Meta-metallic coils and resonators: Methods for high Q -value resonant geometries

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A novel method of decreasing ohmic losses and increasing Q -value in metallic resonators at high frequencies is presented. The method overcomes the skin-depth limitation of rf current flow cross section. The method uses layers of conductive foil of thickness less than a skin depth and capacitive gaps between layers. The capacitive gaps can substantially equalize the rf current flowing in each layer, resulting in a total cross-sectional dimension for rf current flow many times larger than a skin depth. Analytic theory and finite-element simulations indicate that, for a variety of structures, the Q -value enhancement over a single thick conductor approaches the ratio of total conductor thickness to skin depth if the total number of layers is greater than one-third the square of the ratio of total conductor thickness to skin depth. The layer number requirement is due to counter-currents in each foil layer caused by the surrounding rf magnetic fields. We call structures that exhibit this type of Q -enhancement “meta-metallic.” In addition, end effects due to rf magnetic fields wrapping around the ends of the foils can substantially reduce the Q -value for some classes of structures. Foil structures with Q -values that are substantially influenced by such end effects are discussed as are five classes of structures that are not. We focus particularly on 400 MHz, which is the resonant frequency of protons at 9.4 T. Simulations at 400 MHz are shown with comparison to measurements on fabricated structures. The methods and geometries described here are general for magnetic resonance and can be used at frequencies much higher than 400 MHz. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4961573>]

I. INTRODUCTION

In magnetic resonance, the coil or resonator quality factor Q , which is defined as 2π multiplied by the ratio of electromagnetic energy stored to dissipated per cycle,¹ is of great importance. Higher signal can be achieved in coils and resonators with higher Q . It is therefore advantageous to maximize Q for a given coil or resonator design. In addition, for maximum signal-to-noise ratio, rf dissipation in the sample should typically be comparable to the dissipation in the coil or resonator. This can result in an optimum Q -value outside of the range that is possible for typical structures. This paper describes technical advances that lead to higher Q -values for a variety of resonant structures.

In the design of surface coils for human MRI, the free space Q -value is of little importance because the condition of dominant loading, i.e., $Q_{\text{body}} \ll Q_{\text{coil}}$, is readily satisfied.^{2,3} However, for small animal imaging, even at high magnetic fields such as 9.4 T, satisfaction of the dominant loading condition is problematic. This is because body parts are small, and surface coils that correspond to dimensions of body regions are correspondingly small. The purpose of this paper is to present a novel approach to the design of small surface coils. The experimental parameter of interest in the work is the free-space Q -value for a coil resonating at 400 MHz with diameter of the order of 1 cm. Novelty lies in formation of the coil by multiple layers of conducting foil of a thickness that is of the order of the skin depth or less separated by

carefully designed layers of dielectric, which we label with the adjective “meta-metallic.”⁴

As the size of coils and resonators is reduced, the Q -value tends to decrease. This is because the electromagnetic energy is proportional to volume, the dissipated power for metallic structures is proportional to surface area, and the ratio decreases with structure size. The Q -value can also be expressed in terms of the inductance L and resistance R ,

$$Q = \frac{\omega L}{R}, \quad (1)$$

where ω is the radian frequency. For metallic structures the resistance can be expressed as

$$R \simeq \frac{2\pi r_i}{\sigma l T}, \quad (2)$$

where r_i is the inner radius, l is the axial length, σ is the conductivity, and T is the current flow thickness dimension. It is well-known that rf fields and currents tend to reside on the surfaces of metallic conductors with a characteristic exponential decay length with depth called the skin depth,⁵

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}, \quad (3)$$

where μ_0 is the magnetic permeability of free space and f is the rf frequency. For frequencies above several MHz, conductor thicknesses are typically large compared to a skin depth, and $T = \delta$. However, if multiple layers of conducting

foil of thickness less than a skin depth support substantially equal rf currents,

$$T = Nt > \delta, \quad (4)$$

where N is the number of foil layers and foil thickness $t < \delta$. If the foil layers together have a similar inductance as the thick resonator or coil, the Q -value of the foil structure Q_f is enhanced compared to the thick (solid) structure Q_s by the factor T/δ ,

$$\frac{Q_f}{Q_s} \approx \frac{T}{\delta}. \quad (5)$$

At 400 MHz, $\delta = 3.3 \mu\text{m}$ for copper. Consequently, at high frequencies, many thin foil layers can substantially enhance the Q of a typical coil or resonator.

Litz wire has a similar rationale, except that it uses thin strands of non-resonant wire separated by an insulator.⁶ Litz wire is not used at rf frequencies above about 2 MHz because the rf currents are imbalanced due to skin depth effects and capacitance between wires.

Cryo-coil technology is a possible alternative approach to obtain high Q -values. However, this approach is seldom used in fMRI experiments that require a high bandwidth, 400 MHz, for example. In that case, the Q -value of the loaded resonator should not exceed 1000. Other applications that can benefit from the type of Q enhancement discussed in this paper include resonator design for NMR, EPR, rf filters, rf receivers, and low-loss transmission lines.

II. BASIC META-METALLIC STRUCTURAL ELEMENTS

It is found that capacitive gaps between foil layers can result in substantially equal currents in each layer if the overlapping area between layers is substantially equal. Such a structure, which we call a folded-gap loop (FGL), is shown in Fig. 1. The structure consists of 10 sets of 10 foil layers that form a loop. Each foil set wraps 51° and overlaps with the next set on each end for 15° . The capacitance of the overlapping

FIG. 1. An illustration of the folded-gap loop (FGL) foil configuration. For details, see Figs. 2-4.

FIG. 2. RF current density magnitude in each foil layer. The current is zero at the foil edges and maximum where the area of overlap with adjacent foils is maximum. Black lines indicate the PTFE boundary.

regions was designed to resonate⁷ with the inductance of the loop at a frequency of 400 MHz. The structure was simulated using the finite-element computer program Ansys High Frequency Structure Simulator (HFSS) (Canonsburg, PA) version 15. The rf current magnitude inside the foil layer detail of Fig. 1 is shown in Fig. 2. The current in each layer is directed primarily around the loop. The current is maximum in the non-overlapping regions, decreases in the overlapping regions, and goes to zero on the ends of the foils. The current magnitude in a foil layer is substantially proportional to the area of overlap with adjacent foils. This is why the current in the top left and bottom right foil layers in Fig. 2 is about half the values of the others. For illustration purposes, the foil material was chosen to be stainless steel with a conductivity of 1.1 MS/m, which has a skin depth of $24 \mu\text{m}$ at 400 MHz. The foil thickness is $11 \mu\text{m}$. The magnetic field magnitude is shown in Fig. 3. The magnetic field is largest on the inside of the loop and is weaker (and oppositely directed) on the outside. Notice that the magnetic field steps across the foil layers. The magnetic field zero is near the third outermost foil layer. Figure 4 shows the electric field magnitude in the structure. The electric field is nonzero only in the overlapping foil regions. The inner loop diameter is 10 mm, the outer loop diameter is 11.4 mm, and the distance between overlapping foils is $25 \mu\text{m}$. A conducting shield was placed at a diameter of 20 mm. The Q -value of the structure is 587. This can be compared to a simulated Q -value of 242 for

FIG. 3. RF magnetic field magnitude. The magnetic field of the meta-metallic loop is into the page in the center, has a minimum between layers seven and eight, and is out of the page past layer eight. Black lines indicate foil.

