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Nonlinearity measurements of solar cells with an LED-based combinatorial flux addition method

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Abstract

We present a light emitting diode (LED)-based system utilizing a combinatorial flux addition method to investigate the nonlinear relationship in solar cells between the output current of the cell and the incident irradiance level. The magnitude of the light flux is controlled by the supplied currents to two LEDs (or two sets of them) in a combinatorial fashion. The signals measured from the cell are arranged within a related overdetermined linear system of equations derived from an appropriately chosen N^{th} degree polynomial representing the relationship between the measured signals and the incident fluxes. The flux values and the polynomial coefficients are then solved for by linear least squares to obtain the best fit. The technique can be applied to any solar cell, under either monochromatic or broadband spectrum. For the unscaled solution, no reference detectors or prior calibrations of the light flux are required. However, if at least one calibrated irradiance value is known, then the entire curve can be scaled to an appropriate spectral responsivity value. Using this technique, a large number of data points can be obtained in a relatively short time scale over a large signal range.

1. Introduction

Linear response of a solar cell, which refers to whether a linear relationship exists between the incident irradiance flux and the photocurrent output of the cell, is an important device property. From a fundamental physics perspective, knowledge of the linearity of this relationship can be useful in understanding or modeling charge carrier recombination phenomena or the role of defects on device performance. From a practical measurement perspective, reference solar cells or detectors that are used to measure the output of other test cells or modules are typically required to have a linear short circuit current (I_{sc}) output as a function of incident irradiance over the range of interest. Since the plane of incidence irradiance level is set and monitored using I_{sc} measurements of a reference cell, it is imperative that such devices have a linear output with irradiance particularly over the

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standard reporting conditions (SRC). If the reference cell is not linear and no other substitute cells are available, then the irradiance measurements can be corrected if the mathematical relationship between the signal and the flux is known.

Most silicon-based reference solar cells show a good linear behavior at irradiance levels near the SRC air mass (AM) 1.5 spectrum with total irradiance of 1000 Wm^{-2} and temperature fixed at 25°C , but this is not always the case as will be shown later. Moreover, at very low light levels or under very concentrated irradiance conditions, the linearity assumption breaks down in a significant way in many different silicon-based devices [1–3]. Generally speaking, it is useful to not only know that a particular device is nonlinear over a certain irradiance range, but it is also important to know the precise relationship between the photocurrent and the irradiance. Knowing this functional form allows one to determine the exact I_{sc} of the device under a given irradiance level or spectrum [2]. A practical way to express the device nonlinearity over a large range is to plot the ratio of signal (the short circuit current) to irradiance flux as a function of signal. If no nonlinearity is present, this ratio remains constant as a function of signal; however, in nonlinear devices, the ratio will change as a function of signal. In essence, this ratio is related to the spectral responsivity, $R(\lambda)$, of the cell at a given wavelength if the incident radiation is monochromatic. In the case of a broadband spectrum of light such as the AM 1.5 global conditions, then the cell's AM 1.5-weighted spectral response can be obtained and plotted as a function of I_{sc} . In many silicon cells, it is common to see a ratio that is initially changing as a function of current, particularly at low signal levels, and then assumes a constant ratio at higher signal levels, indicating that the device is now operating in the linear regime.

In solar cell and optical detector measurements, there have been reports of several types of nonlinearity verification methods. One involves performing differential spectral responsivity measurements (DSR) as a function of light bias (LB) current or intensity [1–5]. Another involves measurements of nonlinearity with respect to a linear device [6]. A more common method includes the flux addition principle using a dual light source approach, i.e., the two-lamp method [7–9] or the double-aperture method [10]. The first technique, in principle allows for the determination of the actual mathematical relationship between the signal and the flux and can furthermore be used to obtain the I_{sc} under a given light spectrum for nonlinear cells through an iterative integration calculation [2]. However, this method can be time consuming if the DSR is done over all wavelengths for various levels of light bias signal. Furthermore, the AC/DC current separation when that ratio increases can be challenging. The two-lamp method based on comparing the ratio of added photocurrents obtained from two light sources, such as tungsten sources [8] or two LEDs [7,9] to their combined two-source output, can reveal device nonlinearity, but does not provide any insight on the actual nonlinear relationship. This issue will be briefly discussed in this work.

Recently, a combinatorial technique for calibrating indicating instruments was proposed, and its application to mass balances, resistance bridges and optical detectors was demonstrated [11,12]. In particular, a dual light path combinatorial flux addition method was applied to optical detectors for the purpose of verifying the detector linearity with irradiance. This approach consisted of illuminating and measuring the device with a combinatorial flux algorithm that can then be used to construct an overdetermined linear system of equations of

signals and fluxes, to be solved to determine the incident set of unknown fluxes and the fitting coefficients simultaneously. In the case of the optical detectors that were tested (high-quality photodiodes), it was usually found that no higher order correction terms were needed and the detectors exhibited a linear response over a large range.

