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PROBABILISTIC FIBER TRACKING USING A MODIFIED LASSO BOOTSTRAP METHOD

Chuyang Ye, Jeffrey Glaister, and Jerry L. Prince

Department of Electrical and Computer Engineering, Johns Hopkins University, Baltimore, MD, USA

Abstract

Diffusion MRI (dMRI) provides a noninvasive tool for investigating white matter tracts. Probabilistic fiber tracking has been proposed to represent the fiber structures as 3D streamlines while taking the uncertainty introduced by noise into account. In this paper, we propose a probabilistic fiber tracking method based on bootstrapping a multi-tensor model with a fixed tensor basis. The fiber orientation (FO) estimation is formulated as a Lasso problem. Then by resampling the residuals calculated using a modified Lasso estimator to create synthetic diffusion signals, a distribution of FOs is estimated. Probabilistic fiber tracking can then be performed by sampling from the FO distribution. Experiments were performed on a digital crossing phantom and brain dMRI for validation.

Index Terms

Diffusion MRI; probabilistic fiber tracking; Lasso bootstrap

1. INTRODUCTION

Diffusion MRI (dMRI) provides a noninvasive tool for investigating white matter tracts by imaging the anisotropic water diffusion in the fibers. Using dMRI, fiber tracking can be applied to create 3D streamlines that represent fiber structures [1]. Usually, fiber orientations (FOs) are estimated and used to propagate fiber streamlines [1]. Because noise in dMRI introduces uncertainty to FO estimation [2], probabilistic fiber tracking has been proposed [3–5].

Bootstrapping [6] has been widely used for probabilistic representation of FOs [2–5, 7]. It does not require an ad hoc assumption of the relationship between the diffusion profile and the FO uncertainty. Bootstrapping has been applied to the basic dMRI strategy known as diffusion tensor imaging (DTI) [8] and also to the more advanced high angular resolution diffusion imaging (HARDI) [9] which aims to resolve crossing fibers. These methods [2–5, 7] formulate the FO estimation as a linear regression problem and use bootstrapping to estimate the FO distribution.

To reduce the number of diffusion gradient directions required to resolve crossing fibers, methods based on a fixed basis and its sparsity have been proposed [10–12], and FOs can be estimated by solving a Lasso problem [13]. However, previous bootstrap-based probabilistic

fiber tracking methods are based on the linear regression model, and their counterpart on the Lasso model has not been explored.

In this work, we propose a probabilistic fiber tracking method based on bootstrapping the multi-tensor model with a tensor basis. As in [10], the FO estimation is formulated as a Lasso problem. We use a modified Lasso bootstrap method [14] to estimate the distribution of FOs. By resampling the centered residuals calculated by the modified Lasso estimator with replacement to create synthetic diffusion signals, a distribution of FOs can be estimated. Fibers can then be propagated at each step by sampling from the FO distribution. By repeating the fiber tracking process, a probabilistic representation of fiber tracts can be acquired.

2. METHODS

2.1. Signal Model Based on a Tensor Basis

We use N fixed prolate basis tensors to represent FOs. Each basis tensor $\mathbf{D}_i$ represents an FO by its primary eigenvector (PEV) $\mathbf{v}_i$. Then, we have [10]

$$S_k = S_0 \sum_{i=1}^N f_i e^{-b \mathbf{g}_k^T \mathbf{D}_i \mathbf{g}_k} + n_k, \quad (1)$$

where S_k is the diffusion weighted signal in the k -th gradient direction, S_0 is the signal acquired with-out diffusion weighting, f_i is the unknown mixture fractions (MFs) for $\mathbf{D}_i$ and $\sum_i f_i = 1$, b is a constant, $\mathbf{g}_k$ is the k -th gradient direction, and n_k is a noise term. Letting the number of gradient directions be K , by defining $\mathbf{y} = \frac{1}{S_0} (S_1, \dots, S_K)^T$ and $\boldsymbol{\eta} = \frac{1}{S_0} (n_1, \dots, n_K)^T$, we have

$$\mathbf{y} = \mathbf{G} \mathbf{f} + \boldsymbol{\eta}, \quad (2)$$

where $\mathbf{f} = (f_1, \dots, f_N)^T$ and $\mathbf{G}$ is a $K \times N$ matrix comprising the attenuation terms

$$G_{ki} = e^{-b \mathbf{g}_k^T \mathbf{D}_i \mathbf{g}_k}.$$

Because the number of FOs is small in a voxel, sparsity of $\mathbf{f}$ is assumed to reduce the number of gradient directions required for FO estimation [10]. A voxelwise FO estimation is achieved by first relaxing the constraint of $\sum_i f_i = 1$ and then approximating the ℓ_0 -norm with the ℓ_1 -norm [10]:

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f} \geq \mathbf{0}} \|\mathbf{G} \mathbf{f} - \mathbf{y}\|_2^2 + \beta \|\mathbf{f}\|_1. \quad (3)$$

Finally, the MFs are normalized so that they sum to one. The directions represented by the basis tensors with nonzero MFs are interpreted as FOs.