FIG. 4. RF electric field magnitude. The rf electric field magnitude is relatively uniform in the regions of foil overlap and zero elsewhere. Black lines indicate the PTFE boundary.

a one-loop-one-gap loop gap resonator (LGR)^{8,9} of the same inner and outer diameters in the same shield and with a lossless capacitive gap. The ratio of the Q -values is 2.4 and this can be compared to a theoretical enhancement factor, Eq. (5), of $\frac{T}{\delta} = \frac{10(11 \mu\text{m})}{24 \mu\text{m}} = 4.6$. The reason for this discrepancy is given in Sec. III and further described in Sec. IV A.

If the foil thickness is on the order of a skin depth or less, the typical metallic boundary condition relating the current per unit width $\mathbf{J}$ to the surface normal vector $\hat{n}$ and magnetic field $\mathbf{H}$ just outside the conducting surface

$$\mathbf{J} = \hat{n} \times \mathbf{H} \quad (6)$$

no longer applies.⁵ However, this boundary condition is the default for high frequency finite-element computer programs including Ansys HFSS. If this boundary condition is used for the foil structure of Figs. 1-4, a Q -value of 3.4 results. This is because rf current flows in opposite directions on the inner and outer sides of each foil and is much larger in magnitude than the current in each foil in Fig. 2. In order to obtain proper numerical solutions, it is necessary to solve for the fields inside the metal foil and use a mesh with elements of size smaller than a skin depth. Small mesh can make the simulations computationally intense. In addition, for the foil and LGR structures considered here, the ends of the structures were not simulated; a perfect magnetic boundary was used. As such, the resonance frequency and Q -value are independent of axial length, see Appendix and Ref. 8. Further discussion of the simulations appears in Sec. IV. Effects of the ends are simulated and discussed in Sec. IV B.

III. RF CURRENT IN CONDUCTING LAYERS

In order to quantify the Q -factor enhancement in meta-metallic foil structures, an analytic theory of the electromagnetic fields in the foils was developed. In the limit of a single thick foil, the analysis reduces to the treatment of the penetration of electromagnetic fields into a good conductor given in Sec. 4.12 of Ref. 5. Consider a single metallic foil of thickness t with planar surfaces in Cartesian coordinates (x, y, z) occupying the space $0 < x < t$. The fields vary only as a function of x . For z -directed currents, the magnetic field

is y -directed and Ampere's law $\nabla \times \mathbf{H} = \mathbf{J}$ can be written as

$$\frac{dH_y}{dx} = J_z. \quad (7)$$

The conduction current density is related to the electric field in the foil by Ohm's law

$$J_z = \sigma E_z. \quad (8)$$

With time harmonic fields varying as $e^{j\omega t}$, Faraday's law, $\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$, can be combined with Eqs. (7) and (8) to give

$$\frac{d^2 H_y}{dx^2} = j\omega\mu_0\sigma H_y. \quad (9)$$

This equation has solutions of the form

$$H_y = C_1 e^{-\tau x} + C_2 e^{\tau x}, \quad (10)$$

where τ , defined by $\tau^2 \equiv j\omega\mu_0\sigma$, is a complex parameter that can be written in terms of the skin depth,

$$\tau = \frac{1+j}{\delta}, \quad (11)$$

and C_1 and C_2 are integrating constants. Boundary conditions that mimic the stepped magnetic field seen in Sec. II can be imposed, $H_y(x=0) = H_a$, $H_y(x=t) = H_b$. These boundaries on Eq. (10) result in the following expression for the magnetic field inside the foil:

$$H_y = \frac{H_a \sinh \tau(t-x) + H_b \sinh \tau x}{\sinh \tau t}. \quad (12)$$

Substituting this into Eq. (7) gives an equation for the current density in the foil,

$$J_z = \frac{\tau}{\sinh \tau t} [H_b \cosh \tau x - H_a \cosh \tau(t-x)]. \quad (13)$$

It is possible to show, either from Eq. (7) or from Eq. (13), that the net current per unit y -length in the foil is exactly equal to the step (or difference) in magnetic field across the foil,

$$i_t \equiv \int_0^t J_z dx = H_b - H_a. \quad (14)$$

The ohmic power dissipation per unit volume is given by

$$P_V \equiv \frac{J_z J_z^*}{\sigma}, \quad (15)$$

and the power dissipated per unit foil surface area is determined by¹⁰

$$P_A \equiv \int_0^t P_V dx. \quad (16)$$

A. Thick foil limit

In the thick limit $t \gg \delta$, $H_a = 0$, and $H_b = H_{\max}$, using Eq. (13),

$$J_z = H_{\max} \tau \frac{\cosh \tau x}{\sinh \tau t}. \quad (17)$$

Using trigonometric identities, the power dissipation per unit volume can be reduced to

$$P_V = H_{\max}^2 \frac{2}{\sigma \delta^2} e^{2(x-t)/\delta}, \quad (18)$$

and the power dissipation per unit area can be found,

$$P_A \equiv \int_0^t P_V dx = \frac{H_{\max}^2}{\sigma \delta}. \quad (19)$$

Equations (17)-(19) match the standard skin depth results, Ref. 5.

B. Thin foil series expansion

When the foil thickness is less than a skin depth, $t < \delta$, the hyperbolic sine and cosine functions can be expanded in powers of τt , Eq. (11),

$$\sinh \tau t = \tau t + \frac{1}{6}(\tau t)^3 + \dots, \quad (20)$$

$$\cosh \tau t = 1 + \frac{1}{2}(\tau t)^2 + \dots \quad (21)$$

and the current density, Eq. (13), can be written as

$$J_z = \frac{1}{t} (H_b - H_a) + \frac{\tau^2}{2t} \left[H_b \left(x^2 - \frac{1}{3}t^2 \right) - H_a \left((t-x)^2 - \frac{1}{3}t^2 \right) \right] + \dots \quad (22)$$

The first term in Eq. (22) represents a constant current throughout the foil. It is a low-frequency term that persists in the steady-state or direct current (DC) limit. The second term in Eq. (22) is caused by counter-currents (eddy currents). It can be seen that for the second order term, the current density reverses direction on each side of the foil: at $x = 0$ the second term is $-\frac{1}{6}\tau^2 t (2H_a + H_b)$ and at $x = t$ the second term reads $\frac{1}{6}\tau^2 t (H_a + 2H_b)$. The strength of these counter-currents is proportional to the magnetic field magnitude on the surface of the foil. The counter-current strength is also proportional to foil thickness. Consequently, the counter-currents can be reduced compared to the low-frequency term by reducing the foil thickness.

The ohmic power dissipation in a single foil layer caused by each of the current terms can be found from Eqs. (15), (16), and (22),

$$P_V = \frac{1}{\sigma t^2} \left\{ (H_b - H_a)^2 + \frac{1}{\delta^4} \left[\left(x^2 - \frac{1}{3}t^2 \right) H_b - \left((t-x)^2 - \frac{1}{3}t^2 \right) H_a \right]^2 + \dots \right\} \quad (23)$$

and

$$P_A = \frac{1}{\sigma t} \left\{ (H_b - H_a)^2 + \frac{t^4}{180\delta^4} \left[(H_b - H_a)^2 + 15(H_b + H_a)^2 \right] + \dots \right\}. \quad (24)$$

The dissipation in multiple foil layers can now be found by summing Eq. (24) appropriately. Assuming equal current per unit length i_t in each layer with N layers and a total current per unit length in all N layers together,

$$I_T \equiv N i_t = H_{\max}, \quad (25)$$

Eq. (14) can be used to evaluate

$$\sum_{m=1}^N (H_b - H_a)^2 = N \left(\frac{H_{\max}}{N} \right)^2 = \frac{H_{\max}^2}{N}. \quad (26)$$

The last term in Eq. (24) can be evaluated by assuming the magnetic field H_a starts at zero on the first layer and recognizing the incremental increase in H_a (and H_b) over the sum,

$$\begin{aligned} \sum_{m=1}^N (H_b + H_a)^2 &= \sum_{m=1}^N (2m i_t - i_t)^2 \\ &= \left(\frac{H_{\max}}{N} \right)^2 \sum_{m=1}^N (2m - 1)^2 \\ &= \frac{H_{\max}^2}{3N} (4N^2 - 1), \end{aligned} \quad (27)$$

where the finite series was evaluated using Ref. 11. Taking the results of Eqs. (26) and (27) into Eq. (24), we find

$$P_A = \frac{H_{\max}^2}{\sigma T} \left[1 + \left(\frac{T^2}{3N\delta^2} \right) \left(1 - \frac{1}{5N^2} \right) + \dots \right], \quad (28)$$

where $T = Nt$, Eq. (4), has also been used. For many layers, we set $(1 - \frac{1}{5N^2}) \approx 1$. An equation similar to Eq. (28) appears in the literature, Eq. (26) of Ref. 12, in the context of low-frequency (60 Hz) coil design. However, the leading term of Eq. (26) of Ref. 12 scales *proportional* to N rather than inversely with N due to different circuit topology.