In this work, we have extended the combinatorial flux addition technique to solar cell devices that show significant nonlinear response with flux. In order to facilitate fast and reliable measurements, we employed two LED lamps (Note: two sets of LED lamps can also be employed) as flux generating sources that can illuminate the entire cell in overfilled mode while the intensity level can be controlled by the LED drive current. This technique allows for the creation of a large number of flux levels while at the same time eliminating the need for separate optical light paths by use of motorized optical wheels, shutters, and neutral density filters to create a large set of fluxes. It was found that, for several solar cells, higher-order corrections of up to 4th degree were required to obtain the correct irradiance vs. current relationship. The results of the measurements using two different LED wavelengths of 627 nm and 890 nm were compared with light-bias dependent spectral response measurements and found that a good agreement exists between the two. However, the two-lamp method was shown to be ineffective in determining the correct actual amount of nonlinearity in solar cells and should only be used when a pass or fail answer on cell linearity is desired.

2. System description

2A. The setup

A schematic of the LED-based combinatorial nonlinearity setup is illustrated in Fig. 1. A commercial LED controller with at least two independent current source channels controls the operation of two LEDs, producing two distinct light fluxes on the test cell inside a dark enclosure. Although it is possible to operate the two LEDs in DC current mode, we have opted instead to pulse the current to each LED, producing pulsed light modulation and an AC short-circuit current in each solar cell. The output current is first converted to a voltage signal via a current-to-voltage transimpedance amplifier with a fixed gain and is then measured by a lock-in amplifier. The transimpedance amplifier design ensures that the cell is maintained at I_{sc} at all times. Both the LED controller and the lock-in amplifier are triggered by a signal generator. The pulsing frequency was set to 97 Hz for these experiments but it can be changed to any desired level as long as the device under test responds well at that frequency. The duty cycle was set at 50 %. Pulsed excitation measurements provide far greater stability of the LED signal than steady DC operation [13,14] and better signal to noise detection with the lock-in method. Additionally, the LEDs were mounted on water-cooled plates with water temperature of 15 °C to provide a stable output intensity and spectrum. The spectrum changes with supplied current are negligible. The temperature of the cells during all the measurements was monitored and found to remain relatively stable at 22 °C, the lab room temperature. An extensive computer code was written to control all the different components of this measurement. The proof of concept and data analysis were demonstrated with the help of two sets of monochromatic LEDs, specifically two 627 nm-peaked LEDs and two 890 nm-peaked LEDs with full-width at half max (FWHM)

bandwidth of 17 nm and 50 nm respectively, since it has been known that silicon nonlinear response is spectrally dependent [1–3]. Implementation of this technique to combinations of LEDs for construction of a “white” or broadband spectrum is straightforward.

2B. Algorithm for data collection

The algorithm that was used to create the different combination of fluxes is shown in Fig. 2 in tabular form. Many other combinations are possible, as long as the measured signals are properly tracked with the order in which the fluxes were applied to the cell. Flux or intensity level, as mentioned previously, is set by the supplied LED current. For the data in this paper, 15 unique pulsed current levels are sourced to LED A in a sequential way, while the short circuit current signal from the solar cell is measured. This sequence is followed by 15 other current levels to LED B in a similar manner and recording the I_{sc} signal from the cell. Then, the combinations of these currents are applied to *both* LEDs as outlined in Fig. 2. Each supplied current in the left column is paired with 3 adjacent currents in the right column. Values of adjacent currents are generally chosen close together. In this case, $45=15 \times 3$ combination fluxes are supplied and therefore 45 signals are obtained from the cell. So in total, $75=15+15+45$ total data points are measured. These data are then analyzed using the method described in the following section.

2C. Uncertainty

The main uncertainty component in our apparatus is the type B uncertainties associated with the repeatability of the current sourcing to the LEDs and the resulting cell signal. We found this repeatability signal uncertainty to be on the order of 0.5 % to 1 %. Other type B uncertainties such as the transimpedance preamp and the lock-in amplifier linearity were verified over all the gain settings and signal levels and were found to be negligible (< 0.01 %). This was accomplished by performing measurements on a standard reference detector whose response and linearity had been previously verified with calibrated instruments at a different division at NIST. The data provided in section 4 on cell#5 verifies the linear response of the standard reference detector. The LED power stability was measured over a period of minutes and was found to be better than 0.05 % once a few seconds have passed since the initial turn on. The temperature control for heat dissipation is probably important in achieving good stability. For type A uncertainty, each data point was measured at least 100 times for each LED current setting. The relative standard deviation for high gain/low signal regime is about 0.1 % and for higher signal regime is about 0.05 % or better. Therefore, this method can easily be used to probe nonlinearity relations on the order of 1%. A more detailed Monte Carlo-based uncertainty analysis will be provided in a future correspondence.

This uncertainty, however, means that flux combinations between levels that produce differences of more than a factor of 100 in the output signal of the cell should not be used; i.e., a combination of a very large signal with a very small signal by a factor of 1/100 or less will not work due to an overall uncertainty of 1%. Therefore, the solution is to break up the nonlinearity region of interest into multiple signal intervals and perform the combinatorial approach at different overlapping *local* levels. Each level will then have a unique table of singular currents. Lower levels of signals can also be accomplished with only one table of

supplied LED currents combined with use of neutral density filters in front of each LED lamp. These local intervals can later be stitched together to form a *global* curve. The automated measurement time for the 75 averaged signals is about 20 minutes, which includes small wait times after LED current changes for improved stability and 100 measurements at each signal level for calculation of the mean values of the signal.