2.2. FO Estimation by Bootstrapping

The solution to Eq. (3) gives a deterministic estimate of $\mathbf{f}$. To take the uncertainty introduced by noise into account, we use a bootstrap strategy. Due to the sparsity assumption, the FO estimation is not a linear regression problem as in [3, 5], and a Lasso bootstrap strategy [15, 16] should be used for FO estimation. A straightforward option is to use the standard bootstrap strategy where signals are synthesized directly with centered residuals [15]. However, as discussed in [14], such bootstrap strategy fails to provide a valid approximation for an unknown distribution to be estimated. Thus, we adopt a modified Lasso bootstrap approach proposed in [16] for probabilistic FO estimation.

To apply bootstrapping, residuals should first be calculated from the estimate of $\mathbf{f}$. Because of the constraint $\sum_i f_i = 1$, the MFs obtained from solving Eq. (3) are first normalized (denoted by $\hat{\mathbf{f}}$). Then, a modified Lasso estimator is defined as [16]

$$\tilde{\mathbf{f}} = (\tilde{f}_1, \dots, \tilde{f}_N)^T \text{ with } \tilde{f}_i = \hat{f}_i \mathbb{1}_{\hat{f}_i \geq a_K}. \quad (4)$$

Here $\mathbb{1}$ is an indicator function, and a_K is a quantity that satisfies $a_K + (K^{-\frac{1}{2}} \log K) a_K^{-1} \rightarrow \infty$ as $K \rightarrow \infty$. A possible choice of a_K can be $a_K = cK^{-\delta}$ ($c \in (0, \infty)$ and $\delta \in (0, 2^{-1})$) [16]. In this way, small values of $\hat{f}_i$ are set as zero, which solves the inconsistency of the standard bootstrap in [15]. A detailed proof can be found in [16].

Using the modified estimate $\tilde{\mathbf{f}}$, residuals $\mathbf{r} = (r_1, \dots, r_K)^T$ can be calculated: $\mathbf{r} = \mathbf{y} - \mathbf{G}\tilde{\mathbf{f}}$. Then, centered residuals $r_k^* = r_k - \bar{r}_K$ are computed, where $\bar{r}_K = \frac{1}{K} \sum_{k=1}^K r_k$. In each gradient direction k , we randomly sample an r_k^{**} from $\{r_1^*, \dots, r_K^*\}$ (with replacement) and synthesize

$$y_k^* = (\mathbf{G} \tilde{\mathbf{f}})_k + r_k^{**}. \quad (5)$$

A distribution of FOs can then be achieved using the repeatedly synthesized signals

$$\mathbf{y}^* = (y_1^*, \dots, y_K^*)^T.$$

2.3. Probabilistic Fiber Tracking

Each fiber is initiated from a seed voxel with a fractional anisotropy (FA) value larger than a threshold t_{FA} . In the seed voxel, FOs are sampled using the modified bootstrap, and the FO with the largest MF starts the propagation. During the propagation, the FO in the current voxel is used until the fiber reaches the next voxel. Then, the FOs in the new voxel are sampled as proposed, and the basis direction that maximizes $f_{ij} \mathbf{v}_j \cdot \mathbf{v}_{last}^4$ [10] is selected as

the propagation direction. Here $\mathbf{v}_{\text{last}}$ is the unit propagation direction in the previous tracking step. Note that only directions with MFs larger than a threshold t_{mf} are used in the tracking [10]. The fiber is terminated if it reaches a voxel with FA lower than t_{FA} or the angle between the previous and current propagation direction is larger than a threshold θ_t .

3. EXPERIMENTS

3.1. 3D Digital Crossing Phantom

The proposed method was performed (with $\beta = 0.1$) on a 3D digital crossing phantom (see Fig. 1). One b_0 image and 30 diffusion weighted images (DWIs) were simulated and $b = 700 \text{ s}=\text{mm}^2$. Rician noise ($\sigma/S_0 = 0.03$) was added to the DWIs. The proposed method is compared with the CFARI algorithm [10] (deterministic fiber tracking on the multi-tensor model) and the wild bootstrap algorithm [2, 4] (probabilistic fiber tracking on the DTI model). A seed was placed in the noncrossing region (shown in Fig. 1) and 100 fibers were generated for the wild bootstrap and proposed method. The proposed method generates a distribution of fibers that corresponds to the desired anatomy (transverse fibers), while the wild bootstrap method is unable to propagate fibers through the crossing regions.

For quantitative evaluation, we use the maximum of the two FO estimation error measures proposed in [17]. We tested different levels of Rician noise and used a similar evaluation design as in [5]. For each level, we selected a noncrossing voxel and a crossing voxel, and obtained 5000 noisy samples. FOs were estimated and the means and standard deviations (SDs) of the errors were calculated as the gold standard. Then, we created 100 noncrossing and crossing noisy samples for our bootstrap method. For each sample, bootstrapping was performed 100 times to estimate the means and SDs of the error. The 95% confidence intervals (CIs) of the estimates using all 100 samples are plotted in Fig. 2 and compared with the gold standard. The gold standard means and SDs are located within these 95% CIs, indicating the uncertainty estimated by the proposed method is close to the gold standard.