C. The meta-metallic effect: Minimum layer number and maximum thickness

In comparing the first term of Eq. (28) to the thick foil limit Eq. (19), it is seen that, to first order, multiple layers have reduced dissipation by the factor δ/T , compared to a single thick conductor. This is consistent with Eq. (5). However, the second term in Eq. (28) provides the condition for this to be true. In order for the series to converge, the second order term must be smaller than the first. This puts a minimum constraint on the number of layers,

$$1 > \frac{T^2}{3N\delta^2} \quad (29)$$

or

$$N > \frac{1}{3} \left(\frac{T}{\delta} \right)^2. \quad (30)$$

This constraint can be interpreted as the minimum number of layers for a given total conductor thickness T . From Eq. (4), this constraint can also be expressed in terms of the maximum layer thickness for a given number of layers,

$$t < \frac{\delta}{\sqrt{N/3}}. \quad (31)$$

Notice from Eq. (24) that the second order term is squared. This produces rapid decay of the counter-current dissipation as N is increased beyond the minimum (or t is decreased below the maximum).

According to this analysis, there is no theoretical limit to the Q enhancement due to multiple thin conducting layers.

TABLE I. Q -enhancement constraints for copper layers.

$\frac{T}{\delta}$	$N_{\min}$	400 MHz		9.5 GHz	
		δ (μm)	$t_{\max}$ (μm)	δ (μm)	$t_{\max}$ (μm)
5.5	10		1.8		0.37
7.8	20		1.3		0.26
10	33	3.3	0.99	0.68	0.20
20	130		0.50		0.10
100	3300		0.099		0.020

Table I contains some practical cases for copper at different frequencies for a given Q enhancement, T/δ . A major and perhaps counter-intuitive implication of this result is that the Q -value does *not* monotonically increase with the number of foil layers at a fixed foil thickness. Rather, the Q -value will exhibit a *maximum* as the number of layers approaches $3(\delta/t)^2$, Eq. (31). This is because the first- and second-order dissipation terms scale differently with t . Consequently, there is an optimum layer number that produces a maximum Q -value for a given foil thickness. These results have been confirmed by Ansys HFSS simulations. Similarly, at fixed foil layer number, increasing t also produces a maximum in Q -value near $t_{\max}$, Eq. (31). In both cases the maximum occurs when the second-order counter-current dissipation is balanced with the first-order DC dissipation.

From these results, the Q enhancement factor observed in the simulation of Sec. II can be understood. If the foil thickness t is significantly less than the maximum given by Eq. (31), the Q enhancement factor is accurately given by T/δ , Eq. (5) since the counter-current dissipation is small compared to the first-order dissipation. However, if the foil thickness is near the maximum, which is the case in Sec. II, the Q enhancement factor is about one-half of T/δ because the first- and second-order dissipation are nearly equal, doubling the total ohmic dissipation.

By differentiating Eq. (28) with respect to t and setting the result equal to zero, a theoretical foil thickness for minimum ohmic dissipation can be obtained. The result is

$$t_{\text{opt}} = \frac{3^{1/4} \delta}{\sqrt{N}}, \quad (32)$$

as reported in Ref. 12. The foil thickness t_{opt} is about 32% lower than the maximum thickness $t_{\max}$, Eq. (31). Numerical simulations indicate, for structures with minimal foil edge currents, that the foil thickness for maximum Q -value lies between the two thicknesses. This is further discussed in Sec. IV.

IV. SIMULATIONS

In addition to the meta-metallic effect described in Sec. III C, it was found that the geometry of the foils strongly influences the Q -value enhancement. If the rf magnetic fields wrap around the ends of the foil layers and have a significant component perpendicular to the foil edges, intensified rf currents flow along the layer edges, and cause significant additional ohmic dissipation. These end effects are discussed

in Sec. IV B. Structures having these end effects do not follow the predicted meta-metallic Q -value enhancement over thick conducting structures. However, five types of thin foil structures have been envisioned where end effects are minimal and these are described in Sec. IV C. These structures exhibit Q -value enhancements consistent with the theory of Sec. III C.

All simulations were made using Ansys HFSS, Sec. II. The structures have a resonance frequency near 400 MHz, metallic components are copper, and the material between foil layers is polytetrafluoroethylene (PTFE) with a relative dielectric constant ϵ_r of 2.1. In order to isolate the meta-metallic effect, the resistivity of the conducting shield and the loss-tangent of the dielectric were set equal to zero. The influence of dielectric loss is described in Appendix and Sec. IV C 1. A Dell Precision Tower 7910 with 24 Intel Xeon dual-core processors with Hyper-Threading and 512 GB of RAM was used to make the numerical simulations. The foils were drawn using many 0.1 mm or 0.25 mm wide adjacent duplicate structures on axis. The boundaries between adjacent foils facilitated meshing. Most of the simulations were done using the Eigenmode solution type. For some of the larger structures, including the toroidal loop, either the memory limit was reached or solutions were simply not found. In these cases, the driven modal network analysis solution type was used. The structure was coupled using a single mode lumped port defined by a planar face placed symmetrically between two adjacent foils in an overlapping region. The electric field vector integration line was perpendicular to and between the foil surfaces. In Figs. 2-4, 6, 10, 12, 13, 15, and 16, the plotted quantities correspond to a power of 1 W input to the whole structure. If perfect magnetic boundary conditions were used, Figs. 2-4 and 6, the axial length was 1 cm.

A. Axial coils with no end effects

FGL and self-resonant spiral (SRS) structures were simulated using perfect magnetic boundaries at the top and bottom. Both types of structures are found to follow closely the theory of Sec. III C.

1. Folded-gap loop

The foil configuration for an FGL with two sets of 10 foil layers is shown in Fig. 5. Dimensions are shown in Table II with symbol definitions consistent with the Appendix. By constructing the overlapping foil region with a constant distance d_{ov} instead of angle as in Sec. II, the capacitance is constant, producing more uniform currents across the layers. The magnetic field magnitude is shown in Fig. 6(a) and the electric field magnitude in Fig. 6(b). They are similar to those presented in Sec. II. The structure resonates at 393 MHz and, for a foil thickness of 1.6 μm , has a Q -value of 5514. This can be compared to a Q -value of 1407 for an LGR of the same inner and outer radius and the same metal. The resulting Q -enhancement ratio, Eq. (5), is 3.9 and can be compared to $T/\delta = 4.8$, Table II. The eddy current dissipation lowers the Q -enhancement as expected. The inductance of the FGL is approximately $(7.9/5)^2 = 2.5$ times higher than

FIG. 5. FGL with two sets of 10 foil layers.

the inductance of the LGR (Appendix) and this factor has the opposite effect, raising the Q -ratio.