3. Mathematical Framework

This section outlines the mathematical framework for using the combinatorial flux addition data to obtain the relationship between the signal and the flux as well as obtaining a scaled solution. The system of linear equations constructed from the combinatorial measurements is *overdetermined* because the total number of unknowns (i.e., fluxes and the coefficients) are far smaller than the total number of equations. For example, in the algorithm outlined above, the total unknown fluxes are 30 with generally 5 unknown polynomial coefficients, whereas the total number of equations is $15+15+45=75$.

3A. Unscaled solution

Assume an N^{th} -degree polynomial model for flux ϕ (say, in W/m^2) as a function of short-circuit current signal s (say in A DC), i.e.,

$$\phi = g(s) = r_0 + r_1 s + r_2 s^2 + \dots + r_N s^N. \quad (1)$$

Here, what's been referred to as flux ϕ is really irradiance, or the total incident radiant power per unit area of the device. Let $(\phi_{A,k})$, $k = 1, \dots, K_A$, denote a list of K_A distinct fluxes from LED lamp A, and let $(\phi_{B,k})$, $k = 1, \dots, K_B$, denote a list of K_B distinct fluxes from LED lamp B. Let $(\phi_{A,i_m} + \phi_{B,j_m})$, $m = 1, \dots, M$ denote a list of M distinct combinations of the fluxes from lamp A and lamp B, where $i_m \in \{1, \dots, K_A\}$ and $j_m \in \{1, \dots, K_B\}$. Let s_{A,i_m} denote the signal measured for lamp A at flux setting ϕ_{A,i_m} and s_{B,j_m} denote the signal measured for lamp B at flux setting ϕ_{B,j_m} . Let $s_{A,i_m + B,j_m}$ denote the signal measured for lamp A and lamp B with total flux $\phi_{A,i_m} + \phi_{B,j_m} = \phi_{A,i_m} + \phi_{B,j_m}$, because flux addition is linear. Both signals and fluxes are strictly positive. Assuming that no measurements are repeated, the individual and combined measurements of lamp A and lamp B give the following linear system of $K_A + K_B + M$ equations:

$$\left\{ \begin{array}{l} 0=r_0+r_1s_{A_1}+\dots+r_Ns_{A_1}^N-\phi_{A_1}, \\ \vdots \\ 0=r_0+r_1s_{A_{K_A}}+\dots+r_Ns_{A_{K_A}}^N-\phi_{A_{K_A}}, \\ 0=r_0+r_1s_{B_1}+\dots+r_Ns_{B_1}^N-\phi_{B_1}, \\ \vdots \\ 0=r_0+r_1s_{B_{K_B}}+\dots+r_Ns_{B_{K_B}}^N-\phi_{B_{K_B}}, \\ 0=r_0+r_1s_{A_{i_1}+B_{j_1}}+\dots+r_Ns_{A_{i_1}+B_{j_1}}^N-\phi_{A_{i_1}}-\phi_{B_{j_1}}, \\ \vdots \\ 0=r_0+r_1s_{A_{i_M}+B_{j_M}}+\dots+r_Ns_{A_{i_M}+B_{j_M}}^N-\phi_{A_{i_M}}-\phi_{B_{j_M}} \end{array} \right\} \begin{array}{l} K_A \text{ eq's} \\ K_B \text{ eq's} \\ M \text{ eq's} \end{array} \quad (2)$$

with unknown coefficients r_n for $n = 0, \dots, N$, and unknown fluxes ϕ_{A_k} for $k = 1, \dots, K_A$, and ϕ_{B_k} for $k = 1, \dots, K_B$. By inspection, a solution to system (2) is the trivial zero solution, which is not physically reasonable. A non-trivial physical solution to the system will not be unique, because a positive rescaling of the unknowns will also be a solution. ("Solution" in the over-determined case refers to a standard linear-least-squares solution, which must have a zero sum of squared residuals because the zero solution has a zero sum of squared residuals.) Thus, the problem is ill-posed because either (1) a physically relevant solution does not exist, or (2) if a physically relevant solution exists, then it is not unique.

We now make the problem well-posed by assuming that $r_1 \neq 0$. This assumption implies that the flux vs. signal relationship has a significant linear component. Dividing through by $r_1 \neq 0$ gives a *related* linear system, i.e.,

$$\left\{ \begin{array}{l} 0=r_0^*+s_{A_1}+\dots+r_N^*s_{A_1}^N-\phi_{A_1}^*, \\ \vdots \\ 0=r_0^*+s_{A_{K_A}}+\dots+r_N^*s_{A_{K_A}}^N-\phi_{A_{K_A}}^*, \\ 0=r_0^*+s_{B_1}+\dots+r_N^*s_{B_1}^N-\phi_{B_1}^*, \\ \vdots \\ 0=r_0^*+s_{B_{K_B}}+\dots+r_N^*s_{B_{K_B}}^N-\phi_{B_{K_B}}^*, \\ 0=r_0^*+s_{A_{i_1}+B_{j_1}}+\dots+r_N^*s_{A_{i_1}+B_{j_1}}^N-\phi_{A_{i_1}}^*-\phi_{B_{j_1}}^*, \\ \vdots \\ 0=r_0^*+s_{A_{i_M}+B_{j_M}}+\dots+r_N^*s_{A_{i_M}+B_{j_M}}^N-\phi_{A_{i_M}}^*-\phi_{B_{j_M}}^* \end{array} \right\} \quad (3)$$