3.2. Brain dMRI

The proposed method was applied to brain dMRI of a healthy subject. One b_0 image and 30 DWIs ($b = 700 \text{ s}=\text{mm}^2$) were used. Each DWI was acquired on a 3T MR scanner (Intera, Philips, Netherlands) with a multi-slice, single shot EPI sequence. The images were resampled to be 0.828 mm isotropic.

The proposed method (with $\beta = 0.1$) is compared with CFARI [10] and wild bootstrap [2, 4] fiber tracking. Seeds were placed in the thalamus, and the results are shown in Fig. 3 with the cortical surface. The fibers generated by the proposed method are better distributed on the cortical surface.

4. CONCLUSION

In this work, we have proposed a probabilistic fiber tracking method based on a modified Lasso bootstrap strategy. Using the signals synthesized by resampling the modified centered residuals, a distribution of FOs can be generated and probabilistic fiber tracking can be

performed. Experiments on a digital crossing phantom and brain dMRI demonstrate the efficacy of the proposed method.

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References

1. Basser PJ, Pajevic S, Pierpaoli C, Duda J, Aldroubi A. In vivo fiber tractography using DT-MRI data. *Magnetic Resonance in Medicine*. 2000; 44(4):625–632. [PubMed: 11025519]
2. Whitcher B, Tuch DS, Wisco JJ, Sorensen AG, Wang L. Using the wild bootstrap to quantify uncertainty in diffusion tensor imaging. *Human Brain Mapping*. 2008; 29(3):346–362. [PubMed: 17455199]
3. Jeurissen B, Leemans A, Jones DK, Tournier JD, Sijbers J. Probabilistic fiber tracking using the residual bootstrap with constrained spherical deconvolution. *Human Brain Mapping*. 2011; 32(3): 461–479. [PubMed: 21319270]
4. Jones DK. Tractography gone wild: probabilistic fibre tracking using the wild bootstrap with diffusion tensor MRI. *IEEE Transactions on Medical Imaging*. 2008; 27(9):1268–1274. [PubMed: 18779066]
5. Berman JJ, Chung S, Mukherjee P, Hess CP, Han ET, Henry RG. Probabilistic streamline q -ball tractography using the residual bootstrap. *NeuroImage*. 2008; 39(1):215–222. [PubMed: 17911030]
6. Efron B. Bootstrap methods: another look at the jackknife. *The annals of Statistics*. 1979:1–26.
7. Chung S, Lu Y, Henry RG. Comparison of bootstrap approaches for estimation of uncertainties of DTI parameters. *NeuroImage*. 2006; 33(2):531–541. [PubMed: 16938472]
8. Basser PJ, Mattiello J, LeBihan D. MR diffusion tensor spectroscopy and imaging. *Biophysical Journal*. 1994; 66(1):259–267. [PubMed: 8130344]
9. Tuch DS, Reese TG, Wiegell MR, Makris N, Belliveau JW, Wedeen VJ. High angular resolution diffusion imaging reveals intravoxel white matter fiber heterogeneity. *Magnetic Resonance in Medicine*. 2002; 48(4):577–582. [PubMed: 12353272]
10. Landman BA, Bogovic JA, Wan H, ElShahaby FEZ, Bazin P-L, Prince JL. Resolution of crossing fibers with constrained compressed sensing using diffusion tensor MRI. *NeuroImage*. 2012; 59(3): 2175–2186. [PubMed: 22019877]
11. Zhou, Q.; Michailovich, O.; Rathi, Y. SPIE Medical Imaging. International Society for Optics and Photonics; 2014. Resolving complex fibre architecture by means of sparse spherical deconvolution in the presence of isotropic diffusion; p. 903425-903425.
12. Daducci A, Van De Ville D, Thiran JP, Wiaux Y. Sparse regularization for fiber ODF reconstruction: From the suboptimality of ℓ_2 and ℓ_1 priors to ℓ_0 . *Medical Image Analysis*. 2014; 18(6):820–833. [PubMed: 24593935]
13. Tibshirani R. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*. 1996:267–288.
14. Chatterjee A, Lahiri S. Asymptotic properties of the residual bootstrap for Lasso estimators. *Proceedings of the American Mathematical Society*. 2010; 138(12):4497–4509.
15. Knight K, Fu W. Asymptotics for lasso-type estimators. *The Annals of Statistics*. 2000; 28(5): 1356–1378.
16. Chatterjee A, Lahiri S. Bootstrapping Lasso estimators. *Journal of the American Statistical Association*. 2011; 106(494):608–625.
17. Ye, C.; Carass, A.; Murano, E.; Stone, M.; Prince, JL. Bayesian and graphical Models for Biomedical Imaging, vol. 8677 of *Lecture Notes in Computer Science*. Springer; 2014. A Bayesian approach to distinguishing interdigitated muscles in the tongue from limited diffusion weighted imaging; p. 13-24.

Fig. 1.
Fiber tracking results on the FA of a 3D crossing phantom.

Fig. 2.

Means and SDs of estimation errors. 95% CIs are plotted for the proposed method.

Fig. 3. Fiber tracking results on the brain dMRI shown together with the cortical surface.