A numerical study of the Q -value dependence on the foil thickness was made, Fig. 7. Shown are results for the 10-foil FGL, along with several other structures discussed subsequently. Also shown is the maximum conductor thickness given by Eq. (31) for 10 and 20 foil layers. It can be seen that the Q -value peaks about 20% below $t_{\max}$, consistent with theory, and is the thickness used above. It is interesting to note that the optimum foil thickness t_{opt} , Eq. (32) falls about 10% further below the numerically obtained thickness for maximum Q -value. Nonuniform currents across the foil layers are the likely cause.

Ansys HFSS has a minimum physical dimension of $1\ \mu\text{m}$. In order to overcome this limitation, some simulations were done at 10 times the size and one-tenth the conductivity. It can be verified analytically that this produces an identical Q -value for any structure at one-tenth the frequency. This was also verified for several numerical simulations of different structures. The same Q -value to six digits was obtained in each case.

A 20-foil FGL structure was also simulated with dimensions shown in Table II. The structure resonates at 412 MHz and, for a foil thickness of $1.1\ \mu\text{m}$, has a Q -value of 8105. This can be compared to a Q -value of 1403 for an LGR of the same inner and outer radius. The resulting Q -enhancement ratio, Eq. (5), is 5.8 and can be compared to $T/\delta = 6.7$, Table II. A scan of Q -value with thickness for this structure is shown in Fig. 7. The maximum Q -value occurs at a foil thickness about 20% below $t_{\max}$ as for the 10-foil FGL.

2. Self-resonant spiral

Another multilayer foil structure that can show Q -enhancement consistent with the meta-metallic effect, Eq. (5),

FIG. 6. (a) RF magnetic field magnitude and (b) rf electric field magnitude for a 10-foil FGL. The fields resemble those of Figs. 3 and 4. Black lines indicate foil.

is an SRS. Unlike the FGL, the capacitance associated with the overlapping foil area charges to the voltage difference between one foil layer and an adjacent foil layer and, as it discharges, drives a current in parallel with all the layers. The resulting rf current profile in the foil layers produces rf magnetic fields very similar to those of the FGL, Fig. 6(a). Filamentary analogs of the foil SRS are of current interest for meta-material meta-atoms.^{13,14} These references calculate resonance frequency but not Q -value.

Conditions for resonance of single and multiple foil SRS are given in the Appendix. Shown in Fig. 8 is a four-turn spiral foil configuration with dimensions shown in Table II. The SRS was simulated with results also shown in the table. The SRS Q -value can be compared to a Q -value of 809 for an LGR of the same inner and outer radius. This corresponds to an actual Q -enhancement ratio, Eq. (5), of 2.7 compared to $T/\delta = 2.7$, Table II. One might expect that the eddy current dissipation would lower the Q -enhancement for the foil thickness near one skin depth. However, the SRS has an approximately $(5.5/3)^2 = 1.8$ times the inductance of the LGR (Appendix) and this factor raises the Q -ratio. The nearly sinusoidal (nonuniform) current distribution with foil length in the spiral does not have much effect in lowering the Q -enhancement. For the different structure types that have

TABLE II. Dimensions (mm), parameters, and Q enhancement for various structures without and with end effects (see also Fig. 7).

Structure	r_i	r_o	r_s	d	d_{ov}	l	f (MHz)	Q_f	Q_s	N	t (μm)	T/δ	Q_f/Q_s
FGL w/no end effects	5	10.8	28	0.3	1.2	n/a	393	5514	1407	10	1.6	4.8	3.9
	5	11	28.5	0.15	0.18	n/a	412	8105	1403	20	1.1	6.7	5.8
SRS w/no end effects	3	8	20	1.25	104	n/a	407	2169	809	4	2.2	2.7	2.7
	3	8	20	0.36	86	n/a	410	4139	809	14	1.4	5.9	5.1
1 cm length FGL	5	10.9	25	0.3	3	10	374	731	1156	10	3.9	n/a	0.63
1 cm length FGL w/dielectric ends	5	10.8	25	0.3	2.4	10	411	12780	1586	10	1.6	4.8	8.1
Toroidal loop	5	8.5	14	0.076	1.77	3.5	391	1953	1131	10	1.4	4.2	1.7
5 cm length coax	5	7.4	28	0.13	0.9	5	396	19718	5302	10	1.6	4.8	3.7

FIG. 7. Q vs. t for various structures. All data points are from HFSS simulations except for the black dot, which is measured. Blue corresponds to 10-foil structures and red 20-foil. The theoretical maximum foil thickness $t_{\max}$, Eq. (31), is shown for 10 (blue) and 20 foils (red). The lowest and rightmost curve is for a 1 cm length structure with end effects, Fig. 10.

been simulated, it was found that close attention to balancing the currents by equal capacitance does not significantly impact the Q -value.

It is possible to increase the number of foil layers by duplicating the single foil spiral in azimuth as shown in Fig. 9. The addition of the duplicate foils has a small effect on the resonance frequency relative to the single foil resonance frequency, see the Appendix. The structure shown has the same inner and outer radii as the single foil spiral; however, each foil has 3.5 turns and thickness $1.39 \mu\text{m}$. The resonance frequency is 410 MHz and Q -value of 4139. The LGR comparison is the same, giving an actual Q -enhancement ratio, Eq. (5), of 5.1 compared to $T/\delta = 5.9$, Table II.

B. Axial coils: End effects

A 1 cm axial length of an FGL similar to that shown in Sec. IV A 1 was simulated. The perfect magnetic boundary conditions (Sec. IV A) were removed and the structure was centered in a conducting cylindrical boundary of axial length

FIG. 8. Foil configuration for a single-foil four-turn spiral.

FIG. 9. Foil configuration for a four-foil 3.5-turn spiral.

40 mm and a radius 25 mm. A side-view of the magnetic field magnitude for the upper half of the structure is shown in Fig. 10. Intense rf magnetic fields at the axial ends of the foil layers are seen. The intensification is caused by strong rf currents that flow along the foil edges, which, in turn, cause increased dissipation and a significant decrease in Q -value compared to the structure with no end effects. The intensification of rf current on the edges of conductors has been extensively studied in relation to microstrips¹⁵ and is caused by a singularity in the rf fields at sharp edges. The effect has recently been called edge singularity¹⁶ but has also been called strong skin effect at the edge.¹⁷ When the rf current distribution in a conductor is influenced by one or more nearby conductors, there is also typically an intensification of rf currents and an increase in ohmic losses. Proximity effect⁶ has been used in this case. We prefer “end effects.” The structure dimensions are shown in Table II. The larger overlap distance (3 mm vs. 1.2 mm) is needed to compensate for the reduced inductance vs. Eq. (A9) resulting from the finite length. By scanning the foil thickness, a maximum Q -value of 731 was obtained at a thickness of $3.9 \mu\text{m}$. This thickness is about 2.4 times the thickness that gives maximum Q -value for the 10-foil structure with no end effects, Sec. IV A 1. Results are shown in Fig. 7. The theory of Sec. III C cannot be applied due to the end effects. This compares to a Q -value of 1156 for an LGR of the same inner and outer radius and

FIG. 10. Magnetic field magnitude for a 1 cm axial length FGL with two sets of 10 foil layers. Field intensification is seen on the foil edges. Black lines indicate foil.

length and a Q -enhancement ratio, Eq. (5), of 0.63, Table II. Similar results were obtained for 1 cm axial length SRSs.