where $r_0^*=r_0/r_1$, $\phi_{A_1}^*=\phi_{A_1}/r_1$, etc. Here, $M \geq N$ in order to avoid an underdetermined system. This system can be solved by linear least squares to give coefficients and fluxes up to an undetermined linear scale factor r_1 . Alternatively, the system of equations above can be rearranged and placed into matrix-vector form:

$$U \cdot \Phi = S, \quad (4)$$

where the flux vector Φ is composed of first the singular fluxes (from LED A and LED B), and then the coefficients r_0^* , r_2^* , etc. at the very end. The signal vector S , is composed of the singular and combination signals in the same manner as the fluxes. Written in expanded form, the matrix equation looks as follows:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 & -s_{A_1}^2 & \cdots & -s_{A_1}^N \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 & -s_{A_2}^2 & \cdots & -s_{A_2}^N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & -1 & -s_{B_1}^2 & \cdots & -s_{B_1}^N \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & -1 & -s_{B_2}^2 & \cdots & -s_{B_2}^N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & -1 & -s_{A_1+B_1}^2 & \cdots & -s_{A_1+B_1}^N \\ 1 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & -1 & -s_{A_1+B_2}^2 & \cdots & -s_{A_1+B_2}^N \\ 1 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & -1 & -s_{A_1+B_3}^2 & \cdots & -s_{A_1+B_3}^N \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 & \cdots & -1 & -s_{A_2+B_2}^2 & \cdots & -s_{A_2+B_2}^N \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \phi_{A_1}^* \\ \phi_{A_2}^* \\ \vdots \\ \phi_{B_1}^* \\ \phi_{B_2}^* \\ \vdots \\ r_0^* \\ r_2^* \\ \vdots \\ r_N^* \end{bmatrix} = \begin{bmatrix} s_{A_1} \\ s_{A_2} \\ \vdots \\ s_{B_1} \\ s_{B_2} \\ \vdots \\ s_{A_1+B_1} \\ s_{A_1+B_2} \\ s_{A_1+B_3} \\ s_{A_2+B_2} \\ \vdots \end{bmatrix} \quad (5)$$

The large matrix on the left is the U matrix, where rows $1, \dots, K_A+K_B$ are related to the singular flux measurements, and rows $1+K_A+K_B, \dots, M$ are related to combinatorial fluxes. The *linear least squares* solution for the fluxes and the coefficients is given by [15]:

$$\Phi = \left[(U^T \cdot U)^{-1} \cdot U^T \right] S, \quad (6)$$

where symbols T and -1 stand for transpose and inverse of their respective matrices. The analysis is first performed for a linear relationship, i.e., up to the r_0^* coefficient. The residuals of the fit are then considered to determine if this was an adequate fit of the data. The residuals are a comparison between the calculated flux from the regression and the interpolation of the flux from the corresponding signal. The residual for a given combinatorial data point up to the linear term is defined as:

$$\left(\phi_{A_{im}}^* + \phi_{B_{jm}}^* \right) - \left(r_0^* + s_{A_{im}+B_{jm}} \right), \quad (7)$$

A residual value can also be defined for a singular flux/signal data point as well. If the calculated residuals are too large or show a skewed pattern or relationship, then the next

higher order of regression is performed, i.e., up to r_2^* . This approach means that a column of $-s^2$ values is added to the last column of the U matrix. Again, the residuals are analyzed to determine the goodness of the fit. This process can be repeated through r_N^* where we have found that generally $N=4$ suffices even for the most nonlinear cases.

3B. Scaled Solution

The calculation discussed so far allows for determining the relationship between the signals measured and the incident fluxes over a given range; however, the solution remains unscaled. It is possible to obtain a scaled relationship, i.e., the irradiance spectral responsivity plotted as a function of current, if one or more calibrated flux values were known. Flux calibration can be achieved by the use of a reference photodetector with known spectral irradiance response. Suppose one has a list of calibrated fluxes ϕ_{C_k} , $k=1, \dots, K_C$ with a corresponding list of measured signals (s_{C_k}). We now solve for the undetermined linear scale factor r_1 . Using the coefficients $r_0^*, r_2^*, \dots, r_N^*$ from the linear least square fit with the calibrated points gives the linear system:

$$\begin{cases} 0 = r_1(r_0^* + s_{C_1} + \dots + r_N^* s_{C_1}^N) - \phi_{C_1}, \\ \vdots \\ 0 = r_1(r_0^* + s_{C_{K_C}} + \dots + r_N^* s_{C_{K_C}}^N) - \phi_{C_{K_C}}, \end{cases} \quad (8)$$

which is solved for r_1 . Note that it is important for the calibrated signals to lie in the same domain as the signals used to fit the polynomial model (1), because the extrapolated values of the polynomial are not guaranteed to be valid. System (8) has full rank, and solved by linear least squares if $K_C \geq 2$. Finally, the scaled coefficients are given by $r_N = r_1 r_N^*$, $N=1$, and the fluxes are given by $\phi_{A_k} = r_1 \phi_{A_k}^*$ and $\phi_{B_k} = r_1 \phi_{B_k}^*$.

