \|x_i(t) - x_i(t_{k_i}^i)\|$ and the thresholds $\alpha_i \left\| \sum_{j \in N_i} a_{ij} (x_j(t_{k_j}^j) - x_i(t_{k_i}^i)) \right\|$ for all $i = 1, 2, 3, 4, 5, 6$, respectively. According to statistics, before reaching synchronization, the updates for controller inputs are [211, 182, 433, 388, 346, 384] for all the nodes, respectively. At the same time, the iterations of the complex network are [1649, 1408, 3553, 2825, 2508, 2891], respectively. Thus, the update rates for all the controllers are [0.1280, 0.1293, 0.1219, 0.1373, 0.1380, 0.1328].

Fig. 2 The evolution of states x_{ij} under continuous communications $i = 1, 2, 3, 4, 5, 6; j = 1, 2, 3$

Fig. 3 Synchronization errors of all the nodes

Fig. 4 The evolution of measurement error and the threshold for node 1

Fig. 5 The evolution of measurement error and the threshold for node 2

Fig. 6 The evolution of measurement error and the threshold for node 3

Fig. 9 The evolution of measurement error and the threshold for node 6

Fig. 7 The evolution of measurement error and the threshold for node 4

Fig. 8 The evolution of measurement error and the threshold for node 5

Conclusions

We considered exponential synchronization of complex networks for both irreducible and reducible topologies. By employing the M -matrix theory, algebraic graph theory and the Lyapunov method, two kinds of distributed event-triggering laws were designed. Then, several criteria that ensure the event-based exponential synchronization are presented, and the exponential convergence rates are obtained as well. Furthermore, we proved that Zeno behavior of the event-triggering laws can be excluded, that is, the lower bound of inter-event times is strictly positive. Finally, a simulation example was provided to illustrate the effectiveness of theoretical analysis.

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