C. Structures that minimize end effects

In this section, we describe five different structures that have minimal end effects and consequently can have significant meta-metallic Q -enhancement. The first three structures described in Secs. IV C 1-IV C 3 use an additional structure to provide a boundary condition to the meta-metallic structure that mimics a perfect magnetic boundary condition. The fourth and fifth structures described in Secs. IV C 4 and IV C 5 have geometries in which the foil edge currents approach zero.

1. FGL (or SRS) with dielectric ends

By treating an FGL as the central section of a uniform field (UF) resonator,^{18–21} it is found that a dielectric region placed on each end of an FGL can significantly reduce end effects. The physical principle is that a quarter-wavelength thickness of dielectric converts an electric short at the top of the dielectric to an open impedance, which is presented to the foil edges. The rf open is equivalent to a perfect magnetic boundary condition, the same spatial boundary condition required to keep the rf currents uniform along the axial length of the foil. The configuration is shown in Fig. 11. The FGL consists of four sets of 10 foils. The two additional gap regions compared to the FGL of Sec. IV B are found to be needed to couple the foil to the dielectric $TE_{01\delta}$ mode.²² The foil and dielectric are spaced apart by 0.5 mm and placed inside a conducting shield. The rf magnetic field profile is shown in Fig. 12. It is seen that the effect of the dielectric is to nearly eliminate end effects. The combined structure resonates at 411 MHz and, for a foil thickness of $1.6\ \mu\text{m}$, has a Q -value of 12780. The Q -value is maximum at the same foil thickness that produces maximum Q -value for the

FIG. 11. Cutaway view of one-eighth of foil and dielectric of a 1 cm axial length FGL with four sets of 10 foil layers.

FIG. 12. Magnetic field magnitude for the 1 cm axial length FGL with four sets of 10 foil layers and dielectric ends. There are no end effects, compare Fig. 10.

FGL with no end effects of Sec. IV A 1. The Q -value is significantly larger than the FGL of Sec. IV A 1 because the dielectric loss tangent was set equal to zero. In reality, the structure would have a maximum Q -value somewhat larger than the inverse of the dielectric loss tangent, see the Appendix. Dimensions of the FGL are shown in Table II. The dielectric radius is 21.2 mm and length 10 mm. The relative dielectric constant of the dielectric end regions is 760. Dielectrics of larger sizes with relative dielectric constants of 100, 200, and 400 have also been shown to couple to the FGL of the same size. Similar Q -values were obtained. The relative dielectric constant values of 100–200 are similar to some ceramics.²³ Larger diameter foils and higher resonance frequencies can accommodate dielectrics with even lower relative dielectric constant values. The FGL of Figs. 11 and 12 is centered in a conducting cylinder of radius 25 mm and length 31 mm. A one-loop–four-gap LGR of the same inner and outer radius and length as the FGL can be coupled to identical dielectric ends. The resulting Q -value is 1586. The Q -enhancement ratio, Eq. (5), is 8.1 and can be compared to $T/\delta = 4.8$, Table II. The larger enhancement is due to the additional inductance of the FGL compared to the LGR, $(7.9/5)^2 = 2.5$, see the Appendix and Sec. IV A 1, and with this factor is favorably consistent with the theory of Sec. III C. Using low-loss dielectrics, this structure can be used to make resonators with Q -values exceeding 10 000. The foil permits a concentration of the rf magnetic field into much smaller volumes and into shapes that are not possible using dielectrics alone.

SRSs in place of the FGL have been simulated with dielectric ends with similar results. It is found that a larger gap between the dielectric and the foil (1 mm) is required for the SRS than for the FGL to prevent capacitive loading of the foil ends by the dielectric. This is due to the larger voltage between the foil ends of the SRS. The loading causes enhanced rf currents on the foil edges. Such a structure could be used as an NMR resonator, but has limited applicability as an MRI surface coil. We present MRI-suitable structures in Secs. IV C 2 and IV C 3.

2. SRS (or FGL) with dielectric on one side

Surprisingly, it was found that a dielectric region of about twice the size of a uniform field end section described in Sec. IV C 1 placed on *one* side of the SRS or FGL can also suppress the end effects on *both* sides of the foil. This is because the axial length of the foil is much smaller than a wavelength. It is found that when the resonance frequency of the combined structure (dielectric and SRS) is near the resonance frequency of the SRS (or FGL) alone with perfect magnetic boundaries, see Sec. IV A, the end effects are substantially eliminated and the Q -value is maximized. A three-turn spiral of inner radius 4.25 mm, outer radius 7.25 mm, and axial length 2.5 mm embedded in PTFE was found to have a resonant frequency of 402 MHz with perfect magnetic axial boundary conditions. With a $3\ \mu\text{m}$ foil thickness the Q -value is 2506. With a rutile dielectric cylinder, $\epsilon_r = 100$, radius and axial length 53.6 mm placed coaxially to the spiral at a distance 1 mm away from the edge of the foil, the combined structure has a resonance frequency of 409 MHz. The spiral was centered in a conducting boundary of radius 53.6 mm and axial length 111.7 mm. The Q -value of the combined structure was 84 750. The Q -value is higher than the SRS alone because there is a large portion of stored energy in the dielectric. The rf magnetic field strength was about the same in the spiral center as the dielectric center. In reality, the Q -value is limited by the loss tangent of the dielectrics as indicated in the Appendix and Sec. IV C 1.

In the coupled dielectric-meta-metallic structure, the lowest-frequency mode is where the rf magnetic field in the spiral and dielectric is in-phase. This is consistent with the lowest frequency parallel mode described in a dielectric-cavity coupled system.²² We call the dielectric cylinder an “equalization” element for the meta-metallic SRS or FGL component. Because the rf magnetic fields of the dielectric and meta-metallic are in-phase, the rf magnetic fields from each element add constructively, which increases the inductance of both components and lowers the resonance frequency of the combined structure compared to each individually.

3. SRS (or FGL) with equalization coil

It was found that a resonant coil placed near the SRS can produce substantially the same effect as the dielectric equalization element described in Sec. IV C 2. The condition for maximum Q -value is the same. We call such a coil an equalization coil for the meta-metallic component. The rf magnetic fields for the coupled system are shown in Figs. 13(a) and 13(b).

Here, the three-turn copper spiral has an inner radius 4.25 mm, outer radius 7.25 mm, axial length 5 mm, foil thickness $3\ \mu\text{m}$ and is embedded in PTFE. The spiral is coaxial with a toroidal equalization loop made of (thick) silver with major radius 17 mm and minor radius 5 mm. The bottom edges of both coils are coplanar. The toroidal loop has a capacitive gap of thickness 0.6 mm filled with a dielectric of $\epsilon_r = 10$. The capacitive gap thickness was adjusted so that the coupled system resonated near 400 MHz. The Q -value of the coupled system is 2220 at 381 MHz.

FIG. 13. Magnetic field magnitude for (a) a 0.5 cm axial length three-turn SRS inside an equalization coil, (b) expanded view of (a) and (c) the same SRS as in (b) simulated without the equalization coil. The equalization coil suppresses end effects, although not completely, compare Figs. 10 and 12.

Further adjustments would increase the Q -value slightly. This can be compared to a Q -value of 2441 for the three-turn spiral alone with no end effects. Even for only three turns, the theoretical Q -enhancement due to the meta-metallic effect is 2.7, Eqs. (5) and (31), over a thick conducting structure of similar size. With no equalization coil and with end effects, the Q -value becomes 500 at 539 MHz. The enhanced rf magnetic field due to the end effects is shown in Fig. 13(c) and is significantly larger than those with the equalization coil, Fig. 13(b). The Q -value of the equalization coil alone is 3070 with the capacitive gap decreased to resonate at 399 MHz. Therefore, most of the losses are due to the equalization coil and not the SRS. If the equalization coil is made lossless, the Q -value of the coupled system is 4640. This Q -value is higher

than the SRS alone with perfect magnetic boundaries due to the additional stored energy near the equalization coil.