3C. Continuous Solution from Separate Fits on Multiple Overlapping Intervals

As mentioned previously, if s/ϕ vs. s is desired over a large s range, then the combinatorial method has to be performed multiple times over a given signal interval. Sometimes, multiple polynomials are required to achieve good fits on different signal intervals, possibly across multiple decades of signal. If these intervals are ordered with overlaps between them, then one can continuously piece together the unscaled “local” polynomial curve fits on each interval while simultaneously scaling the “global” continuous fit to a list of calibrated fluxes with corresponding measured signals that lie in the overall range of measured signals. Below is a mathematical procedure that describes how to *stitch* these separate curves together.

Let $[s_{\min}, p, s_{\max}, p]$ denote the interval of definition of the p^{th} polynomial curve fit of P ordered polynomial curve fits. For ease of notation, let N be the largest degree among all the local polynomial fits, letting any additional coefficients be zero where necessary. Each of the P unscaled fit curves has one unknown scalar, giving the following P possible functions for ϕ :

$$\begin{aligned}\phi &= g_1(s) = r_{11}(r_{01}^* + s + r_{21}^* s^2 + \dots + r_{N1}^* s^N) \text{ for } s \in [s_{\min,1}, s_{\max,1}], \\ &\vdots \\ \phi &= g_P(s) = r_{1P}(r_{0P}^* + s + r_{2P}^* s^2 + \dots + r_{NP}^* s^N) \text{ for } s \in [s_{\min,P}, s_{\max,P}],\end{aligned}\quad (9)$$

with unknown scalars $r_{11}, \dots, r_{1P}$ that must be determined.

For each of the $P-1$ overlap regions for the P unscaled fit curves, compute the midpoint signal of the overlap. Define the *knot signal* between curves p and $p+1$ as:

$$s_p^* = \frac{s_{\max,p} + s_{\min,p+1}}{2}, \quad p=1, \dots, P-1. \quad (10)$$

Knot selections other than the average value are possible, but with sufficient interval overlaps the simple choice of these middle values should sufficiently reduce any interval edge effects that may occur in typical polynomial fits.

The interval ordering and overlap requirements means that $s_{\min,p+1} < s_{\max,p}$ for all $p \in 1, \dots, P-1$, so that each knot signal is contained in both intervals from which it is constructed. This arrangement gives a partition of new intervals listed in order by

$$([s_{\min,1}, s_1^*], [s_1^*, s_2^*], \dots, [s_{P-1}^*, s_{\max,P}]). \quad (11)$$

The requirement of continuity of flux between any two adjacent segmented curves gives the following system of $P-1$ linear equality constraints for the *knot fluxes* in the P unknown scalars $r_{11}, \dots, r_{1P}$:

$$\begin{cases} r_{11}(r_{01}^* + s_1^* + \dots + r_{N1}^* s_1^{*N}) = r_{12}(r_{02}^* + s_1^* + \dots + r_{N2}^* s_1^{*N}), \\ \vdots \\ r_{1P-1}(r_{0P-1}^* + s_{P-1}^* + \dots + r_{NP-1}^* s_{P-1}^{*N}) = r_{1P}(r_{0P}^* + s_{P-1}^* + \dots + r_{NP}^* s_{P-1}^{*N}), \end{cases} \quad (12)$$

where all the polynomial coefficients and knot signals are known values.

Given the partition (11), one can assign each calibrated flux with its corresponding measured signal to exactly one of the intervals. The scaled polynomial fit corresponding to each interval is evaluated at the measured signal(s) in that interval and set to equal the respective calibrated flux(es). For example, on the p^{th} interval, K_{C_p} calibrated fluxes $\phi_{C_{p1}}, \dots, \phi_{C_{pK_{C_p}}}$ correspond to K_{C_p} measured signals $s_{C_{p1}}, \dots, s_{C_{pK_{C_p}}}$. Furthermore, only one calibration point is required so that some curve intervals may not have any points associated with them, due to the continuity of the relationship between signals and fluxes. This gives a linear system of the following form to solve for $r_{11}, \dots, r_{1P}$

$$\left\{ \begin{array}{l} 0=r_{11}(r_{01}^*+s_{C_{11}}+\dots+r_{N1}^*s_{C_{11}}^N)-\phi_{C_{11}}, \\ \vdots \\ 0=r_{11}(r_{01}^*+s_{C_{1K_{C1}}}+\dots+r_{N1}^*s_{C_{1K_{C1}}}^N)-\phi_{C_{1K_{C1}}}, \\ \vdots \\ 0=r_{1P}(r_{0P}^*+s_{C_{P1}}+\dots+r_{NP}^*s_{C_{P1}}^N)-\phi_{C_{P1}}, \\ \vdots \\ 0=r_{1P}(r_{0P}^*+s_{C_{PK_{CP}}}+\dots+r_{NP}^*s_{C_{PK_{CP}}}^N)-\phi_{C_{PK_{CP}}}, \end{array} \right\} \begin{array}{l} 1^{\text{st}} \text{ curve interval} \\ \\ p^{\text{th}} \text{ curve interval} \end{array} \quad (13)$$

subject to the linear equality constraints (12).