This coupled system can be used as a practical surface coil for MRI. Because the rf magnetic fields of the SRS and the equalization coil are in phase, the depth sensitivity below the SRS is enhanced by the equalization coil. The Q -value can be tailored to whatever it needs to be to produce dominant loading. Dominant loading is where the subject to be imaged absorbs at least as much power from the coil as the power losses in the coil itself. A wide variety of different types of equalization elements could be used. The equalization coil could also be used as a coupling loop.

4. Toroidal loop

Another structure that minimizes end effects is a folded-gap toroidal loop. The structure has an overall shape of a torus (e.g., a thick single loop of wire) but the symmetry of the folded gaps is in the poloidal direction instead of the axial direction of the FGLs. A picture of the foils is shown in Fig. 14. The foils have the shape of concentric rings. The structure has 10 sets of 10 foils with overlapping and non-overlapping regions distributed azimuthally exactly the same as the FGL shown in Fig. 1. The rf currents are directed primarily around the loop.

The rf magnetic field distribution is shown in Fig. 15. Outside of the outermost foil, the rf magnetic field distribution is similar to a thick loop of wire carrying an rf current around the loop. Inside the foils, the rf magnetic field magnitude steps down across the foils similar to that seen for the FGL, e.g., Fig. 6. The magnetic field on the inside of the innermost foil is zero. The major radius of the toroidal loop is 6.78 mm, the minor radius is 1.74 mm, the spacing between foil layers is 76 μm , Table II. The structure was centered in a cylindrical conducting boundary of radius and length 14 mm. Results of a numerical study of the dependence of the Q -value of the structure with foil thickness is shown in Fig. 7. It can be seen that the maximum Q -value is obtained at a foil thickness of about 1.4 μm , nearly the same as for the FGL with no end effects. The maximum Q -value is 1953. This Q -value can be compared to that of a thick conducting loop of copper of the same major and minor radii with a gap and centered

FIG. 14. Cutaway view of the foil for the folded-gap toroidal loop.

FIG. 15. The rf magnetic field magnitude for the folded-gap toroidal loop.

in a conducting boundary of the same size. With the gap capacitance adjusted to produce a resonance frequency of 400 MHz, simulations show a resulting Q -value of 1131. This corresponds to a Q -enhancement factor, Eq. (5), of 1.7 and can be compared to $T/\delta = 4.2$, Table II. The Q enhancement factor is about half of what one would expect based on the theory of Sec. III C.

The reason for this has to do with the rf current distribution in the foils. Examination of the current distribution in the foils of the structure using Ansys HFSS reveals significant poloidally directed currents caused by the relatively large overlapping regions of the outer gaps compared to the inner gaps. The rf currents flow from these capacitive regions poloidally to the innermost regions of the foils and then back poloidally to the next capacitor. The rf current paths are inefficient compared to those in the thick conducting loop and the Q -value enhancement ratio is decreased. This effect can be reduced by reducing the ratio of the minor radius to the major radius.

A simpler method of constructing the toroidal foil configuration than that shown in Fig. 14 is to replace each set of 10 groups of foils with a single poloidally spiraled 10-turn foil. This single foil must then be cut azimuthally in order to break the poloidal currents that tend to flow from inner foil layers to outer foil layers. The cut should be placed where the rf magnetic field is weakest in order to minimize rf currents along the foil edges created by the cut. It can be seen in Fig. 15 that the foils should be cut where they meet the bottom edge of the figure at the largest distance from the axis. This structure has been simulated and produces similar Q -values to the structure of Fig. 14. The data point is shown in Fig. 7. The slightly lower Q -value is due to additional rf currents that flow along the cut edges.

5. Coaxial length

A 5 mm length of coaxial cable was simulated with the inner conductor replaced by two sets of 10 axially overlapping foils. Dimensions are shown in Table II. With each end of the coaxial cable shorted, the capacitance between foils C was designed for a resonance frequency near 400 MHz using the transmission line impedance equation given by Eq. (A22) of the Appendix. A cut view of the cable showing the foils and the rf magnetic field magnitude profile are shown in Fig. 16. At a foil thickness of 1.6 μm , the Q -value of the structure has a maximum value of 19 718. This can be compared to a Q -value

FIG. 16. One-half side view of coaxial cable foil configuration and rf magnetic field magnitude profile. Zero radius is on the left and the outer conductor is on the right. RF currents are zero on the foil edges in this geometry.

of 5302 for the same coaxial length with the foils replaced by a thick inner conductor with an outer radius the same as the *average* radius of the foils, 6.3 mm. In order to resonate the cable, a 0.5 mm gap was created in the center conductor centered at 2.5 mm axial distance and a capacitive element of the same value calculated above (105 pF) was added across the gap. The corresponding Q enhancement factor is 3.7 and can be compared to $T/\delta = 4.8$, Table II, consistent with the theory of Sec. III C. The Q -value could be further improved by adding capacitive gaps to the outer shield. However, the additional Q enhancement would be smaller by the ratio of the inner conductor radius to the outer conductor radius. Such a structure could be used as a low-loss transmission line that would have similar loss characteristics in a much more compact geometry than traditional rectangular waveguide. A small section shorted at both ends could also be used as a resonator for NMR.

A much longer structure with no shorted ends and many capacitive regions could be used as a coaxial cable transmission line. As such, it would exhibit some dispersion due to the capacitive regions. This property is unlike a coaxial cable with a thick inner conductor, which carries a pure TEM mode, but is similar to standard waveguide. The amount of dispersion can be adjusted through the capacitance.

V. RESULTS

A. Fabrication

Spiral foil structures were fabricated by winding strips of foil-dielectric laminate and inserting them into PTFE holders. The laminate can be made by three methods: (1) electrodeposition of copper onto PTFE (CuFlon, Polyflon Company, Norwalk, CT), (2) application of heavy gilding foil onto PTFE, and (3) physical vapor deposition (PVD) onto dielectrics.

In general, dielectric materials used in making the meta-metallic structures must be low-loss, see the Appendix and Sec. IV C. Practical dielectric materials for use between foil layers include PTFE, polyethylene, polypropylene, polystyrene, paraffin wax, silicon dioxide, glass, sapphire, and Rogers RT/duroid® 5880. Materials with large dielectric constants can also be used. Any adhesive used to apply the foil to the substrate is also a dielectric and must therefore also be low loss. Alternatively, if the adhesive is highly conducting it then adds to the foil thickness.

To date, coils have been made by the first two methods. An issue surrounding method 1 is that control of the foil

thickness is poor. The foil thickness tends to run larger than specification by up to a factor of two and vary by up to 50% between different parts of the laminate panels. Foil thickness has been calculated by two techniques: (1) weight, density, and dimensions; and (2) four-point probe voltage and current measurements. Of the two techniques, the four-point determination is more accurate because it is insensitive to any non-conducting adhesive layers between the foil and substrate.

In method 2 (gilding foil), the foil thickness is within a few percent of specification as determined by both weight/dimension and four-point probe measurement techniques. Foil can be ordered in 10 cm² in copper, silver, and gold with any thickness between 2 and 10 μm . Gold can be ordered down to 0.1 μm . However, it is difficult to apply the foil to the dielectric substrate. In traditional gilding, the size (glue) sets up a tack that lasts for at least an hour. This permits time to apply the foil. However, commercial gilding size (both acrylic and oil-based) is too lossy to be used to make meta-material laminates. An adhesive that is known to work is Q-dope (GC Electronics), which is a polystyrene glue. However, Q-dope dries very rapidly. Two methods to deal with this problem have been identified. One is to apply the glue thickly and then press the foil between two blocks. The flow of the glue to the edges straightens the foil and the amount of pressure can be used to control the glue thickness. The other method is to mix the polystyrene glue with a lower vapor pressure solvent such as ethylbenzene or propylbenzene. The thinned glue can be applied more thinly and the solvent evaporates slowly enough for the foil to be applied.