System (13) with linear equality constraints (12) may be solved readily using a linear equation solver program. If the optimization is not constrained to nonzero unknowns $r_{11}, \dots, r_{1P}$, then one should verify the nonzero assumption after a solution is found. However, this outcome is highly unlikely to occur. Additionally, it is noted that if a calibration point is not available, the unscaled solutions can still be stitched together to form a global unscaled solution, as long as an overlap signal interval exists between the local curves. The easiest way to achieve this stitching is to normalize a single local curve, say, the p^{th} curve interval, and then follow the procedure above to stitch the remaining $P-1$ curves sequentially. Lastly, we note that the computational approach presented here can be modified to more elaborate schemes that stitch together curves using, for example, smoothness constraints in addition to the continuity constraint. However, requiring only continuity allows for the solution of system (5) independently from the subsequent curve rescalings.

4. Measurement Results and Discussion

4A. Combinatorial measurements

The measurement method and the analysis as described above were used to study the nonlinear relation or lack thereof in five different types of Si-based solar cells and detectors. For proof of concept and simplicity, only two kinds of LED lamps were used, namely two 627 nm LEDs and two 890 nm LEDs. Furthermore, the results of these measurements were compared with the traditional light-bias dependent differential spectral response measurements in overfilled (irradiance) mode for these two wavelengths. In order for the comparison to be as similar as possible, both the modulated monochromatic light and the DC bias light were of the same kind. The cells tested were as follows: Cell#1 a KG-5 filtered monocrystalline Si cell, Cell#2 a polycrystalline Si cell, Cell#3 a regular monocrystalline Si cell, Cell#4 a second KG-5 filtered Si cell, and Cell#5 a NIST working standard Si photodetector. Cells # 1 to #4 were 2 cm×2 cm World Photovoltaic Scale (WPVS)-designed reference cells, and Cell #5 was a 1 cm-diameter silicon detector.

Figure 3 shows the normalized global unscaled measurement of $R = s/\phi$ vs. s over roughly 5 orders of magnitude at 890 nm for three of the cells. That is, for every signal value, a flux is calculated and the ratio R is plotted vs. s . For the ease of comparison, each global curve was normalized to 1 at the highest R so that the differences between curves at each s level can be

shown more clearly. The different-colored segments correspond to the different intervals of signals in which the measurement and regression analysis were performed. The individual local segments, each consisting of 75 total data points, were stitched together using the method of section 3C. It can be seen that Cell#5 is completely linear over this signal range because the ratio R remains constant over all s , whereas the other two cells show some level of nonlinearity in the signal range below 1 mA, reaching a linear behavior at higher values. Cell#2, for example, shows a dramatic 20 % lower responsivity at the signal level of 100 nA compared to 1 μ A but only 10 % nonlinearity between 1 μ A to 10 μ A, followed by less than 5 % change between 10 μ A and 1 mA. Since R , which is related to the spectral responsivity of the cell, remains constant at signal values greater than ~ 1 mA, this result implies that the total AM 1.5-weighted I_{sc} of the device is also linear with intensity at signal levels above this value. (Note that without at least one calibrated flux measurement, we cannot know the absolute flux value for a given signal; i.e., we can only extract the relationship between them.) This is because the 890 nm wavelength of the LED light is close to the peak responsivity of silicon solar cells and the greatest nonlinear behavior in light bias dependent spectral response measurements have been observed near the peak responsivity [1–3]. It is best, however, to perform this measurement either with broadband light sources or a combination of monochromatic sources in order to know the true nonlinearity of the AM 1.5-weighted I_{sc} . Furthermore, the results of the measurement for these 3 cells imply that: 1. Cell#5 requires no light bias source when performing spectral response or quantum efficiency (QE) measurements, 2. For Cells#2, and #3, it is sufficient to use only a small enough light bias source that produces at least 1 mA of DC current for QE measurements, even if QE of the cell under 1-sun conditions are desired. For these two cells, since the I_{sc} under AM 1.5G is ~ 100 mA, the light bias intensity needed in order for the QE measurement to be valid at SRC conditions is only on the order of 1 % of this total signal value. The bias light spectral composition is generally advocated to be “white” or broadband [3] but it appears to us that light with energy close to the bandgap of the material, i.e., ≈ 900 nm for Si, can provide sufficiently uniform absorption throughout the material to push the cell into the linear regime. This hypothesis will be investigated in more detail in a future work.

Figure 4 shows the results of the unscaled nonlinearity measurements and analysis for Cell #1, #2, #3, and #4 at 627 nm. Cell# 2, 3, and 4 show a similar trend to the measurements in Fig. 3 except that the total change in R over a similar range of s is much smaller than the 890 nm measurements. This finding is consistent with light-bias dependent SR measurements in silicon solar cells [1,2], where UV and visible spectral range show a much weaker light bias dependence. An interesting case here is the behavior shown by Cell#1 (a KG-filtered, $2\text{ cm} \times 2\text{ cm}$ mono-Si reference cell), revealing a very strong nonlinear behavior that persists to 50 mA of signal level (near the 1-Sun I_{sc} for this device). Furthermore, at s levels above 1 Sun, the cell continues to show a non-constant R , although it appears to be leveling off at higher s . This measurement indicates that for certain types of cells such as this one where the nonlinearity is prevalent even up to 1-Sun SRC conditions or higher, significant errors can occur if this cell is used for solar simulator light level setting at conditions even slightly below or above the intended SRC, unless of course this type of data is provided along with SRC calibrations so that corrections can be made. Also, the spectral responsivity for AM 1.5

conditions must be calculated by performing DSR measurements at several light bias levels and an integration over the bias irradiance intensity [1]. It should be mentioned that this type of nonlinearity is very uncommon in silicon cells and we found this cell to be truly an exception than the rule. The origin of nonlinearity is unclear, but most likely originates from significant defects within silicon [1].

To demonstrate this point more clearly, Fig. 5(a) shows a plot of the irradiance spectral responsivity of Cell#1 vs. wavelength λ for several light bias currents (light bias source of white LEDs), revealing a significant bias dependence that is spectrally more severe in the red part of the spectrum, as reported elsewhere [1,2]. The unit of R is the unit of s in amperes divided by the unit of flux intensity in W/m^2 . This measurement was performed using the LED-based integrating sphere method described in a previous work [14], where the pulsed monochromatic probe light is maintained at very low flux levels while the bias light can be changed over the cell to achieve a given light-bias current level. The response of the cell in the near-IR is suppressed due to the KG5 window filter, but the results are consistent with the 627 nm combinatorial measurements for this cell in Fig. 4, in that the responsivity or the ratio R change as a function of the I_{sc} signal. In contrast to these data, Fig. 5(b) shows the light bias dependence of the spectral responsivity for Cell #4 which is also a KG-filtered Si reference cell. This cell, however, shows only a very subtle change in R at low signal levels, even in the red/NIR range, demonstrating a good linear response as was also indicated in the 627 nm combinatorial measurements of Fig. 4.

The measurement results presented in Figs. 3 and 4 indicate that the combinatorial technique is a very useful tool to verify the linearity of solar cells, even before any other type of device characterization such as I - V measurements or spectral response measurements are performed on the cell. Depending on the results, one would know beforehand how to properly perform the spectral response measurements (particularly for I_{sc} determination), or over what signal ranges is the linear assumption valid. Even more importantly, one can actually use the obtained relationship between signals and fluxes to perform intensity measurements without blind assumptions about linearity, or calculate the I_{sc} of the nonlinear device under any given spectrum using the iterative procedure outlined in Ref. [2].

Figure 6 shows the *scaled* global R vs s measurements for Cell #3 at both the 890 nm and 627 nm wavelengths. The flux was measured using the standard working detector, Cell #5, which had previously been calibrated and whose linear behavior was verified over a large s range. In addition to using the calibrated flux measurements to scale the R vs. s plots as outlined in section 3B, we also performed irradiance spectral response measurements as a function of a few select light bias currents as shown by the cross data points. Excellent agreement was observed between the combinatorial technique and the spectral response measurements as indicated in Fig. 6 demonstrating the consistency of our approach and measurement methods. This verification was also performed on all other cells, and they all revealed an agreement between the two techniques. The fact that one does not have to combine AC and DC measurements as is the case in the DSR method, or even use a reference photodetector for obtaining the unscaled relationship, makes the combinatorial measurement approach more attractive.

4B. Comparison to the two-lamp method

Analysis of the combinatorial signal data by the alternative two-lamp flux addition method [7–9] is particularly illuminating, and this small subsection briefly highlights the differences between the two analysis methods. Our main objective here is to show that the simple two-lamp flux addition analysis is inconsistent with the combinatorial results (and by extension the light bias dependent EQE) and does not predict the *true* nonlinearity of the solar cells. In this method, linearity is determined by calculating the parameter, D , defined as a normalized percentage deviation from linearity:

$$D = 100 \times \frac{(s_A + s_B) - s_{AB}}{s_{AB}} \quad (14)$$

where s_A or s_B are the short circuit current signals from each lamp and s_{AB} is the short circuit current signal with both lamps irradiating the cell. In a perfectly linear device, D should be zero simply due to the superposition principle. If we analyze our data using this method, for example for Cells #1 and #2 with the 627 nm LEDs, the results can be seen in Fig. 7 for D vs s_{AB} . Cell #1 which had shown a significant nonlinear signal response with flux when analyzed within the combinatorial method (and also light bias dependent EQE), now only shows deviation from linearity of less than 2.5 % over the given signal range and Cell #2 shows slightly less nonlinearity that is zero at higher signal levels.