In method 3, discussions with scientists in the PVD field indicate that the proper foil thickness can be achieved on up to 15 cm² substrates.

It is found that if the foil is affixed to thin (50 μm) dielectric sheets, buckling or cracking of the foil does not occur when the laminate is bent. Thin foil-PTFE laminate can be cut cleanly into strips using a rotary cutter and cutting mat.

B. Characterization

Q -value measurements were made on SRS foil structures using an Agilent Technologies E8363C PNA Network Analyzer. It was calibrated using Electronic Calibration Module N4691-60001. The network analyzer was connected to a 16 mm diameter coupling loop at the end of a 22 cm length of 3 mm outer diameter 50 Ω semi-rigid coaxial cable with an SMA connector. Critical coupling to the coil was achieved by adjusting the axial distance between the coupling loop and the coil. Q -value measurements were made by observing the frequency of the S_{11} -6 dB points on either side of resonance.

A Q -value of 623 was measured for a stand-alone SRS of copper foil thickness 5.5 μm as determined by a four-point probe. For the measurement, the SRS was centered in a copper shield of diameter 11.4 cm and height 10.2 cm. This Q -value can be compared to the 10-foil FGL simulations as shown in Fig. 7. The spiral is $2\frac{1}{3}$ turns with an outer diameter of 16.8 mm and an inner diameter of 12.6 mm. The structure was made from a CuFlon panel with 51 μm thickness PTFE

and a specified copper cladding weight of 1/16 ounce/square foot, which has a corresponding nominal thickness of 2.2 μm . However, the actual thickness is believed to be 5.5 μm , based on four-point probe measurements. The CuFlon was cut into a 11.5 cm long 1 cm wide strip using a rotary cutter and plastic mat. This strip was sandwiched between two 0.51 mm thickness PTFE strips (with no cladding) of the same length and width, wound and placed inside a cylindrical PTFE holder. The spacing between turns is 1.07 mm. Simulations of the built structure yielded a Q -value of 715 with a PTFE loss tangent of 1.5E-4 and conducting walls. The Q -value of this structure is lower than it can be due to end effects described in Sec. IV B.

The method of using a spiral of a thin sheet of CuFlon sandwiched between two layers of PTFE as described above could also be used to fabricate an FGL. Because CuFlon can be ordered like a printed circuit board, the sizes of clad bars and gaps on the CuFlon panel can be specified such that when cut and spiraled they produce the appropriate overlap of adjacent layers. The non-clad regions would make up almost one full turn of each layer.

VI. DISCUSSION

It is seen that several different types of structures made of many thin layers of metallic foil with capacitive gaps between adjacent layers exhibit enhancement of the Q -value compared to similar structures made with a single layer of thick conductor. The Q enhancement factor is consistent with the meta-metallic effect, given by Eq. (5) and discussed in Sec. III C. Structures that exhibit such enhancement in Q -value have the characteristic that the rf magnetic field lines in the vicinity of the foils are generally parallel to the foil surfaces, particularly the edges. One way this can occur is that the rf currents approach zero near the foil edges as seen in Secs. IV C 4 and IV C 5. Another way this can occur, as described in Secs. IV C 1-IV C 3, is when another nearby structure, such as an equalization coil or dielectric, produces an additional magnetic field parallel to the field generated by the meta-metallic. If the resonance frequency of the coupled system is equal to the resonance frequency of the meta-metallic structure alone with perfect magnetic boundary conditions, the rf current distribution in the foils becomes substantially uniform, eliminating end effects.

A practical coil with a Q -value of 623 was constructed and characterized at 400 MHz. When combined with an equalization coil and run in parallel mode, simulations suggest that significantly higher Q -values can be obtained. We expect the resonance frequency and Q -value of any of these structures to be as stable with temperature as the underlying physical properties of the conductivity of the foil layers and the dielectric constant and loss tangent of the dielectric between layers. The foil should be protected from oxidation using a low-loss coating. The SRS with equalization coil is a promising structure for an MRI surface coil. In the limit of dominant loading of the Q -value of an MRI surface coil by tissue, increase in Q -value of an MRI surface coil tends to be of little benefit. However, for rodent imaging at, for example,

400 MHz using surface coils of 2 cm diameter or smaller, the dominant-loading condition is difficult to achieve. The methods of this paper provide significant advantages for MRI of small animals and also of tissue samples. The problem is acute in murine imaging.²⁴ In principle, the resonance condition can be satisfied over a range of small coil diameters by compensating the decrease in single-turn inductance by increase in the number of metal layers as well as increase in capacitance through use of thin dielectric film between layers. Improved methods of forming the coil, including physical vapor deposition, may be required.

The technology of meta-metallic coils is promising for use in NMR microscopy. Decrease in coil diameter must be offset by increase in capacitance between foil layers in order to achieve resonance. Spacing between layers can be as small as 10 μm , and the dielectric constant can be substantially higher, providing a significant opportunity to decrease the meta-metallic coil diameter. Using advanced manufacturing techniques, a Q -enhancement of 100 seems feasible since the required thickness is 100 nm at 400 MHz and 20 nm at 9.5 GHz, Table I, both of which are considerably larger than the lattice constant of copper, 3.6 Å. As the foil gets thinner, any residual end effects become stronger, and the structure becomes increasingly difficult to fabricate.

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APPENDIX: ANALYTIC FORMULAS FOR COILS AND RESONATORS

One of the most efficient resonators for small samples is the loop-gap.⁸ The inductance of a 1-loop-1-gap LGR is a parallel combination of an inner and outer inductance,^{21,25}

$$L = \frac{1}{\frac{1}{L_i} + \frac{1}{L_o}}, \quad (\text{A1})$$

where, neglecting end effects,

$$L_i = \frac{\mu_0 \pi r_i^2}{l}, \quad (\text{A2})$$

where r_i is the inner radius and l is the axial length,

$$L_o = \frac{\mu_0 \pi (r_s^2 - r_o^2)}{l}, \quad (\text{A3})$$

where r_o is the outer radius of the LGR and r_s is the inner radius of the conducting shield. Similarly, the resistance can be written as

$$R = \frac{1}{\frac{1}{R_i} + \frac{1}{R_o}}, \quad (\text{A4})$$

where

$$R_i = \frac{2\pi r_i}{\sigma l \delta}, \quad (\text{A5})$$

and, neglecting the shield conductivity,

$$R_o = \frac{2\pi r_o}{\sigma l \delta}. \quad (\text{A6})$$

Substituting these equations into Eq. (1) and using Eq. (3), the Q -value can be written as

$$Q_s = \frac{r_i}{\delta} \frac{1 + \frac{r_i}{r_o}}{1 + \frac{r_i^2}{r_s^2 - r_o^2}}. \quad (\text{A7})$$

This equation reduces to Eq. (4) of Ref. 8 in the limit $r_i \ll (r_o, r_s - r_o)$. The Q -value is independent of axial length. For the LGR discussed in Sec. II, Eq. (A7) gives 284, which is about 20% higher than simulation due to ohmic dissipation in the gap. The resonance frequency of a coil or resonator is given by

$$f = \frac{1}{2\pi\sqrt{LC}}, \quad (\text{A8})$$

where L is the inductance and C is the capacitance. For a 1-loop-1-gap LGR, the inductance is given by Eq. (A1) and the capacitance is the gap capacitance.