If the purpose of the linearity measurement is to simply determine whether a particular cell is linear with irradiance or not, then the two-lamp method can be used to analyze the data and provide a pass/fail answer. However, if the correct behavior of signal vs flux is desired, then this technique fails to capture the true device nonlinear behavior. Furthermore, the strength of the inference drawn from a result of the two-lamp method can be quite sensitive to the choice of flux combinations, namely the selected combinations need to be close together in numerical value. If not, D values of 0 can be reported even for a very nonlinear region. In order to demonstrate this point better, we performed the following exercise: Given a particular s vs. ϕ relationship written in terms of ratio R , i.e., $s = R(s)\phi$, two sets of *randomly* chosen signals $\{s_A\}$ and $\{s_B\}$ and their corresponding fluxes $\{\phi_A\}$ and $\{\phi_B\}$ were selected, and from $\{\phi_A + \phi_B = \phi_{AB}\}$ and the function $R(s)$, the corresponding $\{s_{AB}\}$ combined signals were calculated. These signals were then used to calculate the ratio parameter D and plot it as a function of s_{AB} . To make this analysis even more informative, we used the actual measured functional form of R for Cells#1 and 2 as shown in Fig. 4 by first obtaining an interpolating function based on these data and then using it in the above-mentioned calculation. Figures 8(a–d) show the results of these calculations. In Fig. 8(a), the difference between any two randomly selected s_A and s_B signals is not greater than a factor of 3, but in Figs. 8b–d, this difference is allowed to increase up to 10, 100, and 1000 respectively. When the combinations are performed with signal values that are relatively closer together, i.e., Fig. 8(a), the results for D are tighter and in fact very closely matched with the experimental results of Fig. 7. However, as the spread between the signals increase, a larger spread in D is observed and in fact Fig. 8(d) shows that a larger number of points are close to 0.

Mathematically, this finding makes sense because combining very large signal values with very small ones should result in percentage differences close to 0. Obviously, D values of close to 0 on a cell that is purposely defined to be nonlinear over this signal range is meaningless. More importantly, this modeling demonstrates that even if one assumes a well-defined nonlinear relationship between the signal and flux values, the parameter D reveals nothing meaningful regarding that particular relationship and in fact the low values of D here are very misleading when we have deliberately set up a relationship with over 25 % change in responsivity over the given signal range. The values calculated by D should not be used or trusted to define the behavior of a nonlinear cell. However, it is possible that the additional analytical procedure outlined in ref. [9] for a nonlinearity correction factor can be used to determine the relative responsivity change in solar cells where a significant nonlinearity is observed. This will be investigated in more detail at a later time.

5. Conclusions

A combinatorial flux addition technique was used in conjunction with LEDs as light sources to obtain the nonlinear relationship between the output short-circuit current signal of a solar cell with incident flux. The various fluxes were created by controlling the supplied currents to the LEDs in singularity and in combinations over a large range and the unknown fluxes were solved, up to an undeterminable scaling, by a linear least squares fit to the signal data arranged within a linear overdetermined system of equations based on a polynomial relationship of N^{th} degree. It was found that some devices showed nonlinear response with flux particularly over very small signal ranges and the polynomial fits included all terms up to 4th degree in order to obtain the best results. Spectral dependence was also found to be a factor in the severity of nonlinearity in a solar cell. A method for obtaining a global scaled solution was also proposed and it was demonstrated that our technique is equivalent to light-bias dependent spectral response measurements. The shape of the pieced together global solution can be used to hypothesize and validate physically-based I_{sc} to flux functional relationships.

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Figure 1.

The schematic of the nonlinear LED-based tester. The two LEDs are first pulsed individually, followed by a combinatorial algorithm of the singular fluxes. The measurement of the generated signals is achieved by the lock-in amplifier.

Figure 2.

The algorithm for the combinatorial fluxes in our measurement setup. Every current value supplied to LED A is combined with 3 adjacent values of current supplied to LED B.

Therefore, there are a total of $30=15 \cdot 2$ unique fluxes incident on the sample, plus $45=15 \cdot 3$ combinations of them.

Figure 3.

The normalized global unscaled measurement of $R = s/\phi$ vs s over roughly 5 orders of magnitude of signal at 890 nm. The different-colored segments correspond to the different intervals of signals in which the measurement and regression analysis were performed.

Figure 4.
The normalized global unscaled measurement of $R = s/\phi$ vs s over roughly 5 orders of magnitude at 627 nm for a few different cells.

Figure 5. (a) The light bias dependence of the spectral response measurements for Cell#1, (b) The similar measurement for Cell#4. The results indicate that even two nominally prepared reference cells can have a vastly differently spectral response behavior.

Figure 6. The scaled global R vs. s measurements for Cell#3 at both the 890 nm and 627 nm wavelengths. Notice that the units of R are the same as irradiance spectral responsivity.

Figure 7.

The current combinations data for Cell#1 and 2 analyzed using with the alternative two-lamp flux addition method in terms of the deviation parameter D . It was observed that parameter D does not capture the true nonlinear behavior of these devices.

Figure 8.

Findings of a modeling exercise where the actual nonlinear relationships for Cells#1(black) and 2 (red, data in Fig. 4) were used to randomly generate signals and their combinations and plot the results in terms of the parameter D vs signal with the spread for each data pair being limited by a scalar of (a) 3, (b) 10, (c) 100, and (d) 1000. Fig. 8(a) reproduces the results of Fig. 7 and shows that even for highly nonlinear cells, D predicts a small nonlinearity. The rest show that combinations between very large and very small values can result in a large spread in parameter D .