For the FGL with no end effects, Fig. 1, the inductance can be estimated from Eq. (A1) where, instead of Eq. (A2), the average radius is used

$$L_i = \frac{\mu_0 \pi (r_i + r_o)^2}{4l}. \quad (\text{A9})$$

The Q -value is enhanced due to the multiple current paths of thickness t . Equations (4) and (5) apply, although the foil thickness must be thinner than the maximum given by Eq. (31). The Q -value can also be estimated from Eq. (1) with

$$R = \frac{\pi (r_i + r_o)}{\sigma l N t}, \quad (\text{A10})$$

where N is the number of foil layers in the non-overlapping region. Since this resistance does not account for the eddy current dissipation, it can result in an overestimate of the Q -value. The capacitance can be expressed as

$$C = \frac{C_{fov}}{N_{ov}}, \quad (\text{A11})$$

where N_{ov} is the number of azimuthal overlapping regions, see Fig. 1, and

$$C_{fov} = \frac{\epsilon_0 \epsilon_r A}{d}, \quad (\text{A12})$$

where ϵ_0 is the electric permittivity of free space, ϵ_r is the relative dielectric constant of the material between the layers, A is the net area of an overlapping region, and d is the distance between adjacent foil layers in the overlapping region. The area can be approximated as

$$A = (2N - 1) l \theta_{ov} (r_i + r_o) / 2, \quad (\text{A13})$$

where θ_{ov} is the foil overlapping angle in radians.

For a single-foil SRS with no end effects, the inductance, resistance, and Q -value can be estimated using the same equations as the FGL given above, where r_i is the minimum foil radius, r_o is the maximum foil radius, and N is the number of turns of the spiral. For this structure, N need not be an

integer. The resonance frequency is given by Eq. (A8) with C the total capacitance between adjacent layers,

$$C = \frac{\epsilon_0 \epsilon_r (N - 1) \pi (2r_i + Np) l}{p - t}, \quad (\text{A14})$$

where p is the pitch of the spiral. For the multiple foil spiral, Fig. 9, there is surprisingly little interaction between the individual foils. The result is that nearly the same magnetic field is obtained with individual foil currents reduced by the number of individual foils N_f . The equations for inductance, resistance, Q -value, and frequency are the same as for the single foil spiral except the resistance given by Eq. (A10) is divided by the number of foils,

$$R = \frac{\pi (r_i + r_o)}{\sigma l N t N_f}, \quad (\text{A15})$$

where N is the number of turns of the spiral. Numerical results indicate that the frequency is reduced by 26% for a four-foil spiral with the same pitch and number of turns as the one-foil spiral of Fig. 8.

For the toroidal loop, the inductance can be approximated as⁵

$$L = \mu_0 r_M \left[\ln \left(\frac{8r_M}{r_m} \right) - 2 \right], \quad (\text{A16})$$

where r_M is the major radius and r_m is the average minor radius of the torus,

$$r_m = \frac{r_{mi} + r_{mo}}{2}, \quad (\text{A17})$$

where r_{mi} is the inner minor radius of the foil and r_{mo} is the outer minor radius of the foil. The resonance frequency is given by Eq. (A8) with the capacitance given by Eqs. (A11) and (A12) where the overlapping area is

$$A = (2N - 1) \pi \theta_{ov} r_M (r_{mi} + r_{mo}), \quad (\text{A18})$$

where θ_{ov} is the foil overlapping angle in radians.

For the FGL and SRS of length $l \lesssim r_i$, the inductance is smaller than the formulas given above. A very approximate value of inductance is given by Eq. (A16) where the major radius is the average radius and the minor radius is approximately $l/2$.

In any resonant structure, dielectric loss can influence the Q -value. Because the time-average electric and magnetic stored energies are equal, if the electric field is distributed primarily in regions with a dielectric loss tangent $\tan d$, the resonator Q -value can be written as

$$Q_c = \frac{1}{\frac{1}{Q_m} + \tan d}, \quad (\text{A19})$$

where Q_m represents the Q -value of the resonator due to metallic losses alone. Equation (A19) holds for most of the structures in this paper. The loss tangent of PTFE at 400 MHz is 0.000 15.²³ This implies that the upper limit of the Q -value for any of these structures is 6700. If the electric field distribution is not confined to gap regions alone, Eq. (A19) does not apply, Sec. IV C 1.

The impedance of a shorted length l of coaxial line is given by Ref. 5

$$Z_i = jZ_0 \tan \frac{\omega l}{c\sqrt{\epsilon_r}}, \quad (\text{A20})$$

where c is the speed of light in vacuum, ϵ_r is the relative dielectric constant of the cable dielectric, and the characteristic impedance Z_0 of the line is given by

$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} \ln \frac{r_o}{r_i}, \quad (\text{A21})$$

where r_o is the inner radius of the outer conductor and r_i is the outer radius of the inner conductor, or, in the case of the foils, the average radius of the foils. The cable length will resonate with a short on both ends if there is a series capacitance C such that

$$Z_i + \frac{1}{j\omega C} = 0. \quad (\text{A22})$$

The resonator efficiency is given by^{9,22}

$$\Lambda = \frac{B_1}{2\sqrt{P_l}}, \quad (\text{A23})$$

where B_1 is the peak magnetic field at the sample and P_l represents the total power loss in the resonator. If spins are saturated, the signal is directly proportional to Λ and if the spins are not saturated, the signal is proportional to Λ^2 . Unlike the Q -value, Λ depends on the axial length of the structure, even for structures with no end effects where Q is independent of length.

¹This definition of Q , e.g., Chap. 7 of Ref. 26 or Sec. 1.04 of Ref. 5, is often called the unloaded Q because it does not account for the loading of the external circuit, Chap. 7 of Ref. 26, Sec. 10.13 of Ref. 5.

²J. S. Hyde, "Surface and other local coils for *in vivo* nmr studies," in *Encyclopedia of Nuclear Magnetic Resonance*, edited by D. M. Grant and R. K. Harris (John Wiley, 1996), pp. 4656–4664.

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⁴R. R. Mett, J. W. Sidabras, and J. S. Hyde, "High Q -factor magnetic resonance imaging radio frequency coil device and methods," PCT international patent application PCT/US2015/61882.

⁵S. Ramo, J. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics* (John Wiley, 1965).

⁶F. E. Terman, *Radio Engineers' Handbook*, McGraw-Hill Handbooks (McGraw-Hill Book Company, Inc., 1943).

⁷The use of capacitive gaps to produce resonance is one method to achieve equalization of currents among multiple foil layers and appears to be new. Alternative methods, which are applicable at lower frequencies, are discussed by Sullivan.²⁷ Among the methods discussed is "impedance

ballasting," which is the technique of adding an impedance in series in order to limit the current in a leg of a circuit. The technique is commonly used for electrical discharges. Reference 27 shows how to add a different capacitance to each foil layer such that the residual inductive reactance of each layer is the same, thereby equalizing the currents. This "capacitive ballasting" is different than the method of the present work. We create a resonance, which has a phase coherency. As such, capacitance can be added in series or in parallel with the foils, or both. We observe that tailoring the capacitance between layers has little effect on the current distribution among the layers. Every structure presented resonates in a narrow band of frequencies determined by the Q -value, not the impedance of each layer. If the currents in each layer were limited by capacitive ballasting, there would be large variations in current in each layer because the reactance is close to zero near resonance. In addition, the structure would have a broad resonance in which different parts of the structure would carry different currents at different frequencies. This is not observed. All parts of the structure resonate together in a single resonant mode. Feynman, Ref. 28, has an interesting discussion of the nature of electromagnetic resonance for structures of various shapes.

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¹⁰The integral can be expressed in closed form, see Sec. 5.20 of Ref. 5 and Ref. 12.

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